

# Breaking global symmetries with locality-preserving operations

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In the general framework of quantum resource theories, one typically only distinguishes between operations that can or cannot generate the resource of interest. In many-body settings, one can further characterize quantum operations based on underlying geometrical constraints, and a natural question is to understand the power of resource-generating operations that preserve locality. In this work, we address this question within the resource theory of asymmetry, which has recently found applications in the study of many-body symmetry-breaking and symmetry-restoration phenomena. We consider symmetries corresponding to both Abelian and non-Abelian compact groups with a homogeneous action on the space of  $N$  qubits, focusing on the prototypical examples of  $U(1)$  and  $SU(2)$ . We study the so-called  $G$  asymmetry  $\Delta S_N^G$ , and present two main results. First, we derive a general bound on the asymmetry that can be generated by locality-preserving operations acting on product states. We prove that, in any spatial dimension,  $\Delta S_N^G \leq (1/2)\Delta S_N^{G,\max}[1 + o(1)]$ , where  $\Delta S_N^{G,\max}$  is the maximum value of the  $G$  asymmetry in the full many-body Hilbert space. Second, we show that locality-preserving operations can generate maximal asymmetry,  $\Delta S_N^G \sim \Delta S_N^{G,\max}$ , when applied to symmetric states featuring long-range entanglement. Our results provide a unified perspective on recent studies of asymmetry in many-body physics, highlighting a nontrivial interplay between asymmetry, locality, and entanglement.

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**Introduction.** In the past two decades, quantum resource theories (QRTs) [1] have significantly influenced our understanding of different quantum phenomena, with entanglement providing the most notable example [2–5]. A QRT is a mathematical framework in which a property of interest (the resource) is indirectly defined in terms of “free” states and operations. Generally speaking, free states are those that do not display the property of interest, while free operations are those that do not generate the resource [1].

In a many-body setting, one can enrich this mathematical framework and ask about the power of resource-generating operations under geometrical constraints. A natural question pertains to operations that preserve locality, as mathematically captured by the notion of quantum cellular automata [6,7] and their nonunitary generalizations, the so-called locality-preserving (LP) operations [8]. LP operations provide very useful toy models for many-body physics, as their repeated application mimics the evolution of many-body systems described by local Hamiltonians and Lindbladians [6–8]. At the same time, they can be efficiently implemented on quantum computers [6], even in the current era of noisy devices [9], making their study especially timely.

The power of LP operations is well understood in the resource theory of entanglement. For instance, in analogy with established results on the maximal entanglement growth allowed by local-Hamiltonian dynamics [10–16], it was shown that LP operations can only prepare area-law entangled states [8] and are thus limited in the amount of resource they can generate.

Different from entanglement, the power of LP operations remains largely unexplored in other resource theories and, in particular, in the resource theory of asymmetry [17–19]. In this QRT, the free operations are the ones that are symmetric with respect to a given symmetry group  $G$ , while the resources are asymmetric states. This framework is useful when formalizing protocols involving two parties exchanging information that cannot be communicated without some shared reference state [1] (a standard example is the orientation of a spin with respect to a given axis). The two parties can freely perform symmetric operations because these do not require a reference state, while asymmetric states are a resource that can be depleted by (nonunitary) symmetric operations. The asymmetry is quantified by appropriate resource monotones, i.e., state functionals that are nonincreasing under symmetric operations.

The asymmetry QRT can be naturally extended beyond communication protocols, and the past few years have witnessed the development of similar ideas in many-body physics [20,21] and quantum field theory [22–26]. These developments were motivated by the observation that a particular asymmetry monotone, the  $G$  asymmetry  $\Delta S^G$  [17,19] (also known as entanglement asymmetry [21]), can be efficiently computed in interesting physical settings, allowing for

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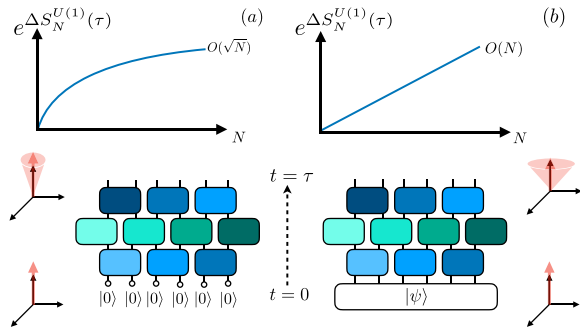


FIG. 1. Sketch of our main results. (a) We consider applying LP operations to a product state and bound the asymmetry of the output state. The cartoon shows a unitary circuit of finite depth  $\tau$  in one dimension ( $\tau$  does not grow with  $N$ ), but our results hold for generic LP channels and arbitrary dimension. The plot shows the bound on the linearized  $G$  asymmetry  $e\Delta S_N^{U(1)} = O(\sqrt{N})$ . (b) We also show that LP operations can generate maximal asymmetry when applied to symmetric states featuring long-range entanglement. The plot shows the linearized asymmetry  $e\Delta S_N^{U(1)} \sim N$  when applying suitably chosen local rotations to an initial Dicke state.

several recent studies of symmetry-breaking and symmetry-restoration phenomena [27–46]. An interesting but puzzling observation of some of these studies is the seemingly universal scaling of the  $G$  asymmetry in the number of qubits. For example, for a  $U(1)$  symmetry acting locally on  $N$  qubits, it was found that the asymmetry scales as  $\Delta S_N^{U(1)} \sim (1/2) \ln N$ , namely, half of its maximum allowed value, in generic translation-invariant matrix product states [47], Gaussian states [21], and Haar random states [48,49].

In this work, we bound the asymmetry generated by LP operations acting on product states, proving that the leading order of  $\Delta S_N^G$  can be at most half its maximum allowed value; cf. Fig. 1 for a sketch of our main results. Conversely, we show that LP operations can generate maximal asymmetry when applied to symmetric states featuring long-range entanglement. The key ingredient to derive our bounds is the light cone of the correlation functions in states prepared by LP operations [6,8], a feature known as the *cluster property*. As the latter is a typical feature of many-body states, our results explain previous observations of the seemingly universal scaling of the  $G$  asymmetry and highlight a nontrivial interplay between asymmetry, locality, and entanglement. Our work may also be relevant for experimental studies of asymmetry generation in digital quantum platforms [50].

**Locality-preserving operations.** We start by briefly reviewing LP operations [8]. We consider a regular lattice in  $d_L$  dimensions, with  $N = M^{d_L}$  qubits placed at the vertices, and periodic boundary conditions. We define the distance between two qubits,  $\delta(n, m)$ , as the minimum number of edges connecting the corresponding vertices. The Hilbert space associated with the set of qubits is  $\mathcal{H} = \otimes_n \mathcal{H}_n$ , where  $\mathcal{H}_n \simeq \mathbb{C}^2$ . Next, given a subset of qubits  $A$ , we define its  $\lambda$  neighborhood,  $a_\lambda = \{n : \delta(n, A) \leq \lambda\}$ , where  $\delta(n, A)$  is the minimum distance between the qubit  $n$  and any qubit in  $A$ . We also set  $\bar{A}_\lambda = A \cup a_\lambda$ . Finally, given a subset  $C$ , we denote by  $\text{Tr}_C$  the trace in  $\mathcal{H}_C = \otimes_{j \in C} \mathcal{H}_j$  and by  $X_C$  an operator supported on  $C$ . We will also denote by  $|0\rangle_j, |1\rangle_j$  a basis for the local Hilbert

space and by  $\sigma_j^\alpha$  the Pauli matrices ( $\alpha = x, y, z$ ). We now introduce LP operations as a special type of quantum channel, i.e., trace-preserving completely positive map [51].

**Definition 1.**  $\mathcal{E}_\lambda$  is an LP operation with range  $\lambda \in \mathbb{N}$  if, for any sets of qubits  $A$  and  $B$ ,

$$\mathcal{E}_\lambda^\dagger(X_A Y_B) = \mathcal{E}_\lambda^\dagger(X_A) \mathcal{E}_\lambda^\dagger(Y_B), \quad (1)$$

with  $\mathcal{E}_\lambda^\dagger(X_A) = X_{\bar{A}_\lambda}$ ,  $\mathcal{E}_\lambda^\dagger(Y_B) = Y_{\bar{B}_\lambda}$ .

Here, we denote by  $\mathcal{E}^\dagger$  the adjoint of the channel  $\mathcal{E}$ , which is defined by the action  $\text{Tr}[\mathcal{E}^\dagger(A)B] = \text{Tr}[A\mathcal{E}(B)]$ . The simplest LP operations are finite-depth local quantum circuits, i.e., unitaries of the form  $U^{(D)} = V_D \cdots V_1$  where  $V_D$  is a product of two-qubit gates acting on disjoint pairs of neighboring qubits ( $D$  is the circuit depth).

In most of this work, we focus on (possibly mixed) initial product states  $\rho_0 = \otimes_j \omega_j$ , where  $\omega_j$  is a state in the local Hilbert space  $\mathcal{H}_j$ . Then, Eq. (1) implies that the output state  $\rho = \mathcal{E}_\lambda(\rho_0)$  satisfies the cluster property

$$\langle O_i O_j \rangle_\rho - \langle O_i \rangle_\rho \langle O_j \rangle_\rho = 0, \quad \text{for } \delta(i, j) > \Lambda = 2\lambda, \quad (2)$$

where  $\langle O \rangle_\rho = \text{Tr}[\rho O]$ , while  $O_j$  is an arbitrary operator supported on site  $j$ . Note that choosing initial product states is very natural both from a theoretical many-body perspective and from the experimental point of view. For instance, current prototypes of quantum computers are typically initialized in unentangled states and evolved by a relatively small number of (noisy) local quantum-circuit steps [9,52], corresponding to LP operations [6,8].

**The  $G$  asymmetry.** We consider symmetries associated with compact Lie groups and local, homogeneous generators of the form  $Q^\alpha = \sum_j q_j^\alpha$ , with  $\alpha$  an integer label. We focus on the prototypical examples of  $U(1)$  and  $SU(2)$ , for which we choose an explicit representation: For  $U(1)$ , we consider the charge  $Q = \sum_j (\sigma_j^z + 1)/2$ , while for  $SU(2)$  we choose the generators  $Q^\alpha = S^\alpha = \sum_j \sigma_j^\alpha/2$ . Now, introducing the  $G$ -twirling operation

$$\mathcal{G}[\rho] := \int_G dg T(g)\rho T(g)^\dagger, \quad (3)$$

where the integral is with respect to the Haar measure [53], the  $G$  asymmetry is defined as [17]

$$\Delta S_N^G(\rho) = S_V(\mathcal{G}[\rho]) - S_V(\rho), \quad (4)$$

where  $S_V(\rho) = -\text{Tr}(\rho \ln \rho)$  is the von Neumann entropy [51].

The  $G$  asymmetry  $\Delta S_N^G(\rho)$  is an asymmetry monotone [17,19,54–56]. That is, given an arbitrary symmetric channel  $\mathcal{E}$ , one has  $\Delta S_N^G(\mathcal{E}[\rho]) \leq \Delta S_N^G(\rho)$  [57]. In fact, the  $G$  asymmetry satisfies a stronger condition: Given a symmetric channel  $\mathcal{E}(\cdot) = \sum_k A_k^\dagger(\cdot)A_k$ , with  $A_k$  symmetric Kraus operators [51], one has  $\Delta S_N^G(\rho) \geq \sum_k p_k \Delta S_N^G(\tilde{\rho}_k)$  where  $\tilde{\rho}_k = (A_k^\dagger \rho A_k)/p_k$  and  $p_k = \text{Tr}(A_k^\dagger \rho A_k)$ . Namely,  $\Delta S_N^G(\rho)$  does not increase on average under symmetric operations. In addition, it is also easy to show that  $\Delta S_N^G(\rho) \geq 0$  and  $\Delta S_N^G(\rho) = 0$  if and only if  $\rho$  is symmetric, making  $\Delta S_N^G$  a good measure of asymmetry [58].

The maximal value of  $\Delta S_N^G$  is logarithmic in  $N$ , with explicit bounds known in the literature [19,56]. For instance, for the  $U(1)$  symmetry, one can show [19]

$$\Delta S_N^{U(1)}(\rho) \leq \ln(N+1). \quad (5)$$

Our goal is to understand how the general bounds are modified for states satisfying Eq. (2).

*The Abelian case.* We start by the Abelian U(1) symmetry and consider a (generally mixed) state  $\rho = \mathcal{E}_\lambda(\otimes \omega_j)$ , satisfying Eq. (2). As a first step, we rewrite

$$\mathcal{G}[\rho] = \sum_{q=0}^N p_q \tilde{\rho}_q, \quad (6)$$

where  $p_q = \text{Tr}[\rho P_q]$ ,  $\tilde{\rho}_q = P_q \rho P_q / p_q$ , while  $P_q$  is the projector onto the  $q$  eigenspace of the charge  $Q$ . Equation (6) follows from the definition (3) and the Haar measure for U(1) [18,56].

Next, we recall the standard inequality [51]:

$$S_V\left(\sum_a p_a \rho_a\right) \leq \sum_a p_a S_V(\rho_a) + H(\{p_a\}), \quad (7)$$

where  $\{p_a\}$  is a discrete probability distribution,  $\rho_a$  are arbitrary mixed states, while  $H(\{p_a\}) = -\sum_j p_j \ln p_j$  is the Shannon entropy. Applying it to Eq. (6), we get

$$\Delta S_N^{U(1)}(\rho) \leq H(\{p_q\}) + \sum_q p_q S_V(\tilde{\rho}_q) - S_V(\rho). \quad (8)$$

Finally, we observe that  $\tilde{\rho}_q$  is obtained from  $\rho$  by measuring the charge operator and postselecting on the outcome  $q$ . On the other hand, the von Neumann entropy does not increase, on average, under measurements [59], and so  $S_V(\rho) \geq \sum_q p_q S_V(\tilde{\rho}_q)$ . Putting all together, we have

$$\Delta S_N^{U(1)}(\rho) \leq H(\{p_q\}). \quad (9)$$

Note that this inequality is saturated in the case  $\rho$  is pure, and that Eq. (9) immediately implies the upper bound (5), because  $q$  can take  $N+1$  values.

In order to derive a tighter bound for clustering states, a key observation is that Eq. (2) constrains the variance of the probability distribution  $p_q$ , which we denote by  $\sigma^2 = \mathbb{E}_q[(q - \bar{q})^2]$ , with  $\bar{q} = \mathbb{E}_q[q]$  and where  $\mathbb{E}[\cdot]$  denotes the expectation value. Indeed, using  $\langle Q^2 \rangle_\rho = \mathbb{E}_q[q^2]$ ,  $\langle Q \rangle_\rho = \bar{q}$ , and Eq. (2) with  $O_j = q_j$ , we obtain [60]

$$\sigma^2 \leq 2z_\Lambda N, \quad (10)$$

where  $z_\Lambda$  is the cardinality of any of the sets  $I_x^\Lambda = \{x' : \delta(x, x') \leq \Lambda\}$  ( $z_\Lambda$  does not depend on  $x$ ). Note that  $z_\Lambda$  only depends on  $\Lambda$  and the dimensionality of the lattice, but not on  $N$ .

Equation (10) allows us to conclude our derivation, using a known bound on the Shannon entropy of discrete random variables. To be precise, let  $x$  be an arbitrary integer-valued random variable, with probability function  $p_x$  and support  $A \subseteq \mathbb{Z}$ , such that  $0 < \sigma^2 < \infty$ . Then, the Shannon entropy is bounded by [61,62]  $H(\{p_x\}) < \frac{1}{2} \ln[2\pi e(\sigma^2 + \frac{1}{12})]$ . Combining the above with Eq. (10), we arrive at our first main result

$$\Delta S_N^{U(1)}(\rho) \leq \frac{1}{2} \ln \left[ 2\pi e \left( 2z_\Lambda N + \frac{1}{12} \right) \right] \sim \frac{1}{2} \ln N, \quad (11)$$

displaying the announced asymptotic scaling in  $N$ .

*The non-Abelian case.* The case of the non-Abelian SU(2) symmetry turns out to be significantly more involved compared to U(1). Still, it is possible to follow a similar logic, which relies on two main ingredients: (1) the asymmetry is

bounded in terms of the Shannon entropy of the probability distribution over the group quantum numbers [cf. Eq. (9)] and (2) the cluster property (2) constrains the latter probability distribution function, resulting in a bound for the corresponding Shannon entropy.

Following Refs. [18,19], we begin by recalling a few facts about the SU(2) representation theory and the SU(2) asymmetry. The eigenvalues of the Casimir operator  $\mathbf{S}^2 = (S^x)^2 + (S^y)^2 + (S^z)^2$  and of  $S^z$  are labeled, respectively, by  $\mathbf{S}^2 |s, m\rangle = s(s+1) |s, m\rangle$  and  $S^z |s, m\rangle = m |s, m\rangle$ . For concreteness, we consider the case of  $N$  even, so that the allowed values for  $s$  and  $m$  are  $s = 0, 1, \dots, N/2$  and  $m = -N/2, \dots, N/2$ . We further denote by  $P_s$  and  $P_m$  the orthogonal projectors onto the subspaces associated with the quantum numbers  $s$  and  $m$ .

The integer  $s$  labels the SU(2) irreducible representations and  $\mathcal{H}$  can be decomposed as

$$\mathcal{H} = \bigoplus_s \mathcal{M}_s \otimes \mathcal{N}_s, \quad (12)$$

where  $\mathcal{M}_s$  is the irreducible representation space corresponding to total spin  $s$ , while  $\mathcal{N}_s$  is the associated multiplicity space. Using Schur's lemma, we can write an explicit form for the symmetrized density matrix [18,19],

$$\mathcal{G}[\rho] = \sum_s p_s \frac{\mathbb{1}_{\mathcal{M}_s}}{\dim \mathcal{M}_s} \otimes \tilde{\rho}_s, \quad (13)$$

where  $p_s = \text{Tr}[\rho P_s]$ , while  $\tilde{\rho}_s = \frac{1}{p_s} \text{Tr}_{\mathcal{M}_s}[\rho P_s]$ . For pure states, inspection of Eq. (13) allows one to bound the dimension of the support of  $\mathcal{G}[\rho]$ , yielding a logarithmic bound [19]  $\Delta S_N^{\text{SU}(2)}(\rho) \leq 3 \ln N(1 + o(1))$ . We now improve this bound assuming the cluster property.

As a first simplifying observation, we note that given a state  $\omega$ , we can always find  $U = u^{\otimes N}$  such that the state  $\rho = U\omega U^\dagger$  satisfies

$$\langle S^x \rangle_\rho = \langle S^y \rangle_\rho = 0. \quad (14)$$

This is because  $U$  can implement arbitrary rotations of the vector  $(\langle S^x \rangle_\omega, \langle S^y \rangle_\omega, \langle S^z \rangle_\omega)$ . Importantly, if  $\omega$  satisfies the cluster property, so does  $\rho$ . Then, we can bound the asymmetry of  $\rho$ , which coincides with that of  $\omega$  (since such  $U$  is in the image of the SU(2) representation [63]).

Now, starting from Eq. (13), exploiting Eq. (7), and the monotonicity of the von Neumann entropy with respect to measurements [59], we derive [60]

$$\Delta S_N^{\text{SU}(2)}(\rho) \leq \sum_s p_s \ln(2s+1) - \sum_{s,m} p_{s,m} \ln p_{s,m}. \quad (15)$$

Equation (15) generalizes Eq. (9) to the non-Abelian case and immediately implies  $\Delta S_N^{\text{SU}(2)}(\rho) \leq 3 \ln N(1 + o(1))$ .

As a final step, we bound the right-hand side (RHS) of Eq. (15), under two constraints induced by the cluster property. The first one is Eq. (10), which bounds the variance of  $Q^z$  and hence of the marginal probability distribution  $\{p_m\}$ . The second constraint is the following: Given a clustering state  $\rho$  satisfying Eq. (14), it is easy to show [60]

$$\mathbb{E}[s(s+1) - m^2] = \langle \mathbf{S}^2 \rangle_\rho - \langle (S^z)^2 \rangle_\rho \leq c(\Lambda)N, \quad (16)$$

where  $c(\Lambda)$  depends on  $\Lambda$  but not on  $N$ . A rigorous bound of the RHS of Eq. (15) under the constraints (10), (14), and (16) can be performed using elementary considerations and inequalities, although the derivation is quite long and technical [60]. As a result, we obtain

$$\Delta S_N^{\text{SU}(2)}(\rho) \leq \frac{3}{2} \ln N + g_\Lambda(N), \quad (17)$$

where  $g_\Lambda(N) = O(\ln \ln N)$  is a subleading correction. Equation (17) states the anticipated SU(2)-asymmetry bound for clustering states. In the rest of this Letter, we further elaborate on our results and discuss their implications.

*Clustering and typicality.* It is important to note that our analysis only requires the cluster property (2), which holds beyond states that are prepared by LP operations. For instance, ground states of local Hamiltonians satisfy Eq. (2), up to exponentially small corrections [64]. As a by-product, our bounds also apply to those states.

More generally, our work provides a unified perspective on previous studies of asymmetry in many-body systems. For instance, we can revisit the fact that the U(1) asymmetry scales as  $\Delta S_N^{\text{U}(1)} \sim (1/2) \ln N$  in generic translation-invariant matrix product states [47], Gaussian states [21], and Haar random states [48,49]. Despite being completely different sets of states, they all satisfy the cluster property (up to subleading corrections), so that their asymmetry is bounded by  $\sim (1/2) \ln N$ . On the other hand, in the absence of additional symmetry constraints, it is natural to expect that the bound induced by the cluster property is saturated. We can appreciate the typicality of the U(1) asymmetry scaling  $\sim (1/2) \ln N$  for clustering states looking at the probability distribution  $p_q$ . For typical short-range correlated states, we expect that  $p_q$  follows the central limit theorem: Heuristically, assuming that the charge densities  $q_i$  are i.i.d. variables with variance  $\text{Var}[q_i] = \sigma^2$ , the total charge  $Q = \sum_i q_i$  is distributed as  $(Q - \langle Q \rangle)/\sqrt{N} \sim \mathcal{N}(0, \sigma^2)$ , with  $\mathcal{N}$  the normal distribution. Such a normal distribution function immediately implies the scaling  $\Delta S_N^{\text{U}(1)} \sim (1/2) \ln N$  [60].

*Rotating long-range entangled states.* In light of our results, a very natural question is whether the resource-generation power of LP operations is increased when acting on *entangled* symmetric states. We answer this question in the affirmative: Focusing on the U(1) symmetry, we show that it is possible to obtain the maximal asymmetry scaling  $\sim \ln N$  when applying LP operations to suitably chosen symmetric entangled states.

To this end, we first characterize pure states with maximal asymmetry scaling. Note that, by our results, any maximally asymmetric state  $|\phi\rangle$  cannot be prepared by a finite-depth circuit, and thus must be long-range entangled [65,66]. Going further, given  $|\phi\rangle$  corresponding to a probability distribution

$\{p_q\}_{q \in A}$ , we find that a maximal asymmetry scaling is achieved if  $\{p_q\}_{q \in A}$  defines a distribution that is given, at leading order in  $N$ , by  $p_q \sim p(q/N)/N$ , where  $p(u) \in L^1([0, 1])$  is a continuous charge density such that  $\int_{[0,1]} du p(u) = 1$ . This condition is better understood in terms of the scaling of the charge generating function  $\langle e^{i\alpha Q} \rangle$ , which is the Fourier transform of  $p_q$ . If  $\lim_{N \rightarrow \infty} \langle e^{i\alpha Q/N} \rangle = f(\alpha)$  is a continuous function with a well-defined Fourier transform, then the leading order in the large- $N$  expansion of the charge distribution function is  $p(u) = N^{-1} \mathcal{F}[f](u)$  and [60]

$$\Delta S_N^{\text{U}(1)} \sim \ln N - \int_0^1 du p(u) \ln p(u). \quad (18)$$

It is then easy to show that LP operations can generate maximal asymmetry scaling when applied to entangled symmetric states. As an example, consider the Dicke states [67]  $|D_k^z\rangle = \binom{N}{k}^{-1/2} (\sum_{j=1}^N \sigma_j^-)^k |0\rangle^{\otimes N}$ . Clearly,  $|D_k^z\rangle$  has zero U(1) asymmetry. However, when  $k$  is extensive, we can map  $|D_k^z\rangle$  to a maximally asymmetric state by acting on every qubit with a Hadamard unitary  $H$ , thus obtaining a Dicke state in the  $x$  direction:  $|D_k^x\rangle = H^{\otimes N} |D_k^z\rangle$ . For  $k = N/2$ , we have computed the charge density  $p(u) = (\pi \sqrt{u(1-u)})^{-1}$ , which is smooth and thus yields the announced maximal scaling. We have verified this scaling by an explicit analytic computation, yielding  $\Delta S_N^{\text{U}(1)}(|D_{N/2}^x\rangle \langle D_{N/2}^x|) \sim \ln N + \pi/4$  [60].

*Outlook.* Our work opens up short-term and long-term research directions. For instance, while we have discussed the case of exponential corrections to the cluster property (2), an immediate question is how our bounds are modified by allowing for power-law violations. It is also very natural to ask whether one can define and characterize a notion of long-range asymmetry, similar to what has been recently done in the resource theory of nonstabilizerness [68–72]. In the long term, extending the setting of the present work, it would be interesting to systematically study the interplay between locality and resource-generating operations in the general context of quantum-resource theories [1]. We hope that our work will motivate further studies in these directions.

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- [1] E. Chitambar and G. Gour, Quantum resource theories, *Rev. Mod. Phys.* **91**, 025001 (2019).  
 [2] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Concentrating partial entanglement by local operations, *Phys. Rev. A* **53**, 2046 (1996).

- [3] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Mixed-state entanglement and quantum error correction, *Phys. Rev. A* **54**, 3824 (1996).  
 [4] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Quantifying entanglement, *Phys. Rev. Lett.* **78**, 2275 (1997).

- [5] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* **81**, 865 (2009).
- [6] T. Farrelly, A review of quantum cellular automata, *Quantum* **4**, 368 (2020).
- [7] P. Arrighi, An overview of quantum cellular automata, *Nat. Comput.* **18**, 885 (2019).
- [8] L. Piroli and J. I. Cirac, Quantum cellular automata, tensor networks, and area laws, *Phys. Rev. Lett.* **125**, 190402 (2020).
- [9] J. Preskill, Quantum computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).
- [10] P. Calabrese and J. Cardy, Evolution of entanglement entropy in one-dimensional systems, *J. Stat. Mech.* (2005) P04010.
- [11] P. Calabrese and J. Cardy, Time dependence of correlation functions following a quantum quench, *Phys. Rev. Lett.* **96**, 136801 (2006).
- [12] T. J. Osborne, Efficient approximation of the dynamics of one-dimensional quantum spin systems, *Phys. Rev. Lett.* **97**, 157202 (2006).
- [13] J. Eisert and T. J. Osborne, General entanglement scaling laws from time evolution, *Phys. Rev. Lett.* **97**, 150404 (2006).
- [14] N. Schuch, M. M. Wolf, K. G. H. Vollbrecht, and J. I. Cirac, On entropy growth and the hardness of simulating time evolution, *New J. Phys.* **10**, 033032 (2008).
- [15] Z. D. Shi, Bounds on Rényi entropy growth in many-body quantum systems, *Phys. Rev. A* **109**, 042404 (2024).
- [16] D. Toniolo and S. Bose, The dynamical  $\alpha$ -Rényi entropies of local Hamiltonians grow at most linearly in time, *Phys. Rev. X* **15**, 031046 (2025).
- [17] J. A. Vaccaro, F. Anselmi, H. M. Wiseman, and K. Jacobs, Tradeoff between extractable mechanical work, accessible entanglement, and ability to act as a reference system, under arbitrary superselection rules, *Phys. Rev. A* **77**, 032114 (2008).
- [18] S. D. Bartlett, T. Rudolph, and R. W. Spekkens, Reference frames, superselection rules, and quantum information, *Rev. Mod. Phys.* **79**, 555 (2007).
- [19] G. Gour, I. Marvian, and R. W. Spekkens, Measuring the quality of a quantum reference frame: The relative entropy of frame-ness, *Phys. Rev. A* **80**, 012307 (2009).
- [20] I. Marvian and R. W. Spekkens, Extending Noether's theorem by quantifying the asymmetry of quantum states, *Nat. Commun.* **5**, 3821 (2014).
- [21] F. Ares, S. Murciano, and P. Calabrese, Entanglement asymmetry as a probe of symmetry breaking, *Nat. Commun.* **14**, 2036 (2023).
- [22] H. Casini, M. Huerta, J. M. Magán, and D. Pontello, Entanglement entropy and superselection sectors. Part I. Global symmetries, *J. High Energy Phys.* **02** (2020) 014.
- [23] H. Casini, M. Huerta, J. M. Magán, and D. Pontello, Entropic order parameters for the phases of QFT, *J. High Energy Phys.* **04** (2021) 277.
- [24] J. M. Magán, Proof of the universal density of charged states in QFT, *J. High Energy Phys.* **12** (2021) 100.
- [25] V. Benedetti, H. Casini, Y. Kawahigashi, R. Longo, and J. M. Magan, Modular invariance as completeness, *Phys. Rev. D* **110**, 125004 (2024).
- [26] L. Capizzi and M. Mazzoni, Entanglement asymmetry in the ordered phase of many-body systems: The Ising field theory, *J. High Energy Phys.* **12** (2023) 144.
- [27] F. Ares, S. Murciano, E. Vernier, and P. Calabrese, Lack of symmetry restoration after a quantum quench: An entanglement asymmetry study, *SciPost Phys.* **15**, 089 (2023).
- [28] C. Rylands, K. Klobas, F. Ares, P. Calabrese, S. Murciano, and B. Bertini, Microscopic origin of the quantum Mpemba effect in integrable systems, *Phys. Rev. Lett.* **133**, 010401 (2024).
- [29] S. Murciano, F. Ares, I. Klich, and P. Calabrese, Entanglement asymmetry and quantum Mpemba effect in the XY spin chain, *J. Stat. Mech.* (2024) 013103.
- [30] K. Chalas, F. Ares, C. Rylands, and P. Calabrese, Multiple crossing during dynamical symmetry restoration and implications for the quantum Mpemba effect, *J. Stat. Mech.* (2024) 103101.
- [31] B. Bertini, K. Klobas, M. Collura, P. Calabrese, and C. Rylands, Dynamics of charge fluctuations from asymmetric initial states, *Phys. Rev. B* **109**, 184312 (2024).
- [32] C. Rylands, E. Vernier, and P. Calabrese, Dynamical symmetry restoration in the Heisenberg spin chain, *J. Stat. Mech.* **2024**, 123102 (2024).
- [33] K. Klobas, Non-equilibrium dynamics of symmetry-resolved entanglement and entanglement asymmetry: Exact asymptotics in rule 54, *J. Phys. A: Math. Theor.* **57**, 505001 (2024).
- [34] F. Caceffo, S. Murciano, and V. Alba, Entangled multiplets, asymmetry, and quantum Mpemba effect in dissipative systems, *J. Stat. Mech.* (2024) 063103.
- [35] F. Ares, V. Vitale, and S. Murciano, Quantum Mpemba effect in free-fermionic mixed states, *Phys. Rev. B* **111**, 104312 (2025).
- [36] A. Foligno, P. Calabrese, and B. Bertini, Non-equilibrium dynamics of charged dual-unitary circuits, *PRX Quantum* **6**, 010324 (2025).
- [37] S. Yamashika, F. Ares, and P. Calabrese, Entanglement asymmetry and quantum Mpemba effect in two-dimensional free-fermion systems, *Phys. Rev. B* **110**, 085126 (2024).
- [38] S. Yamashika, P. Calabrese, and F. Ares, Quenching from superfluid to free bosons in two dimensions: Entanglement, symmetries, and quantum Mpemba effect, *Phys. Rev. A* **111**, 043304 (2025).
- [39] F. Ferro, F. Ares, and P. Calabrese, Non-equilibrium entanglement asymmetry for discrete groups: The example of the XY spin chain, *J. Stat. Mech.* (2024) 023101.
- [40] X. Turkeshi, P. Calabrese, and A. De Luca, Quantum Mpemba effect in random circuits, *Phys. Rev. Lett.* **135**, 040403 (2025).
- [41] S. Liu, H.-K. Zhang, S. Yin, and S.-X. Zhang, Symmetry restoration and quantum Mpemba effect in symmetric random circuits, *Phys. Rev. Lett.* **133**, 140405 (2024).
- [42] S. Liu, H.-K. Zhang, S. Yin, S.-X. Zhang, and H. Yao, Quantum Mpemba effects in many-body localization systems, *Sci. Bullet.* **70**, 3991 (2025).
- [43] T. Banerjee, S. Das, and K. Sengupta, Entanglement asymmetry in periodically driven quantum systems, *SciPost Physics* **19**, 051 (2025).
- [44] K. Klobas, C. Rylands, and B. Bertini, Translation symmetry restoration under random unitary dynamics, *Phys. Rev. B* **111**, L140304 (2025).
- [45] A. Summer, M. Moroder, L. P. Bettmann, X. Turkeshi, I. Marvian, and J. Goold, A resource theoretical unification of Mpemba effects: Classical and quantum, *arXiv:2507.16976*.
- [46] H. Yu, Z.-X. Li, and S.-X. Zhang, Symmetry breaking dynamics in quantum many-body systems, *Chin. Phys. Lett.* **42**, 110602 (2025).

- [47] L. Capizzi and V. Vitale, A universal formula for the entanglement asymmetry of matrix product states, *J. Phys. A: Math. Theor.* **57**, 45LT01 (2024).
- [48] F. Ares, S. Murciano, L. Piroli, and P. Calabrese, Entanglement asymmetry study of black hole radiation, *Phys. Rev. D* **110**, L061901 (2024).
- [49] F. Ares, S. Murciano, P. Calabrese, and L. Piroli, Entanglement asymmetry dynamics in random quantum circuits, *Phys. Rev. Res.* **7**, 033135 (2025).
- [50] L. K. Joshi, J. Franke, A. Rath, F. Ares, S. Murciano, F. Kranzl, R. Blatt, P. Zoller, B. Vermersch, P. Calabrese, C. F. Roos, and M. K. Joshi, Observing the quantum Mpemba effect in quantum simulations, *Phys. Rev. Lett.* **133**, 010402 (2024).
- [51] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2010).
- [52] E. Altman, K. R. Brown, G. Carleo, L. D. Carr, E. Demler, C. Chin, B. DeMarco, S. E. Economou, M. A. Eriksson, K.-M. C. Fu, *et al.*, Quantum simulators: Architectures and opportunities, *PRX Quantum* **2**, 017003 (2021).
- [53] A. A. Mele, Introduction to Haar measure tools in quantum information: A beginner's tutorial, *Quantum* **8**, 1340 (2024).
- [54] N. Schuch, F. Verstraete, and J. I. Cirac, Nonlocal resources in the presence of superselection rules, *Phys. Rev. Lett.* **92**, 087904 (2004).
- [55] N. Schuch, F. Verstraete, and J. I. Cirac, Quantum entanglement theory in the presence of superselection rules, *Phys. Rev. A* **70**, 042310 (2004).
- [56] G. Gour and R. W. Spekkens, The resource theory of quantum reference frames: Manipulations and monotones, *New J. Phys.* **10**, 033023 (2008).
- [57] We say that a channel is symmetric if  $\mathcal{E}(U_g(\cdot)U_g^\dagger) = U_g\mathcal{E}(\cdot)U_g^\dagger$  for any element  $U_g$  of the symmetry group.
- [58] We mention that in a QRT, one is also typically interested in the connections between a resource monotone and the so-called rates of convertibility between states [1,73]. In the case of the asymmetry, partial results are known; see, e.g., Refs. [56,74–78].
- [59] G. Lindblad, An entropy inequality for quantum measurements, *Commun. Math. Phys.* **28**, 245 (1972).
- [60] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/gcjj-88j6> for further details, including Refs. [79–85].
- [61] J. L. Massey, On the entropy of integer-valued random variables, in *Proceedings of 1988 Beijing International Workshop of Information Theory, July 4–7 1988* (International Academic Publishers, 1989).
- [62] O. Rioul, Gaussian bounds for discrete entropies, in *IEEE Information Theory and Applications Workshop (ITA 2022)* (San Diego, United States, 2022).
- [63] Namely, from the right invariance of the SU(2) Haar measure,  $\mathcal{G}[U\omega U^\dagger] = \mathcal{G}[\omega]$  [cf. Eq. (3)]. Since  $S_V(\rho) = S_V(\omega)$ ,  $\Delta S_N^{\text{SU}(2)}(\rho) = \Delta S_N^{\text{SU}(2)}(\omega)$ .
- [64] By inspection, such exponentially small corrections are seen to only give subleading corrections to our inequalities, which continue to hold at the leading order in  $N$ .
- [65] X. Chen, Z.-C. Gu, and X.-G. Wen, Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order, *Phys. Rev. B* **82**, 155138 (2010).
- [66] B. Zeng, X. Chen, D.-L. Zhou, and X.-G. Wen, Quantum information meets quantum matter—From quantum entanglement to topological phase in many-body systems, [arXiv:1508.02595](https://arxiv.org/abs/1508.02595).
- [67] R. H. Dicke, Coherence in spontaneous radiation processes, *Phys. Rev.* **93**, 99 (1954).
- [68] S. Bravyi and A. Kitaev, Universal quantum computation with ideal Clifford gates and noisy ancillas, *Phys. Rev. A* **71**, 022316 (2005).
- [69] V. Veitch, S. H. Mousavian, D. Gottesman, and J. Emerson, The resource theory of stabilizer quantum computation, *New J. Phys.* **16**, 013009 (2014).
- [70] C. D. White, C. Cao, and B. Swingle, Conformal field theories are magical, *Phys. Rev. B* **103**, 075145 (2021).
- [71] T. D. Ellison, K. Kato, Z.-W. Liu, and T. H. Hsieh, Symmetry-protected sign problem and magic in quantum phases of matter, *Quantum* **5**, 612 (2021).
- [72] D. A. Korbany, M. J. Gullans, and L. Piroli, Long-range nonstabilizerness and phases of matter, *Phys. Rev. Lett.* **135**, 160404 (2025).
- [73] F. G. S. L. Brandão and G. Gour, Reversible framework for quantum resource theories, *Phys. Rev. Lett.* **115**, 070503 (2015).
- [74] I. Marvian and R. W. Spekkens, Asymmetry properties of pure quantum states, *Phys. Rev. A* **90**, 014102 (2014).
- [75] Y. Yang, G. Chiribella, and Q. Hu, Units of rotational information, *New J. Phys.* **19**, 123003 (2017).
- [76] I. Marvian, Coherence distillation machines are impossible in quantum thermodynamics, *Nat. Commun.* **11**, 25 (2020).
- [77] I. Marvian, Operational interpretation of quantum Fisher information in quantum thermodynamics, *Phys. Rev. Lett.* **129**, 190502 (2022).
- [78] K. Yamaguchi, Y. Mitsuhashi, T. Shitara, and H. Tajima, Quantum geometric tensor determines the i.i.d. conversion rate in the resource theory of asymmetry for any compact Lie group, [arXiv:2411.04766](https://arxiv.org/abs/2411.04766).
- [79] P.-A. Bernard and L. Vinet, A dynamical algebra of protocol-induced transformations on Dicke states, *Philos. Trans. A Math. Phys. Eng. Sci.* **383**, 20240415 (2025).
- [80] L. A. Shepp and I. Olkin, Entropy of the sum of independent Bernoulli random variables and of the multinomial distribution, in *Contributions to Probability*, edited by J. Gani and V. K. Rohatgi (Elsevier, 1981), p. 201.
- [81] E. Hillion and O. Johnson, A proof of the Shepp–Olkin entropy concavity conjecture, *Bernoulli* **23**, 3638 (2017).
- [82] N. Defenu, A. Lerose, and S. Pappalardi, Out-of-equilibrium dynamics of quantum many-body systems with long-range interactions, *Phys. Rep.* **1074**, 1 (2024).
- [83] X.-C. Li and R. Wong, A uniform asymptotic expansion for Krawtchouk polynomials, *J. Approx. Theory* **106**, 155 (2000).
- [84] D. Dominici, Asymptotic analysis of the Krawtchouk polynomials by the WKB method, *Ramanujan J.* **15**, 303 (2008).
- [85] G. Morettini, L. Capizzi, M. Fagotti, and L. Mazza, Unconventional transport in a system with a tower of quantum many-body scars, *Phys. Rev. B* **112**, 134314 (2025).