

Feedback-enhanced driven-dissipative quantum batteries in waveguide QED systems

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Quantum batteries (QBs), acting as energy storage devices, have potential applications in future quantum science and technology. However, the QBs inevitably lose energy due to their interaction with the environment. How to enhance the performance of the QBs in the open-system scenario remains an important challenge. Here, we propose a scheme to realize driven-dissipative QBs in atom-waveguide-QED systems and demonstrate significant improvements in both the stored energy and extractable work (ergotropy) of the QBs via measurement feedback (MFB). For a single-atom QB, we show that MFB alone can realize perfect charging without coherent driving, with the sign of the feedback strength determining whether dissipation is suppressed or enhanced. For a many-body QB array, MFB and collective effects generate three distinct dynamical phases, including a boundary time-crystal phase with persistent energy oscillations and cyclic charging dynamics, as well as two stationary phases with low- and high-energy storage. By analyzing the Liouvillian spectrum, charging power, heat currents, and integrated energetic cost, we identify the dynamical origin of these phases and quantify the energy investment required for charging. This work broadens the scope of driven-dissipative QBs and provides practical strategies for enhancing their performance.

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Introduction. The rapid development of quantum technologies has created an urgent demand for efficient quantum energy storage devices [1–4]. Quantum batteries (QBs), first proposed by Alicki and Fannes [5], utilize quantum mechanical principles with the potential to impact key performance metrics such as charging power and work extraction [6–16]. Early theoretical studies in closed systems have revealed remarkable advantages through unitary charging protocols and collective quantum effects [17–28]. However, the practical implementation of QBs remains hindered by the decoherence-induced energy loss and dissipative charging dynamics in open quantum environments. To address these challenges, various approaches have been proposed [29–49], such as utilization of non-Markovian effects [38–42], Floquet engineering [43], dark states [44], and feedback control [45–47]. Experimental realizations of QBs have also been demonstrated across diverse platforms, including superconducting circuits [50,51], quantum dots [52], organic microcavities [53], and nuclear spins [54].

Waveguide quantum electrodynamics (QED) systems [55–58] have emerged as a promising platform for studying and realizing QBs. Existing implementations typically rely on passive energy transfer mechanisms, such as remote wireless charging mediated by atom-photon bound states [41,42] or collective effect against the energy loss of the QBs [59]. These passive mechanisms, however, fundamentally differ from the driven-dissipative regime [29–37], where the external laser field continuously pumps the charger or QBs while energy is simultaneously dissipated into the waveguide. Therefore, this operating regime describes actively maintained nonequilibrium energy storage, which is distinct from passive energy storage protocols. The competition between energy injection and dissipative loss may create rich nonequilibrium dynamics [60–62] that render conventional protocols ineffective. To address this, feedback control [63–68] offers a pathway not only to mitigate the decoherence effects but also to actively exploit the driven-dissipative environment and thus enhance the performance of the QBs. The key open question we address is how feedback control can regulate the balance between energy injection and dissipative loss and thereby enhance the charging performance of QBs under driven-dissipative conditions.

In this Letter, we propose a driven-dissipative QB scheme by coupling two-level atoms to a semi-infinite waveguide terminated by a mirror, which introduces coherent feedback (CFB) [63–65]. To control the charging performance, we employ measurement feedback (MFB) [66–68]. We find that adjusting the MFB parameter controls the direction of the

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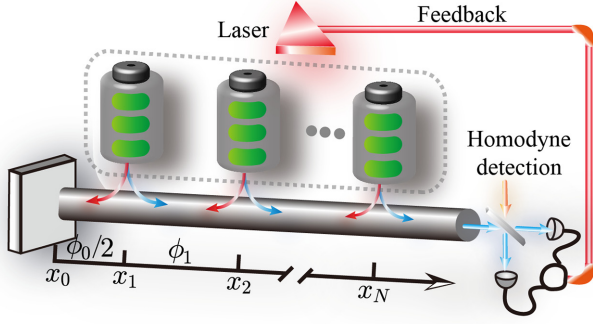


FIG. 1. Schematic of the QB array (two-level atoms) coupled to a semi-infinite waveguide terminated by a mirror. An external laser resonantly drives the atoms, and the right-propagating output field is monitored via homodyne detection. The measured photocurrent is fed back to modulate the driving field.

feedback driving, which allows one to switch between the dissipation suppression and enhancement. For a single-atom QB, perfect charging can be achieved even without an external drive. For a many-body QB array, MFB together with collective effects generates three distinct dynamical phases. These are a boundary time-crystal (BTC) phase [60] with persistent energy oscillations, a high-energy stationary phase, and a low-energy stationary phase. We also analyze the energy investment and dissipation during the charging process. Our results provide insights into the design of QBs in open quantum systems.

Driven-dissipative-charging QB model. We consider a QB array of two-level atoms with transition frequency ω_a , coupled to a semi-infinite waveguide, as shown in Fig. 1. The single-atom spontaneous decay rates into the right- and left-propagating modes are γ_R and γ_L , respectively, with a total decay rate $\gamma = \gamma_R + \gamma_L$. The chiral factor is defined as $\Delta\gamma = \gamma_R - \gamma_L$, with $\Delta\gamma = 0$ and $\Delta\gamma = \gamma$ corresponding to the achiral and perfect-chiral couplings, respectively. An external laser with amplitude Ω resonantly drives the QB array to compensate for dissipation and store a finite amount of energy. The right-propagating output field is continuously monitored via homodyne detection of the phase quadrature $P(t)$ of the photocurrent, and the measurement record is instantaneously fed back to modulate the driving amplitude. The mirror at the waveguide end ($x_0 = 0$) reflects left-propagating photons back to the atoms, thereby introducing the CFB control [63–65]. Within the Born-Markov approximation framework and using the scattering matrix, Lindblad operators, and Hamiltonian (SLH) formalism [69–72], the quantum master equation for the QB array is given by [73]

$$\dot{\rho} = -i[H_d + H_R + H_L + H_M + H_F, \rho] + D[\tilde{L} - iF]\rho, \quad (1)$$

where the coherent driving term is $H_d = \Omega \sum_{j=1}^N \sigma_j^x$, and the waveguide-mediated exchange interactions read

$$H_R = \frac{\gamma_R}{2i} \sum_{j>l} e^{i \sum_{s=l}^{j-1} \phi_s} \sigma_j^+ \sigma_l^- + \text{H.c.}, \quad (2a)$$

$$H_L = \frac{\gamma_L}{2i} \sum_{j<l} e^{i \sum_{s=j}^{l-1} \phi_s} \sigma_j^+ \sigma_l^- + \text{H.c.}, \quad (2b)$$

with $\phi_s = k_0(x_{s+1} - x_s)$ the phase accumulated between neighboring coupling points, k_0 the photon wave vector, and $\sigma_j^\pm$ the raising/lowering operators. The CFB from the mirror and the MFB are described by

$$H_M = e^{i \sum_{j=0}^{N-1} \phi_j} L_R^\dagger L_L / 2i + \text{H.c.}, \quad (3a)$$

$$H_F = \frac{1}{2} (F^\dagger \tilde{L} + \text{H.c.}), \quad (3b)$$

where $\phi_0 = 2k_0 x_1$ is the round-trip phase from the first atom to the mirror and back, $F = i\sqrt{\gamma_R g} \sum_j \sigma_j^x$ is the feedback-related operator, and g is the feedback strength. The sign of g determines the direction of the feedback driving. Flipping the sign is equivalent to a π phase shift in the feedback loop, leading to different charging dynamics. The dissipator $D[o] \bullet \equiv o \bullet o^\dagger - \frac{1}{2} \{o^\dagger o, \bullet\}$ accounts for the waveguide losses, with $\tilde{L} = e^{i \sum_{j=0}^{N-1} \phi_j} L_L + L_R$. The collective jump operators are $L_{R(L)} = \sqrt{\gamma_{R(L)}} \sum_{j=1}^N e^{i \phi_{\Sigma_{R(L)}}} \sigma_j^-$, with the phases $\phi_{\Sigma_R} = \sum_{s=j}^{N-1} \phi_s$ and $\phi_{\Sigma_L} = \sum_{s=j-1}^{j-1} \phi_s$.

QB performance metrics. To characterize the charging performance of the QB, we introduce the stored energy $\mathcal{E}_b(t) = \text{Tr}[H_b \rho(t)]$, where $H_b = \omega_a \sum_{j=1}^N \sigma_j^+ \sigma_j^-$ is the bare atomic Hamiltonian. This definition is justified in the typical condition $\omega_a \gg \gamma, \Omega$ [74–79]. The maximum extractable work from $\rho(t)$ is the ergotropy $\mathcal{W}(t) = \text{Tr}[\rho(t) H_b] - \text{Tr}[\tilde{\rho}(t) H_b]$ [80,81], where $\tilde{\rho}(t) = \sum_n r_n(t) |\epsilon_n\rangle \langle \epsilon_n|$ is the passive state formed by the eigenvalues $r_n(t)$ of $\rho(t)$ (sorted decreasingly) and the eigenstates $|\epsilon_n\rangle$ (sorted increasingly). Based on the stored energy, we define the instantaneous charging power $\mathcal{P}_b(t) = \dot{\mathcal{E}}_b(t)$ [4]. Rewriting Eq. (1) to separate the MFB term [73], we decompose $\mathcal{P}_b(t)$ into three contributions, including the driving power $\mathcal{P}_d(t) = -i \text{Tr}[\{H_b, H_d\} \rho(t)]$, the MFB power $\mathcal{P}_{fb}(t) = \text{Tr}\{H_b \mathcal{F} \rho(t)\}$, with $\mathcal{F} \rho(t) = D[F] \rho(t) - i([F, \rho(t) \tilde{L}^\dagger] + \text{H.c.})$, and the intrinsic heat current $\dot{\mathcal{Q}}_{\text{itr}}(t) = \text{Tr}\{H_b D[\tilde{L}] \rho(t)\}$. These quantities satisfy $\mathcal{P}_b(t) = \mathcal{P}_d(t) + \mathcal{P}_{fb}(t) + \dot{\mathcal{Q}}_{\text{itr}}(t)$, with $\dot{\mathcal{Q}}_{\text{itr}}(t) < 0$ reflecting energy loss to the waveguide. To quantify the net dissipation under MFB, we define an effective heat current $\dot{\mathcal{Q}}_{\text{eff}}(t) = \dot{\mathcal{Q}}_{\text{itr}}(t) + \dot{\mathcal{Q}}_{fb}(t)$ [73], where $\dot{\mathcal{Q}}_{fb}(t)$ denotes the heat current induced by MFB. The integrated effective heat is $\mathcal{Q}_{\text{eff}}(t) = \int_0^t \dot{\mathcal{Q}}_{\text{eff}}(t') dt'$, and the effective energy dissipated up to time t is $-\mathcal{Q}_{\text{eff}}(t)$. We define the instantaneous charging cost power as $\mathcal{P}_c(t) = \mathcal{P}_d(t) + \mathcal{P}_{fb}(t)$. The total charging cost $\mathcal{E}_c(t) = \int_0^t \mathcal{P}_c(t') dt'$ then quantifies the energy supplied by the driving and MFB.

Feedback-enhanced single-atom QB. We first consider a single-atom QB to understand the charging dynamics with MFB. In this case, Eq. (1) reduces to

$$\dot{\rho} = -i[\Omega \sigma^x + \tilde{\Delta} \sigma^+ \sigma^-, \rho] + \sum_{\mu, \nu=\pm} \Gamma_{\mu\nu} D_{\sigma^\mu, \sigma^\nu} \rho, \quad (4)$$

with the frequency shift $\tilde{\Delta} = \sqrt{\gamma_R \gamma_L} (g+1) \sin \phi_0$ and decay rates $\Gamma_{-+} = \gamma + \gamma_R g (g+2) + 2\sqrt{\gamma_R \gamma_L} (g+1) \cos \phi_0$, $\Gamma_{+-} = \gamma_R g^2$, and $\Gamma_{++} = (\Gamma_{--})^* = \gamma_R g (g+1) + \sqrt{\gamma_R \gamma_L} g e^{-i\phi_0}$, where $D_{\sigma^\mu, \sigma^\nu} \bullet \equiv \sigma^\mu \bullet \sigma^\nu - \{\sigma^\nu \sigma^\mu, \bullet\}/2$. The master equation (4) has the same form as that of a driven two-level atom in a squeezed vacuum reservoir [82–85], with g playing the role of the squeezing parameter that controls the decay rates.

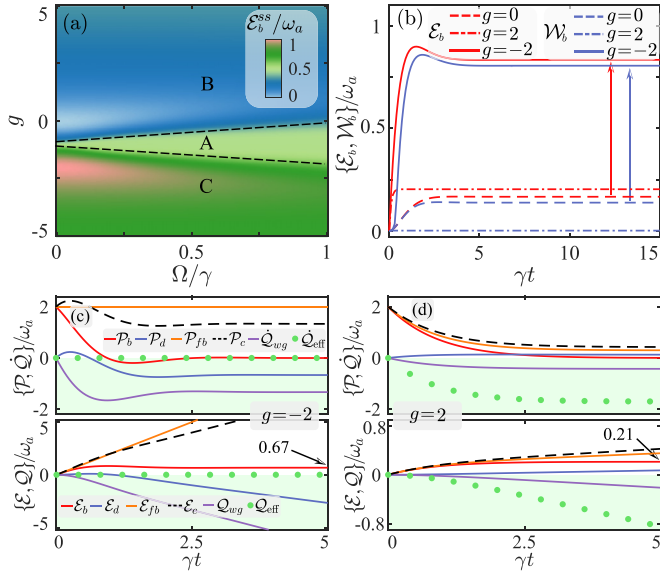


FIG. 2. (a) $\mathcal{E}_b^{ss}/\omega_a$ vs Ω and g . (b) Dynamics of $\mathcal{E}_b(t)/\omega_a$ and $\mathcal{W}_b(t)/\omega_a$ for different g at $\Omega/\gamma = 0.5$. (c), (d) Power and heat-current decomposition (upper) and their integrated quantities (lower) for $g = -2$ and $g = 2$, respectively, with $\Omega/\gamma = 1$. Other parameters are $\Delta\gamma = 0$ and $\phi_0 = 2\pi$.

We study the charging behavior of the single-atom QB in Fig. 2. the normalized steady-state energy $\mathcal{E}_b^{ss}/\omega_a$ in the (Ω, g) plane [Fig. 2(a)] can be approximately divided into three regions. In region C, the steady-state energy is highest, but it decreases as Ω increases. In particular, perfect charging ($\mathcal{E}_b^{ss}/\omega_a = 1$) occurs even at $\Omega = 0$, where MFB alone charges the QB via an effective pumping process with rate $\Gamma_{+-} = \gamma_R g^2$ [73]. Figure 2(b) reveals a strong dependence on the sign of g . At $g = 0$, both energy and ergotropy remain low. For $g = -2$, they are strongly enhanced with $\mathcal{W}_b(t) \simeq \mathcal{E}_b(t)$, whereas for $g = 2$ the charging is weak and $\mathcal{W}_b(t) = 0$.

Figures 2(c) and 2(d) show the time evolution of the power decomposition, heat currents, and integrated quantities. For $g = -2$ with MFB driving opposite to σ_x , $\mathcal{P}_b(t)$ dominates, while $\mathcal{P}_d(t)$ changes sign from positive to negative (green shaded area). Thus, the coherent driving initially injects energy but later extracts it. This explains why increasing Ω reduces $\mathcal{E}_b^{ss}/\omega_a$ in region C. Despite this, the effective heat current vanishes (green dots), indicating complete suppression of dissipation by MFB. Accordingly, the effective energy dissipated up to time t is zero. The integrated quantities show that $\mathcal{E}_b(t) > \mathcal{E}_c(t)$, as MFB compensates for both dissipation and energy extraction by the driving. Nevertheless, the normalized steady-state energy exceeds the $1/2$ limit achievable with coherent driving alone. For $g = 2$ with MFB driving along σ_x , $\mathcal{P}_b(t)$ decays to a finite value and remains below $\mathcal{P}_c(t)$. The effective heat current is significantly enhanced as MFB introduces additional dissipation. As a result, the charging performance is inferior to that for $g = -2$, and the effective energy dissipated up to time t is $-\mathcal{Q}_{eff}(t)$, which exceeds the intrinsic loss $-\mathcal{Q}_{itr}(t)$. This reveals that whether MFB drives along or opposite to σ_x direction critically affects the effective heat current and the charging performance.

Phase-controlled many-body QB array. For a QB array with N atoms, the MFB control and collective effects can engineer rich dynamical phases [68,86]. Setting $\gamma_R = \gamma_L = \gamma/2$ and $\phi_j = 2m\pi$ ($m \in \mathbb{Z}$), Eq. (1) reduces to

$$\dot{\rho} = -i[2\Omega J_x + \frac{i\gamma g}{2}(J_+^2 - J_-^2), \rho] + \sum_{\mu, \nu=\pm} \Gamma_{\mu\nu} D_{J_{\mu\nu}} \rho, \quad (5)$$

where $J_{\alpha=x,y,z} = \sum_{j=1}^N \sigma_j^\alpha/2$ and $J_{\pm} = J_x \pm iJ_y$ are the collective operators. The decay rates are $\Gamma_{+-} = \gamma(g+2)^2/2$, $\Gamma_{++} = \gamma g^2/2$, and $\Gamma_{--} = \gamma g(g+2)/2$. Therefore, the constructive interference of propagating photons in the waveguide yields a collective spin mode of length $N/2$ [87–89].

In the QB array, the normalized stored energy is defined as $\mathcal{E}_b(t)/N\omega_a = \langle J_z \rangle/N + 1/2$, with the expectation value defined as $\langle \bullet \rangle = \text{Tr}[\bullet \rho]$. We consider the thermodynamic limit $N \rightarrow \infty$ and introduce the expectation values for the magnetization vector components $m_\alpha = \langle J_\alpha \rangle/N$ [60]. When $N \rightarrow \infty$, the dynamics of the expectation values is exactly governed by the mean-field equations [73]: $\dot{m}_x = 2\Gamma m_z m_x$, $\dot{m}_y = -2\Omega m_z + 2\Gamma \xi m_y m_z$, and $\dot{m}_z = 2\Omega m_y - 2\Gamma m_x^2 - 2\Gamma \xi m_y^2$, with $\xi = 2g + 1$. To ensure a well-defined thermodynamic limit, we introduce the rescaled decay rate $\Gamma = N\gamma$ for the $N/2$ spin. The mean-field equations conserve the length j of the average magnetization vector $\vec{m} = [m_x(t), m_y(t), m_z(t)]$. For the many-body charging process, we consider that the N atoms are initially in their ground state, with the initial condition $m_x(0) = m_y(0) = 0$ and $m_z(0) = -1/2$. The first mean-field equation immediately implies that $\dot{m}_x = 0$ at $t = 0$, and a direct integration shows that $m_x(t)$ remains zero for all times. Thus, the dynamics is restricted to the (m_y, m_z) plane. Consequently, $j^2 = m_y^2 + m_z^2 = 1/4$ holds throughout the evolution. It is found that there exists two stationary phases and a BTC phase in this system, separated by the critical driving amplitude $\Omega_{\text{cri}} = \Gamma|\xi|/2$. When $\Omega > \Omega_{\text{cri}}$, the energy stored in the QB array can exhibit persistent oscillations despite coupling to the fields in the waveguide.

In contrast, when $\Omega \leq \Omega_{\text{cri}}$, the normalized stored energy in the QB array approaches two stationary values $\mathcal{E}_b^{ss}/N\omega_a = -\text{sgn}(\xi)\sqrt{1/4 - \Omega^2/\Gamma^2\xi^2} + 1/2$ [73]. The sign of g determines the direction of the feedback driving along J_x ($g > 0$) or opposite to it ($g < 0$), and this sign dependence is captured by $\xi = 2g + 1$. For $\xi \geq 0$ (i.e., $g \geq -1/2$), the normalized steady-state energy satisfies $\mathcal{E}_b^{ss}/N\omega_a \leq 1/2$, which agrees with the steady-state charging energy achieved via collective effects in the driven-dissipative regime without MFB [32]. For $\xi < 0$ (i.e., $g < -1/2$), we have $\mathcal{E}_b^{ss}/N\omega_a > 1/2$. In this regime, for fixed g (with $\xi < 0$), the normalized steady-state energy attains its maximum $\mathcal{E}_b^{ss}/N\omega_a = 1$ at $\Omega = 0$ and then decreases with increasing Ω . The stability of these two stationary solutions can be checked by examining the eigenvalues of the Jacobian matrix [90].

Figure 3(a) shows that $\mathcal{E}_b^{ss}/N\omega_a$ is clearly divided into three distinct dynamical regions by Ω_{cri} (black dashed lines): the BTC phase and two stationary phases I and II. In stationary phase II (with negative g), perfect steady-state energy can be achieved even for $\Omega = 0$. Figures 3(b) and 3(c) compare the

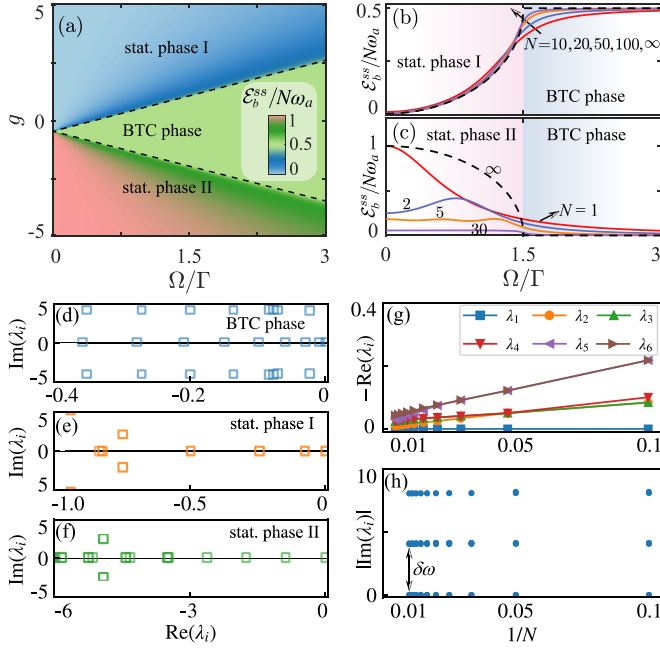


FIG. 3. (a) Phase diagram of $E_b^{ss}/N\omega_a$. (b), (c) $E_b^{ss}/N\omega_a$ vs Ω for $g = 1$ and $g = -2$, respectively, with different N . The black dashed curve is the mean-field result. (d)–(f) Liouvillian spectra for (all with $\Omega/\Gamma = 2$, $N = 100$) BTC phase ($g = -1/2$) and stationary phases I ($g = 4$) and II ($g = -4$). (g), (h) Finite-size scaling of the Liouvillian eigenvalues in the BTC phase vs $1/N$ [same parameters as in panel (d)]. The first 6 real parts and 12 imaginary parts are shown, ordered by increasing $|\text{Re}(\lambda_i)|$ and normalized by Γ . The imaginary parts approximately form a band structure with characteristic spacing $\delta\omega$.

mean-field predictions with exact numerical results for finite N obtained from Eq. (5). As shown in Fig. 3(b), the mean-field results converge to the exact numerics as N increases, and $E_b^{ss}/N\omega_a$ approaches $1/2$ with increasing Ω . In contrast, Fig. 3(c) reveals that while $E_b^{ss}/N\omega_a$ converges to the mean-field prediction in the BTC phase, its behavior in stationary phase II deviates significantly for accessible system sizes. This deviation is expected to vanish as N increases because the mean-field result becomes exact in the thermodynamic limit [91].

To understand the origin of these dynamical phases, we examine the Liouvillian spectrum. Writing the quantum master equation (5) as $\dot{\rho} = \mathcal{L}\rho$, the formal solution is $\rho(t) = e^{\mathcal{L}t}\rho(0) = \sum_i c_i e^{\lambda_i t} \rho_i$, where λ_i are the eigenvalues of the Liouvillian superoperator $\mathcal{L}$. Each eigenmode ρ_i decays at rate $-\text{Re}(\lambda_i)$ and oscillates with frequency $\text{Im}(\lambda_i)$. In the BTC phase [Fig. 3(d)], the Liouvillian gap closes. The real parts of the leading eigenvalues approach zero, indicating nearly nondecaying modes. These modes carry nonzero imaginary parts, which produce sustained coherent oscillations. In stationary phases I and II [Figs. 3(e) and 3(f)], however, the gap is finite, with the leading eigenvalues being purely real and relatively large in magnitude. Accordingly, the dynamics in the stationary phases relax quickly to the steady state, with any oscillatory components decaying on a short timescale. The BTC phase is further confirmed by finite-size scaling [Figs. 3(g) and 3(h)]. As N increases, the real parts of eigenvalues with nonzero imaginary parts tend to zero, signaling the emergence

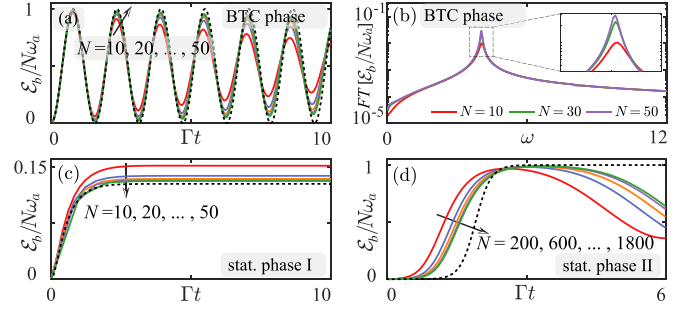


FIG. 4. (a) Time evolution of $E_b(t)/N\omega_a$ in the BTC phase ($g = -1/2$, $\Omega/\Gamma = 2$). (b) Fourier spectrum of panel (a). (c), (d) Same as panel (a) but for stationary phases I ($g = 1$, $\Omega/\Gamma = 1$) and II ($g = -2$, $\Omega/\Gamma = 0.01$), respectively. System sizes are indicated in each panel. The black dashed curves in panels (a), (c), and (d) are the mean-field results.

of persistent energy oscillations in the thermodynamic limit, while the imaginary parts approach a finite band structure with spacing $\delta\omega$.

The finite-time evolution of the normalized stored energy is studied in Fig. 4. In the BTC phase [Fig. 4(a)], $E_b(t)/N\omega_a$ exhibits damped oscillations with a decay rate that decreases as N increases, consistent with the finite-size scaling of the Liouvillian eigenvalues [Fig. 3(g)]. In the thermodynamic limit, the mean-field solution yields persistent periodic oscillations. For $g = -1/2$ ($\xi = 0$), the mean-field equations give $E_b(t)/N\omega_a = [1 - \cos(2\Omega t)]/2$. The Fourier spectrum [Fig. 4(b)] shows a single dominant peak at $\omega = 4.02$, matching the spectral spacing in Fig. 3(h). This peak frequency approaches the mean-field oscillation frequency 2Ω as N increases. For $g \neq -1/2$ within the BTC phase, the oscillations become tilted and the Fourier spectrum exhibits multiple peaks [73]. The persistent oscillatory behavior enables controlled cyclic energy storage and release by providing predictable optimal charging and discharging times. In stationary phase I [Fig. 4(c)], $E_b(t)/N\omega_a$ rapidly relaxes to a steady state without oscillations, and the normalized stored energy remains far below $1/2$. In stationary phase II [Fig. 4(d)], a larger value of N is required to approach the mean-field prediction, where perfect steady-state charging can be achieved. As N increases, the plateau near unit energy becomes broader, indicating enhanced stability of the fully charged state. Thus, MFB enables different dynamical phases and flexible control over the charging behavior of the many-body QB array.

The charging power and cost for the QB array are presented in Fig. 5. In the BTC phase for $g = 0$ [Figs. 5(a) and 5(b)], the coherent driving alone governs the dynamics, with $\mathcal{P}_c(t) = \mathcal{P}_d(t)$ and $\mathcal{Q}_{\text{eff}}(t)$ tracking $\mathcal{Q}_{\text{itr}}(t)$, as expected in the absence of MFB-induced dissipation. The stored energy $E_b(t)$ exhibits damped oscillations, while $E_c(t)$ increases continuously, reflecting the energetic cost required to compensate intrinsic losses. In the stationary phases, we fix $\Omega = 0$ so that the charging relies solely on MFB, with $\mathcal{P}_c(t) = \mathcal{P}_{\text{fb}}(t)$. In stationary phase I for $g = 2$ [Figs. 5(c) and 5(d)], $\mathcal{P}_{\text{fb}}(t)$ decays monotonically to a finite value, and the magnitude of $\mathcal{Q}_{\text{eff}}(t)$ exceeds that of $\mathcal{Q}_{\text{itr}}(t)$. This means that MFB introduces extra dissipation, keeping the stored energy as low

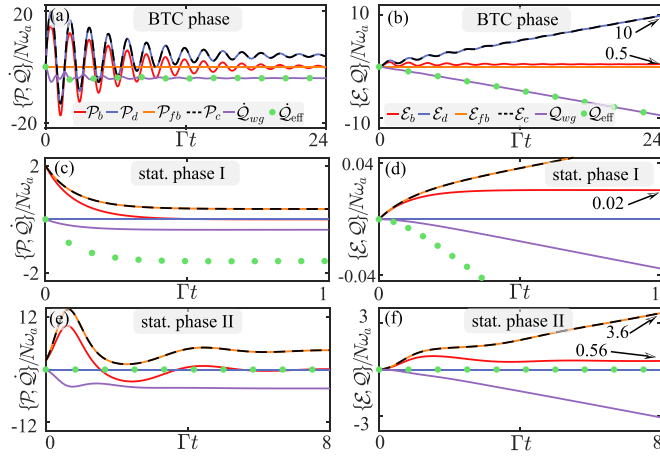


FIG. 5. (a) Power decomposition and heat currents in the BTC phase ($g = 0$, $\Omega/\Gamma = 2$). (c), (e) Same for stationary phases I ($g = 2$) and II ($g = -2$) with $\Omega = 0$. (b), (d), (f) Corresponding integrated quantities for panels (a), (c), and (e), respectively. The atom number is $N = 10$.

as $\mathcal{E}_b^{\text{ss}}/N\omega_a \approx 0.02$. In contrast, in stationary phase II for $g = -2$ [Figs. 5(e) and 5(f)], $\mathcal{P}_{\text{fb}}(t)$ first increases and then decays with oscillations to a steady-state value larger than that in phase I. The effective heat current at $g = -2$ vanishes [73], which implies zero accumulated effective dissipation and the steady-state balance $\mathcal{P}_{\text{fb}}(t) = -\dot{Q}_{\text{ir}}(t)$. In this case, MFB compensates intrinsic dissipation rather than enhancing it, allowing the QB array to rapidly reach a stored energy comparable to that in the BTC phase but at a lower charging cost. This dissipation-compensating mechanism enables more efficient energy transfer than purely driven schemes [32].

Conclusion and outlook. In conclusion, we have proposed a driven-dissipative QB scheme in waveguide-QED systems with the MFB control. For a single-atom QB, MFB alone achieves perfect charging without an external coherent drive. For a many-body QB array, MFB generates a BTC phase and two stationary phases, with the sign of g controlling dissipation suppression or enhancement. In the Supplemental Material [73], we have further shown that for the single-atom QB, perfect charging can be maintained across the chiral factor by tuning g , and that the overall scheme for both single-atom and many-body QBs remains robust against detection inefficiency, measurement noise, and finite feedback time delays. These results position MFB as a promising approach for high-performance QBs in open quantum systems. Future work could investigate physically richer settings for QB charging in the driven-dissipative regime, such as exploring the role of time delays between coupling points of emitters [92–95], using the rich interference effects in giant atoms [96], and extending the scheme to multilevel atoms [97,98].

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Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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