

Reply to “Comment on “Experimental demonstration of electric power generation from Earth’s rotation through its own magnetic field””

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We argued in 2016 that an emf could be generated from Earth’s rotation through its own magnetic field using a magnetically permeable hollow cylinder with a low magnetic Reynolds number R_m [Chyba *et al.*, *Phys. Rev. Appl.* **6**, 014017 (2016)]. Experiments have now generated the appropriate emf and verified five predictions of the theory [Chyba *et al.*, *Phys. Rev. Res.* **7**, 013285 (2025)]. Wijngaarden [*Phys. Rev. Res.* **8**, 028001 (2026)] argues that our 2016 argument is in error. In his model, the magnetization of the cylindrical shell is due to a magnetic field that translates exactly with the velocity of the shell, giving $\text{emf} = 0$ always. However, we show that this only holds if magnetic diffusion is negligible, whereas in fact it is significant in the low- R_m case. We do find $\text{emf} = 0$ in agreement with Wijngaarden for the special cases of a cylindrical shell operating at high R_m , or for a solid cylinder of any R_m . In these cases, the shell’s field does translate perfectly with the body, there is no diffusion, and current density $\mathbf{J} = \mathbf{0}$. In contrast, our device is a hollow cylinder with low R_m so magnetic diffusion contributes alongside advection. For this case, the magnetic field does not rigidly translate with the shell, neither curl $\mathbf{J}$ nor $\mathbf{J}$ need vanish, and the emf need not be zero.

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I. INTRODUCTION

Earth rotates through the axisymmetric part of its own magnetic field [1–3], but a simple argument shows that this cannot be used to generate electricity in a conductor rotating with Earth. However, in a theoretical paper published in 2016 [2], we showed how to circumvent this conclusion using a soft magnetic device of low magnetic Reynolds number R_m and topology chosen to ensure $\nabla \times (\mathbf{v} \times \mathbf{B}) \neq \mathbf{0}$, where $\mathbf{v}$ is Earth’s rotation velocity and $\mathbf{B}$ the magnetic flux density within the object. In 2025, we published results in which we realized the low- R_m and topological requirements determined in Ref. [2], using a hollow cylindrical shell of manganese-zinc ferrite [4]. Controlling for thermoelectric and other confounding effects, we generated a continuous DC voltage and tested and verified five other predictions of the theory, including changes in the emf’s sign and magnitude under rotation, and $\text{emf} = 0$ for an otherwise identical solid cylinder. We also

reproduced the effect at a second location 5.5 km from our primary laboratory. Wijngaarden in his Comment [5] argues that we made a theoretical error in our 2016 paper [2]. Similarly to Jeener [3,6], he considers the integral giving the emf around a path C with line element $d\mathbf{l}$ (with corresponding surface S and area element $d\mathbf{a}$), viz.,

$$\text{emf} = \oint_C (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}, \quad (1)$$

and argues that for our device the two terms in the integrand in Eq. (1) cancel when properly chosen, so that $\text{emf} = 0$ always. He does not refute our experimental results [4], but concludes with a brief section discussing ways in which our experimental system could be further tested.

We agree with Wijngaarden that further experimental tests of our predictions should be carried out; this is why we concluded our experimental paper [4] with the statement that “The next step would be for an independent group to reproduce (or contradict) our results under experimental conditions closely similar to those used here.” However, Wijngaarden immunizes his objection against experimental refutation, concluding his Comment with the remarkable statement that “Should future experiments be able to prove that the present device is indeed capable of extracting energy from Earth’s rotation, then my detailed theoretical objections . . . will still

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hold. Thus also new experiments cannot be used by themselves as refutation of my objections.”

We disagree with this view of the role of experiment in evaluating theoretical disagreements. However, given that Wijngaarden has not explicitly challenged our experimental results [4], in this Reply our focus must be on his theoretical argument. We show that his approach to the problem excludes magnetic diffusion for low- R_m cylindrical shells, and that this exclusion guarantees his conclusion that $\text{emf} = 0$. We then show that not ruling out diffusion *ab initio* in Wijngaarden’s model leads to predictions identical to our own.

II. TWO MODELS FOR A SOFT MAGNET

We first quickly describe the system under discussion and establish notation. We are concerned with the magnetic flux density $\mathbf{B}$ within and exterior to a magnetically permeable cylindrical shell with inner radius $\rho = a$ and outer radius $\rho = b$, oriented along the z axis perpendicular to a background constant magnetic flux density $\mathbf{B}_\infty = B_\infty \hat{\mathbf{x}}$. Here, (ρ, ϕ, z) are the usual cylindrical coordinates; the device is illustrated in Fig. 1 in either Ref. [2] or [4]. We work first in a frame K in which the shell is moving with velocity $\mathbf{v} = v\hat{\mathbf{y}}$. The background field $\mathbf{B}_\infty$ is the axially symmetric part of Earth’s field, with components written in detail in Refs. [3,4]. This system can be treated, to an extremely good approximation, as a shell moving rectilinearly through a constant background field [2].

First, consider the special case in which the shell is at rest ($\mathbf{v} = \mathbf{0}$) in K . As illustrated in Fig. 1, the magnetically permeable shell distorts the field within and outside the shell. There are two approaches to understanding these field-distorting effects.

A. Field channeling

The first approach, common in the electromagnetism and magnetic shielding literature [7–10], is to picture the shell as channeling or shunting $\mathbf{B}_\infty$ through its magnetically permeable layer $a \leq \rho \leq b$. Our intuition has been guided by this picture, in which the channeling is due to the requirement from Maxwell’s equations that the normal component of $\mathbf{B}$ and the tangential component of the magnetic field $\mathbf{H}$ must be continuous across the boundaries at $\rho = a$ and $\rho = b$ where the relative magnetic permeability μ_r shifts discontinuously [7]. For the materials of interest here, the magnetic field $\mathbf{H}$ is related to $\mathbf{B}$ by $\mathbf{H} = \mathbf{B}/\mu$, where $\mu = \mu_r\mu_0$ with μ_0 the permeability of free space.

This picture of a permeable shell channeling and distorting $\mathbf{B}_\infty$ is used to derive the resulting field $\mathbf{B}$ everywhere in space [7–9]. When the shell is at rest relative to $\mathbf{B}_\infty$, Ampère’s law is $\nabla \times \mathbf{H} = \mathbf{J} = \mathbf{0}$. We designate the magnetic field in this special case as $\mathbf{H}_0 = \mathbf{B}_0/\mu$. Then, $\mathbf{H}_0 = -\nabla W$, where W is a magnetic potential and $\nabla \cdot \mathbf{H}_0 = \nabla^2 W = 0$. The solution for $\mathbf{B}_0$ is well known [8,9] and is given by Eqs. (9)–(15) in Ref. [2]. For example, $\mathbf{B}_0$ within the walls of the cylindrical shell is given by

$$B_{0x}(a \leq \rho \leq b) = \beta_1 - \beta_2(a/\rho)^2 \cos 2\phi, \quad (2)$$

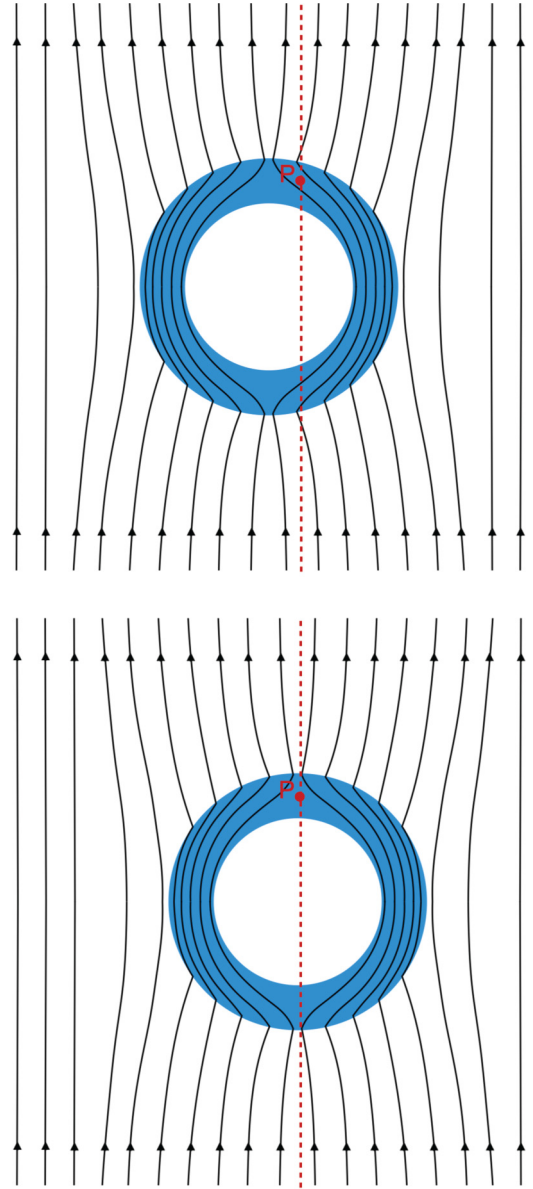


FIG. 1. Distortion of magnetic flux density by a translating magnetically permeable cylindrical shell (shown in cross section). Point P is stationary in the frame K , across which the cylindrical shell is moving at velocity $\mathbf{v} = v\hat{\mathbf{y}}$ to the right, orthogonal to both the long axis of the cylinder and to the background field $\mathbf{B}_\infty = B_\infty \hat{\mathbf{x}}$. The upper panel shows field lines at some time t_0 ; the lower panel shows field lines at a subsequent time $t_0 + \Delta t$. The dotted red line fixes the location of P with respect to the field $\mathbf{B}$ inside the shell as it translates. When R_m is small and $\mathbf{v} \neq \mathbf{0}$, the field lines are distorted slightly away from those of the static case [Eq. (13); see Figs. 2 and 3 in Ref. [2]], and the change in the field lines at P is due both to the advection of the field at velocity $\mathbf{v}$ with the shell, and also to the diffusion of the field at a velocity $\mathbf{v}_D \neq \mathbf{v}$ (see text for discussion).

where

$$\beta_1 = 2B_\infty\mu_r(\mu_r + 1)\zeta, \quad (3)$$

$$\beta_2 = 2B_\infty\mu_r(\mu_r - 1)\zeta, \quad (4)$$

$$\zeta = [(\mu_r + 1)^2 - (\mu_r - 1)^2(a/b)^2]^{-1}, \quad (5)$$

and a similar expression for $B_{0y}(a \leq \rho \leq b)$. Figure 1 sketches the field lines corresponding to $\mathbf{B}_0$ throughout space. The special case of a solid rod corresponds to setting $a = 0$ in Eq. (2), which becomes

$$B_{0x}(a \leq \rho \leq b) = 2B_\infty \frac{\mu_r}{\mu_r + 1}, \quad (6)$$

while $B_{0y} = 0$. When $a = 0$, the field in the rod is spatially uniform, parallel to $\mathbf{B}_\infty$, and double its magnitude provided $\mu_r \gg 1$. The corresponding field lines are sketched in Fig. 2.

B. Magnetization

The second approach to understanding the fields depicted in Figs. 1 and 2 appeals to the magnetization of the (magnetically soft) cylindrical shell that occurs when the shell is placed in the background field. The magnetization $\mathbf{M}$ is the material effect of the magnetic field $\mathbf{H}$ [11], and is given by

$$\mathbf{M} = \chi_m \mathbf{H}, \quad (7)$$

where magnetic susceptibility $\chi_m = \mu_r - 1$. The flux density $\mathbf{B}$ is then

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \quad (8)$$

The distortion of the field $\mathbf{B}$ away from $\mathbf{B}_\infty$ as depicted in Fig. 1 is thereby understood as resulting from the vector sum of the field $\mathbf{H}$ and the magnetization $\mathbf{M}$ that $\mathbf{H}$ itself produces in the soft magnetic material. This picture is discussed for soft magnets by Cullity and Graham [12]. Cullwick [13] extends this approach to the case depicted in Fig. 2, viz., a solid cylindrical rod made of soft magnetic material with long axis perpendicular to $\mathbf{B}_\infty$, translating at velocity $\mathbf{v}$ in a direction orthogonal to both its long axis and $\mathbf{B}_\infty$.

III. THE TRANSLATING MAGNET

Wijngaarden adopts Cullwick's approach for the soft magnetic solid rod and attempts to extend it to our system where the translating soft magnet is instead a hollow cylindrical shell. Wijngaarden treats the $\mathbf{B}_\infty$ field separately from the field he terms $\mathbf{B}_{\text{tube}}$, taken to be that generated by the shell itself. In his notation, $\mathbf{B}_\infty = \mathbf{B}_{\text{Earth}}$ so

$$\mathbf{B}_{\text{tube}} = \mathbf{B} - \mathbf{B}_\infty. \quad (9)$$

Wijngaarden argues that $\mathbf{B}_{\text{tube}}$ translates together with the permeable shell at velocity $\mathbf{v}$, yielding an electric field $\mathbf{E}_K$ in K according to the usual relativistic field transformation:

$$\mathbf{E}_K = -\mathbf{v} \times \mathbf{B}_{\text{tube}}. \quad (10)$$

He states (his Sec. VII) that we incorrectly obtained a nonzero emf in Eq. (1) by "ignoring" the cancellation of $\mathbf{v} \times \mathbf{B}$ by $\mathbf{E}_K$ (at least, cancellation to within a potential term that yields zero upon integration) in Eq. (1). If this cancellation by $\mathbf{E}_K$ is not ignored, he claims $\text{emf} = 0$ always.

However, our model includes the identical electric field. Our analysis [2] begins with Ohm's law for a moving conductor in K :

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{J}/\sigma, \quad (11)$$

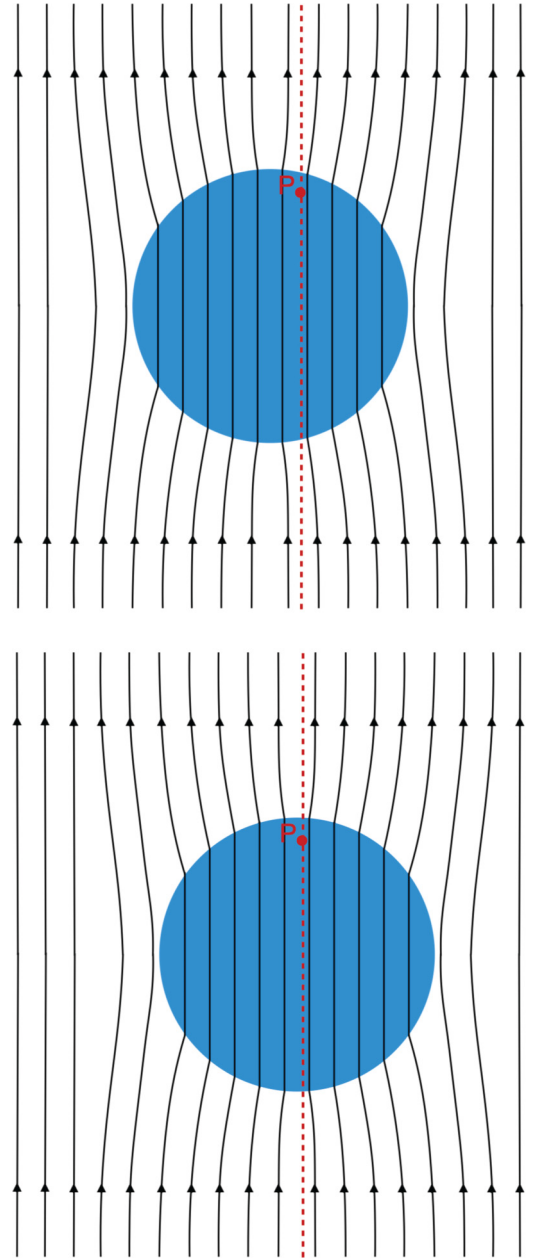


FIG. 2. Distortion of magnetic flux density by a translating magnetically permeable solid cylindrical rod (shown in cross section). Point P is stationary in the frame K , across which the rod is moving at velocity $\mathbf{v} = v\hat{\mathbf{y}}$ to the right, orthogonal to both the long axis of the cylinder and to the background field $\mathbf{B}_\infty = B_\infty\hat{\mathbf{x}}$. The upper panel shows field lines at time t_0 ; the lower panel shows field lines at a subsequent time $t_0 + \Delta t$. The dotted red line visually fixes the location of P with respect to the field $\mathbf{B}$ inside the rod as it translates. The field within a solid soft magnetic rod is spatially uniform and parallel to $\mathbf{B}_\infty$, so that P experiences no change in $\mathbf{B}$ while the rod moves across it. Therefore, there is no diffusion of the field, only advection with the translating rod.

where $\mathbf{J}$ is the current density and σ the conductivity. If in Eq. (11) we have $\mathbf{J} = \nabla \times \mathbf{H} = \mathbf{0}$, then $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, identical to Wijngaarden's Eq. (10). So, our results agree for the special case $\mathbf{J} = \mathbf{0}$. The electric field he obtains from a relativistic transformation from the frame K' moving at $\mathbf{v}$

relative to K we obtain from Ohm's law for a moving conductor in K .

Why, then, does Wijngaarden find $\text{emf} = 0$ always, whereas we predict (and measure [4]) $\text{emf} \neq 0$ in some special cases? Wijngaarden claims that we do so by "assuming" (his Sec. VII) that $\mathbf{J} \neq \mathbf{0}$ in Eq. 11, but that is not what we do. Rather, we first recognize that for a motionless ($\mathbf{v} = \mathbf{0}$) cylinder, we just have a stationary soft magnet in a background field, so in this case of course $\mathbf{J} = \mathbf{0}$ in Eq. (11), and Eq. (10) follows. However, we then *ask* whether this necessarily continues to hold when $\mathbf{v} \neq \mathbf{0}$. We neither assume for the case $\mathbf{v} \neq \mathbf{0}$ that $\mathbf{J} = \mathbf{0}$ nor assume that $\mathbf{J} \neq \mathbf{0}$.

Wijngaarden argues that there is no other possible choice but $\mathbf{J} = \mathbf{0}$ because by special relativity, given his assertion that $\mathbf{B}_{\text{tube}}$ translates at $\mathbf{v}$ with the cylindrical shell, Eq. (10) unavoidably holds, so Eq. (11) must give $\mathbf{J} = \mathbf{0}$. This argument appears powerful because it seems to be an unavoidable consequence of relativity. However, it is not. If the translating magnet is a permanent (hard) magnet, this analysis is valid [14]. However, for a soft magnet, the argument holds if $a = 0$, or if $a \neq 0$ and the magnetic Reynolds number of the translating cylindrical shell $R_m \gg 1$. The first of these special cases is the one considered by Cullwick [13]. The second of these special cases is the one implicitly assumed by Wijngaarden. However, for a translating soft magnetic cylindrical shell with $a \neq 0$ and R_m small, the magnetic field does not necessarily simply advect perfectly with the translating shell at velocity $\mathbf{v}$, because in this case the field also diffuses with a non-negligible velocity $\mathbf{v}_d \neq \mathbf{v}$. This in turn would mean that Eq. (10) is incorrect, so that $\mathbf{J} \neq \mathbf{0}$ in Eq. (11). In Sec. IV, we show how this conclusion arises in our treatment of the translating magnetically permeable shell. In Sec. V, we show that the same effect arises in Wijngaarden's magnetization model when R_m is small. With this effect included, the Cullwick-Wijngaarden approach gives results identical to our own, including $\text{emf} \neq 0$ when $a \neq 0$ and $R_m \lesssim 1$.

IV. MAGNETIC DIFFUSION IN OUR MODEL

Our intuitive picture is straightforward (see Fig. 1). First, we leave open the *possibility* that the field $\mathbf{B}$ within the shell when $\mathbf{v} \neq \mathbf{0}$ in K may not be identical to $\mathbf{B}_0$. (If $\mathbf{B} = \mathbf{B}_0$, then because $\nabla \times \mathbf{B}_0 = \mathbf{0}$ there can be no diffusion.) We do not make this assumption for the $\mathbf{v} \neq \mathbf{0}$ case *ab initio*. Consider a fixed point P in K across which the interior ($a \leq \rho \leq b$) of the cylindrical shell is moving. As the shell moves, the field changes at P . The special case where $a = 0$ (see Fig. 2) is an exception; in this case, the field in the shell is uniform so $\mathbf{B}$ at P does not change as the rod moves across P . However, if $a \neq 0$, as the shell translates, then $\mathbf{B}$ at P changes to take on the value appropriate to its new position relative to the cylindrical shell. The field $\mathbf{B}$ changes in two ways. It advects with the shell at the shell's velocity $\mathbf{v}$, corresponding to an advection or transport time $\tau_v = b/v$ for a system with a characteristic length scale b (here, the shell's radius). However, provided $\mathbf{B} \neq \mathbf{B}_0$, the field also diffuses with a diffusion velocity given by $v_D = b/\tau_D$, where $\tau_D = b^2/\eta$ is the magnetic diffusion time and $\eta = (\sigma\mu)^{-1}$ the magnetic diffusivity.

If we do not assume that $\mathbf{J} = \mathbf{0}$, then taking the curl of Ohm's law [Eq. (11)] and applying Maxwell's

equations yields the advection-diffusion equation for $\mathbf{B}$ within the shell:

$$\partial\mathbf{B}/\partial t = \nabla \times \mathbf{v} \times \mathbf{B} + \eta \nabla^2 \mathbf{B}. \quad (12)$$

In general, $\partial\mathbf{B}/\partial t$ is due to both the advection and diffusion terms. Whether $\eta \nabla^2 \mathbf{B}$ is negligible in Eq. (12) depends on $R_m = \tau_D/\tau_v = \sigma\mu vb$. If $R_m \gg 1$, then $\tau_D \gg \tau_v$ and the much more slowly acting diffusion effect is negligible. In this case, $\eta \nabla^2 \mathbf{B}$ is negligible in Eq. (12) compared to the advection term, and indeed $\text{emf} = 0$. If $R_m \lesssim 1$, the diffusion term is comparable to or greater than the advection term, and we must solve Eq. (12) in full. We find that the solution $\mathbf{B}$ is [2]

$$\mathbf{B}(a \leq \rho \leq b) = \mathbf{B}_0 + \Sigma_i^n \mathbf{B}_i + O(R_m)^{n+1}, \quad (13)$$

where, for example,

$$B_{1x} = -R_m \beta_2 (a/b) (a/\rho) \sin \phi \cos^2 \phi, \quad (14)$$

with $R_m = v\sigma\mu b$ and β_2 given by Eq. (4), and similarly for B_{1y} . When $R_m < 1$, $\mathbf{B}$ for $\mathbf{v} \neq \mathbf{0}$ is perturbed away from the magnetostatic $\mathbf{v} = \mathbf{0}$ solution $\mathbf{B}_0$ by a series whose terms are scaled by successive powers of R_m . With $\mathbf{B}$ given by Eq. (13), $\nabla \times \mathbf{B} \neq \mathbf{0}$, so $\mathbf{J} \neq \mathbf{0}$ in Eq. 11 and $\text{emf} \neq 0$. This is not because we have forced that result, but because we have discovered it by not excluding it *ab initio* in requiring $\mathbf{J} = \mathbf{0}$.

V. MAGNETIC DIFFUSION IN WIJNGAARDEN'S MODEL

Equation (10) in Wijngaarden's model guarantees that $\mathbf{J} = \mathbf{0}$ in Eq. (11). However, Eq. (10) holds by the relativistic field transformations only if it is correct that $\mathbf{B}_{\text{tube}}$ moves at $\mathbf{v}$ perfectly together with the moving cylinder. This would be true for a translating hard magnet [14]. As we saw in Sec. IV, for a soft magnet that is a solid rod (the case Cullwick [13] examined) this is also correct and Eq. (10) holds. However, for $a \neq 0$ and R_m small, $\mathbf{B}$ diffuses as well as advects. Then, because $\mathbf{H} = \mathbf{B}/\mu$, by Eq. (12) we also have

$$\partial\mathbf{H}/\partial t = \nabla \times \mathbf{v} \times \mathbf{H} + \eta \nabla^2 \mathbf{H}, \quad (15)$$

so that the magnetic field $\mathbf{H}$ diffuses together with $\mathbf{B}$. Equation (9) may be written as

$$\mathbf{B}_{\text{tube}} = \mu\mathbf{H} - \mathbf{B}_\infty. \quad (16)$$

For a soft magnet, the magnetization is a direct result of the magnetic field: $\mathbf{M} = (\mu_r - 1)\mathbf{H}$ by Eq. (7), with $\chi_m = \mu_r - 1$. Then, Eq. (16) implies

$$\mathbf{B}_{\text{tube}} = \frac{\mu_0\mu_r}{\mu_r - 1} \mathbf{M} - \mathbf{B}_\infty. \quad (17)$$

Equation (17) simply writes Wijngaarden's expression for $\mathbf{B}_{\text{tube}}$ in terms of the more familiar magnetization $\mathbf{M}$ used in the soft and hard magnet literature.

If $\mathbf{H}$ is diffusing according to Eq. (15) when $R_m \lesssim 1$, then by Eq. (16), $\mathbf{B}_{\text{tube}}$ must in effect diffuse as well. Equation (17) provides a more physical picture: In K , $\mathbf{M}$ at a point P within the cylindrical shell does not change solely due to the translation of the magnet, because $\mathbf{H}$ does not merely advect with the translating shell. The magnetization $\mathbf{M}$ itself is not diffusing as such (it is a macroscopic sum over many magnetic dipoles that are responding to $\mathbf{H}$ in the shell), but the field $\mathbf{H}$ that determines $\mathbf{M}$ within the shell varies at P due to both advection

and diffusion, so $\mathbf{M}$ too behaves as if it were diffusing as well as advecting. Then, by Eq. (17) this is true for $\mathbf{B}_{\text{tube}}$ as well, which in turn means that $\mathbf{B}_{\text{tube}}$ is not simply moving at $\mathbf{v}$ in K . It is moving with both $\mathbf{v}$ and $\mathbf{v}_D \neq \mathbf{v}$, so Eq. (10) does not exactly hold. The higher-order terms in $\mathbf{H}$ given by Eq. (13) diffuse as well as advect.

VI. THE ELECTROMOTIVE FORCE IN EXTENDED CONDUCTORS

Wijngaarden's argument does not depend on details of how the emf is calculated within the shell. His argument rather is that because (in his view) $\mathbf{B}_{\text{tube}}$ is translating perfectly with the shell at velocity $\mathbf{v}$ but not also diffusing, the emf calculated by *any* means must be zero. We demonstrated in Sec. V why this is incorrect. Nevertheless, it is perhaps illuminating to review briefly how the emf calculation is treated in our work and how that relates to the more familiar example of the Faraday disk.

Our system (Fig. 1 in Refs. [2,4]) involves an extended conductor where current presumably flows through the conductor's entire volume, rather than along any single linear path C . In this respect, our system is reminiscent of the Faraday disk or homopolar generator, for which different choices of C lead to different predictions for the generated emf. The Faraday disk has drawn special attention because it seemingly presents a "paradox" of a system that generates an emf even though the flux rule

$$\text{emf} = -d\Phi/dt' = - \iint_S \partial \mathbf{B} / \partial t' \cdot d\mathbf{a} \quad (18)$$

[where the surface S is that corresponding to the complete path C in Eq. (1) and Φ is the magnetic flux through S] appears to yield $\text{emf} = 0$, because Φ through the entire rotating conducting disk does not change with time [15–21]. However, since the Faraday disk produces a constant emf, Eq. (18) would seem to imply that $d\Phi/dt$ is a constant and Φ must increase indefinitely (or decrease to zero). Pugh [16] argued that in this and similar cases, Eq. (18) "is only valid in its differential form, since direct integration produces the nonsensical result $\Phi = -\mathcal{E}_m t + C$," where C is an integration constant and $\mathcal{E}_m = \text{emf}$. Wijngaarden (his Sec. VI) makes just this objection for our device when analyzed in the laboratory frame.

In the case of the Faraday disk, this issue may be addressed by choosing the segment of C in the disk to rotate with the disk, so that the electrons are taken to share the linear velocity $\mathbf{v}$ of the rotating disk. This choice gives the experimentally correct result and is consistent with Eq. 18, provided one argues that the flux increases (or decreases) continuously even while $\mathbf{B}$ remains constant because the area defined by a rotating segment of C relative to its initial position increases (or decreases) [17,19], seemingly indefinitely. Instead of saving the flux rule in this way, Griffiths and others join Pugh and Feynman in arguing that the flux rule does not apply for the Faraday disk (since by common sense the flux through the entire disk does not change as the disk rotates) or for extended conductors in general [15,16,18,20]. These authors emphasize that the "flux rule" [Eq. (18)] does not have the status of Maxwell's equations and the Lorentz force law, which together always give the correct result, predicting the correct Faraday disk emf immediately [15–21].

We showed in 2015 that the correct emf and power produced by the Faraday disk could also be calculated without choosing a specific linear path C within the volume of the disk, but rather by integrating through the disk's entire volume [22]. In our 2016 paper, we expressed analogous concern over making any given choice for C within our extended magnetically permeable cylindrical shell conductor. In that paper, we first demonstrated the existence of a nonzero emf around a particular path C within the shell (Sec. IX in Ref. [2]), but also suggested an alternative approach to calculating the emf by performing a volume integration (Sec. XIII). In our experimental 2025 paper, this is indeed what we do, averaging over the entire shell volume (Appendix A in Ref. [4]): It is the emf predicted in this way that we test against experiment, finding agreement.

For the Faraday disk, the current generates secondary magnetic fields [1], but only the primary field contributes to the magnetic braking of the disk [22]. In our system, $\mathbf{B}(\mathbf{v} \neq \mathbf{0})$ is perturbed away from its $\mathbf{v} = \mathbf{0}$ value $\mathbf{B}_0$ to include small correction terms $\mathbf{B}_i$, such as $\mathbf{B}_1$ in Eqs. (13) and (14). It is these terms that generate the nonzero current $\mathbf{J} = \mu^{-1} \nabla \times \mathbf{B}_i$ and therefore the emf, whereas $\nabla \times \mathbf{B}_0 = \mathbf{0}$. The nonzero $\mathbf{J}$ that flows when $\mathbf{v} \neq \mathbf{0}$ and the perturbation terms $\mathbf{B}_i$ are mutually consistent both with respect to Eq. (12) and also with Poynting's theorem [2]. By Eq. (14), the leading perturbations of $\mathbf{B}$ are of $O(R_m) \sim 0.1 \mathbf{B}$ for our experimental system [4].

Current flow modifies $\mathbf{v}$ because the charge carriers acquire a drift velocity $\mathbf{v}_q$ given by $\mathbf{J} = -n_q e \mathbf{v}_q$, where n_q is the charge carrier number density and $e = 1.6 \times 10^{-19} \text{ C}$ is the elementary charge [1]. Does $\mathbf{v}_q$ significantly modify $\mathbf{v}$? At Princeton's latitude, $v = 354 \text{ m s}^{-1}$. MnZn ferrites have $n_q \sim 10^{24} - 10^{28} \text{ m}^{-3}$ [23–25], compared to n_q as high as $\sim 10^{29} \text{ m}^{-3}$ for Cu, say [1]. We calculate using the lowest MnZn ferrite value for n_q to maximize $v_q = J/n_q e$ but still find a negligible effect. For our MnZn system, $J \sim \sigma v B = 3 \times 10^{-2} \text{ A m}^{-2}$ [4], so $v_q \sim 10^{-7} \text{ m s}^{-1}$ or $v_q/v \sim 10^{-9}$. Even increasing J or decreasing n_q by orders of magnitude would not change the conclusion that $\mathbf{v}_q$ may be disregarded in our calculations.

In the case of the Faraday disk, we showed that the power absorbed by the disk according to Poynting's theorem is identical to the loss in kinetic energy due to magnetic braking of the disk's rotation, calculating each quantity by volume averaging over the disk [22]. Similarly, for our device, we calculated the power absorbed by our cylindrical shell according to Poynting's theorem (Sec. XI in Ref. [2]). We then showed that this equation for the power absorbed by the shell is identical to the kinetic energy we calculate to be lost from Earth's rotation due to $\mathbf{J} \times \mathbf{B}$ magnetic braking in the device (Sec. XII in Ref. [2]). These quantities are determined by volume averaging over the cylindrical shell, but unlike in the Faraday disk case are only nonzero because of the secondary fields $\mathbf{B}_i$.

Our analysis of our device in the laboratory frame K' began with Ohm's law $\mathbf{E}' = \mathbf{J}'/\sigma$. Applying Maxwell's equations gives Eq. (65) in Ref. [2]:

$$\partial \mathbf{B} / \partial t' = \eta \nabla^2 \mathbf{B}. \quad (19)$$

We showed in Sec. XI in Ref. [2] that this equation leads to the same results as the analogous equation in frame K . However,

Wijngaarden (his Sec. VI) argues that the flux rule [Eq. (18)] implies that our result $\text{emf} \neq 0$ leads to a physically absurd conclusion that magnetic flux Φ “should increase linearly with time indefinitely,” and that therefore emf must equal zero—similar to the claims of “paradox” arising from the Faraday disk experiment [15,17,19,20].

We first note that by the Maxwell-Faraday equation, Stokes’ theorem, and the relativistic field transformation $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$, the surface integral [Eq. (18)] is just identical to Eq. (1), the usual equation for the emf . Maxwell’s equations and the Lorentz force law give the correct answer, whatever the status of the “flux rule.” But second, we note that a constant emf does not lead to $\mathbf{B}$, or even Φ , increasing to infinity for our device. Because the power flowing into the low- R_m cylindrical shell comes from the power lost from Earth’s rotational kinetic energy due to magnetic braking, Earth despins and $\partial v / \partial t' < 0$. However, the $\mathbf{B}_i$ perturbation terms in Eq. (13) are all proportional to powers of $R_m = v\sigma\mu b$, and these are the only terms that contribute to the power that drives the current we measure. That is, Wijngaarden’s objection must ultimately fail because over an extremely long period of time, $\mathbf{B}$ decreases in magnitude as Earth despins.

VII. EXPERIMENTAL DISCUSSION

We stand by our view that the most important next step would be for independent groups to test our experiments [4] by attempting to reproduce them. In that context, it should be remembered that parameters such as μ_r , σ (and so R_m), and likely the Seebeck coefficient may vary significantly from production run to production run, even for material (such as M100 MnZn) under the same product name [2,4]. We emphasize that measuring voltages in the $\sim 10\ \mu\text{V}$ range requires care, including with potentially confounding thermoelectric (Seebeck) voltages. If there is a temperature gradient in the laboratory (as there will be), Seebeck voltages may change sign under rotation just as does the emf expected from the predicted effect.

Many manufacturers of MnZn ferrites do not produce long hollow cylinders, but only annular disks. We are skeptical that such disks can be joined together, even by pressure alone, without altering σ and therefore R_m in unpredictable ways, and also without allowing “leaking” of the external $\mathbf{B}$ field into the shell wall in an unpredictable manner, destroying the sought-after effect. Our theoretical results were derived assuming an infinite cylindrical shell. The shell must be long enough compared to its internal radius to render end effects unimportant; we give a criterion for this in Ref. [4].

We agree with Wijngaarden that measurements at substantially different latitudes should vary with $\mathbf{v}$, thereby providing an additional test of the theory. A perhaps simpler-to-execute test that (if successful) would also demonstrate the possibility of scaling to higher voltages would be to take advantage of the strong dependence of the predicted emf [Eq. (25) in Ref. [4]] on R_m and a/b , with emf increasing as $a/b \rightarrow 1$. Our MnZn ferrite shells were produced by drilling out the centers of solid cylinders [4], placing a practical limit on how close a/b could be to 1—but there are likely other ways to reach

this goal. Still, it should be possible with drilled ferrites to reach voltages of $\sim 100\ \mu\text{V}$, a voltage magnitude more robust against confounding effects.

We are skeptical that placing the shell between laboratory charged capacitor plates perpendicular to the shell’s z axis will generate a voltage. This is because of the difference between the field $\mathbf{v} \times \mathbf{B}_0$ [for which $\nabla \times (\mathbf{v} \times \mathbf{B}_0) \neq \mathbf{0}$] and an applied spatially constant electric field. In distinguishing between these two fields by generating an emf in the one case but not the other, our device would not act as an inertial frame detector (which would seem to violate special relativity) because in neither case could the experiment determine the laboratory’s absolute velocity.

VIII. CONCLUSION

Experimental results provide a way to choose between Wijngaarden’s model and ours. Here in closing, we summarize our replies to Wijngaarden’s theoretical objections.

In Sec. III of Ref. [5], it is argued that we have ignored the canceling effect of the electric field caused by the moving tube. In our Sec. III in this Reply, we show that we have not ignored this field nor have we forced $\mathbf{J} \neq \mathbf{0}$; our theory predicts the same result $\mathbf{J} = \mathbf{0}$ as that of Ref. [5] for the cases of $a = 0$ with $\mathbf{v} = \mathbf{0}$ and for the case $a \neq 0$ with $\mathbf{v} \neq \mathbf{0}$ and $R_m \gg 1$. Had the electric field been truly ignored or $\mathbf{J}$ been forced to be nonzero, this agreement would not be possible. It is only in the special case with $a \neq 0$, $\mathbf{v} \neq \mathbf{0}$, and $R_m \lesssim 1$ does our theory disagree with Ref. [5].

Wijngaarden’s result arises from the assumption that $\mathbf{B}_{\text{tube}}$ simply translates in K (advects without diffusion), which is equivalent to the assumption that $R_m \gg 1$. This results in his conclusion that $\nabla \times (\mathbf{v} \times \mathbf{B})$, $\mathbf{J}$, $\nabla \times \mathbf{J}$, and the emf are all zero. In Sec. IV of this Reply, we show that there is diffusion of $\mathbf{B}$ in our case and so these quantities are not zero.

A related objection in Ref. [5] is that the Lorentz transformation of $\mathbf{B}$ from K' to K necessarily results in $\text{emf} = 0$. We agree that if $\mathbf{B}_{\text{tube}}$ is assumed to simply translate without diffusion as in Ref. [5], the Lorentz transformation of $\mathbf{B}$ from K' to K must result in $\mathbf{J} = \mathbf{0}$ and $\text{emf} = 0$. However, we have shown in Secs. III and IV of this Reply that a small component of $\mathbf{B}_{\text{tube}}$ diffuses and does not translate with the velocity of the tube, resulting in $\text{emf} \neq 0$.

In Sec. IV of Ref. [5], it is argued that the $\text{emf} = 0$ in K' , but this again is a result of the putative translation of $\mathbf{B}_{\text{tube}}$ with velocity $\mathbf{v}$. As we show in Sec. IV of this Reply, diffusion occurs so that a component of $\mathbf{B}_{\text{tube}}$ moves with velocity $\mathbf{v}_D \neq \mathbf{v}$. Hence, $\text{emf} \neq 0$.

In Sec. V of Ref. [5], the objection is made that we have assumed *ab initio* that $\mathbf{J}$ is nonzero, which then automatically forces emf to be nonzero. We demonstrate in Secs. III and IV of this Reply that if we do not make the *ab initio* assumption as done in Ref. [5] that $\mathbf{B}$ only advects, then we calculate that $\mathbf{J} \neq \mathbf{0}$ in the special case described above. We do not force $\mathbf{J}$ to be nonzero; to the contrary, we find that under many circumstances it is equal to zero.

In Sec. VI of Ref. [5], it is argued that Cullwick’s results for a solid moving magnetic rod [13,21,26] proves that the emf must be zero for our experimental system. As stated in

Secs. IV and V of our Reply, our results agree with Cullwick's for a solid rod. However, as stated in Sec. IV, for the specific case of our device with $\mathbf{v} \neq \mathbf{0}$, $a \neq 0$, and $R_m \lesssim 1$, Cullwick's treatment must be modified to allow for the diffusion of $\mathbf{B}$. When so done, as described in Sec. V of this Reply, the Cullwick-Wijngaarden approach agrees with our result that $\text{emf} \neq 0$.

In Sec. VI of Ref. [5], it is argued that an ever-increasing (and therefore unphysical) magnetic flux Φ in the device is required for a persistent nonzero emf, analogous to the long-standing problem that arises when applying the "flux rule" [Eq. (18)] to the Faraday disk [15,16,18]. In Sec. VI of this Reply, we recalled that regardless of the applicability of this "flux rule," in Sec. XII in Ref. [2] we showed that applying Maxwell's equations and the Lorentz force law in K' give the same nonzero emf that we found in K .

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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