

Emerging structures at the linear stage of evolution of an unstable dispersive modulated wave field

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We study the spatiotemporal evolution of small disturbances developing in an unstable dispersive system. At the initial linear stage, the slow modulation of an unstable harmonic is analyzed subject to prescribed initial and boundary conditions. A theoretical model based on a modified linearized Ginzburg-Landau equation predicts the emergence of effectively localized envelope structures that can strongly affect the subsequent nonlinear stage of evolution and provides a simple criterion for the onset of absolute instability. As an illustrative example, we consider wind waves generated by an impulsive forcing in a semi-infinite domain over a viscous shear flow.

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I. INTRODUCTION

Gravity waves on the free surface of a heavy liquid provide an example of a system that is, on one hand, accessible to a range of macroscopic measurement techniques, while on the other hand is described by equations common to systems across diverse areas of physics, including optics, plasma physics, and quantum mechanics [1–5]. Irrespective of the physical setting, appropriately designed external forcing can excite waves with localized in time and/or space shapes, such as solitons, breathers, or kinks. Examples include stratified fluids [6] where significantly nonequilibrium initial spatial inhomogeneity can cause generation of solitonlike waves, and solitary wave propagation imposed by time-dependent boundary conditions in the standing wave envelopes [7,8] or in a discrete microelectromechanical system [9].

Alternatively, finite-amplitude structures can emerge through the evolution of small disturbances in intrinsically unstable dispersive systems that are ubiquitous in nature, such as shear flows [10], plasma [11], and optically nonlinear media [12]. Zakharov [11] showed that the disturbance dynamics at the initial stage is fully defined by complex-valued dispersive relation, which also uniquely determines higher-order nonlinear terms for both hydrodynamic and plasma systems.

Temporal linear stability analysis describes the time evolution of initial disturbances at a fixed wave number $k \in \mathbb{R}$ in

an unbounded spatial domain; the dispersion relation yields the frequency $\Omega(k) \in \mathbb{C}$. In contrast, spatial stability analysis considers disturbances with fixed frequency $\Omega \in \mathbb{R}$ that are time independent but vary in space propagating from some reference point. The dispersion relation defines wave numbers $k(\Omega) \in \mathbb{C}$. Gaster [13] demonstrated that linear temporal and spatial forms of the dispersion relation are related through analytic continuation via the Cauchy-Riemann conditions.

A disturbance localized in both space x and time t evolves in x and in t even when the base steady shear flow is spatially uniform; the problem becomes further complicated when the background flow evolves in space, as in boundary layers [14], mixing layers [15,16], jets [17], and liquid films [18,19]. Following Huerre and Monkewitz [20], disturbances that form wave packets that move away from the excitation site and eventually leave the flow unaltered are classified as convectively unstable; in contrast, under absolute instability, a localized disturbance grows in space and time and eventually affects the entire flow. The *a priori* distinction between convective and absolute instability is challenging as it requires the knowledge of the full dispersion relation with both $k, \Omega \in \mathbb{C}$.

Following the initial localized excitation, the slow evolution of the disturbance envelope is governed by the Ginzburg-Landau equation, which applies to a wide range of physical systems, including turbulence [21], wind waves [22], nonlinear optics [23,24], and plasma [25]. The coefficients of this amplitude equation are derived from the applicable dispersion relation and its derivatives; they determine the envelope dynamics.

The present extension of the approach developed in those studies is motivated by investigations into the initial stages of water-surface-wave excitation by turbulent wind (see Sec. II). A key feature of this system, which is semi-infinite in both time and space, is the intrinsic slow modulation of any harmonic disturbance. The aim of the present work is to model the initial linear stage of the wave-field evolution and thereby

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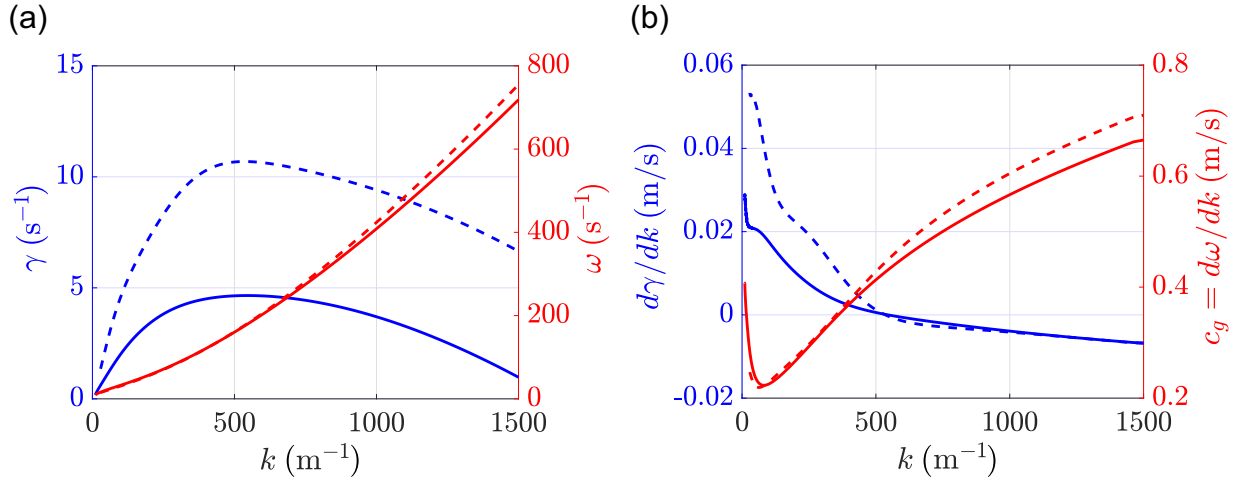


FIG. 1. OS-derived dispersion relation for wind-water waves (solid lines: $u_* = 0.35$ m/s; dashed lines: $u_* = 0.45$ m/s): (a) eigenfrequencies and growth rates and (b) their derivatives.

establish the initial conditions for its subsequent nonlinear dynamics. The governing equations are introduced in Sec. III. The resulting wave-envelope shapes are presented in Sec. IV, where the properties of the solution and the range of applicability of the model are also discussed. The main findings are summarized in Sec. V.

II. MOTIVATION

For viscous shear flows in general, the dispersion relation is a solution of the eigenvalue problem for the Orr-Sommerfeld (OS) equation for the prescribed operational conditions [26]. Excitation of waves by wind is governed by two coupled OS equations in air and water with proper boundary condition at the interface [27]. In experiments, a steady unidirectional mean wind flow is applied nearly impulsively at $t = 0$ over an initially calm water surface [28–31]. The airflow enters the test section at a fixed location $x = 0$. Measurements show that the ensuing wind-wave field develops a broad wave-number spectrum that varies in time and space. Geva and Shemer [32] further demonstrated that at the early stages of the wave-field evolution it can be represented as a linear superposition of multiple modulated, unstable wave-number harmonics, governed by the coupled OS equations with prescribed velocity profiles in air and water.

We thus consider the evolution of the unstable disturbance at $x > 0$ and $t > 0$ represented by the modulated in time and space complex envelope $C(x, t)$ of the wave packet localized in the wave-number space in the vicinity of a real wave number k_0 and real angular frequency ω_0 :

$$\eta(x, t) = \text{Re}\{C(x, t) \exp[i(k_0 x - \omega_0 t)]\}. \quad (1)$$

For the wind-wave problem, η is surface elevation, and the dispersion relation is obtained by solving the coupled OS problem for two-phase flows [27, 29]. The OS eigenfrequency $\Omega(k)$ is complex in the temporal-evolution formulation:

$$\Omega(k) = \omega(k) + i\gamma(k), \quad \omega, \gamma \in \mathbb{R}, \quad \omega_0 = \omega(k_0), \quad (2)$$

$\gamma > 0$ corresponds to instability, and only wave numbers k_0 such that $\gamma(k_0) > 0$ are considered. Assumption of weak

instability allows introduction of the small parameter ε :

$$\varepsilon = \frac{\gamma_0}{\omega_0} \ll 1, \quad \gamma_0 = \gamma(k_0). \quad (3)$$

An example of the OS-derived dispersion relation for waves excited by wind with linear-logarithmic air velocity profile [33] is shown in Fig. 1. The experimental studies [34] confirmed the adopted shape of the air velocity and establish the relation between the friction velocity u_* (the main governing parameter of the OS problem) and the wind velocity in the facility. The coupled OS equations in air and water were solved under the following assumptions: (1) the friction velocity u_* that characterizes momentum transfer between wind and waves remains constant for a given air flow rate and (2) the air shear flow induces water drift flow with velocity profile exponentially decaying with depth from its wind-shear-induced value at the surface $U_d \approx 0.5u_*$ [29, 35]. The values presented in Fig. 1 correspond to recent calculations [36] that demonstrated that for disturbances over a smooth water surface, the coupled OS solution is only weakly sensitive to the precise shapes of the air and water velocity profiles; however, it depends both quantitatively and qualitatively on the adopted value of u_* .

Note that the temporal-evolution formulation assumes spatial homogeneity, which holds neither in laboratory experiments nor in field measurements. This assumption precludes direct comparison between theoretical predictions and measurements and motivates the present study, in which the effects of the initial and boundary conditions, as well as the imposed modulation, are incorporated into the modeling of disturbance evolution in an unstable dispersive system.

III. MODEL EQUATIONS

We now assume that the complex dispersion (2) is prescribed and the weak instability condition (3) holds; the subsequent analysis is not restricted to the particular case considered in Sec. II or to any specific physical system.

Adopting the approach of Sutherland [37], a second-order Taylor expansion of the complex-valued dispersion relation

(2) about the carrier wave number k_0 yields the envelope evolution equation for $x > 0, t > 0$:

$$i \left[\frac{\partial C}{\partial t} - \gamma_0 C + \frac{d\Omega}{dk} \frac{\partial C}{\partial x} \right] + \frac{1}{2} \frac{d^2 \Omega}{dk^2} \frac{\partial^2 C}{\partial x^2} = 0. \quad (4)$$

This equation differs from linearized Schrödinger equation [37,38] or linearized Ginzburg-Landau equation [20,22,39] only by the complex group velocity $d\Omega/dk$ in Eq. (4), in contrast to the real one in the conventional Ginzburg-Landau equation applied in the vicinity on the most unstable mode.

The initial and boundary conditions correspond to the initially small disturbance distributed uniformly over all $x > 0$, with the disturbance at $x = 0$ retained at this level; the radiation condition is stated at infinity:

$$C(x, 0) = C_0, \quad C(0, t) = C_0, \quad \left. \frac{\partial C}{\partial x} \right|_{+\infty} = 0. \quad (5)$$

The problem is well posed if $\text{Im}(d^2 \Omega / dk^2) < 0$ [40].

The dimensionless amplitude Ψ and the slow time τ and coordinate ξ are now introduced:

$$C = C_0 \Psi, \quad \tau = \varepsilon \omega_0 t = \gamma_0 t, \quad \xi = \delta k_0 x, \quad (6)$$

where

$$c_g = \frac{d\omega}{dk}(k_0), \quad \delta = \frac{\varepsilon \omega_0}{c_g k_0} \ll 1. \quad (7)$$

Introducing three real dimensionless parameters,

$$v_i = \frac{1}{c_g} \frac{d\gamma}{dk}(k_0), \quad d_r = -\frac{\gamma_0}{2c_g^2} \frac{d^2 \gamma}{dk^2}(k_0), \quad d_i = \frac{\gamma_0}{2c_g^2} \frac{d^2 \omega}{dk^2}(k_0), \quad (8)$$

allows to present Eq. (4) in dimensionless form

$$\frac{\partial \Psi}{\partial \tau} + (1 + iv_i) \frac{\partial \Psi}{\partial \xi} - (d_r + id_i) \frac{\partial^2 \Psi}{\partial \xi^2} = \Psi, \quad (9)$$

subject to dimensionless initial and boundary conditions

$$\Psi(\xi, 0) = 1, \quad \Psi(0, \tau) = 1, \quad \left. \frac{\partial \Psi}{\partial \xi} \right|_{+\infty} = 0. \quad (10)$$

The well-posedness condition reads $d_r > 0$ [40].

Assuming that

$$|d_r| \ll |d_i| \ll 1, \quad |v_i| \ll 1, \quad (11)$$

which holds for the wind-wave problem [36] (see Sec. II and Fig. 1). Neglecting all terms with small coefficients in Eq. (9) reduces it to the transport equation (TE) for the slow spatiotemporal modulation of the wave envelope [38]:

$$\frac{\partial \Psi}{\partial \tau} + \frac{\partial \Psi}{\partial \xi} = \Psi, \quad (12)$$

with the solution for the scaled boundary conditions (5)

$$\Psi_0 = \begin{cases} \exp(\xi), & \xi < \tau, \\ \exp(\tau), & \xi \geq \tau. \end{cases} \quad (13)$$

The two parts of the solution are separated by a weak discontinuity at the characteristic $\xi = \tau$ corresponding to the real part

of the group velocity. The boundary conditions at $\xi = 0$ do not affect the solution to the right of this line due to the finite velocity of the information propagation. The initial conditions are irrelevant for $\xi < \tau$; thus, the steady boundary condition at $\xi = 0$ results in τ -independent solution in this region.

This solution structure guides the construction of an analytical approximate solution of Eq. (9): The envelope Ψ remains independent of ξ for $\xi \gg \tau$ and is effectively independent of τ for $\xi \ll \tau$. These asymptotics need to be smoothly connected in the vicinity of $\xi = \tau$. While the asymptotics of Eqs. (9) and (12) collapse away from the origin, the terms of Eq. (9) that were neglected in Eq. (12) modify the τ -independent asymptotics to $\exp(\lambda \xi)$, where

$$\lambda = \frac{v - \sqrt{v^2 - 4d}}{2d} \approx 1 - iv_i \quad (14)$$

is a root of the corresponding characteristic equation:

$$1 - v\lambda + d\lambda^2 = 0, \quad v = 1 + iv_i, \quad d = d_r + id_i. \quad (15)$$

Note that at the leading order, Eq. (14) is identical to the Gaster [13] relation between the spatial and temporal growth rates. The solution of Eq. (9),

$$\Psi = \frac{1}{2} \exp(\tau) \text{erfc} \left[-\frac{\xi - v\tau}{2\sqrt{d\tau}} \right] + \frac{1}{2} \exp(\lambda \xi) \text{erfc} \left[\frac{\xi - v\tau + 2d\lambda\tau}{2\sqrt{d\tau}} \right], \quad (16)$$

where erfc is the complementary complex error function, tends to the asymptotics at $\xi \rightarrow \pm\infty$. The boundary condition $\Psi(0, \tau) = 1$ is not satisfied exactly; however, the error is negligible, provided the complementary error functions in Eq. (16) reach their asymptotic limit at $\xi = 0$ [41]:

$$\left| \frac{v}{2\sqrt{d}} \sqrt{\tau} \right| \gg 1, \quad \left| \arg \left(\frac{v}{2\sqrt{d}} \right) \right| < \frac{\pi}{4}. \quad (17)$$

The complementary complex error function is computed using the MATLAB Faddeeva library [42]:

$$\text{erfc}(z) = \exp(-z^2) w(iz), \quad w(\zeta) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{\zeta - t} dt. \quad (18)$$

The implementation combines several algorithms to ensure fast, accurate evaluation of the Faddeeva function w . The problem is also solved numerically using Crank-Nicolson scheme on uniform grids in ξ and τ , with the grid spacings $\Delta \xi = 10^{-4}$ and $\Delta \tau = 10^{-3}$.

IV. RESULTS AND DISCUSSION

A. Emerging envelope structures

The typical evolution of the envelope shape $|\Psi(x)|$ is shown in Fig. 2 (see the Supplemental Material [43] for the animated version of the plots); the parameters for the calculations were taken in general agreement with Fig. 1. The numerical solution Ψ_n (not shown in Fig. 2) deviates from the analytical solution Ψ by less than 1% for the used parameter set as long as $\tau < 2$. After that, the relative discrepancy $|\Psi_n - \Psi|/|\Psi|$ grows rapidly and can approach the values of

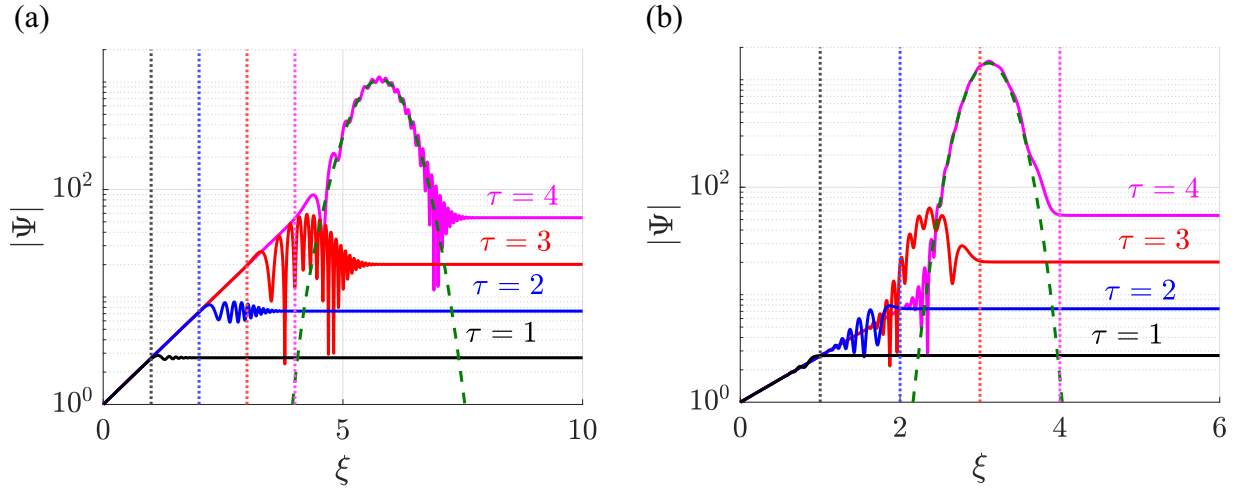


FIG. 2. Evolution of the envelope (solid lines: analytical solution (16); dashed lines: asymptotics (19) for $\tau = 4$; dotted lines: $\xi = \tau$) for different sets of parameters: (a) $v_i = 10^{-1}$, $d_i = 0.5 \times 10^{-2}$, $d_r = 10^{-3}$ and (b) $v_i = 10^{-1}$, $d_i = -0.25 \times 10^{-2}$, $d_r = 10^{-3}$.

the order of unity; nevertheless, the global envelope parameters (center position, peak height, and width) coincide within 5% for $\tau < 4$.

The piecewise exponential solution (13) is valid for ξ away from $\xi = \tau$. In the vicinity of this line, oscillations with growing in time amplitude appear. Those oscillations then evolve into a localized “bump” that grows in both amplitude and width while moving relative to the characteristic $\xi = \tau$. Its profile closely resembles the envelope solitons observed experimentally and recovered in models based on the nonlinear Schrödinger equation, e.g., sloshing-wave studies [7,44] and parametrically excited cross-wave work [8]. We therefore for brevity refer to these structures hereafter as solitonlike envelopes (SLEs). Unlike nonlinear solitons, for these essentially linear structures, the magnitude of the envelope $|\Psi|$ varies in time and does not affect its velocity.

The cases shown in Figs. 2(a) and 2(b) differ by the sign of d_i . For $d_i > 0$ in Fig. 2(a), the oscillations and the SLE lag the front $\xi = \tau$ (dotted lines) predicted by the TE solution (12); for $d_i < 0$ [Fig. 2(b)], they precede that front. In terms of wind waves, Fig. 2(a) corresponds to short gravity-capillary waves with wave number $k > k_{\min}$ ($k_{\min}$ corresponds to the minimum of the group velocity c_g), while Fig. 2(b) corresponds to longer, gravity-dominated waves with $k < k_{\min}$.

The asymptotic envelope shape (16) for $|\Psi| \gg \exp(\tau)$ is [41]

$$\Psi = \frac{2}{\sqrt{\pi}} \frac{\lambda \sqrt{(d\tau)^3}}{\xi^2} \exp\left(-\frac{\xi^2}{4d\tau}\right), \quad \xi = \xi - v\tau, \quad (19)$$

allowing estimation of its properties: the width and the location (Fig. 3), and the height (Fig. 4). The envelope magnitude attains its maximum at

$$\xi_* = \tau + \frac{v_i d_i}{d_r} \tau - \frac{4d_i}{v_i}. \quad (20)$$

The shape of $|\Psi|$ in Fig. 2 differs significantly from that given by the TE solution (13)

$$|\xi - \xi_*| \lesssim \frac{v_i d_i}{d_r} \tau. \quad (21)$$

For $\tau > 3$, the values of $|\Psi|$ may exceed Ψ_0 in Eq. (12) by orders of magnitude. Under assumptions (11), the height of the envelope $H = |\Psi(\xi_*)|$ can be estimated as

$$H = \frac{2}{\sqrt{\pi}} \frac{d_r^2}{v_i^2 \sqrt{|d_i|} \tau} \exp\left[\left(\frac{v_i^2}{4d_r} + 1\right)\tau\right]. \quad (22)$$

The proposed model suggests a clear physical explanation for the formation of the SLE and its motion. In the presence of slow spatial modulation, the wave-number spectrum has a small but finite width, peaked at the carrier wave number k_0 with weak tails on either side. Because the growth rate depends on wave number, the symmetry of these tails is broken: For $v_i > 0$, the shorter-wavelength tail grows faster, and for $v_i < 0$, the longer-wavelength tail does. Dispersion causes a difference in group velocities, $c_g(k) - c_g(k_0) \sim d_i(k - k_0)$. The resulting spectral asymmetry thus shifts relative to the characteristic $\xi = \tau$ at the rate $\sim v_i d_i$, in accordance with Eq. (20) (note that $d_r > 0$ due to the well-posedness condition).

The height H of the envelope is mostly governed by the interplay of two concurrent factors: the spectral asymmetry, controlled by v_i , and its diffusion, controlled by d_r . The growth of H in time is independent of the SLE position relative to $\xi = \tau$: The curves in Fig. 4 that correspond to parameter sets (Table I) that differ only by the sign of d_i collapse once the envelope shape is established. The rate of growth $dH/d\tau$ significantly exceeds unity in agreement with measurements [31] where sharp increase in the wind-wave amplitude was reported to occur in the vicinity of $t = x/c_g$.

TABLE I. Parameters in Fig. 4.

Color	$10^1 v_i$	$10^2 d_i$	$10^3 d_r$
Blue	1	0.5	1
Red	1	-0.5	1
Magenta	-0.5	0.5	0.5
Green	-0.5	-0.5	0.5

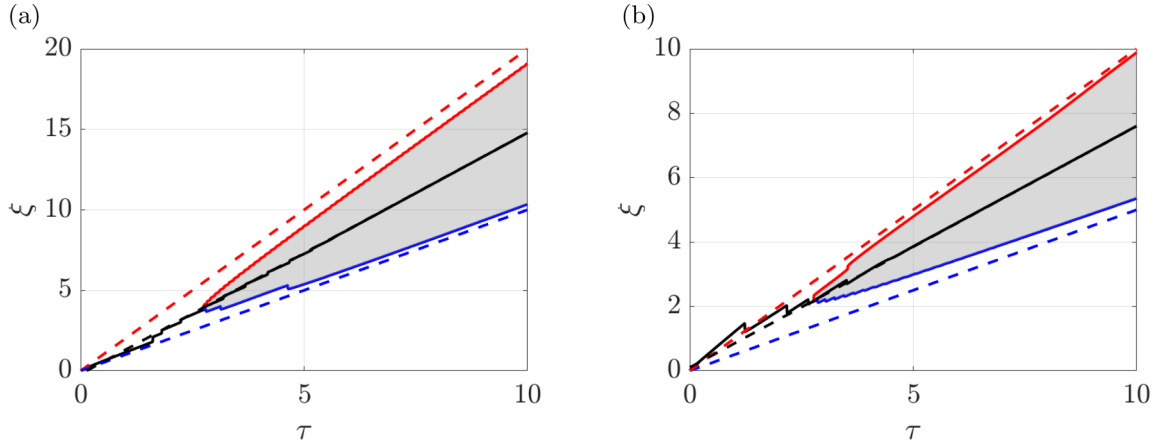


FIG. 3. Envelope motion for different sets of parameters: (a) $v_i = 10^{-1}$, $d_i = 0.5 \times 10^{-2}$, $d_r = 10^{-3}$ and (b) $v_i = 10^{-1}$, $d_i = -0.25 \times 10^{-2}$, $d_r = 10^{-3}$. Shaded area: existence domain $|\Psi|/|\Psi_0| > 1.1$; black lines: location of the maximum ξ_* ; red and blue lines: right and left boundaries; solid lines: analytical solution (16); dashed lines: asymptotics (20) and (21).

B. Limitations on the model's applicability

Under assumptions (11), the validity condition for the analytic solution given by the second equation in Eq. (17) can be rewritten as

$$\frac{v_i d_i}{d_r} > -\frac{1}{2}. \quad (23)$$

A typical numerical solution for the case when this condition is violated is shown in Fig. 5. In the vicinity of $\xi = 0$, the envelope exhibits an abrupt rise from the boundary condition $\Psi(0, \tau) = 1$ to the asymptotic solution (16) at $\xi \sim 1$ and for larger ξ closely follows the analytical solution. Such a pattern can be interpreted as an indication of the absolute instability of the flow [19].

The criterion of the convective/absolute instability [20] can be applied for the dimensionless simplified form of the dispersion relation (2) valid to the second

order:

$$\tilde{\Omega}(\tilde{k}) = 1 + i\varepsilon + \frac{\varepsilon}{\delta}v(\tilde{k} - 1) - i\frac{\varepsilon}{\delta^2}d(\tilde{k} - 1)^2, \quad (24)$$

$$\tilde{\Omega} = \frac{\Omega}{\omega_0}, \quad \tilde{k} = \frac{k}{k_0},$$

which is analytically continued for the complex $\tilde{k}$. To establish the type of instability, the complex wave number $\tilde{k}_*$ corresponding to zero group velocity is calculated:

$$\tilde{k}_* - 1 = \frac{\delta(1 + iv_i)}{2d}. \quad (25)$$

The general condition for the convective instability, $\text{Im } \tilde{\Omega}(\tilde{k}_*) < 0$, for the dispersion relation (24) reads

$$\frac{d_r + 2v_i d_i - d_r v_i^2}{4(d_r^2 + d_i^2)} > 1. \quad (26)$$

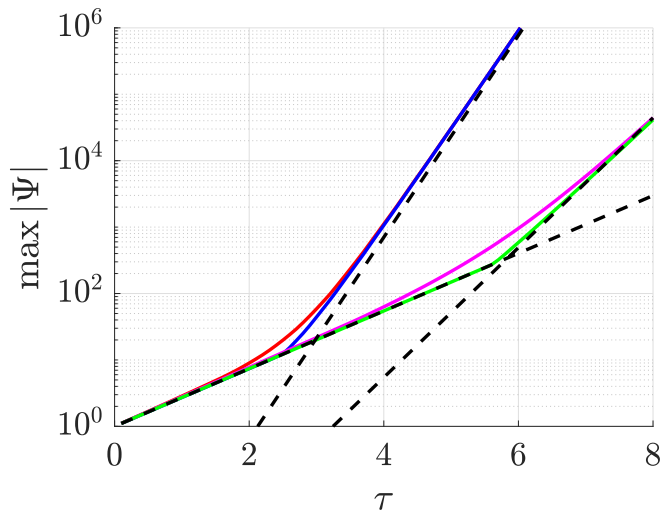


FIG. 4. Maximum envelope height: Solid lines correspond to analytical solution (16), and dashed lines correspond to asymptotics (13) and (22); colors are defined in Table I.

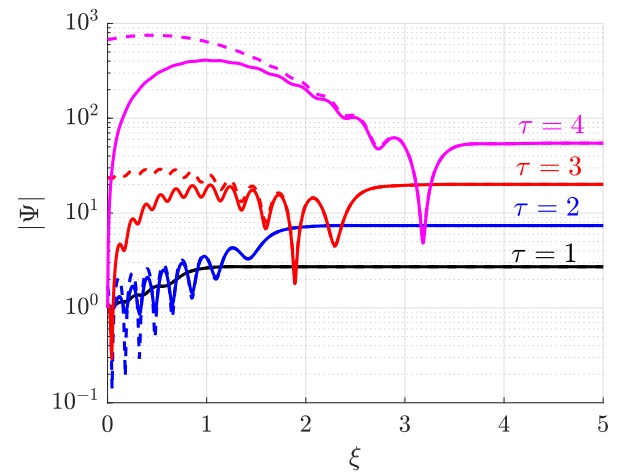


FIG. 5. Evolution of the envelope beyond the domain of analytical solution validity (23): $v_i = 10^{-1}$, $d_i = -10^{-2}$, $d_r = 10^{-3}$; solid and dashed lines correspond to numerical and analytical solutions (16), respectively. For $\tau = 1$, solid and dashed lines collapse.

The principal terms in the criterion (26) correspond to the analytical solution validity condition (23), due to the assumption (11), implying that the SLE formation occurs in the convectively unstable systems. This criterion can be reduced to

$$\left| \frac{\delta}{2d_i} \right| \ll 1, \quad (27)$$

in view of condition (25).

The adopted assumption of the slow modulation implies that Ψ does not change significantly at the length scale of the carrier wave δ . Within SLE, the real and imaginary parts of Ψ vary much faster than $|\Psi|$ and the characteristic wave number corresponding to this variation can be estimated using the asymptotic expression (19) as

$$\frac{\partial}{\partial \xi} \operatorname{Im} \frac{(\xi - v\tau)^2}{4d\tau} \Big|_{\xi=\xi_*} = \frac{1}{2} \frac{v_i}{d_r}. \quad (28)$$

Thus, the applicability criterion for the proposed model reads

$$\left| \frac{\delta v_i}{2d_r} \right| \ll 1, \quad (29)$$

ensuring long and slow modulations.

V. CONCLUSIONS

The presented model predicts the formation of localized structures within narrow-banded wave packets that are modulated in both time and space during the linear stage of evolution. Their emergence is governed by the interplay between instability and dispersion, as described by the modified linearized Ginzburg-Landau equation (9). The cubic dissipation term appearing in the full equation [22] is omitted, making the resulting equation resemble the linear Schrödinger equation [37]. The principal difference between the two equations is the presence of a source term on the right-hand side of Eq. (9). It should be noted that the cubic terms in the

Ginzburg-Landau and cubic Schrödinger equations arise from different physical mechanisms [38].

The resulting structures resemble solitary waves, which are commonly associated with solutions of nonlinear equations subject to specific initial and boundary conditions [45,46] and arise from a balance between nonlinearity and dispersion. In contrast, the present model describes the linear stage of an initially infinitesimal disturbance, which grows through instability and develops distinctive patterns as a result of modulation and dispersion. A localized structure emerges after a finite evolution time and, once formed, grows faster than the corresponding nonmodulated unstable wave, as observed in wind-wave experiments [30]. The resulting envelope can subsequently serve as an initial condition for the analysis of the nonlinear stage of evolution.

The principal physical effects incorporated into the model are dispersion, linear instability, and the nontrivial dependence of the growth rate (γ) and angular frequency (ω) on the wave number (k). These effects are reflected in the complex-valued coefficients of Eq. (9), given by Eq. (8). Their specific values can be obtained from a linear stability analysis, such as the solution of the Orr-Sommerfeld spectral problem for viscous shear flows. The concurrence between the evolution of the spectral shape, governed by the wave number dependence of γ , and the corresponding variation of the group velocity $c_g(k)$ causes the velocity of the emerging structures to deviate from the carrier-wave group velocity $c_g(k_0)$. Accounting for a complex-valued group velocity extends the applicability of the model to harmonics away from the most unstable mode.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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