

Floquet polaron and molecule in a highly polarized Fermi gas with periodically modulated interactions

Ming-Yue Yang, An-Chun Ji,^{*} and Qing Sun[†]

Department of Physics, *Capital Normal University*, Beijing 100048, China



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We investigate the impurity problem of a highly polarized Fermi gas with periodically modulated atom-atom interactions. We find that generalized time-periodic Floquet polaron and molecule emerge as quasistationary quasiparticle states of the system, which exhibit peculiar features distinct from those in a static system. Even in the high-frequency limit, the periodic driving can renormalize the scattering length of the static (effective) Hamiltonian and thus modify the polaron-to-molecule transition. Remarkably, due to the folding of the quasienergy of the Floquet states in the Floquet Brillouin zone, unusual energy-level crossings beyond the conventional polaron-to-molecule transition occur. In particular, when the driving frequency is in resonance with the quasienergy difference, both branches of the Floquet polaron can intersect with the molecular state or even themselves under weak driving. The unique physics of these Floquet impurity states is further characterized and elucidated by the experimentally measurable spectral functions and the radio-frequency spectra. Our results pave a way to explore the intriguing nonequilibrium impurity physics beyond a static system via “Floquet engineering.”

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I. INTRODUCTION

The polaron [1,2], known as a quasiparticle emerging from the interaction between a moving impurity and the quantum environment, plays a fundamental role toward the understanding of intriguing but complicated many-body physics [3–5]. In the diverse realizations of polaron-type states [6,7], the ultracold quantum gases [8,9], being of high controllability and extreme cleanness, provide an ideal platform to study the polaron and related impurity physics [10–13]. Significant advances include the Fermi polaron [14–21] and its observations in ultracold Fermi gases [22–28], the Bose polarons in bosonic gases [29–36], and the polaron-to-molecule transitions [17,22,27,37–40], which can be readily addressed by tuning, e.g., the atom-atom interaction between the impurity and its environment via Feshbach resonance [41].

In recent years, the periodically driven quantum systems, whose stroboscopic dynamics can be described by time-independent Hamiltonians [42–44], are widely utilized to engineer exotic Hamiltonians [45,46] and nontrivial quantum matter states [47–49] beyond a static systems. For example, the density-dependent tunnelings [50,51] and synthetic gauge fields [52–57], anomalous Floquet topological phases

[58–64], and discrete-time crystals [65–70] can be implemented by the so-called “Floquet engineering.” One thus could expect that the impurity problem would also be significantly influenced in a periodically driven system. On one hand, the periodic driving may renormalize the parameters of the static Floquet Hamiltonian [51] and thus affect the underlying impurity physics. On the other hand, nontrivial generalized time-dependent Floquet impurity states beyond the static counterparts may appear. These interesting issues, however, have not ever been studied so far.

To reveal the unique impurity physics in the periodically driven systems, in this work, we consider a highly polarized two-component Fermi gas with a periodically modulated contact interaction between the impurity atom and the majority atoms. We find that the periodic driving can dramatically change the impurity states, giving rise to remarkable Floquet polaron and molecular states with unique features beyond static system. In the high-frequency limit (HFL), the system can be described by a static Hamiltonian with effective s -wave scattering length $a_{\text{eff}} = a_0 \sqrt{1 - (a_1/a_0)^2}$, suggesting that the usual polaron-to-molecule transition in the Bose-Einstein condensate (BEC) side ($a_0 > 0$) can be addressed by increasing the driving strength a_1 . However, for a finite but large driving frequency, generalized time-periodic Floquet polaron and molecular states are developed, with the quasienergy levels being folded and intersecting in the Floquet Brillouin zone. Remarkably, unprecedented energy-level crossings may appear even for a weak driving field when the driving frequency is in resonance with the quasienergy difference between two impurity states. The underlying physics is further characterized and elucidated by the spectral functions and the radio-frequency (rf) spectra of the Floquet polaron and molecule, which can be measured

^{*}Contact author: andrewjee@sina.com

[†]Contact author: sunqing@cnu.edu.cn

in experiments. Our research opens an avenue to study the exotic Floquet-engineered impurity physics as well as the unconventional quasistationary many-body states beyond a static system.

II. THE MODEL

We consider a single fermionic impurity atom (labeled as $\sigma = \downarrow$) immersed in a Fermi sea formed by N atoms (labeled as $\sigma = \uparrow$), with a periodical modulating s -wave interaction between the impurity and the majority atoms. In the momentum space, the Hamiltonian of the system is given by

$$\hat{H}(t) = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma}^\dagger \hat{c}_{\mathbf{k}\sigma} + \frac{g(t)}{V} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \hat{c}_{\mathbf{k}\uparrow}^\dagger \hat{c}_{-\mathbf{k}+\mathbf{q}\downarrow}^\dagger \hat{c}_{-\mathbf{k}'+\mathbf{q}\downarrow} \hat{c}_{\mathbf{k}'\uparrow}. \quad (1)$$

Here, $\hat{c}_{\mathbf{k}\sigma}^\dagger$ ($\hat{c}_{\mathbf{k}\sigma}$) creates (annihilates) a Fermi atom of spin σ with momentum $\mathbf{k}$. $\epsilon_{\mathbf{k}\uparrow} = \epsilon_{\mathbf{k}\downarrow} = \frac{\hbar^2 \mathbf{k}^2}{2M} \equiv \epsilon_{\mathbf{k}}$ denote the energy dispersions, which are considered to be degenerate for both components with equal mass M . $g(t)$ describes a time-dependent contact interaction, which relates to the time-dependent s -wave scattering length $a_s(t)$ via the renormalization relation $\frac{1}{g(t)} = \frac{M}{4\pi\hbar^2 a_s(t)} - \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} = \frac{M}{4\pi\hbar^2 a_s(t)} - \frac{M\Lambda}{2\pi^2\hbar^2}$, with Λ being the momentum cutoff [71]. For a periodical modulation, we have $g(t+T) = g(t)$, i.e., $a_s(t+T) = a_s(t)$, with T the period. In experiments, such an $a_s(t)$ can be achieved by tuning, e.g., the magnetic field $B(t)$ [50,72,73] via the Feshbach resonance. Without loss of generality, we consider a single-mode modulation of frequency $\Omega = \frac{2\pi}{T}$, i.e., $a_s(t) = a_0 + a_1 \cos(\Omega t)$, where a_0 and a_1 characterize the static and time-dependent components of the scattering length, respectively.

III. FLOQUET POLARON

We generalize the Chevy's ansatz [14] for a Fermi polaron state with center-of-mass (c.m.) momentum $\mathbf{Q}$ to a time-dependent case, with the form of

$$|P(t)\rangle_{\mathbf{Q}} = e^{-i\mathcal{E}_P t/\hbar} \left[\phi_0(t) \hat{c}_{\mathbf{Q}\downarrow}^\dagger + \sum_{\mathbf{k}\mathbf{q}} \phi_{\mathbf{k}\mathbf{q}}(t) \hat{c}_{\mathbf{Q}-\mathbf{k}+\mathbf{q}\downarrow}^\dagger \hat{c}_{\mathbf{k}\uparrow}^\dagger \hat{c}_{\mathbf{q}\uparrow} \right] |N\rangle. \quad (2)$$

Here, $\phi_0(t) = \phi_0(t+T)$ and $\phi_{\mathbf{k}\mathbf{q}}(t) = \phi_{\mathbf{k}\mathbf{q}}(t+T)$ are the time-periodic amplitudes for the bare impurity and particle-hole dressing, respectively, satisfying the normalization condition $\frac{1}{T} \int_0^T dt \{ |\phi_0|^2 + \sum_{\mathbf{k}\mathbf{q}} |\phi_{\mathbf{k}\mathbf{q}}|^2 \} = 1$. $|N\rangle$ denotes the Fermi sea formed by N atoms of spin $\uparrow$, and the summations over $\mathbf{k}$ and $\mathbf{q}$ are limited to $|\mathbf{k}| > k_F$ (particle) and $|\mathbf{q}| < k_F$ (hole), where $k_F = |\mathbf{k}_F|$, with $\mathbf{k}_F$ being the Fermi wave vector. $\mathcal{E}_P$ is the quasienergy of such Floquet state defined in a Floquet Brillouin zone (FBZ) with the energy interval spanned by $\hbar\Omega$.

Expanding $\phi_0(t) = \sum_n \phi_0^{(n)} \exp(in\Omega t)$ and $\phi_{\mathbf{k}\mathbf{q}}(t) = \sum_n \phi_{\mathbf{k}\mathbf{q}}^{(n)} \exp(in\Omega t)$ ($n \in \mathbb{Z}$), and substituting them into the Schrödinger equation $i\hbar \frac{\partial}{\partial t} |P(t)\rangle = \hat{H}(t) |P(t)\rangle$, we obtain a couple of recursion equations for the time-independent

coefficients $\{\phi_0^{(n)}, \phi_{\mathbf{k}\mathbf{q}}^{(n)}\}$, which are

$$(\mathcal{E}_P - n\hbar\Omega - \epsilon_{\mathbf{Q}}) \phi_0^{(n)} = \frac{1}{V} \sum_{m,\mathbf{q}} g^{(m)} \left[\phi_0^{(n-m)} + \sum_{\mathbf{k}} \phi_{\mathbf{k}\mathbf{q}}^{(n-m)} \right], \quad (3a)$$

$$(\mathcal{E}_P - n\hbar\Omega - E_{\mathbf{k}\mathbf{q}}) \phi_{\mathbf{k}\mathbf{q}}^{(n)} = \frac{1}{V} \sum_m g^{(m)} \left[\phi_0^{(n-m)} + \sum_{\mathbf{k}} \phi_{\mathbf{k}\mathbf{q}}^{(n-m)} \right]. \quad (3b)$$

Here, $E_{\mathbf{k}\mathbf{q}} = \epsilon_{\mathbf{k}\uparrow} + \epsilon_{\mathbf{Q}-\mathbf{k}+\mathbf{q}\downarrow} - \epsilon_{\mathbf{q}\uparrow}$ and $g^{(m)} \equiv \frac{1}{T} \int_0^T g(t) e^{-im\Omega t} dt$, which for $a_1 < |a_0|$ are given by

$$g^{(0)} = -\frac{2\pi^2}{M\Lambda} \left[1 + \text{sgn}(a_0) \frac{\frac{\pi}{\Lambda}}{\sqrt{(\frac{\pi}{\Lambda} - 2a_0)^2 - 4a_1^2}} \right], \quad (4)$$

$$g^{(m>0)} = g^{(-m)} = -\text{sgn}(a_0) \frac{2\pi^3}{M\Lambda^2 \sqrt{(\frac{\pi}{\Lambda} - 2a_0)^2 - 4a_1^2}} \times \left[\frac{(\frac{\pi}{\Lambda} - 2a_0) + \text{sgn}(a_0) \sqrt{(\frac{\pi}{\Lambda} - 2a_0)^2 - 4a_1^2}}{2a_1} \right]^m. \quad (5)$$

$\text{sgn}(a_0) = 1$ (-1) for $a_0 > 0$ (< 0). We have kept Λ explicitly, which is taken to infinity afterward. Notice that while $g^{(m)}$ retains an explicit dependence on Λ , the final results are convergent and independent of Λ (see the Supplemental Material [74] for details). After a tedious but straightforward derivation, we obtain the following equation (see the Supplemental Material [74] for more details):

$$\left| \hat{I} - \frac{1}{V} \sum_{\mathbf{q}} \hat{\mathcal{P}}(\mathcal{E}_P - \epsilon_{\mathbf{Q}}) \left[\hat{I} - \frac{1}{V} \sum_{\mathbf{k}} \hat{\mathcal{P}}(\mathcal{E}_P - E_{\mathbf{k}\mathbf{q}}) \right]^{-1} \right| = 0, \quad (6)$$

where $|\dots|$ denotes the determinant of “ $\dots$ ”, $\hat{I}$ is the identity matrix, and

$$\hat{\mathcal{P}}(x) = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & \frac{g^{(0)}}{x - \hbar\Omega} & \frac{g^{(-1)}}{x - \hbar\Omega} & \frac{g^{(-2)}}{x - \hbar\Omega} & \cdots \\ \cdots & \frac{g^{(1)}}{x} & \frac{g^{(0)}}{x} & \frac{g^{(-1)}}{x} & \cdots \\ \cdots & \frac{g^{(2)}}{x + \hbar\Omega} & \frac{g^{(1)}}{x + \hbar\Omega} & \frac{g^{(0)}}{x + \hbar\Omega} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (7)$$

Notice that for $a_1 = 0$, we have $g^{(m \neq 0)} = 0$ and thus a diagonal $\hat{\mathcal{P}}$, from which Eq. (6) reduces to the familiar result of a static Fermi polaron [14,17]. However, for $a_1 \neq 0$, $g^{(m)}$ would mix the different frequency components in the Floquet space via m -“photon” processes [45], giving rise to unique physics of the Floquet polaron state.

Numerically solving Eq. (6) in a truncated Floquet space [74], one can obtain the quasienergy of the Floquet polaron, which as a function of the dimensionless driving strength a_1/a_0 is shown in Fig. 1(a). In general, there are two branches of the Floquet polaron state, labeled as $\mathcal{E}_P^\pm$, which can be viewed as the periodical generalization of the usual “repulsive” and “attractive” polarons of the static system at $a_1 = 0$.

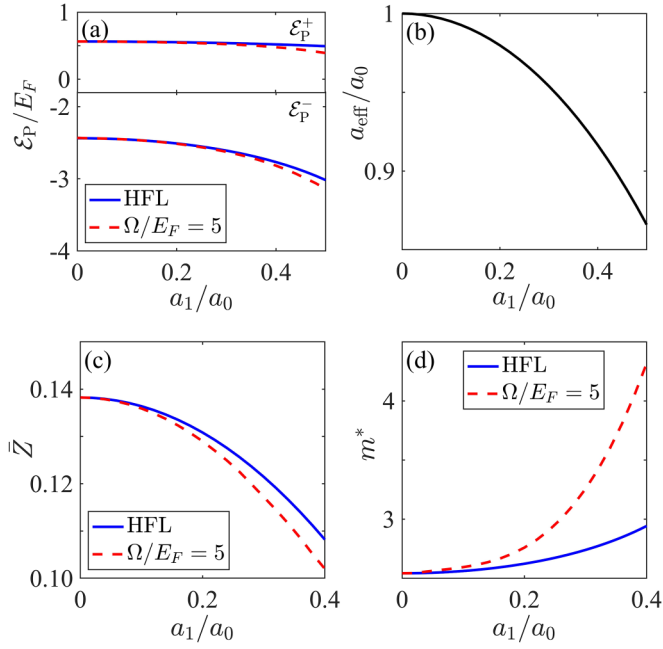


FIG. 1. The evolutions of the two quasienergy branches of the Floquet Polaron ($\mathcal{E}_P^\pm$) [panel (a)] and the time-averaged quasiparticle residue $\bar{Z}$ [panel (c)] and effective mass m^* [panel (d)] of the $\mathcal{E}_P^-$ branch with respect to the dimensionless driving strength a_1/a_0 for the HFL (blue solid lines) and $\Omega/E_F = 5$ (red dashed lines). The effective scattering length a_{eff}/a_0 in the HFL is plotted in panel (b). Here, $(k_F a_0)^{-1} = 1$.

Here, for convenience, we have chosen the FBZ such that the static limit is continuously approached.

In the high-frequency limit, i.e., $\Omega/E_F \rightarrow \infty$, the high-frequency components in Eq. (1) can be neglected and the system is well captured by a static Hamiltonian described by $g^{(0)}$ [74], with an effective scattering length $a_{\text{eff}} = \text{sgn}(a_0)\sqrt{a_0^2 - a_1^2}$ shown in Fig. 1(b). This coincides with the Floquet scattering theory [75] and is different from the perturbative result [72,73], where $g^{(0)}$ is solely given by a_0 and thus not affected by the driving in the HFL. As a result, the quasienergy $\mathcal{E}_P$ decreases (increases) with the driving strength a_1 in the BEC Bardeen-Cooper-Schrieffer (BCS) side with $a_0 > 0$ ($a_0 < 0$); see the solid lines in Fig. 1(a) (not shown). However, for a finite but large Ω , different frequency components in the Floquet space couple to each other by exchanging photons, which renormalizes the polaron states with lower energies [the dashed lines in Fig. 1(a)]. We would notice here that in the static limit, the energy difference $\Delta \equiv \mathcal{E}_P^+(a_1 = 0) - \mathcal{E}_P^-(a_1 = 0)$ of two Floquet polarons is quite large and thus well separated. However, in the resonant regime, i.e., $\Delta \sim n\Omega$ ($n \in \mathbb{Z}$), as we will see in the following, the two Floquet polarons are comparable to each other by absorbing or releasing photon(s), giving rise to unusual energy-level crossings even for a weak driving strength.

The properties of the Floquet polaron are further revealed by the effective mass $m^* \equiv \frac{1}{2}(\frac{\partial^2 \mathcal{E}_P}{\partial \mathbf{Q}^2})_{\mathbf{Q}=0}^{-1}$ and the time-averaged quasiparticle residue $\bar{Z}$, which is defined as $\bar{Z} \equiv \frac{1}{T} \int_0^T Z_t dt$, with $Z_t = |\phi_0(t)|^2$ being the temporal residue. Physically, the

time-averaged (in a cycle) quantity defines the correspondence in the time-independent Floquet Hamiltonian, which naturally reduces to the results in the static limit [22,23]. Taking the $\mathcal{E}_P^-$ branch as an example shown in Figs. 1(c) and 1(d), we find that $\bar{Z}$ decreases while m^* increases with a_1/a_0 . Moreover, away from the HFL (solid lines), the Floquet polaron for a finite driving frequency has an even smaller $\bar{Z}$ and a larger m^* (dashed lines). This suggests that compared to the static limit, the periodical modulation of the interactions tends to enhance the particle-hole fluctuations, which may drive the transition to a (Floquet) molecule-type state.

IV. FLOQUET MOLECULE

Analogously to the Floquet polaron, the molecular state with one particle-hole excitation in the presence of periodic driving can be constructed as

$$|M_{\text{ph}}(t)\rangle_{\mathbf{Q}} = e^{-i\mathcal{E}_M t/\hbar} \left[\sum_{\mathbf{k}} \psi_{\mathbf{k}}(t) \hat{c}_{\mathbf{Q}-\mathbf{k}\downarrow}^\dagger \hat{c}_{\mathbf{k}\uparrow}^\dagger + \frac{1}{2} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}(t) \hat{c}_{\mathbf{Q}+\mathbf{q}-\mathbf{k}-\mathbf{k}'\downarrow}^\dagger \hat{c}_{\mathbf{k}\uparrow}^\dagger \hat{c}_{\mathbf{k}'\uparrow}^\dagger \hat{c}_{\mathbf{q}\uparrow}^\dagger \right] |N-1\rangle, \quad (8)$$

where $\mathcal{E}_M$ is the quasienergy of the Floquet molecule with c.m. momentum $\mathbf{Q}$. Here, $\psi_{\mathbf{k}}(t) = \psi_{\mathbf{k}}(t+T)$ and $\psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}(t) = \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}(t+T)$ are the time-periodic amplitudes for the bare impurity-majority pair and those of particle-hole dressing, respectively. By writing $\psi_{\mathbf{k}}(t) = \sum_n \psi_{\mathbf{k}}^{(n)} e^{in\Omega t}$, $\psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}(t) = \sum_n \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n)} e^{in\Omega t}$ and following procedures as those of the Floquet polaron, we can derive the recursion equations for $\{\psi_{\mathbf{k}}^{(n)}, \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n)}\}$, which are

$$[\mathcal{E}_M + n\hbar\Omega - E_{\mathbf{k}}] \psi_{\mathbf{k}}^{(n)} = \sum_m \frac{g^{(m)}}{V} \left[\sum_{\mathbf{k}} \psi_{\mathbf{k}}^{(n-m)} + \sum_{\mathbf{k}'\mathbf{q}} \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n-m)} \right], \quad (9a)$$

$$[\mathcal{E}_M + n\hbar\Omega - E_{\mathbf{k}\mathbf{k}'\mathbf{q}}] \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n)} = \frac{1}{V} \sum_m g^{(m)} \left[\psi_{\mathbf{k}}^{(n-m)} + \sum_{\mathbf{k}} \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n-m)} + \sum_{\mathbf{k}'} \psi_{\mathbf{k}\mathbf{k}'\mathbf{q}}^{(n-m)} \right], \quad (9b)$$

with $E_{\mathbf{k}} = \epsilon_{\mathbf{k}\uparrow} + \epsilon_{\mathbf{Q}-\mathbf{k}\downarrow}$ and $E_{\mathbf{k}\mathbf{k}'\mathbf{q}} = \epsilon_{\mathbf{Q}+\mathbf{q}-\mathbf{k}-\mathbf{k}'\downarrow} + \epsilon_{\mathbf{k}\uparrow} + \epsilon_{\mathbf{k}'\uparrow} - \epsilon_{\mathbf{q}\uparrow}$. In general, one needs to solve a couple of integral equations in the extended Floquet space to obtain $\mathcal{E}_M$ (see the Supplemental Material [74] for more details), which for a bare molecule, i.e., $\psi_{\mathbf{k}\mathbf{k}'\mathbf{q}} = 0$, can be simplified to the following equation:

$$\left| \hat{I} - \frac{1}{V} \sum_{\mathbf{k}} \hat{\mathcal{P}}(\mathcal{E}_M - \epsilon_{\mathbf{k}\uparrow} - \epsilon_{\mathbf{Q}-\mathbf{k}\downarrow}) \right| = 0. \quad (10)$$

Figure 2(a) shows the quasienergies of the bare Floquet molecular state $\mathcal{E}_M$ and that with a particle-hole dressing $\mathcal{E}_{M_{\text{ph}}}$ with respect to the driving strength a_1/a_0 for $\mathbf{Q} = 0$. Similar to the behavior of $\mathcal{E}_P$, both $\mathcal{E}_M$ and $\mathcal{E}_{M_{\text{ph}}}$ decrease with a_1/a_0 in the HFL (solid lines) and become even lower for a finite

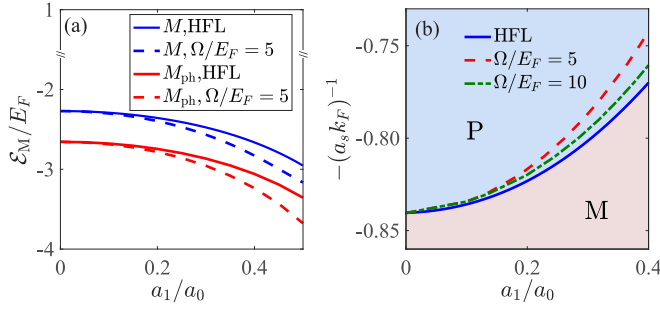


FIG. 2. (a) The quasienergies of the bare Floquet molecule $\mathcal{E}_M$ (blue) and that with one particle-hole excitation $\mathcal{E}_{M_{ph}}$ (red) as a function of the dimensionless driving amplitude a_1/a_0 with $(k_F a_0)^{-1} = 1$ for the HFL (solid lines) and $\Omega/E_F = 5$ (dashed lines). (b) The phase boundary between the polaron (P) and molecule (M) states of the effective model in the HFL (blue solid lines). In comparison, the red dashed ($\Omega/E_F = 5$) and green dash-dotted ($\Omega/E_F = 10$) lines label the energy crossing point between Floquet polaron ($\mathcal{E}_P^-$) and Floquet molecule with one particle-hole excitation ($\mathcal{E}_{M_{ph}}$) of dimensionless driving strength a_1/a_0 .

Ω (dashed lines). It is well known that in the static limit, a polaron-to-molecule transition happens at $\mathcal{E}_{M_{ph}} = \mathcal{E}_P^-$ for sufficient $a_0 > 0$ [15]. An important consequence of the periodic modulation is that such energy crossing point can be achieved by varying the driving strength, depicted in Fig. 2(b). In the HFL, as $a_{\text{eff}} = a_0 \sqrt{1 - (a_1/a_0)^2}$, the crossing point of two states described by the effective model moves to the polaron side with the increase of a_1/a_0 . Such tendency becomes more significant when away from the HFL [see the green and red lines in Fig. 2(b)]. Compared to the static case, the transition between the Floquet polaron and molecule remains first order and discontinuous. The main difference is that in the Floquet system such transition is of dynamical type rather than the equilibrium one.

The periodically modulated interaction can not only renormalize the scattering length and thus the quasienergies of the above Floquet impurity states, but also lead to the folding of quasienergy spectrum in the FBZ. Such unique feature may have significant effects especially when the driving frequency Ω is in resonance with the quasienergy difference between two Floquet impurity states, e.g., $\Delta \sim n\Omega$ ($n \in \mathbb{Z}$). In Fig. 3, we show a typical configuration of the quasienergy spectra of the Floquet impurity states in such a regime with $\Omega/E_F = 1.3$. For the chosen parameters, $\Delta \approx 2.56E_F$, which is close to $2\Omega = 2.6E_F$, and both Floquet polaron branches get close to each other in the FBZ via a two-photon process. For $(k_F a_0)^{-1} = 0.81$ near the static polaron-to-molecule transition $(k_F a_0)^{-1} \simeq 0.84$ [17], we can see that besides the usual crossing point of $\mathcal{E}_P^-$ and $\mathcal{E}_{M_{ph}}$ (marked as “•”) discussed above, two other crossing points (marked as “★” and “▲”) where the $\mathcal{E}_P^+$ branch intersects with $\mathcal{E}_P^-$ and $\mathcal{E}_{M_{ph}}$ branches, respectively, may appear at a weak driving strength. This indicates that unusual (dynamical) transitions between the “repulsive” Floquet polaron and the “attractive” one or a molecular state may be driven by a periodic modulation, which have no counterparts in a static system. We emphasize here that though the folding type of energy spectrum is dependent on the choice of FBZ, the energy-level crossing is robust.

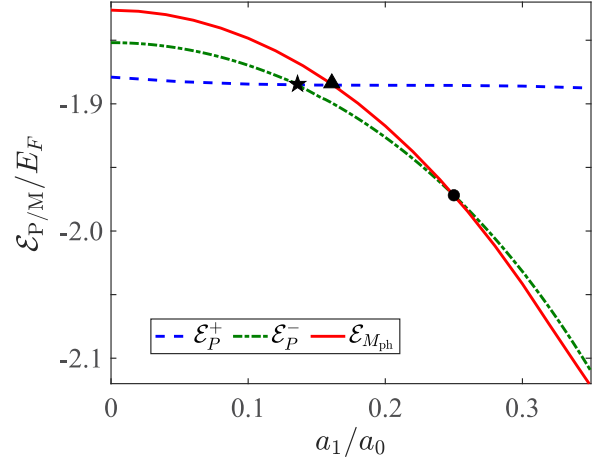


FIG. 3. The quasienergy spectra of the Floquet polaron $\mathcal{E}_P^+$ (blue dashed line), $\mathcal{E}_P^-$ (green dash dotted line), and the Floquet molecule $\mathcal{E}_{M_{ph}}$ (red solid line) with zero c.m. momentum. Here, $(k_F a_0)^{-1} = 0.81$ and $\Omega/E_F = 1.3$. “★”, “▲”, and “•” label the crossing points.

Besides the Floquet polaron and molecule, we have also considered the possible competition from a Floquet trimer state (see the Supplemental Material [74] for more details). In a static system with mass imbalance, the trimer state may become the ground state in competition with the molecule and polaron states [11, 76–78]. However, for the case of equal mass, the energy of the trimer state is quite high-lying [79]. In the presence of periodical driving, we find that in the parameter regime considered in this work, the quasienergy of the Floquet trimer is still well separated from that of Floquet molecule and polaron states in the FBZ, and thus would not give rise to other energy-level crossing points. It would be interesting to consider the competition of Floquet polaron, molecule, and trimer states in a mass-imbalance system, which, however, is beyond the scope of this work and we would leave it in the future work.

In experiments, the polaron and molecule states have already been observed in static systems [22–28]. Recently, the periodical modulation of the interactions has been created via tuning the magnetic field of a Feshbach resonance in Fermi gases [80–82]. To detect the intriguing properties of the Floquet impurity states discussed above, in the following, we turn to the spectral functions [83] and the rf spectroscopy [22, 23], which provide unique measurable signatures of the impurity states [22–24, 26, 27].

V. SPECTRAL FUNCTION

For a time-periodic system, the spectral function is also time-periodic and it is useful to introduce the generalized time-averaged spectral function $\bar{A} \equiv \frac{1}{T} \int_0^T A(t) dt$ [84, 85], which in terms of the Floquet Green’s function method [86–89] is given by

$$\bar{A}_P(\mathbf{Q}, \omega') = -\frac{1}{\pi} \text{Im}\{G_{nn}(\mathbf{Q}, \omega + i0^+)\}. \quad (11)$$

Here, G_{nn} is the n th diagonal element of the single-particle retarded Green’s function of the impurity atom in the Floquet representation, which for the Floquet polaron [Eq. (2)] can

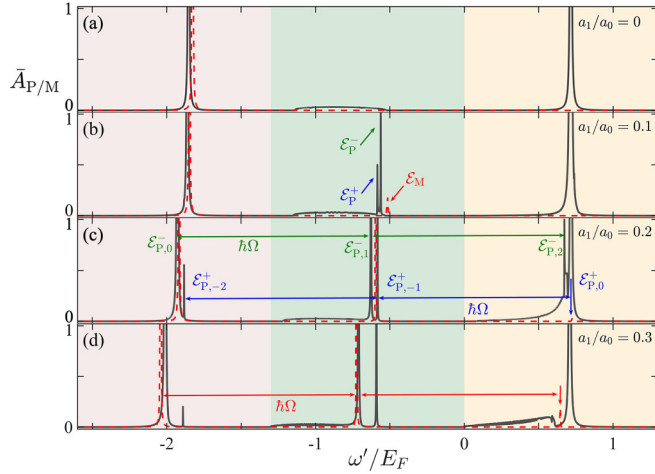


FIG. 4. The time-averaged spectral functions of Floquet polaron $\bar{A}_P(\mathbf{Q}, \omega')$ (black solid lines) and Floquet molecule $\bar{A}_M(\mathbf{Q}, \omega')$ (red dashed lines) at $\mathbf{Q} = 0$ for dimensionless driving strengths $a_1/a_0 = 0$ [panel (a)], 0.1 [panel (b)], 0.2 [panel (c)], and 0.3 [panel (d)]. The background colors mark the different FBZs spanned $\hbar\Omega$. $\mathcal{E}_{P,n}^\pm$ labels the n th (side) peak of $\bar{A}_P$. Other parameters are the same as those in Fig. 3.

be obtained from the Dyson's equation [90] $G^{-1} = G_0^{-1} - \Sigma$ with the noninteracting part $G_{0,mn} = \frac{1}{\omega + n\Omega - \epsilon_{\mathbf{Q}}} \delta_{mn}$ and the self-energy $\Sigma = \frac{\mathcal{G}}{V} \sum_{\mathbf{q}} 1/[\hat{I} - \frac{1}{V} \sum_{\mathbf{k}} \hat{P}(\omega - E_{\mathbf{kq}})]$ (here, $\mathcal{G}$ is the Floquet representation of the contact interaction $g(t)$ with $\mathcal{G}_{mn} = g^{(m-n)}$; see the Supplemental Material [74] for more details). The physical frequency ω' relates to the quasifrequency $\omega \in (-\frac{\Omega}{2}, \frac{\Omega}{2}]$ via $\omega' = \omega + n\Omega$. However, for the molecular state, the spectral function is related to the two-particle correlation and can be defined as

$$\bar{A}_M(\mathbf{Q}, \omega') = -\frac{1}{\pi} \text{Im}\{\Gamma_{mn}(\mathbf{Q}, \omega + i0^+)\}, \quad (12)$$

where Γ is the Floquet vertex function that encodes the same poles of the two-particle propagator [21]. According to the Bethe-Salpeter equation [90], we have $\Gamma^{-1} = \mathcal{G}^{-1} - \Pi$ with the two-particle self-energy $\Pi_{mn} \simeq \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{\omega + n\Omega - (\epsilon_{\mathbf{k}+\mathbf{Q}} - \epsilon_{\mathbf{k}})} \delta_{mn}$ for the bare Floquet molecule [Eq. (10)].

In Fig. 4, we plot the spectral function $\bar{A}_{P/M}(\mathbf{Q} = 0, \omega')$ for different driving strength at $\Omega/E_F = 1.3$. In the static limit $a_1/a_0 = 0$ [Fig. 4(a)], we can see two isolated peaks corresponding to the attractive and repulsive polarons in $\bar{A}_P$ and a single molecular peak in $\bar{A}_M$ as expected. Notice here that we have chosen $(k_F a_0)^{-1} = 0.81$ with the energy of the attractive polaron slightly smaller than the molecule with particle-hole excitation. For $a_1/a_0 > 0$ [Figs. 4(b)–4(d)], besides the main peaks given by $n = 0$, a series of small side peaks separated by $\hbar\Omega$ appear due to the population of $n \neq 0$ Floquet modes, signaling the formation of the aforementioned Floquet impurity states. Meanwhile, as the quasienergies of the above Floquet states become lower with the increase of a_1 , all peaks move to the left side gradually. Moreover, when the driving strength a_1 surpasses the crossing points (see Fig. 3), the relative positions of the peaks interchange continuously in each FBZ. Taking the middle FBZ in Fig. 4 as an

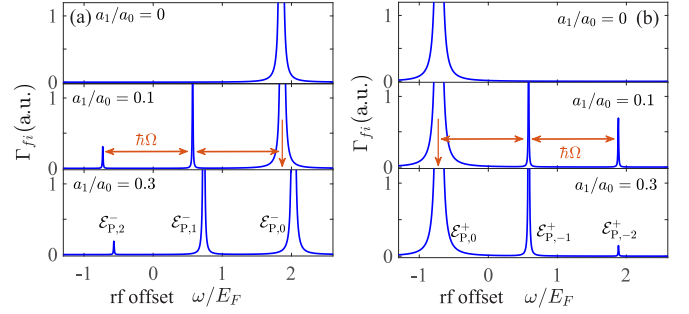


FIG. 5. The rf spectra of (a) $\mathcal{E}_P^-$ and (b) $\mathcal{E}_P^+$ branches at different driving strengths $a_1/a_0 = 0, 0.1$, and 0.3 . $\mathcal{E}_{P,n}^\pm$ labels the n th (side) peak. Here, $(k_F a_0)^{-1} = 0.81$ and $\Omega/E_F = 1.3$.

example, the rightmost side peak of the Floquet molecule at $a_1/a_0 = 0.1$ [Fig. 4(b)] becomes the leftmost one at $a_1/a_0 = 0.3$ [Fig. 4(d)] via interchanging twice with the two side peaks of the Floquet polaron (“▲” and “●” in Fig. 3), the positions of which have already interchanged in Fig. 4(c) (“★” in Fig. 3).

In general, the Floquet-engineered impurity states are quasi-stationary, which may be heated up to a thermal state at “infinite temperature” eventually via absorbing energy from the external driving [91,92]. Nevertheless, for an interacting Fermi gas, recent experiments show that the heating rate could be largely suppressed by a weak periodic driving [80–82]. In this work, $k_F a_1/(\Omega/E_F) \ll 1$ and we can observe in Fig. 4 that the half-height width (HHW) of the main peaks is almost not changed by the periodical driving compared to the static limit, suggesting that the lifetimes of the Floquet impurity states are still large with small decay rates in this regime, which enables the observation of such Floquet states prethermalized within the experimental timescales. It is also interesting to notice that in Fig. 4 the side peaks have even narrower HHWs than the main peaks, a hint of the heating suppression.

VI. rf SPECTROSCOPY

In experiments, the rf absorption spectrum provides another useful and direct signature of the impurity states, which for the Floquet polaron can be derived as (see the Supplemental Material for more details [74])

$$\Gamma_{fi}(\omega) = 2\pi \hbar \Omega_R^2 \sum_n \left[|\phi_0^{(n)}|^2 \delta(\omega + \mathcal{E}_P + n\hbar\Omega) + \sum_{\mathbf{kq}} |\phi_{\mathbf{kq}}^{(n)}|^2 \delta(\omega + \mathcal{E}_P - E_{\mathbf{kq}} + n\hbar\Omega) \right]. \quad (13)$$

Here, the total weight of the coherent peaks $\sum_n |\phi_0^{(n)}|^2$ is directly related to the time-averaged quasiparticle residue $\bar{Z}$, which measures the polaron's coherence. The calculated spectra for the $\mathcal{E}_P^-$ and $\mathcal{E}_P^+$ are shown in Fig. 5. In the static limit ($a_1/a_0 = 0$), each branch exhibits a single peak at the energy of the usual attractive or repulsive polaron. Under periodical driving ($a_1/a_0 > 0$), a series of Floquet sidebands emerge at frequencies $\omega = -(\mathcal{E}_P + n\hbar\Omega)$ ($n \in \mathbb{Z}$), mapping to the

positions of the coherent peaks in the spectral function $\bar{A}_p$. This multipeak structure, spaced by $\hbar\Omega$, serves as a spectroscopic signature of the Floquet polaron, which can be readily observed via rf spectroscopy [22,93,94].

In conclusion, we have investigated the impurity problem of a highly polarized Fermi gas with periodically driven interactions. A Floquet Hamiltonian with renormalized interaction strength is obtained in the high-frequency limit. However, in the high-frequency regime, generalized Floquet polaron and molecular states are constructed and show unique features. Unusual energy-level crossings between these Floquet impurity states beyond the static system are revealed and characterized by the spectral functions and rf spectra, which can be measured in experiments. Future studies may include the

quench dynamics [25,95–98] of the Floquet impurity states and the “Floquet engineering” to other few-body and many-body states, such as the Efimov state [79], Fermi superfluid [99], etc.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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