

Quantum simulation of two-level PT -symmetric systems using Hermitian Hamiltonians

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Parity-time (PT)-symmetric Hamiltonians exhibit nonunitary dynamical evolution while maintaining real spectra, and offer unique approaches to quantum sensing and entanglement generation. Here, we present a method for simulating the quantum dynamics of PT -symmetric systems on unitary-gate-based quantum computers by leveraging Hermitian equivalents through similarity transformations. We introduce two algorithms for simulating PT -symmetric systems near exceptional points where eigenvalues and eigenstates coalesce. The first is a hybrid classical-quantum algorithm and the second reduces the classical component by using an ancilla qubit. We then use perturbation theory to extend this method to consider the dynamics of two weakly interacting PT -symmetric systems. Demonstrations on a quantum device and noisy simulators validate the algorithm on current quantum devices, offering potential for scalable simulations of PT -symmetric systems and pseudo-Hermitian operators in quantum computation.

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I. INTRODUCTION

Understanding the evolution of open quantum systems (OQSs), quantum systems that interact with an environment, is crucial for fundamental research and technological advancements in quantum information and sensing [1–6]. Simulating these systems efficiently on quantum computers remains a challenge due to their nonunitary time propagators, which cannot be mapped directly into the unitary framework common in current quantum computers. Recent progress in open quantum simulation offers approaches to address this challenge with different methods, including quantum imaginary time evolution (QITE) [7–11] and block encodings [12–19], among others [20–50].

A particularly intriguing class of open quantum systems with non-Hermitian dynamics is parity-time (PT -) symmetric quantum systems [51], which exhibit unique properties not observed in standard Hermitian quantum mechanics [52–78]. PT -symmetry refers to systems where the Hamiltonian remains invariant under combined parity- (P) and time-reversal (T) transformations. These systems possess real energy spectra in the so-called “unbroken” PT -symmetric phase, despite the Hamiltonian being non-Hermitian. PT -symmetric systems possess a variety of interesting quantum properties, including the ability to tune the evolution time between two desired states such that the resulting time evolution is faster than a Hermitian system with the same spectrum

[66,79,80]. This phenomenon is illustrated in Fig. 1(a), which compares the time evolution of the ground-state population under PT -symmetric (solid line) and Hermitian (dashed line) Hamiltonians with the same spectrum.

The real spectrum in the unbroken regime allows PT -symmetric systems to be mapped to an equivalent Hermitian form [81], permitting simulation with unitary quantum algorithms. Here, we explore two approaches to simulating PT -symmetric systems on quantum computers by constructing an equivalent Hermitian Hamiltonian through similarity transformations. The similarity transformation allows for the offloading of the nonunitary dynamics to either classical pre/postprocessing or additional quantum resources via dilation. Here, we explore both possible offloadings, referring to the algorithm, which relies on classical postprocessing as the hybrid algorithm, and the other dilation algorithm. To examine the potential scaling of the hybrid algorithm, we extend beyond a single PT -symmetric system to consider two weakly interacting spins [59,60,82,83], where the weak interaction between the two PT -symmetric systems results in entanglement generation that is proposed to be faster than in Hermitian systems. We successfully simulate this entanglement generation by extending the one-qubit method via perturbation theory using a quantum simulator. Finally, we perform an error analysis on the single PT -symmetric algorithms to further assess their scalability. These results offer a scalable approach for simulating weakly coupled PT -symmetric qubits with unitary quantum algorithms.

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II. THEORY AND METHODS

A. Single PT -symmetric system

We consider a two-level system with balanced gain and loss that can be effectively described by a PT -symmetric

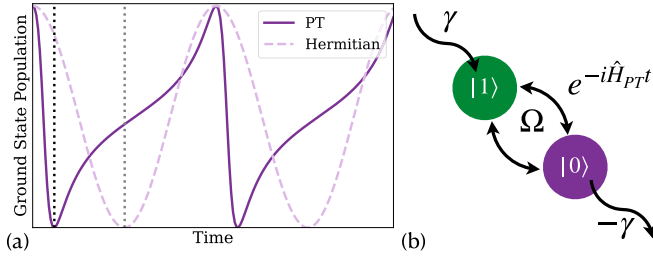


FIG. 1. (a) Comparison of the ground-state population dynamics of a Hermitian (dashed line) and a PT -symmetric system (solid line). Dotted vertical lines show the time it takes for the ground state to evolve to the excited state for the PT (black) and Hermitian dynamics (gray), demonstrating faster evolution on the PT curve. (b) Schematic of a two-level system with balanced gain and loss, where Ω is the coupling rate between the two levels and γ is the gain and loss rate.

Hamiltonian,

$$\hat{H}_{PT} = \begin{pmatrix} -i\gamma & \Omega \\ \Omega & i\gamma \end{pmatrix}, \quad (1)$$

where Ω is the coupling rate between the two states of the system and γ is the rate of gain and loss, as depicted schematically in Fig. 1(b). When $\Omega > \gamma$, this Hamiltonian is in the PT -symmetric unbroken phase and has real eigenvalues with nonorthogonal eigenstates. When $\Omega = \gamma$, the system reaches an exceptional point where eigenvalues are degenerate and the eigenstates coalesce.

To explore the PT -symmetric region with real spectra, we investigate its Hermitian equivalent, which can be derived through a similarity transformation [81,84–88]. In the unbroken regime, PT -symmetric systems are a category of pseudo-Hermitian systems [84,89], so there exists a Hermitian, invertible, linear operator η such that $\hat{H}_{PT}^\dagger = \eta \hat{H}_{PT} \eta^{-1}$. This pseudo-Hermiticity allows for unitary evolution by redefining the inner product [81,84]. In the unbroken regime, Eq. (1) is parametrized as

$$\hat{H}_{PT} = \begin{pmatrix} -i\Omega \sin \alpha & \Omega \\ \Omega & i\Omega \sin \alpha \end{pmatrix}, \quad (2)$$

where $\sin \alpha = \frac{\gamma}{\Omega}$. The corresponding eigenstates are

$$|\psi_+\rangle = \frac{1}{\sqrt{2 \cos \alpha}} (e^{-\frac{i\alpha}{2}}, e^{\frac{i\alpha}{2}})^T, \quad (3)$$

$$|\psi_-\rangle = \frac{1}{\sqrt{2 \cos \alpha}} (e^{\frac{i\alpha}{2}}, -e^{-\frac{i\alpha}{2}})^T.$$

The similarity transformation, η , which connects $\hat{H}_{PT}$ with $\hat{H}_{PT}^\dagger$ can be constructed from eigenstates of $\hat{H}_{PT}^\dagger$, which are complex conjugates of the original eigenstates, $|\phi_\pm\rangle = |\psi_\pm\rangle^*$. This similarity transformation takes the form

$$\eta = |\phi_+\rangle\langle\phi_+| + |\phi_-\rangle\langle\phi_-| \quad (4)$$

$$= \begin{pmatrix} \sec \alpha & i \tan \alpha \\ -i \tan \alpha & \sec \alpha \end{pmatrix}. \quad (5)$$

Since η is positive semidefinite in the unbroken regime, we define its square root [81],

$$\sqrt{\eta} = \tau = \frac{1}{2} \begin{pmatrix} a+b & -i(a-b) \\ i(a-b) & a+b \end{pmatrix}, \quad (6)$$

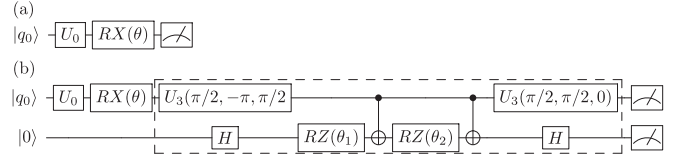


FIG. 2. Quantum circuits corresponding to the [panel (a)] hybrid algorithm with $e^{-i\hat{H}_{PT}t} \tau|0\rangle$ on a single qubit with tomography and [panel (b)] dilation algorithm with $\tau^{-1} \exp(-i\hat{H}_{PT}t) \tau|0\rangle$ using an ancilla qubit to implement the dilated τ^{-1} . Here, U_0 is the single-qubit rotation to prepare the initial state, RX a single-qubit rotation about the x axis, RZ a single-qubit rotation about the z axis, H the Hadamard gate, and U_3 a universal single-qubit rotation gate with three angles.

where τ is a positive-definite matrix, $a = \sqrt{\sec \alpha - \tan \alpha}$, and $b = \sqrt{\tan \alpha + \sec \alpha}$. This matrix is used in a similarity transformation, resulting in a Hermitian equivalent of the PT -symmetric Hamiltonian, $\hat{h} = \tau \hat{H}_{PT} \tau^{-1}$,

$$\hat{h} = \begin{pmatrix} 0 & \Omega \cos \alpha \\ \Omega \cos \alpha & 0 \end{pmatrix}. \quad (7)$$

In other words, $\hat{H}_{PT}$ is similar to $\hat{H}_{PT}^\dagger$ through η , and $\hat{H}_{PT}$ is similar to $\hat{h}$ through $\tau = \sqrt{\eta}$. Notably, for a time-independent Hamiltonian, τ is a Hermitian matrix that does not depend on time.

The time-evolution operator for $\hat{H}_{PT}$ is

$$e^{-i\hat{H}_{PT}t} |\psi_0\rangle = e^{-it(\tau^{-1}\hat{h}\tau)} |\psi_0\rangle = \tau^{-1} e^{-i\hat{h}t} \tau |\psi_0\rangle. \quad (8)$$

This suggests that the simulation of the PT -symmetric dynamics can be done in three parts: initialization by $\tau|\psi_0\rangle$, unitary time evolution of the single-qubit Hamiltonian $e^{-i\hat{h}t}$, and the final Hermitian operator τ^{-1} to complete the similarity transformation. We prepare the initial state with a one-qubit gate, U_0 , that results in $\frac{1}{\mathcal{N}} \tau |\psi_0\rangle$ with normalization $\mathcal{N}$. After initialization, we apply the unitary time evolution with an $R_x(\theta)$ gate. For the final step, we implement τ^{-1} in one of two ways: first, using a one-qubit hybrid classical-quantum algorithm, and second by dilation and simulation in a two-qubit setting. We distinguish the two algorithms as either the hybrid or dilation algorithm. We show how the hybrid algorithm can be extended to weakly coupled PT -symmetric qubits via perturbation theory, without incurring significant additional classical costs.

In the hybrid approach, we perform the initialization and time propagation as shown in the circuit in Fig. 2(a), followed by tomography of the density matrix. We then multiply the τ^{-1} matrix classically to postprocess the result and find the evolution of the PT -symmetric system in the initial basis. The classical multiplication of the matrix τ^{-1} requires measuring every element of the time-evolved density matrix, known as full tomography, which significantly limits the scalability of this algorithm beyond noisy-intermediate scale quantum (NISQ) devices, where the number of qubits is limited and prone to error.

In the dilation approach, we realize the time evolution by implementing τ^{-1} as a dilated unitary, eliminating the need for full tomography of the density matrix to complete the similarity transform. In dilation approaches, an additional, or

ancilla, qubit is used in addition to the qubits spanning the system Hilbert space. Since τ^{-1} is nonunitary, we employ a unitary dilation of the operator, using the spectral decomposition of this Hermitian matrix, $\tau = UDU^\dagger$ and $\tau^{-1} = UD^{-1}U^\dagger$, where U is unitary and D is diagonal. The unitary dilation of D results in a two-qubit unitary operator, U_D ,

$$U_D = \begin{pmatrix} D_+ & 0 \\ 0 & D_- \end{pmatrix}, \quad (9)$$

where $D_\pm$ are diagonal matrices with entries,

$$D_{\pm,jj} = D_{jj} \pm iD_{jj}\sqrt{\frac{1-D_{jj}}{\|D_{jj}\|}}, \quad (10)$$

and D_{jj} are diagonal elements of D . This diagonal operator can be implemented with the combination of one-qubit rotation, $R_z(\theta)$, and two-qubit controlled-not, CNOT, gates [90] as shown in Fig. 2(b), where the angles are defined as

$$\theta_1 = \frac{1}{2}(\arctan(D_{\pm,11}) + \arctan(D_{\pm,22})), \quad (11)$$

$$\theta_2 = \frac{1}{2}(\arctan(D_{\pm,11}) - \arctan(D_{\pm,22})). \quad (12)$$

The two-qubit algorithm results in the probabilistic evolution of the state, conditioned on the ancilla being in the state $|0\rangle$; however, the dilation requires only one qubit, regardless of the size of the PT -symmetric Hamiltonian. Both optimal and efficient approximate algorithms are known for implementing unitary diagonal operators, although in this work we implement all diagonal operators exactly [91].

B. Two weakly coupled PT -symmetric systems

Using first-order perturbation theory, we use the single-qubit eigenstates to simulate the dynamics of two weakly coupled PT -symmetric systems. We consider a weak interaction term of the form

$$\hat{H}_{\text{int}} = J(\sigma_+^1 \sigma_-^2 + \sigma_-^1 \sigma_+^2), \quad (13)$$

where $\sigma_+ = (\sigma_x + i\sigma_y)/2$, $\sigma_- = (\sigma_x - i\sigma_y)/2$, and σ_i are the Pauli operators [37,60]. In this regime, the system retains a real spectrum within the perturbative approximation. The total Hamiltonian describing this system is given by

$$\hat{H}_{\text{PT}}^{12} = \hat{H}_{\text{PT}}^1 \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}_{\text{PT}}^2 + \hat{H}_{\text{int}}^{12}, \quad (14)$$

where the superscripts list the qubit(s) on which the operator is supported. To find the eigenstates of a system of two noninteracting PT -symmetric systems, we can use the eigenstates of the one-qubit Hamiltonian from Eq. (3). These eigenstates are

$$\begin{aligned} |\psi_{--}\rangle &= |\psi_{-}\rangle \otimes |\psi_{-}\rangle, \\ |\psi_{++}\rangle &= |\psi_{+}\rangle \otimes |\psi_{+}\rangle, \\ |\psi_{+-}\rangle &= |\psi_{+}\rangle \otimes |\psi_{-}\rangle, \\ |\psi_{-+}\rangle &= |\psi_{-}\rangle \otimes |\psi_{+}\rangle. \end{aligned} \quad (15)$$

Using first-order perturbation theory, we modify these states and generate a similarity transformation to capture the dynamics of two weakly coupled PT -symmetric systems [59,60],

$$\tau = \{|\psi_{--}^{(1)}\rangle, |\psi_{-+}^{(1)}\rangle, |\psi_{+-}^{(1)}\rangle, |\psi_{++}^{(1)}\rangle\}, \quad (16)$$

where the superscript (1) refers to first-order perturbation and

$$|\psi_{1,2}^{(1)}\rangle = |\psi_{-+}^{(1)}\rangle \mp |\psi_{+-}^{(1)}\rangle. \quad (17)$$

Using the identity $e^{-i\hat{H}_{\text{PT}}^{12}t} = e^{-i\tau E^{12}\tau^{-1}t} = \tau e^{-iE^{12}t}\tau^{-1}$, we note that E^{12} is the diagonal matrix of the perturbed eigenvalues. We implement the evolution of this system using the hybrid algorithm approach introduced for the single PT -symmetric system.

For a system of this size, we can measure the full density matrix via tomography, allowing calculation of the unique entanglement behavior of PT -symmetric systems near their exceptional points. We explore the entanglement dynamics using the concurrence, which is an entanglement measure for two-qubit systems given by [92,93]

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \quad (18)$$

where ρ is the density matrix, and $\lambda_1, \lambda_2, \lambda_3$, and λ_4 are the square roots of the eigenvalues, in decreasing order, of the non-Hermitian matrix

$$R = \rho \tilde{\rho} \quad (19)$$

with

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y). \quad (20)$$

Note that in the regime where the interaction is weak, the eigenvalues are real and the dynamics are now mapped on the dynamics of a Hermitian Hamiltonian evolution with a unitary time operator. The similarity transformation, τ^{-1} , is implemented on the initial state with single- and two-qubit gates. Here, following the hybrid algorithm approach, the final τ is implemented as a classical postprocessing; however, future work could investigate the explicit decomposition of these circuits using the dilated three-qubit algorithm.

III. RESULTS

A. Single PT -symmetric system

To demonstrate the two approaches for the time evolution of a single PT -symmetric system, we select parameters near the exceptional points, $\Omega = 7.5 \text{ rad}/\mu\text{s}$ and $\gamma = 7 \mu\text{s}^{-1}$, which result in distorted oscillatory dynamics, similar to those in Fig. 1(a). We first identify the unitary operator $U_3(\theta, \phi, \lambda)$ that implements τ on the state $|0\rangle$. For the unitary time evolution, we note that $R_x(\theta) = e^{-i\hat{H}_t}$, where $\theta = \Omega t/2 \cos(\alpha)$. The derived quantum circuits are implemented on IBM-Sherbrooke quantum processing unit (QPU), as well as a simulator with the IBM-Sherbrooke noise model using 2^{12} shots [94]. The device data were acquired over multiple days, with more details discussed in the Appendix.

The evolution of the ground- and excited-state populations on a single qubit initialized as $\tau|0\rangle$ is shown [Fig. 3(a)] in purple and green, respectively. The resulting evolution is a unitary evolution of a Hermitian Hamiltonian on a single qubit with the expected sinusoidal behavior. Figure 3(b) shows the ground- and excited-state populations from the hybrid algorithm considering the full density matrix measured by tomography in the Pauli basis, then classically multiplied by τ^{-1} at each time step, resulting in a PT -symmetric time evolution. Both the QPU (circles) and the simulator results

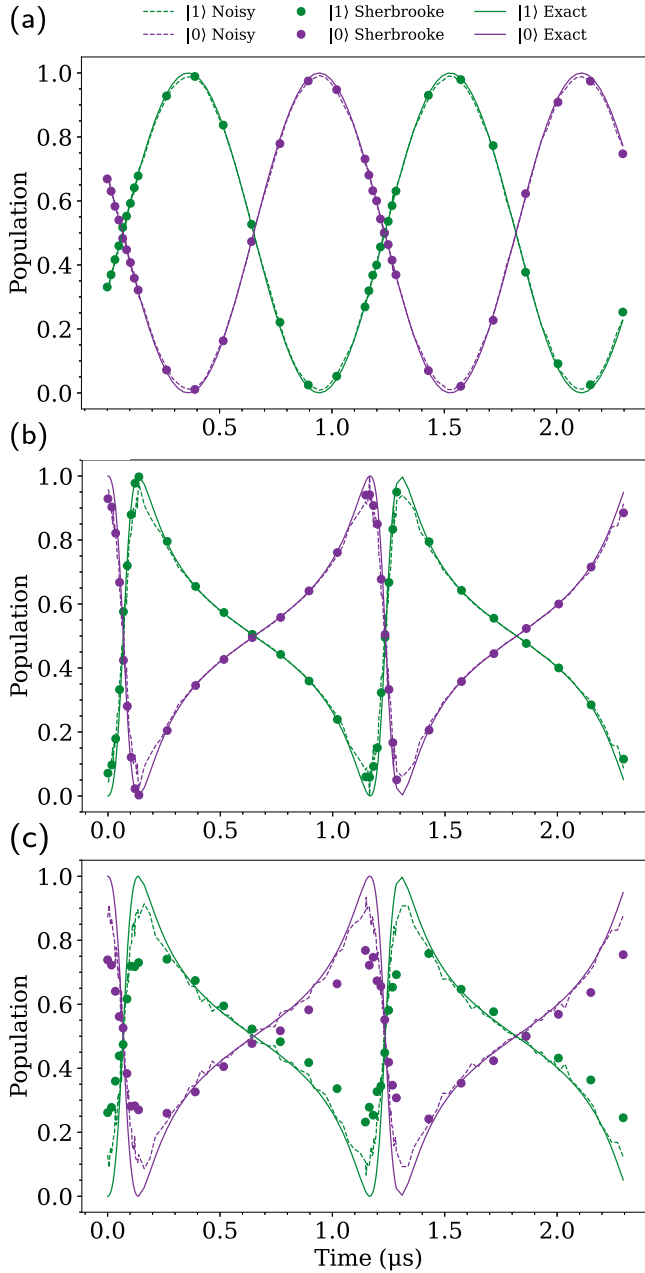


FIG. 3. Ground (purple) and excited (green) state populations for the evolution of the system with (a) $e^{-iHt} \tau|0\rangle$, (b) the one-qubit hybrid algorithm, and (c) the two-qubit dilation algorithm. Solid lines indicate the exact solutions, circles represent experimental data from the IBM Sherbrooke quantum processor, and dashed lines represent the noise-model data, each with 2^{12} shots.

with the Sherbrooke noise model (dashed lines) agree well with the exact classical solution (solid lines). The dilation algorithm approach implements τ^{-1} with final postselection on the $|0\rangle$ state of the ancilla qubit. The normalized populations of ground and excited states, $|00\rangle$ and $|01\rangle$, respectively, are shown in Fig. 3(c), with QPU results as circles. The QPU results are compared with IBM Sherbrooke noise model (dashed lines) as well as the exact classical solution (solid lines). This method remains NISQ friendly as the ancilla qubit is used only for the sparse diagonal operator $\hat{D}$, as seen by

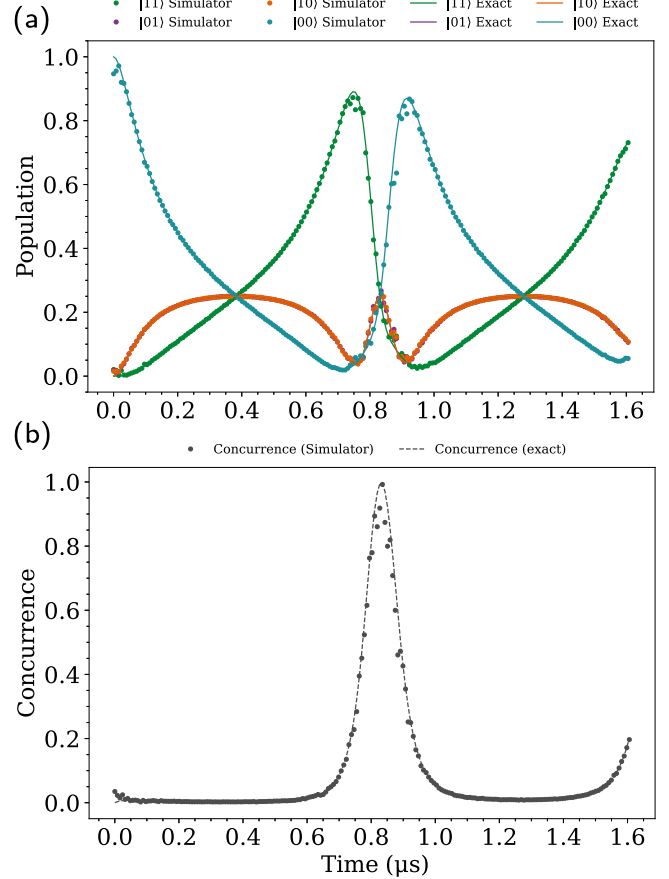


FIG. 4. Time evolution of the [panel (a)] populations in the two-qubit basis states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ over a duration of 1.6 μs and [panel (b)] concurrence, which reaches its maximum when the populations of the basis states become approximately equal.

the agreement of the QPU results with the classical solution. This approach is naturally more sensitive to device noise due to the additional circuit volume; however, this approach can be applicable to coupled PT -symmetric qubits in the fault-tolerant regime via perturbation theory, because the diagonal operator can be implemented efficiently [91].

B. Two weakly coupled PT -symmetric systems

To assess the success of the perturbative treatment of two coupled PT -symmetric systems, we explore these dynamics with the parameters $\gamma = 7 \mu s^{-1}$, $J = 0.019 \text{ rad}/\mu s$, and $\Omega = 7.82 \text{ rad}/\mu s$, and from an initially excited state, $|11\rangle$. We implement the first approach, the hybrid algorithm, from the single PT -symmetric system. Using the Aer simulator, we show the time evolution of the populations for two qubits in Fig. 4(a). Additionally, we compute the concurrence of this system using the density matrix of the evolved system, shown in Fig. 4(b). This shows that the system achieves a maximally entangled state when all populations are equal.

Noting here that we demonstrate the perturbative generalization using the first (hybrid) approach, but we could also use the second approach by diagonalizing τ_2^{-1} and dilating with one ancilla qubit. This method can be used to further simulate interaction Hamiltonians for multiqubit systems with

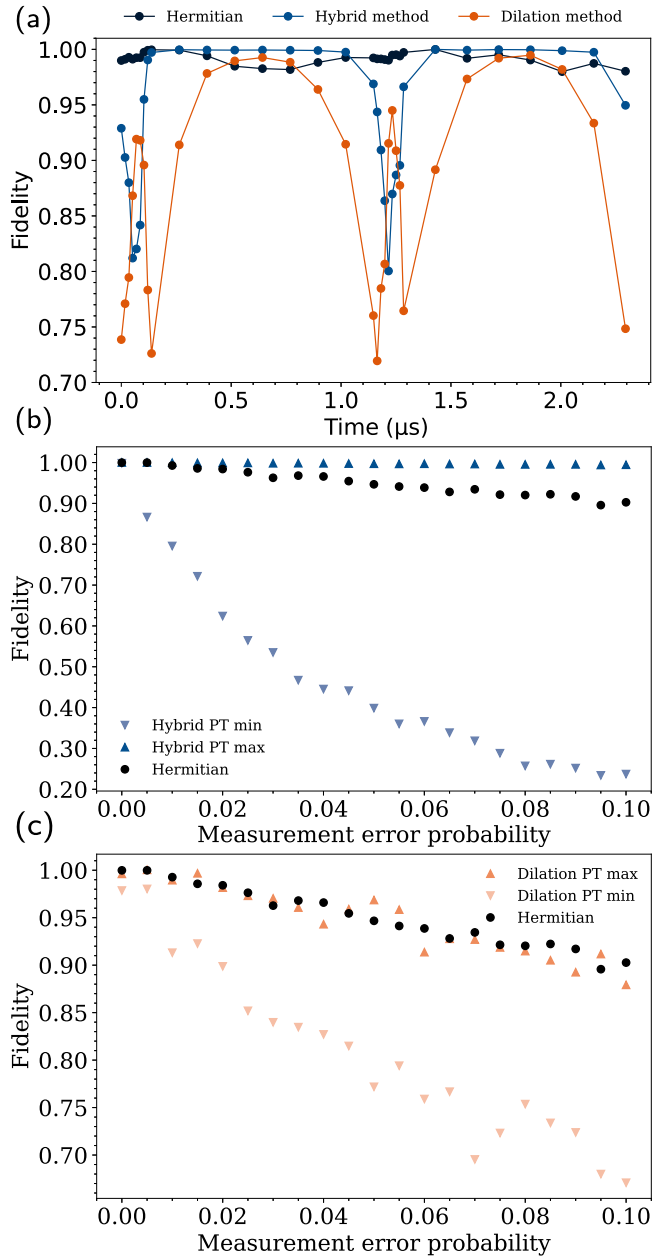


FIG. 5. (a) The fidelity between the exact density matrix and the experimental results. Blue is the hybrid algorithm, orange shows the dilation algorithm results, and black shows the Hermitian evolution of a single qubit. (b) Simulated fidelity with respect to measurement error for the hybrid algorithm. (c) Simulated fidelity with respect to measurement error for the dilation algorithm.

this symmetry [60,82], and would allow us to work with larger systems since it no longer requires full-state tomography.

C. Single PT -symmetric qubit error analysis

Scaling these algorithms beyond two coupled PT -symmetric systems requires a thorough understanding of how the algorithms for a single PT -symmetric system respond to noisy conditions. Returning to the single PT -symmetric system, Fig. 5(a) shows the fidelity between the experimental data and the exact solution for three cases: the

Hermitian single-qubit evolution (black), the hybrid PT -symmetric algorithm (blue), and the dilation algorithm (orange). In the PT -symmetric single-qubit case, errors are amplified in the fast-evolution regime, leading to a decrease in fidelity, whereas in the slow-evolution regime, the fidelity remains stable. The difference in error behavior between Hermitian and PT -symmetric evolution arises due to classical postprocessing, which can either suppress or amplify errors. In particular, matrix multiplication with the density matrix redistributes the error across its elements, so that, for example, X -type Pauli noise no longer corresponds to the same noise model after postprocessing. In the dilation method, postselection is applied independently at each time step, and the use of the CNOT gate contributes to the error characteristics of this approach.

To further assess the impact of noise on fidelity in each algorithm, we analyze the fidelity at two points in the dynamics under varying measurement error using a Qiskit noise model [94]. From the results in Fig. 5(a) for the hybrid algorithm, we choose time $t = 1.22 \mu\text{s}$, where the fidelity is low and $t = 1.77 \mu\text{s}$, where the fidelity is high. Here, the measurement error corresponds to the Pauli error X , where the state of the qubit flips with probability P [90]. As shown in Fig. 5(b), the fidelity at the minimum from $t = 1.22 \mu\text{s}$ (light blue down arrows) exhibits a nonlinear dependence on measurement error, while it remains nearly constant at the maximum of the fidelity (dark blue up arrows) from $t = 1.77 \mu\text{s}$. In contrast, the Hermitian Hamiltonian demonstrates a linear dependence on measurement error (black stars).

For the dilation approach depicted by the orange line in Fig. 5(a), we observe a more complex behavior due to the presence of two-qubit measurement errors and entangling gates. In general, the dilated algorithm is considerably less sensitive to the measurement error, with around 70% fidelity at the highest rate, compared to 20% for the hybrid algorithm. Examining the effect of measurement error on fidelity at different points in the absence of other errors, we find that at the maximum $t = 0.07 \mu\text{s}$, the fidelity depends linearly on noise as shown in Fig. 5(c). However, at the minimum fidelity $t = 0.01 \mu\text{s}$, the dependence on measurement error increases. Because of increased quantum volume requirements, the dilated algorithm may incur larger error from two-qubit gate coherence limitations, which could explain the overall worse fidelity of the dilated algorithm; however, the classical postprocessing in the hybrid algorithm is highly sensitive to the measurement error.

IV. DISCUSSION AND CONCLUSIONS

Here, we have demonstrated that pseudo-Hermiticity can be harnessed to develop perturbatively generalizable quantum algorithms for PT -symmetric dynamics in the PT -unbroken phase. In the unbroken regime, there exists a similarity transformation, which allows the non-Hermitian Hamiltonian to be mapped onto a Hermitian Hamiltonian with the same spectrum. Using this similarity transformation, these dynamics can be simulated using both classical-quantum hybrid and ancilla-assisted algorithms. Both algorithms suggest the possibility to simulate PT -symmetric systems through real-time evolution

by using the Hermitian-equivalent Hamiltonian rather than individual unitary gates.

We then considered the evolution of two weakly interacting PT -symmetric systems to demonstrate the algorithm extension to larger systems utilizing perturbation theory. Larger systems of equivalent weakly interacting PT -symmetric qubits can further be simulated in this way. The weakly coupled qubits can be perturbatively treated in the basis of eigenstates of a single qubit, which allows for an efficient mapping of the coupled-qubit dynamics to a quantum algorithm. Here, we simulated the two-qubit dynamics using a hybrid algorithm, which effectively requires tomography of the density matrix. Alternatively, the coupled-qubit dynamics could be simulated with the fully quantum dilated algorithm, which would bypass the costly tomography.

Additionally, we analyzed the error of the hybrid and fully quantum algorithms to understand how they affect the fidelity of the PT dynamics. By simulating varying measurement errors, we found that the hybrid algorithm is significantly more sensitive to error, compared to the fully quantum algorithm. The largest errors occur during the fast part of the dynamics, and are absent from the Hermitian dynamics, highlighting the unique aspects of PT -symmetric systems, which are potentially useful for quantum sensing applications.

We note that the decompositions presented here are formally exact, but rely on probabilistic sampling of the quantum circuits. The complexity of the circuit for the fully quantum algorithm is dominated by the application of the similarity transformation τ over the dilated circuit. The similarity transformation may retain physical symmetries of the eigenvectors of H ; however, in general, this operator may not be simple or efficient to decompose. Importantly, the similarity transformation is time independent, so the decomposition only needs to be performed once. The qubit count of the algorithm scales directly with the size of the system, plus one for the ancilla qubit.

Other algorithms for PT -symmetric systems include QITE and quantum trajectories (QT) methods [8,50]. In this context, QITE iteratively approximates the state at each time step of the dynamics, although to our knowledge, QITE has not been used to simulate PT dynamics. The recently reported QT algorithm leverages an important connection between the Lindblad equation and PT -symmetric dynamics described here. In the QT algorithm, the PT dynamics would be implemented as an example of nonlinear Lindblad dynamics using two ancilla qubits. An interesting open question is the error sensitivity near the fast regime in other quantum algorithms for nonunitary dynamics.

Our results describe a mapping of weakly coupled PT -symmetric qubits to a unitary gate-based quantum algorithm. The results suggest a scalable approach for simulating these dynamics on both near-term and fault-tolerant quantum computers. Error analysis highlights the unique challenges of simulating PT -symmetric systems on noisy quantum devices, which mirrors the unique behavior of PT -symmetric dynamics near exceptional points. This work opens an avenue for further investigation of non-Hermitian Hamiltonians on quantum computers. Future works will explore methods that enhance the efficiency of these algorithms as well as focusing on the potential application of these non-Hermitian systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: DEVICE DETAIL

We used the default quantum state tomography in Qiskit experiments version 0.8 for the hybrid algorithm, and we performed tomography with projective measurement with three tomographic gates on qubit 0 for the dilation algorithm. Both methods used samplerV2 with no error mitigation and suppression enabled. The QPU used here is `ibm_sherbrooke`, which is a 127-qubit quantum platform, where the processor type is specified as Eagle r3 superconducting qubit chip. The basis gates for this QPU are {ECR, ID, RZ, SX, X}. The median of respective T_1 and T_2 times are $\sim 267 \mu\text{s}$, and $\sim 171 \mu\text{s}$. The median error of each ID, SX, and X gates is 2.42×10^{-4} , whereas the error of RZ gate is 0. In terms of two-qubit gate error, the median of the ECR gate error is 8.02×10^{-3} . The median of the readout error is 1.66×10^{-2} , the state preparation and measurement (SPAM) error is ~ 0.015 .

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- [1] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, UK, 2007).
 - [2] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Preparation of entangled states by quantum Markov processes, *Phys. Rev. A* **78**, 042307 (2008).
 - [3] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, Quantum states and phases in driven open quantum systems with cold atoms, *Nat. Phys.* **4**, 878 (2008).
 - [4] F. Verstraete, M. M. Wolf, and J. I. Cirac, Quantum computation and quantum-state engineering driven by dissipation, *Nat. Phys.* **5**, 633 (2009).
 - [5] K. Head-Marsden, J. Flick, C. J. Ciccarino, and P. Narang, Quantum information and algorithms for correlated quantum matter, *Chem. Rev.* **121**, 3061 (2021).
 - [6] M. L. Olivera-Atencio, L. Lamata, and J. Casado-Pascual, Benefits of open quantum systems for quantum machine learning, *Adv. Quantum Technol.* **8**, 2300247 (2023).

- [7] S. McArdle, T. Jones, S. Endo, Y. Li, S. C. Benjamin, and X. Yuan, Variational ansatz-based quantum simulation of imaginary time evolution, *npj Quantum Inf.* **5**, 75 (2019).
- [8] H. Kamakari, S.-N. Sun, M. Motta, and A. J. Minnich, Digital quantum simulation of open quantum systems using quantum imaginary-time evolution, *PRX Quantum* **3**, 010320 (2022).
- [9] T. Shirakawa, K. Seki, and S. Yunoki, Discretized quantum adiabatic process for free fermions and comparison with the imaginary-time evolution, *Phys. Rev. Res.* **3**, 013004 (2021).
- [10] H. Nishi, T. Kosugi, and Y.-i. Matsushita, Implementation of quantum imaginary-time evolution method on NISQ devices by introducing nonlocal approximation, *npj Quantum Inf.* **7**, 85 (2021).
- [11] S.-H. Lin, R. Dilip, A. G. Green, A. Smith, and F. Pollmann, Real- and imaginary-time evolution with compressed quantum circuits, *PRX Quantum* **2**, 010342 (2021).
- [12] Z. Hu, R. Xia, and S. Kais, A quantum algorithm for evolving open quantum dynamics on quantum computing devices, *Sci. Rep.* **10**, 3301 (2020).
- [13] A. W. Schlimgen, K. Head-Marsden, L. M. Sager, P. Narang, and D. A. Mazziotti, Quantum simulation of open quantum systems using a unitary decomposition of operators, *Phys. Rev. Lett.* **127**, 270503 (2021).
- [14] A. W. Schlimgen, K. Head-Marsden, L. M. Sager-Smith, P. Narang, and D. A. Mazziotti, Quantum state preparation and nonunitary evolution with diagonal operators, *Phys. Rev. A* **106**, 022414 (2022).
- [15] A. Gaikwad, Arvind, and K. Dorai, Simulating open quantum dynamics on an NMR quantum processor using the Sz.-Nagy dilation algorithm, *Phys. Rev. A* **106**, 022424 (2022).
- [16] N. Suri, J. Barreto, S. Hadfield, N. Wiebe, F. Wudarski, and J. Marshall, Two-unitary decomposition algorithm and open quantum system simulation, *Quantum* **7**, 1002 (2023).
- [17] Z. Ding, X. Li, and L. Lin, Simulating open quantum systems using Hamiltonian simulations, *PRX Quantum* **5**, 020332 (2024).
- [18] T. Basile and C. Pineda, Quantum simulation of Pauli channels and dynamical maps: Algorithm and implementation, *PLoS One* **19**, e0297210 (2024).
- [19] J. Xuereb, S. Campbell, J. Goold, and A. Xuereb, Deterministic quantum computation with one-clean-qubit model as an open quantum system, *Phys. Rev. A* **107**, 042222 (2023).
- [20] P. Gupta and C. M. Chandrashekar, Optimal quantum simulation of open quantum systems, *arXiv:2012.07540*.
- [21] J. T. Barreiro, M. Mueller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, An open-system quantum simulator with trapped ions, *Nature (London)* **470**, 486 (2011).
- [22] S.-J. Wei, D. Ruan, and G.-L. Long, Duality quantum algorithm efficiently simulates open quantum systems, *Sci. Rep.* **6**, 30727 (2016).
- [23] S. Mostame, J. Huh, C. Kreisbeck, A. J. Kerman, T. Fujita, A. Eisfeld, and A. Aspuru-Guzik, Emulation of complex open quantum systems using superconducting qubits, *Quantum Info. Proc.* **16**, 44 (2017).
- [24] G. Garcia-Perez, M. A. C. Rossi, and S. Maniscalco, IBM Q experience as a versatile experimental testbed for simulating open quantum systems, *npj Quantum Inf.* **6**, 1 (2020).
- [25] S. Patsch, S. Maniscalco, and C. P. Koch, Simulation of open-quantum-system dynamics using the quantum zeno effect, *Phys. Rev. Lett.* **2**, 023133 (2020).
- [26] H.-Y. Su and Y. Li, Quantum algorithm for the simulation of open-system dynamics and thermalization, *Phys. Rev. A* **101**, 012328 (2020).
- [27] S. Sun, Li-C. Shih, and Y.-C. Cheng, Efficient quantum simulation of open quantum system dynamics on noisy quantum computers, *Phys. Scr.* **99**, 035101 (2024).
- [28] A. Burger, L. C. Kwek, and D. Poletti, Digital quantum simulation of the spin-Boson model under Markovian open-system dynamics, *Entropy* **24**, 1766 (2022).
- [29] A. Wang, J. Zhang, and Y. Li, Error-mitigated deep-circuit quantum simulation of open systems: Steady state and relaxation rate problems, *Phys. Rev. Res.* **4**, 043140 (2022).
- [30] M. Cattaneo, M. A. C. Rossi, G. García-Pérez, R. Zambrini, and S. Maniscalco, Quantum simulation of dissipative collective effects on noisy quantum computers, *PRX Quantum* **4**, 010324 (2023).
- [31] M. S. Zanetti, D. F. Pinto, M. L. W. Basso, and J. Maziero, Simulating noisy quantum channels via quantum state preparation algorithms, *J. Phys. B: At. Mol. Opt. Phys.* **56**, 115501 (2023).
- [32] J. Leppaakangas, N. Vogt, K. R. Fratus, K. Bark, J. A. Vaitkus, P. Stadler, J.-M. Reiner, S. Zanker, and M. Marthaler, Quantum algorithm for solving open-system dynamics on quantum computers using noise, *Phys. Rev. A* **108**, 062424 (2023).
- [33] J. D. Guimarães, J. Lim, M. I. Vasilevskiy, S. F. Huelga, and M. B. Plenio, Noise-assisted digital quantum simulation of open systems using partial probabilistic error cancellation, *PRX Quantum* **4**, 040329 (2023).
- [34] J. D. Guimarães, M. I. Vasilevskiy, and L. S. Barbosa, Digital quantum simulation of non-perturbative dynamics of open systems with orthogonal polynomials, *Quantum* **8**, 1242 (2024).
- [35] L. Del Re, B. Rost, M. Foss-Feig, A. F. Kemper, and J. K. Freericks, Robust measurements of n -point correlation functions of driven-dissipative quantum systems on a digital quantum computer, *Phys. Rev. Lett.* **132**, 100601 (2024).
- [36] A. M. Childs and T. Li, Efficient simulation of sparse Markovian quantum dynamics, *Quantum Info. Comput.* **17**, 901 (2017).
- [37] X. Li and C. Wang, Succinct description and efficient simulation of non-Markovian open quantum systems, *Commun. Math. Phys.* **401**, 147 (2023).
- [38] X. Li and C. Wang, Simulating Markovian open quantum systems using higher-order series expansion, *Leibniz Int. Proc. Inf.* **261**, 87 (2023).
- [39] M. Cosacchi, M. Cygorek, F. Ungar, A. M. Barth, A. Vagov, and V. M. Axt, Path-integral approach for nonequilibrium multitime correlation functions of open quantum systems coupled to Markovian and non-Markovian environments, *Phys. Rev. B* **98**, 125302 (2018).
- [40] J. Liu, D. Segal, and G. Hanna, Hybrid quantum-classical simulation of quantum speed limits in open quantum systems, *J. Phys. A: Math. Theor.* **52**, 215301 (2019).
- [41] M. Cygorek, M. Cosacchi, A. Vagov, V. M. Axt, B. W. Lovett, J. Keeling, and E. M. Gauger, Simulation of open quantum systems by automated compression of arbitrary environments, *Nat. Phys.* **18**, 662 (2022).

- [42] F. Andreadakis, N. Anand, and P. Zanardi, Scrambling of algebras in open quantum systems, *Phys. Rev. A* **107**, 042217 (2023).
- [43] F.-M. L. Régent and P. Rouchon, Adiabatic elimination for composite open quantum systems: Reduced-model formulation and numerical simulations, *Phys. Rev. A* **109**, 032603 (2024).
- [44] M. Müller, K. Hammerer, Y. L. Zhou, C. F. Roos, and P. Zoller, Simulating open quantum systems: From many-body interactions to stabilizer pumping, *New J. Phys.* **13**, 085007 (2011).
- [45] N. Ramusat and V. Savona, A quantum algorithm for the direct estimation of the steady state of open quantum systems, *Quantum* **5**, 399 (2021).
- [46] M. Mahdian and H. D. Yeganeh, Incoherent quantum algorithm dynamics of an open system with near-term devices, *Quantum Info. Proc.* **19**, 285 (2020).
- [47] C. S. Wang, N. E. Frattini, B. J. Chapman, S. Puri, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Observation of wave-packet branching through an engineered conical intersection, *Phys. Rev. X* **13**, 011008 (2023).
- [48] R. Dutta, *et al.*, Simulating chemistry on bosonic quantum devices, *J. Chem. Theory Comput.* **20**, 6426 (2024).
- [49] L. H. Delgado-Granados, T. J. Krogmeier, L. M. Sager-Smith, I. Avdic, Z. Hu, M. Sajjan, M. Abbasi, S. E. Smart, P. Narang, S. Kais, A. W. Schlimgen, K. Head-Marsden, and D. A. Mazziotti, Quantum algorithms and applications for open quantum systems, *Chem. Rev.* **125**, 1823 (2025).
- [50] Y.-G. Liu, H. Fan, and S. Chen, Digital quantum simulation of the nonlinear Lindblad master equation based on quantum trajectory averaging, [arXiv:2504.00121](https://arxiv.org/abs/2504.00121).
- [51] C. M. Bender and S. Boettcher, Real spectra in non-Hermitian Hamiltonians having $\mathcal{PT}$ symmetry, *Phys. Rev. Lett.* **80**, 5243 (1998).
- [52] M. Naghiloo, M. Abbasi, Y. N. Joglekar, and K. W. Murch, Quantum state tomography across the exceptional point in a single dissipative qubit, *Nat. Phys.* **15**, 1232 (2019).
- [53] M. Abbasi, W. Chen, M. Naghiloo, Y. N. Joglekar, and K. W. Murch, Topological quantum state control through exceptional-point proximity, *Phys. Rev. Lett.* **128**, 160401 (2022).
- [54] W. Chen, M. Abbasi, Y. N. Joglekar, and K. W. Murch, Quantum jumps in the non-Hermitian dynamics of a superconducting qubit, *Phys. Rev. Lett.* **127**, 140504 (2021).
- [55] W. Chen, M. Abbasi, B. Ha, S. Erdamar, Y. N. Joglekar, and K. W. Murch, Decoherence-induced exceptional points in a dissipative superconducting qubit, *Phys. Rev. Lett.* **128**, 110402 (2022).
- [56] Y. Wu, W. Liu, J. Geng, X. Song, X. Ye, Chang-K. Duan, X. Rong, and J. Du, Observation of parity-time symmetry breaking in a single-spin system, *Science* **364**, 878 (2019).
- [57] J. Höller, N. Read, and J. G. E. Harris, Non-Hermitian adiabatic transport in spaces of exceptional points, *Phys. Rev. A* **102**, 032216 (2020).
- [58] C. Guria, Q. Zhong, S. K. Ozdemir, Y. S. S. Patil, R. El-Ganainy, and J. G. E. Harris, Resolving the topology of encircling multiple exceptional points, *Nat. Commun.* **15**, 1369 (2024).
- [59] Z.-Z. Li, W. Chen, M. Abbasi, K. W. Murch, and K. B. Whaley, Speeding up entanglement generation by proximity to higher-order exceptional points, *Phys. Rev. Lett.* **131**, 100202 (2023).
- [60] B. B. Liu, S.-L. Su, Y. L. Zuo, Q. He, G. Chen, F. Nori, and H. Jing, Using PT -symmetric qubits to break the trade-off between fidelity and the degree of quantum entanglement, [arXiv:2407.08525](https://arxiv.org/abs/2407.08525).
- [61] F. Quijandria, U. Naether, S. K. Özdemir, F. Nori, and D. Zueco, PT -symmetric circuit qed, *Phys. Rev. A* **97**, 053846 (2018).
- [62] J. Zhang, Y. L. Zhou, Y. L. Zuo, P. X. Chen, H. Jing, and L. M. Kuang, Parity-time-symmetric two-qubit system: Entanglement and sensing, [arXiv:2305.19034](https://arxiv.org/abs/2305.19034).
- [63] A. S. Kazmina, *et al.*, Demonstration of a parity-time-symmetry-breaking phase transition using superconducting and trapped-ion qutrits, *Phys. Rev. A* **109**, 032619 (2024).
- [64] J. Wen, C. Zheng, X. Kong, S. Wei, T. Xin, and G. Long, Experimental demonstration of a digital quantum simulation of a general PT -symmetric system, *Phys. Rev. A* **99**, 062122 (2019).
- [65] S. Dogra, A. A. Melnikov, and G. S. Paraoanu, Quantum simulation of parity-time symmetry breaking with a superconducting quantum processor, *Commun. Phys.* **4**, 26 (2021).
- [66] U. Günther and B. F. Samsonov, Naimark-dilated PT -symmetric brachistochrone, *Phys. Rev. Lett.* **101**, 230404 (2008).
- [67] K. Kawabata, Y. Ashida, and M. Ueda, Information retrieval and criticality in parity-time-symmetric systems, *Phys. Rev. Lett.* **119**, 190401 (2017).
- [68] C. M. Bender, D. C. Brody, J. Caldeira, U. Günther, B. K. Meister, and B. F. Samsonov, PT -symmetric quantum state discrimination, *Phil. Trans. R. Soc. A* **371**, 20120160 (2013).
- [69] Y.-C. Lee, M.-H. Hsieh, S. T. Flammia, and R.-K. Lee, Local PT symmetry violates the no-signaling principle, *Phys. Rev. Lett.* **112**, 130404 (2014).
- [70] S.-L. Chen, G.-Y. Chen, and Y.-N. Chen, Increase of entanglement by local PT -symmetric operations, *Phys. Rev. A* **90**, 054301 (2014).
- [71] J. Wiersig, Sensors operating at exceptional points: General theory, *Phys. Rev. A* **93**, 033809 (2016).
- [72] Z.-P. Liu, J. Zhang, Ş. Kaya Özdemir, B. Peng, H. Jing, X.-Y. Lü, C.-W. Li, L. Yang, F. Nori, and Y.-X. Liu, Metrology with PT -symmetric cavities: Enhanced sensitivity near the PT -phase transition, *Phys. Rev. Lett.* **117**, 110802 (2016).
- [73] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Non-Hermitian physics and PT symmetry, *Nat. Phys.* **14**, 11 (2018).
- [74] G. Yoo, H.-S. Sim, and H. Schomerus, Quantum noise and mode nonorthogonality in non-Hermitian PT -symmetric optical resonators, *Phys. Rev. A* **84**, 063833 (2011).
- [75] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Observation of parity-time symmetry in optics, *Nat. Phys.* **6**, 192 (2010).
- [76] F. Klauck, L. Teuber, M. Ornigotti, M. Heinrich, S. Scheel, and A. Szameit, Observation of PT -symmetric quantum interference, *Nat. Photon.* **13**, 883 (2019).
- [77] L. Xiao, T. Deng, K. Wang, Z. Wang, W. Yi, and P. Xue, Observation of non-Bloch parity-time symmetry and exceptional points, *Phys. Rev. Lett.* **126**, 230402 (2021).
- [78] J. Li, A. K. Harter, J. Liu, L. de Melo, Y. N. Joglekar, and L. Luo, Observation of parity-time symmetry breaking transitions in a dissipative Floquet system of ultracold atoms, *Nat. Commun.* **10**, 855 (2019).

- [79] C. M. Bender, D. C. Brody, H. F. Jones, and B. K. Meister, Faster than Hermitian quantum mechanics, *Phys. Rev. Lett.* **98**, 040403 (2007).
- [80] C. Zheng, L. Hao, and G. L. Long, Observation of a fast evolution in a parity-time-symmetric system, *Phil. Trans. R. Soc. A.* **371**, 20120053 (2013).
- [81] A. Mostafazadeh, Exact PT -symmetry is equivalent to hermiticity, *J. Phys. A: Math. Gen.* **36**, 7081 (2003).
- [82] C. G. Feyisa, J.-S. You, H.-Y. Ku, and H. H. Jen, Accelerating multipartite entanglement generation in non-Hermitian superconducting qubits, *Quantum Sci. Technol.* **10**, 025021 (2025).
- [83] Z. Li, X. Huang, H. Zhu, G. Zhang, F. Wang, and X. Zhong, Multitype entanglement dynamics induced by exceptional points, *Phys. Rev. A* **111**, 022213 (2025).
- [84] A. Mostafazadeh, Pseudo-Hermiticity versus PT symmetry: The necessary condition for the reality of the spectrum of a non-Hermitian Hamiltonian, *J. Math. Phys.* **43**, 205 (2002).
- [85] P. D. Mannheim, PT symmetry as a necessary and sufficient condition for unitary time evolution, *Philos. Trans. R. Soc. A*, **371**, 20120060 (2013).
- [86] C. Zheng, Quantum simulation of pseudo-Hermitian- ϕ -symmetric two-level systems, *Entropy* **24**, 867 (2022).
- [87] S. Croke, $\mathcal{PT}$ -symmetric Hamiltonians and their application in quantum information, *Phys. Rev. A* **91**, 052113 (2015).
- [88] Q.-H. Wang, S.-Z. Chia, and J.-H. Zhang, $\mathcal{PT}$ symmetry as a generalization of Hermiticity, *J. Phys. A: Math. Theor.* **43**, 295301 (2010).
- [89] C. Zheng, Universal quantum simulation of single-qubit nonunitary operators using duality quantum algorithm, *Sci. Rep.* **11**, 3960 (2021).
- [90] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, 10th anniversary ed. (Cambridge University Press, Cambridge, UK, 2010).
- [91] J. Welch, D. Greenbaum, S. Mostame, and A. Aspuru-Guzik, Efficient quantum circuits for diagonal unitaries without ancillas, *New J. Phys.* **16**, 033040 (2014).
- [92] S. A. Hill and W. K. Wootters, Entanglement of a pair of quantum bits, *Phys. Rev. Lett.* **78**, 5022 (1997).
- [93] W. K. Wootters, Entanglement of formation of an arbitrary state of two qubits, *Phys. Rev. Lett.* **80**, 2245 (1998).
- [94] A. Javadi-Abhari, M. Treinish, K. Krsulich, C. J. Wood, J. Lishman, J. Gacon, S. Martiel, P. D. Nation, L. S. Bishop, A. W. Cross, B. R. Johnson, and J. M. Gambetta, Quantum computing with Qiskit, [arXiv:2405.08810](https://arxiv.org/abs/2405.08810).