

# Nonlinear dynamics of one-dimensional bright solitonlike state in polariton condensates induced by a PT-symmetric periodic potential

Ming Chen<sup>1,\*</sup>, Muhammad Idrees<sup>2,\*</sup>, Kun Zhang<sup>1</sup>, Zheng Sun<sup>3</sup>, Hui-jun Li<sup>1,†</sup> and Alexey Kavokin<sup>4,5,6</sup>

<sup>1</sup>*Institute of Nonlinear Physics and Department of Physics, Zhejiang Normal University, Jinhua 321004, Zhejiang, China*

<sup>2</sup>*Department of Physics, Jiangsu University, Zhenjiang 212013, China*

<sup>3</sup>*State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200241, China*

<sup>4</sup>*Key Laboratory for Quantum Materials of Zhejiang Province, School of Science, Westlake University, Hangzhou 310024, Zhejiang, China*

<sup>5</sup>*Department of Physics, Saint Petersburg State University, University Embankment, 7/9, Saint Petersburg 199034, Russia*

<sup>6</sup>*Abrikosov Center for Theoretical Physics, Moscow Institute of Physics and Technology, Dolgoprudny 141701, Moscow Region, Russia*



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We investigate the dynamics of one-dimensional bright solitonlike state in exciton-polariton condensates under a parity-time (PT) symmetric periodic potential. For the static potential case, we demonstrate the formation of stable gap solitonlike state arising from the combined action of the potential and a composite Gaussian pump. Notably, these gap solitonlike state differ from those in the conventional nonlinear Schrödinger equation in two key ways. First, their existence and stability depend not only on the balance between nonlinearity and dispersion but also critically on the system's gain-loss balance. Second, their profiles (e.g., main-peak amplitude) can be directly controlled by the intensity of the composite Gaussian pump. Furthermore, when the PT-symmetric potential is designed to include a barrier with exponentially varying intensity, we observe that a stable incident bright solitonlike state exhibits controllable switching dynamics, in stark contrast to the unidirectional transmission seen in conservative systems. During this process, the bright solitonlike state transforms into a polariton wave, temporarily disappears, and subsequently re-emerges. This behavior is governed by the interaction between the PT-symmetric barrier and the intrinsic gain-loss mechanism of the system. Additionally, we propose an active control scheme based on nonresonant optical Gaussian pump pulses to precisely manipulate the disappearance time of the bright solitonlike state. Our findings establish a promising platform for soliton-based photonic devices in nonequilibrium systems.

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## I. INTRODUCTION

Non-Hermitian Hamiltonians are widely used to model open quantum systems that interact with their environment, often resulting in complex eigenvalues due to gain and loss mechanisms. However, Bender *et al.* demonstrated that a non-Hermitian Hamiltonian possessing parity-time (PT) symmetry can retain a completely real eigenvalue spectrum below a certain symmetry-breaking threshold [1,2]. Since then, PT symmetry has become a central topic in non-Hermitian research and has been successfully realized in various physical systems, including optics [3–27], inductor–resistor–capacitor (LRC) circuits [28,29], metamaterials [30,31], cold atomic

systems [32–36], and topological phase transitions [37,38]. Particularly in linear optical systems, PT symmetry has led to a range of physical phenomena, including double refraction, power oscillations [4], nonreciprocal propagation [5], optical transparency [6], unidirectional transmission [7], and perfect absorption [25]. In nonlinear optical systems, PT-symmetric potentials have been found to support stable soliton propagation [8]. Moreover, under the combined effects of PT-symmetric potentials and strong nonlinearity, various soliton structures have been discovered, including fundamental solitons [10–12], dipole solitons [10,12], tripole solitons [10], gap solitons [13,14,23], defect solitons [15,16], and vortices [17,18]. Additionally, beam propagation in PT-symmetric optical lattices can exhibit reflection and diffraction phenomena [21–23], and longitudinally modulated PT-symmetric potential barriers can induce transverse drift of solitons [24]. Recently, asymmetric scattering of soliton molecules has also been demonstrated in a Rydberg atomic system [35]. These findings not only deepen the understanding of light propagation in non-Hermitian systems but also enhance the controllability of soliton dynamics.

\*These authors contributed equally to this work.

†Contact author: [hjli@zjnu.cn](mailto:hjli@zjnu.cn)

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In recent years, a distinct class of quasibosonic particle, exciton-polariton, has been observed to form Bose-Einstein condensates (BECs) at temperatures ranging from a few kelvin up to room temperature in semiconductor microcavities. This discovery has attracted significant interest as a promising alternative to ultracold atomic BECs [39–46]. Exciton-polaritons arise from the strong coupling between excitons and photons, and the system exhibits distinctive features such as a short lifetime, strong nonlinear effects (self-interaction and cross-interaction), and the coexistence of gain and loss, making it a unique platform for exploring quantum and nonequilibrium physics. To sustain stable condensates, an incoherent pump is used to compensate for these losses. However, nonlinear states obtained using only a uniform pump, such as dark solitons, spinor vortices, and spiral waves [47–49], are generally unstable because a uniform pump cannot adequately balance gain and loss locally. To achieve nonlinear steady states, researchers have proposed various approaches. The first involves designing spatially structured pump profiles, including periodic pumps [50], symmetric pumps [51], ring-shaped pumps [52,53], and Gaussian-shaped pumps [54–58]. These configurations have successfully stabilized diverse nonlinear states, such as parity solitons [51], bright solitons and vortices [54], gap solitons [55], spatial patterns [56], petal states [57], and dark soliton chains [58]. The second approach utilizes composite pump schemes—combining a homogeneous pump with a Gaussian pump [59] or a Bessel pump [60], which have been demonstrated to support stable brightlike solitons, darklike solitons, dark vortices, and dark soliton cloning [61]. The third and more recently reported method employs a combination of spatial external potentials with either homogeneous or composite pumps to achieve nonlinear steady states. For instance, one-dimensional (1D) stable multipole dark solitons [62] and various two-dimensional non-PT-symmetric steady states [63] have been realized.

Several studies have proposed the construction of PT-symmetric exciton-polariton systems [63–69]. However, due to the inherently strong nonlinear effects and the coexistence of non-Hermitian gain and loss in this system, it remains extremely challenging to construct a strictly PT-symmetric polariton condensate model. In previous work, we achieved transitions from non-PT-symmetric Gaussian steady states to near-PT-symmetric Gaussian steady states using a composite Gaussian pump [63]. Moreover, the interaction between an artificially constructed PT-symmetric potential and the intrinsic gain and loss of the polariton system is itself a topic worthy of study. Although this constitutes a non-PT-symmetric model, it can still reveal interesting physical phenomena. For example, a natural question arises: In optics, optical solitons exhibit scattering and diffraction during propagation in PT-symmetric optical lattices [21–24]. How would the dynamics of solitons evolve in a PT-symmetric periodic lattice within a nonequilibrium system such as an exciton-polariton condensate?

To address this question, in this work we observe that one-dimensional bright solitonlike state in exciton-polariton condensates exhibits switching evolution dynamics under a PT-symmetric periodic lattice potential with intensity varying exponentially over time. This behavior is reminiscent of that seen in optical couplers [3,7,26,27]. Furthermore, we

numerically demonstrate that this dynamic behavior arises from the combined effects of the PT-symmetric periodic lattice potential and the nonequilibrium characteristics (gain and loss) of the system. Importantly, we propose an experimentally feasible and convenient scheme using injected Gaussian pump pulses to control the switching dynamics (i.e., the disappearance time) of such bright solitonlike state.

The structure of this paper is outlined with Sec. II introducing the model that describes the dynamics of the polariton field. Within the mean-field formalism, the polariton field dynamics are governed by a generalized Gross-Pitaevskii (GP) equation that incorporates both gain and dissipation. Given the inherently nonequilibrium nature of the system, we also employ a rate equation to account for the noncondensed thermal reservoir. Next, in Sec. III, under weak nonlinearity, we calculate the band structures of the one-dimensional potential. Under strong nonlinearity, stable gap solitonlike state are obtained through the combined action of the PT-symmetric potential and a composite Gaussian pump. Then, in Sec. IV, with a fixed composite pump intensity, we investigate the case in which the strength of the PT-symmetric potential varies exponentially in time. Unlike the unidirectional transmission behavior observed in conventional nonlinear Schrödinger systems [24], one-dimensional bright solitonlike state exhibits switching dynamics: They first transform into extended states (oscillating polariton waves), whose amplitude decays to zero, leading to a temporary disappearance of the solitonlike state before its recovery. This phenomenon occurs only under the joint influence of the spatially periodic, temporally localized PT-symmetric potential and the gain-loss mechanism. The disappearance duration of the bright solitonlike state is strongly influenced by the imaginary strength of the potential, the exponential variation rate, and the type of incident solitonlike state. Furthermore, in Sec. V, we introduce a more practical control scheme using high-intensity nonresonant optical Gaussian pump pulses, allowing for real-time experimental tuning. After the solitonlike state vanishes, such a pulse can rapidly restore the gain-loss balance. Specifically, for a fixed injection time within a certain range, higher pulse intensity and longer duration both shorten the disappearance time. Additionally, we demonstrate that the disappearance time can be controlled by adjusting the pulse injection time. Numerical simulations further reveal a one-to-one correspondence between the pulse duration and the threshold intensity required for saturation; beyond this threshold, the control effect becomes insensitive to further increases in pulse duration. Finally, in Sec. VI, we conclude the paper by summarizing the key findings and discussing the implications of our work for the development of soliton-based active photonic devices in nonequilibrium systems.

## II. MODEL

Within the mean-field formalism, the dynamics of the polariton field can be described by a generalized GP equation that incorporates both gain and dissipation. Owing to the inherently nonequilibrium nature of the system, a rate equation is also required to account for the noncondensed thermal reservoir [46,54]. Consequently, the one-dimensional polariton Bose-Einstein condensates in a 1D liquid crystal

microcavity stripe are governed by the following coupled equations, which describe the temporal evolution of the polariton field  $\Psi$  and the reservoir density  $n_R$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m^*} \frac{\partial \Psi}{\partial x^2} + V_{\text{ext}} \Psi + g_c |\Psi|^2 \Psi + g_R n_R \Psi + i\frac{\hbar}{2} (R n_R - \gamma_c) \Psi, \quad (1)$$

$$\frac{\partial n_R}{\partial t} = P_u(x) - (\gamma_R + R |\Psi|^2) n_R, \quad (2)$$

where  $m^*$  denotes the effective mass of the lower polaritons, with a magnitude approximately  $10^{-4}$  times the free electron mass.  $V_{\text{ext}}$  is the external potential experienced by the polariton condensates,  $g_c$  represents the self-interaction of polaritons,  $g_R$  is the cross-interaction between polaritons and reservoir density,  $\gamma_c$  and  $\gamma_R$  are the polariton and reservoir loss rates, respectively,  $R$  denotes the condensation rate,  $n_R$  is the density of the reservoir, and  $P_u(x)$  is the incoherent composite pump of the system, given by  $P_u(x) = P_0 + P_1 e^{-\frac{x^2}{w^2}}$ . The former one is homogeneous pump and the latter one is nonhomogeneous Gaussian pump. After introducing the characteristic time  $\tau_0$ , the width of the quantum well in the  $x$  direction  $R_x$ , the density of the polariton BEC  $\psi_0^0$ , and the density of the reservoir  $n_R^0$ , Eqs. (1) and (2) can be rewritten as the dimensionless coupled equations

$$i \frac{\partial u}{\partial s} = -\frac{\partial^2 u}{\partial \xi^2} - V u - \sigma_1 |u|^2 u - \sigma_2 n u + i(\sigma_3 n - \sigma_4) u, \quad (3)$$

$$\frac{\partial n}{\partial s} = P(\xi) - \sigma_5 (1 + \sigma_6 |u|^2) n, \quad (4)$$

where  $s = t/\tau_0$ ,  $\xi = x/R_x$ ,  $u = \Psi/\Psi_0$ ,  $V = -V_{\text{ext}}\tau_0/\hbar$ ,  $n = n_R/n_R^0$ ,  $w_0 = w/R_x$ ,  $P(\xi) = P_u(x)\tau_0/n_R^0 = \sigma_7 + \sigma_8 e^{-\frac{\xi^2}{w_0^2}}$ , and  $\tau_0 = 2m^*R_x^2/\hbar$ . The coefficients in Eqs. (1) and (2) are given by  $\sigma_1 = -g_c\Psi_0^2\tau_0/\hbar$ ,  $\sigma_2 = -g_R n_R^0\tau_0/\hbar$ ,  $\sigma_3 = R n_R^0\tau_0/2$ ,  $\sigma_4 = \gamma_c\tau_0/2$ ,  $\sigma_5 = \gamma_R\tau_0$ ,  $\sigma_6 = R\Psi_0^2/\gamma_R$ ,  $\sigma_7 = P_0\tau_0/n_R^0$ , and  $\sigma_8 = P_1\tau_0/n_R^0$ . Here,  $\sigma_1$  represents the self-interaction of the condensate,  $\sigma_2$  is the cross-interaction between the condensate and the reservoir,  $\sigma_3$  is the nonlinear gain, and  $\sigma_4$  is the linear loss.  $\sigma_5$  and  $\sigma_6$  correspond to the reservoir loss rate and the condensation rate, respectively, while  $\sigma_7$  and  $\sigma_8$  are the intensities of the homogeneous and Gaussian pumps.  $w_0$  denotes the width of the Gaussian pump. According to Ref. [59], the parameters adopted in Secs. IV and V are as follows:  $\sigma_1 = -1$ ,  $\sigma_2 = 0.3$ ,  $\sigma_3 = 0.20$ ,  $\sigma_4 = 0.3423$ ,  $\sigma_5 = 1$ , and  $\sigma_6 = 4$ . The experimentally relevant parameters used in this study correspond to  $g_c = 3.29 \times 10^{-3}$  meV  $\mu\text{m}^2$ ,  $g_R = -4.93 \times 10^{-3}$  meV  $\mu\text{m}^2$ ,  $\gamma_c = 0.3423$  ps $^{-1}$ ,  $\gamma_R = 0.5$  ps $^{-1}$ , and  $R = 0.01$   $\mu\text{m}^2$  ps $^{-1}$ . These values are of the same order of magnitude as those reported in Refs. [50,70], based on gallium arsenide (GaAs) microcavity samples. In Sec. III, to obtain stable one-dimensional gapsolitonlike states of different branches, parameter  $\sigma_1$  is slightly modified.

Generally, external potentials satisfying PT symmetry can be classified into two categories. The first category includes spatially periodic potentials, such as optical lattice potentials [5] and Moiré lattice potentials [71]. The second category consists of spatially localized potentials, such as the Scarff-II potential [20,72] and Gaussian potentials [10,63,73]. The

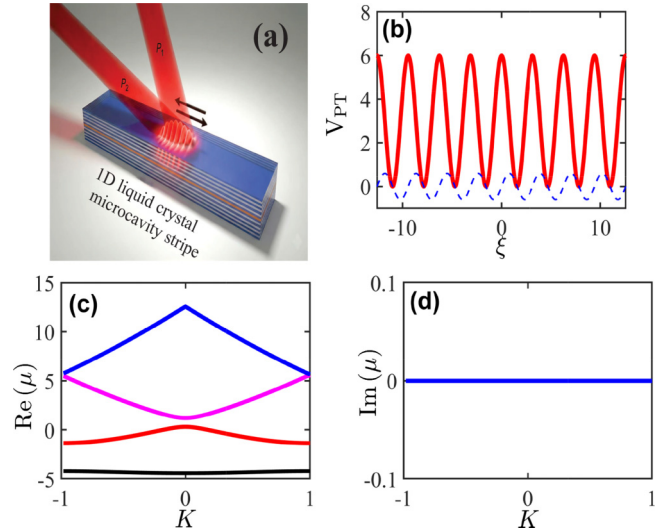


FIG. 1. (a) Schematic of the experimental setup for realizing a PT-symmetric periodic lattice potential with an exponentially time-varying barrier [Eq. (10)]. Two nonresonant laser beams are incident on the same side of the liquid crystal microcavity stripe at specific oblique angles and form interference fringes. (b) Real (red solid line) and imaginary (blue dashed line) component of the PT-symmetric periodic lattice potential with real part strength of PT-symmetric potential  $\sigma_9 = 6$  (this parameter values remain unchanged in figures below), intensity ratio of imaginary/real parts of PT-symmetric potential  $W = 0.1$  and spatial modulation frequency of the PT-symmetric potential  $k = 1$  [these two parameter value remains unchanged in Figs. 2(c) and 2(d)]. (c), (d) Real and imaginary parts of the first four energy band structures corresponding to Eq. (12) (here,  $K$  is the Bloch wave number). Other parameters: cross-interaction coefficient  $\sigma_2 = 0.3$  (this parameter value also applies to all figures below), nonlinear gain coefficient  $\sigma_3 = 0.2$ , and linear loss coefficient  $\sigma_4 = 0.3423$  (these two parameter values remain unchanged in all figures except Fig. 4).

PT-symmetric periodic lattice potential adopted in this work is  $V_{\text{ext}} = V_R \cos^2(k_x x) + iV_I \sin(2k_x x)$ , and its corresponding dimensionless form is

$$V_{\text{PT}} = \sigma_9 [\cos^2(k\xi) + iW \sin(2k\xi)], \quad (5)$$

where  $\sigma_9$  is the real-part strength of the PT-symmetric complex potential,  $W$  represents the intensity ratio of the imaginary to real parts of the PT-symmetric complex potential, and  $k$  is the spatial modulation frequency.  $\sigma_9 = -V_R\tau_0/\hbar$ ,  $W = V_I/V_R$ , and  $k = k_x/k_0$ , where  $k_0$  is the characteristic frequency used for normalization. The real and imaginary components of the PT-symmetric potential in one dimension are illustrated in Fig. 1(b), where the real part is represented by a red solid line and the imaginary part by a blue dashed line. In optics, there have been extensive reports on the study of PT-symmetric optical lattice potentials [4,5,8,9,12–14,21–24], yet such research remains scarce in polariton systems. The real part of this potential can be experimentally implemented using the interference of optical waves [74] and counter-propagating acoustic waves [68,75], or by employing a metallic grating on a microcavity [76]. The imaginary part, corresponding to the spatially varying loss distribution, can be achieved by depositing a dielectric mask with a

sinusoidally patterned profile onto the sample surface [77] or, alternatively, by structuring the gain through the induction of a periodic exciton-density distribution [78]. Here, we focus on systems based on liquid crystal microcavities [79,80], where optical control of the gain and the exciton-induced repulsive potential can be complemented by exploiting the effective stationary spin-orbit Hamiltonian imposed by the birefringence of the liquid crystal layer. The inherent tunability of liquid crystal microcavities, together with the non-Hermitian potential terms imposed by two-beam optical pumping, provides the versatility required to optimize the complex potential [Eq. (5)]. In Sec. III, when searching for stable gap solitonlike state,  $V = V_{PT}$ , meaning the external potential does not change over time. To obtain solitonlike solutions of stable states, we assume that the system satisfies the adiabatic approximation condition, i.e.,  $\frac{\partial n}{\partial s} = 0$ , meaning that the density of the excitonic reservoir does not change with time. We substitute  $u = \psi(\xi)e^{-i\mu s}$  and  $n(\xi, \eta, s) = n'(\xi)$  into Eqs. (3) and (4), and obtain the steady-state equation

$$\mu\psi + \frac{\partial^2 \psi}{\partial \xi^2} + V_{PT}\psi + \sigma_1|\psi|^2\psi + \sigma_2 n'\psi - i(\sigma_3 n' - \sigma_4)\psi = 0, \quad (6)$$

where  $n' = \frac{P(\bar{r})}{1+\sigma_6|\psi|^2}$ , and after selecting the appropriate initial solution, one-dimensional solitonlike state solutions can be obtained in this paper through the Squared-Operator Iteration Method [81]. The relative norm  $N_R = \int_{-\infty}^{\infty} (|\psi|^2 - |\psi_0|^2)d\xi$  (where  $|\psi|$  is the profile of the solitonlike state and  $|\psi_0|$  is the background field) of the solitonlike state solutions is obtained in the following section. After obtaining the solitonlike state solution, we introduce two perturbation modes  $v_1(\xi)$  and  $v_2(\xi)$  on the original solitonlike state solution  $\psi(\xi)$  to assess its stability as

$$u = [\psi(\xi) + \varepsilon v_1(\xi)e^{\lambda s} + \varepsilon v_2^*(\xi)e^{\lambda^* s}]e^{-i\mu s}, \quad (7)$$

where  $\varepsilon \ll 1$ . Considering that the density of the excitonic reservoir  $n'$  also changes with time in real situations, a perturbation mode is introduced on  $n'$  corresponding to the original solitonlike state solution  $\psi(\xi)$ ,

$$n = n' + \varepsilon(v_3(\xi)e^{\lambda s} + v_3^*(\xi)e^{\lambda^* s}), \quad (8)$$

by substituting Eqs. (7) and (8) back into Eqs. (3) and (4), respectively, and extracting the coefficients before  $e^{\lambda s}$  and  $e^{\lambda^* s}$ , we obtain the linear equations for  $v_1(\xi)$ ,  $v_2(\xi)$ , and  $v_3(\xi)$ ,

$$i \begin{pmatrix} -\hat{L}_1 & -\hat{L}_2 & -\hat{L}_3 \\ \hat{L}_2^* & \hat{L}_1^* & \hat{L}_3^* \\ -\hat{L}_4 & \hat{L}_4^* & -\hat{L}_5 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad (9)$$

where  $\hat{L}_1 = -\mu - V_{PT} - \frac{\partial^2}{\partial \xi^2} - 2\sigma_1|\psi|^2 - \sigma_2 n' + i(\sigma_3 n' - \sigma_4)$ ,  $\hat{L}_2 = -\sigma_1\psi^2$ ,  $\hat{L}_3 = i\sigma_3\psi - \sigma_2\psi$ ,  $\hat{L}_4 = -i\sigma_5\sigma_6 n'\psi^*$ , and  $\hat{L}_5 = -i\sigma_5(1 + \sigma_6|\psi|^2)$ . The eigenvalue  $\lambda$  of the perturbation eigenmodes can be solved numerically using the Fourier collocation method. The stable solitonlike state solution  $\psi$  is characterized by all the real parts of the eigenvalues being negative or zero. Finally, we add 1% random noise to the solitonlike state obtained and verify its stability by evolving it through the split-step Fourier method. During this process,

some of the solitonlike states' dynamical behaviors can be observed as it evolves.

According to Ref. [24], in the conventional nonlinear Schrödinger equation (NLSE) model, bright soliton exhibits refractive dynamics when propagating through a PT-symmetric periodic lattice potential with an exponentially time-varying barrier. The expression for this behavior is given as follows:

$$V(\xi, s) = \begin{cases} V_{PT}e^{\delta(s-200)}, & s < 200, \\ V_{PT}e^{-\delta(s-200)}, & 200 < s < 400, \end{cases} \quad (10)$$

where  $\delta$  is the barrier length in the temporal dimension, determining the exponential variation rate of the potential strength of the PT-symmetric periodic lattice along the propagation direction (time). We suggest that by combining a stationary potential imposed on a liquid crystal microcavity through the application of an external bias [79,80] and an optically induced complex potential generated by two interfering laser pulses [74], it should be possible to achieve the required temporal and spatial dependences in both the real and imaginary parts of the potential [Eq. (10)]. The real and imaginary parts of the periodic potential [Eq. (5)] can be modulated by two interfering nonresonant laser beams incident on the microcavity sample at different oblique angles, as in Ref. [74]. A chopper applied to the two interfering lasers would ensure the abrupt temporal variation of the potential described by Eq. (10). It is important to note that a third laser would be required to replenish polaritons in the system. The key to this experimental scheme is the use of two excitation lasers incident at different oblique angles. Their interference creates a periodic spatial dependence of the real part of the potential, caused by the blueshift of the exciton-polariton mode. Moreover, the same interference pattern generates a periodically modulated gain distribution in the system. By carefully choosing the oblique incidence angles, one can shift the periodic gain distribution with respect to the periodic blueshift-induced potential, which is essential for the realization of potential (5); see also the schematic in Fig. 1(a). Importantly, the propagation of incoherent excitons in the direction imposed by the in-plane wave vectors of the incident laser beams results in a shift of the imaginary part of the effective potential acting upon polaritons with respect to its real part. Other elements of the suggested setup allow for its tunability and temporal control. The stationary (quasi-stationary) potential imposed by the liquid crystal helps adjust the shapes of the real and imaginary parts of the potential (5). The frequency detuning of the two interfering lasers enables a smooth temporal variation of the potential, which facilitates reaching the PT-symmetric regime. The real and imaginary components of the PT-symmetric periodic lattice potential with an exponentially time-varying barrier are, respectively, illustrated in Figs. 5(e) and 5(f), with the real-part intensity  $\sigma_9 = 6$ , the intensity ratio of the imaginary to real part  $W = 1$ , and barrier length  $\delta = 0.1$ . This potential possesses a distinct characteristic: an exponential time-dependent variation in the strength of the external potential. However, it remains an open question whether such refractive dynamics persist or evolve into more complex phenomena in nonequilibrium systems like exciton-polariton condensates, which are inherently governed

by gain and loss. It is worth noting that, in describing the time-varying PT-symmetric potential barrier in Eq. (10), we adopt the same simplifying assumptions commonly used in conventional photonics: Namely, that the modulation of the potential is instantaneous in time [24], and that the amplitudes of its real and imaginary parts maintain a fixed proportional relationship, characterized by the parameter  $W = \sigma_{10}/\sigma_9$  [4,8,12]. This approach allows us to focus on the influence of linear PT symmetry on the nonlinear dynamics. However, unlike conventional optical systems, our model explicitly incorporates the feedback effect of the exciton reservoir through Eq. (4) and accounts for the nonequilibrium nature of the system via the gain-loss term  $i(\sigma_3 n - \sigma_4)\psi$ . It is precisely due to the dynamic feedback of the reservoir and the intrinsic gain-loss mechanism that solitonlike states in the exciton-polariton system can exhibit dynamical behaviors, even when the modulation protocol of the PT-symmetric potential barrier is similar to that employed in traditional optical settings.

Therefore, in Secs. IV and V, we adopt Eq. (10) as the configuration of the external potential and investigate the switching dynamics of bright solitonlike state induced by the combined effects of this PT-symmetric periodic lattice potential with an exponentially time-varying barrier and the gain-loss mechanism. All dimensionless physical quantities and parameters introduced in this section are summarized in Table I of Appendix A, which provides their corresponding physical meanings and definitions for clarity. In addition, since the model presented in this paper involves a large number of physical quantities, whose values vary across different figures, we provide a summary of these parameters in two tables in Appendix C. Table II lists the fixed parameters and their values in each figure, whereas Table III lists the variable parameters, their specific values in each figure, and the physical motivations behind their variations.

### III. STABLE GAP SOLITONLIKE STATE EXCITED BY A COMPOSITE GAUSSIAN PUMP AND THE PT-SYMMETRIC PERIODIC LATTICE POTENTIAL

Initially, to lay the groundwork for obtaining stable gap solitonlike state, we first calculate the underlying band structure. For this linear analysis, we consider the weak nonlinear case near the condensation threshold under a homogeneous pump configuration. When the homogeneous pump intensity reaches its critical value  $\sigma_7 = \sigma_{th} = \sigma_4/\sigma_3$ , the polariton field  $\psi$  remains weak. Under this weak-field condition, the nonlinear self-interaction term  $\sigma_1|\psi|^2\psi$  can be neglected. Furthermore, the reservoir density  $n'$  is predominantly governed by the pump and its decay, with negligible feedback from the condensate. It can therefore be approximated as a fixed background density:  $n' = n_0 \approx P(x) = \sigma_7 = \sigma_4/\sigma_3$ . In this limit, Eq. (7) reduces to a linear Schrödinger-like equation with the PT-symmetric periodic lattice potential

$$\mu\psi + \frac{\partial^2\psi}{\partial\xi^2} + V_{PT}\psi + \sigma_2 n_0\psi = 0, \quad (11)$$

which can be used to calculate the band structure of the system. According to Bloch's theorem, let  $\psi = \varphi e^{iK\xi}$ , where  $K$  is the Bloch wave number and  $\varphi$  is a periodic function with period  $\frac{\pi}{k}$ , satisfying  $\varphi(\xi + \frac{\pi}{k}) = \varphi(\xi)$ . Substituting this

expression into Eq. (11) yields the following Bloch eigenvalue equation:

$$\mu\varphi = -\frac{\partial^2\varphi}{\partial\xi^2} - 2iK\frac{\partial\varphi}{\partial\xi} + (K^2 - V_{PT} - \sigma_2 n')\varphi. \quad (12)$$

Figure 1(b) illustrates the even-symmetric real part (red solid line) and odd-symmetric imaginary part (blue dotted line) of the PT-symmetric periodic lattice potential with  $\sigma_9 = 6$ ,  $W = 0.1$ , and  $k = 1$  in this paper [Eq. (5)]. Figure 1(c) shows the real parts of the eigenvalue  $\mu$  corresponding to the first four energy band structures in the first Brillouin zone when PT symmetry is not broken, obtained by solving Eq. (12) under the same parameters. In fact, the resulting band structures are similar to those reported in Refs. [4,5,8,12], since Eq. (11) differs from the standard Schrödinger equation only by an additional term  $\sigma_2 n_0\psi$  (which can be regarded as a constant external potential). This term only shifts the energy bands vertically, without altering their shapes or affecting the PT symmetry-breaking point (phase transition point). The phase transition of this PT-symmetric periodic lattice potential occurs at  $W = 0.5$ , as has been consistently reported in previous studies [4,5,8,12]. Below the phase transition point, all energy bands exhibit purely real characteristics. Figure 1(d) shows the imaginary part of the eigenvalue  $\mu$  corresponding to the four energy band structures, which coincide with each other and are all zero. After the breaking of PT symmetry, not only do complex eigenvalues emerge, but the energy bands also become degenerate and the band gaps close. Figure 2(a) shows the curve of the stable gap solitonlike state relative norm  $N_R$  versus the chemical potential  $\mu$ . The red and blue curves correspond to self-interaction coefficients  $\sigma_1 = -1$  and  $-3$ , respectively. A key distinction from conservative systems is that these two branches do not converge at the edge of the left band, which arises from the direct control of the solitonlike states' amplitude and relative norm by the Gaussian pump, as demonstrated below. The gray regions on the left and right sides indicate the value ranges of the ground-state energy band (black) and the first excited-state energy band (red) in Fig. 1(c), respectively, while the white area corresponds to the bandgap region. Compared with the multipeak gap solitonlike state in the conventional NLSE [13], the formation of gap solitonlike state in polariton condensates is influenced not only by self-interaction (Kerr nonlinearity  $\sigma_1|u|^2u$ ) but also by cross-interaction (saturated nonlinearity  $\sigma_2 nu$ ) and the non-Hermitian characteristics of the system [gain-loss term  $i(\sigma_3 n - \sigma_4)u$ ]. Specifically, as a stable dissipative state, the gain-loss intensity of the system must approach zero [63]. In fact, using only a homogeneous pump and the PT-symmetric periodic lattice potential cannot locally balance gain and loss to achieve a stable localized state. Therefore, the introduction of a composite Gaussian pump is essential to obtain stable gap solitonlike state. The profiles of gap solitonlike state obtained by solving Eq. (6) are shown in Figs. 2(b1) and 2(c1) with a self-interaction coefficient  $\sigma_1 = -1$  and Gaussian pump intensities  $\sigma_8 = 5.2$  and 10, respectively. Figure 2(d1) shows the corresponding profile with  $\sigma_1 = -3$  and  $\sigma_8 = 10$ . The insets on the right side of Figs. 2(b1)–2(d1) illustrate the real part (solid red line) and the imaginary part (dashed blue line) of the gap solitonlike state. By comparing the solitonlike state

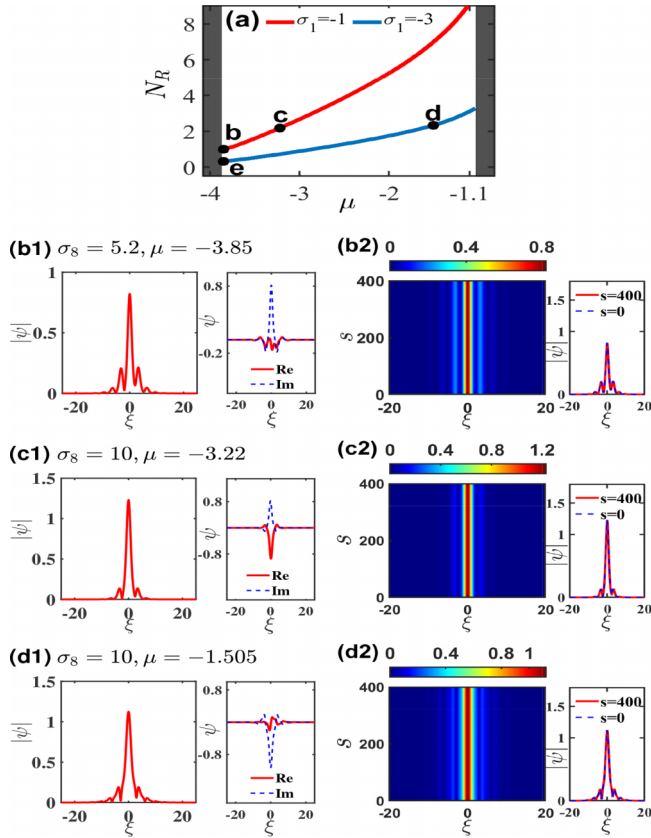


FIG. 2. Gap solitonlike state excited by the PT-symmetric periodic lattice potential and the composite Gaussian pump in one-dimensional case. (a) Relative norm  $N_R$  curves of the gap solitonlike state as a function of chemical potential  $\mu$ . The red and blue curves correspond to the cases of self-interaction coefficients  $\sigma_1 = -1$  and  $\sigma_1 = -3$ , respectively. The gray regions on the left and right sides represent the energy bands of the ground state (black) and the first excited state (red) shown in Fig. 1(c). (b1), (c1) Profiles of the gap solitonlike states with Gaussian pump intensity  $\sigma_8 = 5.2$ ,  $\mu = -3.85$ ,  $\sigma_8 = 10$ ,  $\mu = -3.22$ , and  $\sigma_1 = -1$ . (d1) Profile of the gap solitonlike state with  $\sigma_8 = 10$ ,  $\mu = -1.505$ , and  $\sigma_1 = -3$ , which are marked by the black dots in panel (a). The right insets show the real parts (red solid line) and imaginary parts (blue dashed line) of the gap solitonlike states. (b2)–(d2) Evolution results of these solitonlike states. In the right insets, the blue dotted lines and the red solid lines, respectively, represent the profiles of solitonlike state when  $s = 0$  and  $s = 400$ . Other parameters: reservoir loss rate  $\sigma_5 = 1$  and homogeneous pump intensity  $\sigma_7 = 1$  (these two parameter values remain unchanged in all figures below), condensation rate  $\sigma_6 = 4$  (this parameter values remain unchanged in all figures except Fig. 4), and width of Gaussian pump  $w_0 = 1$ .

profiles in Figs. 2(b1) and 2(c1), where the self-interaction coefficient is identical but the Gaussian pump intensity differs, it can be observed that the amplitude of the main peak of the gap solitonlike state is determined by the Gaussian pump intensity: A stronger pump intensity results in a larger main-peak amplitude. In contrast, a comparison between Figs. 2(c1) and 2(d1), where the Gaussian pump intensity is the same but  $\sigma_1$  differs, reveals that they possess main peaks of identical amplitude and similar relative norm  $N_R$ . These results demonstrate a notable distinction from gap solitonlike state in the

conventional NLSE [13]: Relative norm  $N_R$  of the solitonlike state is primarily determined by the Gaussian pump intensity. This mechanism explains why the  $N_R$ – $\mu$  curves at the edge of the left band in Fig. 2(a) fail to converge, as exemplified by point b with  $\sigma_8 = 5.2$  lying above point e with  $\sigma_8 = 2.39$ . Figures 2(b2)–2(d2) show the corresponding evolution results. The insets on the right display the profiles at  $s = 0$  (blue dashed line) and  $s = 400$  (red solid line), respectively. These gap solitonlike states remain stable, exhibiting no significant changes in their profiles before and after evolution. Overall, in this section, we discuss one-dimensional stable gap solitonlike state excited by a PT-symmetric lattice potential and a composite Gaussian pump. The profiles of these gap solitonlike state differ significantly from those in the NLSE [13]. Their stability arises not only from the balance between nonlinear and dispersive effects but also from the equilibrium between the gain and loss mechanisms in the system.

#### IV. SWITCHING DYNAMICS OF BRIGHT SOLITONLIKE STATE INDUCED BY THE PT-SYMMETRIC PERIODIC LATTICE POTENTIAL WITH AN EXPONENTIALLY TIME-VARYING BARRIER

In this section, we investigate solitonlike state dynamics in polariton condensates, focusing on behaviors that may differ from the scattering and diffraction dynamics typically observed in the NLSE [21–24]. Specifically, we consider a scenario where the strength of the PT-symmetric lattice potential varies exponentially over time—first increasing and then decaying—following the configuration described in Ref. [24]. This corresponds to a PT-symmetric periodic lattice potential with an exponentially time-varying barrier. To begin, we generate a stable bright solitonlike state with zero background using a composite Gaussian pump, without considering any external potential, as outlined in Ref. [59]. The relevant parameters for this setup are as follows:  $\sigma_1 = -1$ ,  $\sigma_2 = 0.3$ ,  $\sigma_3 = 0.20$ ,  $\sigma_4 = 0.3423$ ,  $\sigma_5 = 1$ ,  $\sigma_6 = 4$ ,  $\sigma_7 = 1$ ,  $\sigma_8 = 2.33$ , and  $w_0 = 5$ . We then use this stable bright solitonlike state as the initial condition for evolution, set the external potential in Eq. (3) to the PT-symmetric periodic lattice potential with an exponentially time-varying barrier [Eq. (10)], and observe the dynamic behavior of the bright solitonlike state within this potential barrier.

Figure 3(a) shows the regions where switching dynamics can or cannot occur for the bright solitonlike state under the real-part intensity ( $\sigma_9 = 6$ ) and barrier length ( $\delta = 0.1$ ) of the PT-symmetric periodic lattice potential with an exponentially time-varying barrier. The red regions represent areas where switching dynamics can occur, while the blue regions represent areas where they cannot. As shown in the figure, switching dynamics can only occur in regions resembling a fragmented island chain near the intensity ratio of the imaginary to real parts,  $W \approx 1$ , of the PT-symmetric periodic lattice potential. This corresponds to the PT-symmetry-breaking region in the linear model. In contrast, such dynamics do not occur in regions of the traditional lattice potential ( $W = 0$ ). The evolution results in Figs. 3(b)–3(d) correspond to the points marked with red labels (b)–(d) in Fig. 3(a). Figures 3(b) and 3(c) demonstrate the evolution of stable bright solitonlike state in a traditional lattice potential with an exponentially

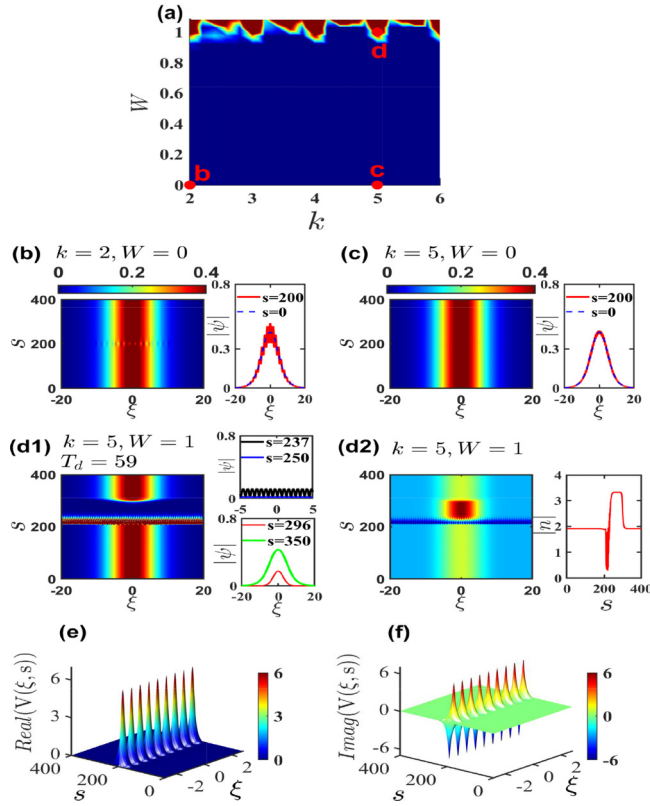


FIG. 3. (a) Region diagram of bright solitonlike state switching dynamics existence conditions as a function of the spatial modulation frequency  $k$  and the ratio of the imaginary part intensity to the real-part intensity  $W$  for the PT-symmetric periodic lattice with an exponentially time-varying barrier. The red area represents the region where it can occur, and the blue area represents the region where it cannot occur. The red points marked with labels (b)–(d) correspond to the evolution results with the same serial numbers in the following figures. (b), (c) The evolution results of stable bright solitonlike state in conventional periodic lattice potentials with exponentially time-varying barriers ( $W = 0$ ) at modulation frequencies  $k = 2$  and  $k = 5$ , respectively. The blue dashed line and red solid line in the right inset represent the profiles when  $s = 0$  and  $s = 200$ . (d1), (d2) The evolution results of the stable bright solitonlike state and reservoir density in the PT-symmetric periodic lattice potential with an exponentially time-varying barrier (here,  $k = 5$ ,  $W = 1$ ). In the two insets on the right side of panel (d1), the black, blue, red, and green lines represent the profile when  $s = 237$ , 250, 296, and 350, respectively. The inset on the right side of panel (d2) shows the cross-sectional view of the absolute value of the reservoir density evolving over time. (e), (f) The real and imaginary parts of the PT-symmetric periodic lattice potential with an exponentially time-varying barrier ( $W = 1$ ). Other parameters: self-interaction coefficients  $\sigma_1 = -1$  (this parameter value remains unchanged in all figures below), Gaussian pump intensity  $\sigma_8 = 2.33$  (this parameter value remains unchanged in Figs. 3–5), width of Gaussian pump  $w_0 = 5$ , and the temporal length of the PT-symmetric potential barrier  $\delta = 0.1$  (this parameter value remains unchanged in Figs. 3, 4, and 6(c)–6(f)).

time-varying barrier under modulation frequencies of  $k = 2$  and  $k = 5$ , respectively. The blue dashed and red solid lines in the right insets represent the profiles at  $s = 0$  and  $s = 200$ , respectively. It can be observed that during transmission through

the potential barrier, the bright solitonlike state temporarily transforms into a lattice solitonlike state before recovering its original state, without exhibiting switching dynamics. Figure 3(d1) presents the evolution of the bright solitonlike state in this PT-symmetric lattice potential with an exponentially time-varying barrier, under parameters  $\sigma_9 = 6$ ,  $W = 1$ ,  $k = 2$ , and  $\delta = 0.1$ . The black, blue, red, and green lines in the two right insets correspond to profiles at  $s = 237$ ,  $s = 250$ ,  $s = 296$ , and  $s = 350$ , respectively. Figure 3(d2) shows the corresponding evolution of the reservoir density  $n$  over time, with the inset on the right displaying the cross-sectional view of its absolute value evolving with time. Combining the results from Figs. 3(d1) and 3(d2), an intriguing switching dynamic of the bright solitonlike state can be observed. In Fig. 3(d1), after the stable bright solitonlike state is injected, it becomes unstable when approaching the PT-symmetric barrier. Initially, it transforms into a high-amplitude polarized wave spreading across the entire space, resembling the wave function in the linear model. This high-amplitude polarized wave then oscillates and decays to zero, as indicated by the blue line ( $s = 250$ ). Subsequently, no solitonlike state solution emerges for a period, despite the continuous action of the composite Gaussian pump (with  $\sigma_8 = 2.33$  remaining constant throughout). Eventually, the bright solitonlike state regenerates at a certain time point and fully recovers its original state, as shown by the green line at  $s = 350$ .

To quantitatively record the disappearance time of the bright solitonlike state, we establish a criterion, when the maximum amplitude of  $|\psi|$  is less than or equal to 0.2 (black line), the solitonlike state is considered to have disappeared. This time point is recorded as the disappearance time  $S_1$ . When the maximum amplitude of  $|\psi|$  becomes greater than or equal to 0.2 again (red line), the solitonlike state is considered to have reappeared, and this time point is recorded as  $S_2$ . Thus, the disappearance time is given by  $T_d = S_2 - S_1$ . Therefore, the disappearance time of the solitonlike state in Fig. 3(d1) is calculated as  $T_d = 296(S_2) - 237(S_1) = 59$  (118 ps). The evolution of the reservoir density  $n$  in Fig. 3(d2) further corroborates this dynamic behavior. As shown in the right inset, when the stable bright solitonlike state initially transforms into a high-amplitude polarized wave, the amplitude of the reservoir density decreases rapidly (indicating the release of high-energy excitons). When the high-amplitude polarized wave decays to zero, the reservoir density amplitude increases sharply (reflecting condensate rupture and exciton storage). By comparing Figs. 3(d1) and 3(d2), it can be observed that after the solitonlike state vanishes, it effectively returns to the reservoir density. This switching dynamic between the solitonlike state and the reservoir density resembles the behavior observed in PT-symmetric couplers. Figures 3(e) and 3(f), respectively, display the real and imaginary parts of the PT-symmetric periodic lattice potential with an exponentially time-varying barrier, under the parameters  $\sigma_9 = 6$ ,  $W = 1$ , and  $\delta = 0.1$ , corresponding to the conditions where bright solitonlike state switching dynamics occur. When the intensity of the PT-symmetric periodic potential varies temporally as a Gaussian function, the bright solitonlike state incident upon this potential barrier also exhibits switching dynamics. The relevant results are presented in Appendix B.

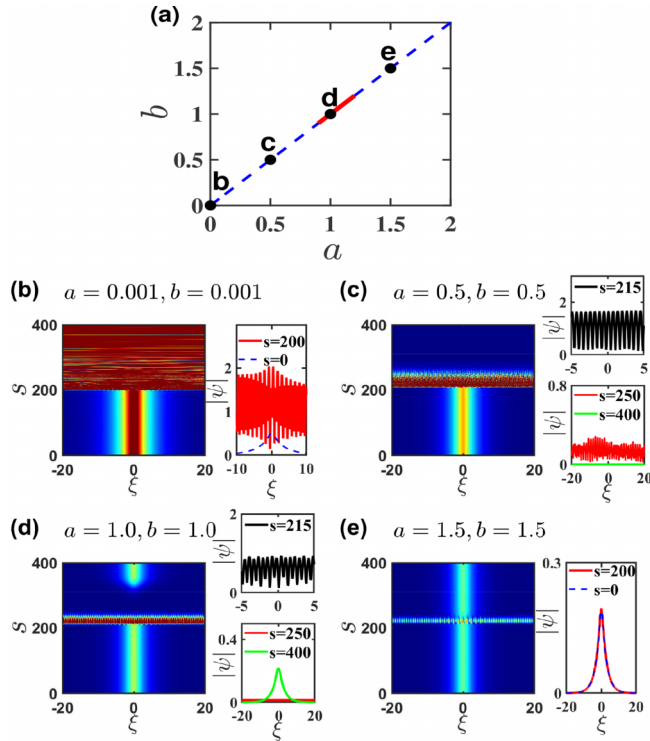


FIG. 4. (a) Region diagram for the occurrence/nonoccurrence of bright solitonlike state switching dynamics vs the magnitude of the scaling factors  $a$  and  $b$  of the nonlinear gain coefficient  $\sigma_3$  and the linear loss coefficient  $\sigma_4$ , under the condition of  $a = b$ . The blue dashed lines represent regions where the switching dynamics cannot occur, while the red solid line indicates regions where it can occur. (b)–(e) The evolution results of bright solitonlike state under the scaling factors  $a = 0.001, b = 0.001$ ;  $a = 0.5, b = 0.5$ ;  $a = 1.0, b = 1.0$ ;  $a = 1.5, b = 1.5$ , which correspond to the black points marked with labels in panel (a). In the insets on the right of panels (b) and (e), the blue dashed line and the red solid line represent the profiles when  $s = 0$  and  $s = 200$ , respectively, while in the insets on the right of panels (c) and (d), the black, red, and green lines represent the profiles when  $s = 215$ ,  $s = 250$ , and  $s = 400$ , respectively. Other parameters: nonlinear gain coefficient  $\sigma_3 = 0.20a$ , linear loss coefficient  $\sigma_4 = 0.3423b$ , condensation rate  $\sigma_6 = 4a$ , width of Gaussian pump  $w_0 = 1$ , intensity ratio of imaginary/real parts of PT-symmetric potential  $W = 1$  (this parameter value remains unchanged in all figures below except Fig. 5), and spatial modulation frequency of the PT-symmetric potential  $k = 5$  (this parameter value remains unchanged in all figures below).

Figure 3 shows that the bright solitonlike state switching dynamics are closely linked to the PT-symmetric periodic lattice (a behavior that does not occur in traditional lattices). This raises the question of whether the disappearance and reappearance of bright solitonlike state are related to the inherent nonequilibrium characteristics of the exciton-polariton system. To investigate this, we scale the nonlinear gain coefficient  $\sigma_3$  and the linear loss coefficient  $\sigma_4$  by factors  $a$  and  $b$ , respectively. This corresponds experimentally to tuning the condensation rate  $R$  and the polariton loss rate  $\gamma_c$ . Since  $\sigma_6 = R\Psi_0^2/\gamma_R$ , the modified coefficients in Eqs. (3) and (4) become  $\sigma_3 = 0.20a$ ,  $\sigma_4 = 0.3423b$ , and  $\sigma_6 = 4a$ . Figure 4(a) displays the parameter-space diagram indicating whether bright

solitonlike state switching dynamics can occur under the condition  $a = b$  (as cases with  $a \neq b$  rarely support stable bright solitonlike state), plotted with respect to the scaling factors  $a$  and  $b$ . The blue dashed curves denote regions where switching dynamics do not occur, while the red solid curve marks the region where switching dynamics are present. The black points labeled (b)–(e) correspond to the evolution results shown in the subsequent panels. When  $a = 0.001$  and  $b = 0.001$ , the system approaches a nearly conservative regime. As shown in Fig. 4(b), the bright solitonlike state evolves into a chaotic state after passing through the PT-symmetric potential barrier. For  $a = 0.5$  and  $b = 0.5$ , Fig. 4(c) demonstrates that the bright solitonlike state converts into a polariton wave, disappears after crossing the PT-symmetric barrier, and does not reappear. In contrast, when  $a = 1.5$  and  $b = 1.5$ , Fig. 4(e) shows that although the bright solitonlike state briefly transforms into a polariton wave, it successfully traverses the barrier without undergoing disappearance. Only in the parameter region near  $a = 1$  and  $b = 1$  does the bright solitonlike state exhibit clear switching dynamics—disappearing and subsequently reappearing—during its interaction with the PT-symmetric potential barrier. These results demonstrate that the switching dynamics of bright solitonlike state depend not only on the PT-symmetric periodic lattice but also critically on the nonequilibrium nature of the exciton-polariton system (i.e., the gain and loss terms). Neither excessively strong nor excessively weak gain/loss effects are capable of inducing such switching behavior.

Next, we demonstrate that the switching dynamics of these bright solitonlike state are controllable, as the disappearance time  $T_d$  is affected by various system parameters. The control

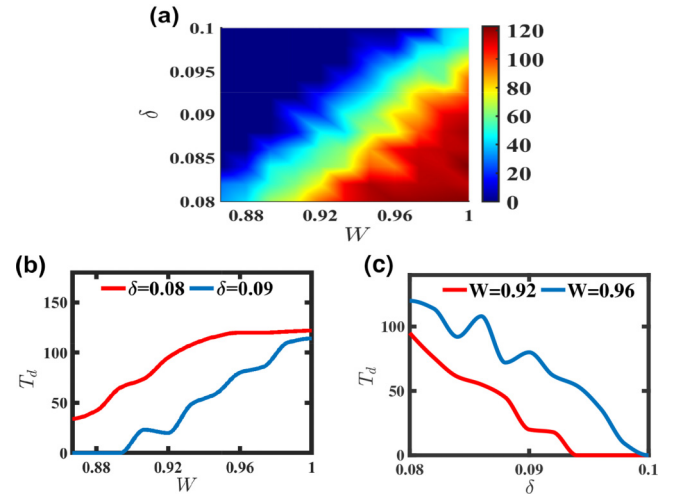


FIG. 5. (a) Contour plot of the disappearance time of bright solitonlike state vs the ratio of the imaginary part intensity to the real-part intensity  $W$  and the temporal length of the PT-symmetric barrier  $\delta$  when switching dynamics occur. The blue regions represent areas with a short disappearance time, while the red regions represent areas with a long disappearance time. (b) The curves of disappearance time vs  $W$  with  $\delta = 0.08$  (red) and  $\delta = 0.09$  (blue). (c) The curves of disappearance time vs  $\delta$  with  $W = 0.92$  (red) and  $W = 0.96$  (blue). Other parameters: width of Gaussian pump  $w_0 = 5$  (this parameter value remains unchanged in all figures below).

strategies can be broadly divided into two categories. The first is passive control, which involves preparing or adjusting parameters related to the experimental materials or to the intrinsic properties of the bright solitonlike state, such as the number of peaks or the amplitude. Figure 5(a) shows a contour plot of the bright solitonlike state disappearance time  $T_d$  as a function of  $W$  (the ratio of the imaginary to real-part intensities) and  $\delta$  (the barrier length) under the PT-symmetric barrier. Blue regions correspond to shorter disappearance times, while red regions indicate longer disappearance times. A clear pattern is observed: As  $W$  increases and  $\delta$  decreases, the disappearance time  $T_d$  becomes longer. Figure 5(b) shows the dependence of  $T_d$  on  $W$ , where the red and blue curves correspond to  $\delta = 0.08$  and  $\delta = 0.09$ , respectively [i.e., horizontal cross-sections of Fig. 5(a) at these  $\delta$  values]. For both curves,  $T_d$  increases with  $W$ ; additionally, for a fixed  $W$ , a smaller  $\delta$  leads to a larger disappearance time. Figure 5(c) displays the dependence of  $T_d$  on  $\delta$ , with the red and blue curves corresponding to  $W = 0.92$  and  $W = 0.96$ , respectively [i.e., vertical cross-sections of Fig. 5(a) at these  $W$  values]. In both cases,  $T_d$  decreases as  $\delta$  increases; moreover, for a fixed  $\delta$ , a larger  $W$  results in a longer disappearance time. In fact, the disappearance time of bright solitonlike state depends not

only on the parameters of the experimental setup—such as the intensity ratio  $W$  of the imaginary to real part of the PT-symmetric barrier, which is determined by the thickness of the sinusoidally patterned dielectric mask—but also on the characteristics of the incident bright solitonlike state. On one hand, the disappearance time varies with the incident amplitude of single-peak bright solitonlike state, which is controlled by the strength of the composite Gaussian pump  $\sigma_8$ . Figure 6(a) shows the profiles of stable single-peak bright solitonlike state prepared with  $\sigma_8 = 2.1$  (black line),  $\sigma_8 = 3.5$  (blue line), and  $\sigma_8 = 5.0$  (red line), under a fixed Gaussian pump width of  $w_0 = 5$ . The amplitude magnitudes increase with  $\sigma_8$ . Figure 6(b) shows the disappearance time  $T_d$  of these single-peak bright solitonlike state as a function of  $\sigma_8$  when incident on the PT-symmetric barrier. The red and blue curves correspond to barrier lengths  $\delta = 0.08$  and  $\delta = 0.10$ , respectively. As shown,  $T_d$  gradually decreases with increasing  $\sigma_8$ . On the other hand, the disappearance time of multipeak bright solitonlike state with the same amplitude depends on both the number of peaks and the spacing between them. Stable multipeak bright solitonlike state can be generated using a composite Gaussian pump described by the following expression:

$$P = \begin{cases} \sigma_7 + \sigma_8 e^{-\frac{\xi^2}{w_0^2}} + \sigma_8 \sum_{l=1}^{\lfloor \frac{N}{2} \rfloor} \left[ e^{-\frac{(\xi - \xi_{2l+1})^2}{w_0^2}} + e^{-\frac{(\xi + \xi_{2l+1})^2}{w_0^2}} \right], & N = 1, 3, \dots, \\ \sigma_7 + \sigma_8 \sum_{l=1}^{\lfloor \frac{N}{2} \rfloor} \left[ e^{-\frac{(\xi - \xi_{2l})^2}{w_0^2}} + e^{-\frac{(\xi + \xi_{2l})^2}{w_0^2}} \right], & N = 2, 4, \dots, \end{cases} \quad (13)$$

where  $N$  denotes the total number of peaks and  $d$  represents the distance between adjacent peaks. The peak positions  $\xi_m$  are symmetrically distributed around  $\xi = 0$  and are given by

$$\xi_m = \pm \frac{m-1}{2}d. \quad (14)$$

After obtaining stable multipeak bright solitonlike state, they can similarly be injected into the PT-symmetric potential barrier. Figure 6(c) shows the variation of the disappearance time of multipeak bright solitonlike state with the total number of peaks  $N$ , under a fixed Gaussian pump intensity  $\sigma_8 = 2.33$  and  $w_0 = 5$ . The green, blue, and red curves correspond to peak distances  $d = 10, 20$ , and  $30$ , respectively. Figures 6(d)–6(f) present the evolution results for a fixed peak distance  $d = 30$  as the number of peaks increases from 2 to 4, corresponding to the red points marked in Fig. 6(c). These results demonstrate that the switching dynamics of bright solitonlike state are indeed controllable. The disappearance time depends on both the experimental parameters ( $\sigma_3$ ,  $\sigma_4$ ,  $W$ , and  $\delta$ ) and intrinsic solitonlike state properties (amplitude, number of peaks, and peak spacing). However, because these parameters and solitonlike state types must be prepared or adjusted prior to the experiment, this control method is passive and inconvenient. Since our observations reveal an intrinsic connection between the switching dynamics and the gain-loss effects in the polariton system, we propose a more efficient and convenient active control method in the following section.

Specifically, during the experiment, after observing the bright solitonlike state incident on the PT-symmetric potential barrier and its subsequent disappearance, a high-intensity nonresonant optical Gaussian pump pulse can be injected to rapidly rebalance the system's gain and loss, thereby inducing an earlier recovery of the bright solitonlike state. This provides a practical scheme for actively regulating the switching dynamics in nonequilibrium systems.

## V. SWITCHING DYNAMICS OF BRIGHT SOLITONLIKE STATE CONTROLLED BY A NONRESONANT OPTICAL GAUSSIAN PUMP PULSE

The passive control strategies discussed in Sec. IV, which rely on pre-experiment adjustments of material parameters and solitonlike state properties, lack flexibility for real-time manipulation. To address this limitation, we implement an active control scheme using injected Gaussian pump pulses to dynamically regulate the disappearance time of bright solitonlike state. Since the switching dynamics of bright solitonlike state are inherently linked to the gain-loss effects in the polariton system, this section focuses on the experimentally feasible implementation of this active control method. Specifically, we design nonresonant optical Gaussian pump pulses and perform numerical simulations to investigate (1) the relationship between the disappearance time of the bright solitonlike state and the pulse parameters (intensity  $\sigma_8'$  and duration

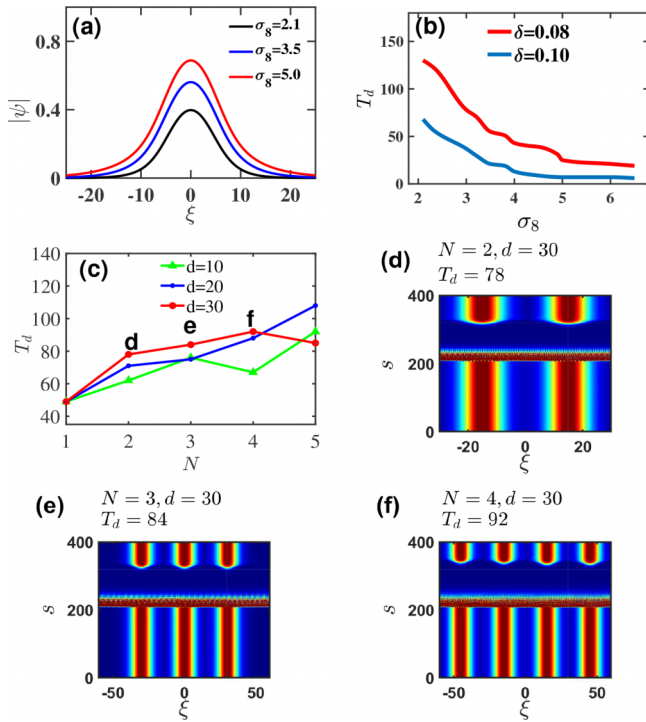


FIG. 6. (a) Stable single-peak bright solitonlike state profiles excited by composite Gaussian pumps at  $\sigma_8 = 2.1$  (black),  $3.5$  (blue), and  $5.0$  (red). (b) Curves of the disappearance time of the single-peak bright solitonlike state as a function of the Gaussian pump strength  $\sigma_8$  for the temporal length of the PT-symmetric potential barrier  $\delta = 0.08$  (red) and  $0.10$  (blue). (c) Curves of the disappearance time of the bright solitonlike state as a function of the number of peaks  $N$  of multipeak bright soliton under the fixed conditions of Gaussian pump strength  $\sigma_8 = 2.33$  and PT-symmetric barrier length  $\delta = 0.10$ , where green, blue, and red correspond to the cases with bright solitonlike state peak distances  $d = 10, 20$ , and  $30$ , respectively. (d)–(f) Evolution results for the number of peaks  $N = 2, 3$ , and  $4$ , respectively, under the fixed peak distance  $d = 30$ .

$\Delta s$ ) for a fixed pulse injection time, and (2) the relationship between the disappearance time and the injection time point  $s_{\text{int}}$  under varying pulse intensities and durations. Moreover, the numerical results reveal a saturation effect in this active control scheme: Beyond a certain intensity threshold, which shows a one-to-one correspondence with the pulse duration, the disappearance time becomes independent of the pulse duration for any fixed injection time.

Figure 7(a) shows the temporal profile of the Gaussian pump intensity. Prior to the injection time point of the Gaussian pump pulse,  $s_{\text{int}} = 240$  (indicated by the red arrow), the Gaussian pump intensity  $\sigma_8$  is maintained at  $2.33$  to ensure the stability of the bright solitonlike state. The rectangular wave form appearing at  $s_{\text{int}}$  represents the injected nonresonant optical Gaussian pump pulse, with  $\sigma_8'$  denotes the pulse intensity and  $\Delta s$  indicates its duration. The blue dashed line marks  $s = 244$ , corresponding to the moment when the maximum amplitude of  $|\psi| \leq 0.2$  after the bright solitonlike state has traversed the PT-symmetric barrier and transformed into a polariton wave in the absence of the injected Gaussian pump pulse. This instant corresponds to the black line in the right

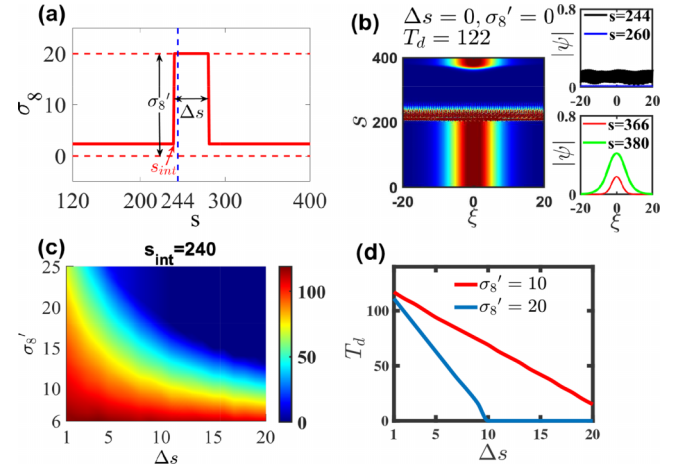


FIG. 7. (a) Schematic of the nonresonant optical Gaussian pump pulse injected after the bright solitonlike state vanishes, where  $\sigma_8'$  represents the intensity of the Gaussian pump pulse and  $\Delta s$  denotes the duration of the pulse action. The red arrow indicates the injection time  $s_{\text{int}}$  of the Gaussian pump pulse. The blue dashed line marks the time  $s = 244$ , which corresponds to the moment when the bright solitonlike state disappears after transforming into a polariton wave in the absence of the injected pulse. (b) Evolution results of the bright solitonlike state incident on the PT-symmetric barrier without pulse injection. (c) Contour plot of the disappearance time of the bright solitonlike state as a function of  $\sigma_8'$  and  $\Delta s$  when the pulse is injected at a fixed time  $s_{\text{int}} = 240$ . Blue regions indicate shorter disappearance time, while red regions represent longer disappearance time. (d) Curves of the disappearance time of the bright solitonlike state as a function of the pulse duration  $\Delta s$  for pump pulse strengths  $\sigma_8' = 10$  (red) and  $20$  (blue). Here, the temporal length of the PT-symmetric potential barrier  $\delta = 0.08$  (this parameter value remains unchanged in the figure below).

inset of Fig. 7(b). Figure 7(b) depicts the evolution of the bright solitonlike state without a Gaussian pump pulse. In the right insets, the black line represents the profile at  $s = 244$  ( $S_1$ ), the blue line at  $s = 260$ , the red line at  $s = 366$  ( $S_2$ ), and the green line at  $s = 380$ , after the bright solitonlike state fully recovers. The disappearance time is therefore  $T_d = 366$  ( $S_2$ )  $- 244$  ( $S_1$ ) =  $122$  (244 ps). Figure 7(c) presents a contour plot of the bright solitonlike state disappearance time as a function of Gaussian pump pulse intensity  $\sigma_8'$  and pulse duration  $\Delta s$ , with the pulse injected at a fixed time  $s_{\text{int}} = 240$ . Blue regions correspond to shorter disappearance times, while red regions indicate longer times. From Fig. 7(c), it can be seen that as both the Gaussian pump pulse intensity and duration increase, the disappearance time of the bright solitonlike state gradually decreases, implying a shorter recovery time. Figure 7(d) shows the disappearance time  $T_d$  as a function of pulse duration  $\Delta s$ , where the red and blue curves correspond to pulse intensities  $\sigma_8' = 10$  and  $20$ , respectively. These curves represent horizontal cross-sections of Fig. 7(c) at the respective pulse intensities. It is observed that  $T_d$  decreases with increasing  $\Delta s$  for both curves, and at the same  $\Delta s$ , a higher pulse intensity results in a smaller  $T_d$ , as evidenced by the blue curve lying below the red curve.

In addition, the disappearance time of the bright solitonlike state depends on the injection time point of the Gaussian pump

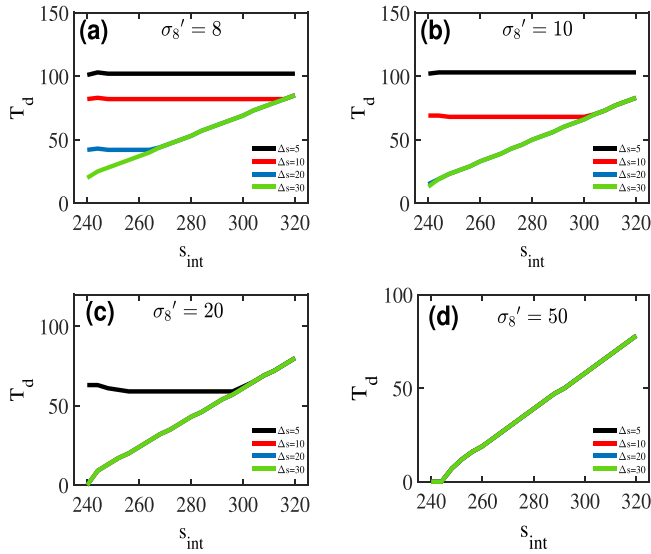


FIG. 8. (a)–(d) The curves of the disappearance time of the bright solitonlike state as a function of the Gaussian pump pulse injection time point with fixed Gaussian pump pulse intensities of  $\sigma_g' = 8, 10, 20$ , and  $50$ . The black, red, blue, and green curves represent the results for pulse durations of  $\Delta s = 5, \Delta s = 10, \Delta s = 20$ , and  $\Delta s = 30$ , respectively.

pulse. More importantly, we identify a threshold in the pulse intensity: Beyond this threshold, for a fixed injection time, the disappearance time of the bright solitonlike state becomes largely insensitive to the pulse duration. Figures 8(a)–8(d) show the disappearance time of the bright solitonlike state  $T_d$  as a function of the Gaussian pump pulse injection time  $s_{\text{int}}$  for pulse intensities  $\sigma_g' = 8, 10, 20$ , and  $50$ , respectively. The black, red, blue, and green curves correspond to pulse durations  $\Delta s = 5, 10, 20$ , and  $30$ . As seen in Figs. 8(a) and 8(b), under relatively low pulse intensity and short pulse duration (black curves), delaying the injection time has only a minor effect on the disappearance time. In contrast, for the same low intensity but longer pulse duration (green curves), delaying the injection significantly increases  $T_d$ . A more intriguing phenomenon emerges when comparing Figs. 8(a)–8(d): As the Gaussian pump pulse intensity increases, the curves for different pulse durations progressively converge. For example, in Fig. 8(a) ( $\sigma_g' = 8$ ), the blue ( $\Delta s = 20$ ) and green ( $\Delta s = 30$ ) curves begin to overlap after  $s_{\text{int}} \approx 268$ , whereas they are distinct before this point. Similarly, the red ( $\Delta s = 10$ ) and green ( $\Delta s = 30$ ) curves converge around  $s_{\text{int}} \approx 316$ . As the pulse intensity increases, this convergence occurs at earlier injection times. In Fig. 8(b) ( $\sigma_g' = 10$ ), the blue and green curves coincide from  $s_{\text{int}} \approx 244$ , and the red and green curves converge near  $s_{\text{int}} \approx 304$ . In Fig. 8(c) ( $\sigma_g' = 20$ ), the red, blue, and green curves ( $\Delta s = 10, 20, 30$ ) completely overlap, and even the black curve ( $\Delta s = 5$ ) aligns with them over a significant portion. Finally, Fig. 8(d) ( $\sigma_g' = 50$ ) demonstrates that at sufficiently high pulse intensity, all curves for different durations coincide entirely. It is noteworthy that there exists a one-to-one correspondence between pulse duration and its threshold intensity for complete convergence: Longer pulses require lower threshold intensity [ $(\sigma_g')_{\text{th}} \approx 20$  for  $\Delta s = 10$ ], while shorter pulses require higher intensity [ $(\sigma_g')_{\text{th}} \approx$

$50$  for  $\Delta s = 5$ ]. This indicates that once the intensity exceeds the critical value for each duration, varying the pulse duration produces identical effects on the bright solitonlike state disappearance time at a fixed injection point. Overall, in this section, we propose a scheme for injecting Gaussian pump pulses to achieve more convenient and flexible control over the switching dynamics of bright solitonlike state. This control is primarily manifested in the dependence of the disappearance time on the pulse parameters, including intensity, duration, and injection time. A key finding is the one-to-one correspondence between pulse duration and threshold intensity, which ensures that all pulse durations produce the same regulatory effect once their respective intensity thresholds are exceeded.

By applying a Gaussian pulse control method with the typical microcavity parameters presented in Sec. II, we can regulate the disappearance time of the bright solitonlike state within the range of 0–244 ps. The corresponding system response time is 24 ps.

## VI. CONCLUSION

In conclusion, we have demonstrated the formation of stable gap solitonlike state and, more importantly, the controllable on-off switching dynamics of bright solitonlike state in exciton-polariton condensates under a PT-symmetric periodic potential. Unlike the unidirectional transmission or scattering dynamics observed in conventional nonlinear Schrödinger equations, this switching phenomenon characterized by the solitonlike state's temporary disappearance and subsequent recovery is governed by the interplay between the PT-symmetric potential and the system's intrinsic gain-loss effects. Furthermore, we established a versatile control scheme, showing that the disappearance duration of bright solitonlike state can be regulated not only passively through experimental material parameters but also actively via nonresonant optical Gaussian pump pulses. Our work provides a promising platform for developing soliton-based active photonic devices in nonequilibrium systems.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

## APPENDIX A: DIMENSIONLESS QUANTITIES AND PARAMETERS

Table I lists the dimensionless physical quantities and parameters used in the main text.

TABLE I. Physical meanings and definition locations of key quantities.

| Symbol             | Physical meaning                                                         | Definition location |
|--------------------|--------------------------------------------------------------------------|---------------------|
| $s$                | Dimensionless time                                                       | Eq. (3)             |
| $\xi$              | Dimensionless spatial coordinates                                        | Eq. (3)             |
| $u$                | Dimensionless polariton field                                            | Eq. (3)             |
| $\nabla_{\perp}^2$ | Dimensionless Laplacian operator                                         | Eq. (3)             |
| $n$                | Dimensionless reservoir density                                          | Eq. (4)             |
| $\sigma_1$         | Self-interaction coefficient of polaritons                               | Eq. (3)             |
| $\sigma_2$         | Cross-interaction coefficient (condensate-reservoir)                     | Eq. (3)             |
| $\sigma_3$         | Nonlinear gain coefficient                                               | Eq. (3)             |
| $\sigma_4$         | Linear loss coefficient                                                  | Eq. (3)             |
| $\sigma_5$         | Reservoir loss rate                                                      | Eq. (4)             |
| $\sigma_6$         | Condensation rate                                                        | Eq. (4)             |
| $P(\vec{r})$       | Dimensionless pump distribution                                          | Eq. (4)             |
| $\sigma_7$         | Homogeneous pump intensity                                               | Eq. (4)             |
| $\sigma_8$         | Gaussian pump intensity                                                  | Eq. (4)             |
| $w_0$              | Width of Gaussian pump                                                   | Eq. (4)             |
| $V_{PT}$           | PT-symmetric complex potential                                           | Eq. (5)             |
| $\sigma_9$         | Real-part strength of PT-symmetric potential                             | Eq. (5)             |
| $k$                | Dimensionless spatial modulation frequency of the PT-symmetric potential | Eq. (5)             |
| $W$                | Intensity ratio of imaginary/real parts of PT-symmetric potential        | Eq. (5)             |
| $\delta$           | The temporal length of the PT-symmetric potential barrier                | Eq. (10)            |
| $\mu$              | Chemical potential/eigenvalue                                            | Eq. (6)/Eq. (12)    |
| $\psi$             | Steady-state wave function                                               | Eq. (6)             |
| $N_R$              | Relative norm of steady state                                            | Sec. II             |
| $n'$               | Time-independent reservoir density                                       | Eq. (6)             |
| $K$                | Bloch wave number                                                        | Eq. (12)            |
| $S_1, S_2$         | Disappearance and reappearance time points                               | Sec. IV             |
| $T_d$              | Disappearance duration of bright solitonlike state ( $S_2 - S_1$ )       | Sec. IV             |
| $\sigma_8'$        | Intensity of Gaussian pump pulse                                         | Sec. V              |
| $\Delta s$         | Duration of Gaussian pump pulse action                                   | Sec. V              |
| $s_{int}$          | Injection time point of Gaussian pump pulse                              | Sec. V              |

## APPENDIX B: BRIGHT SOLITONLIKE STATE SWITCHING DYNAMICS UNDER A GAUSSIAN TIME-VARYING PT-SYMMETRIC BARRIER

In the main text, we primarily investigated the case where the strength of the PT-symmetric periodic barrier first increases and then decays exponentially over time. Additionally, we now consider the scenario where the barrier strength varies as a Gaussian function in time. In this case, Eq. (10) is modified to the following expression:

$$V = V_{PT} e^{-\frac{(\xi-200)^2}{w_s^2}}, \quad (B1)$$

where  $w_s$  is the width of the Gaussian function in time. When a bright solitonlike state is incident on this potential, it can also exhibit switching dynamics. Figure 9(a) presents a contour plot of the bright solitonlike state disappearance

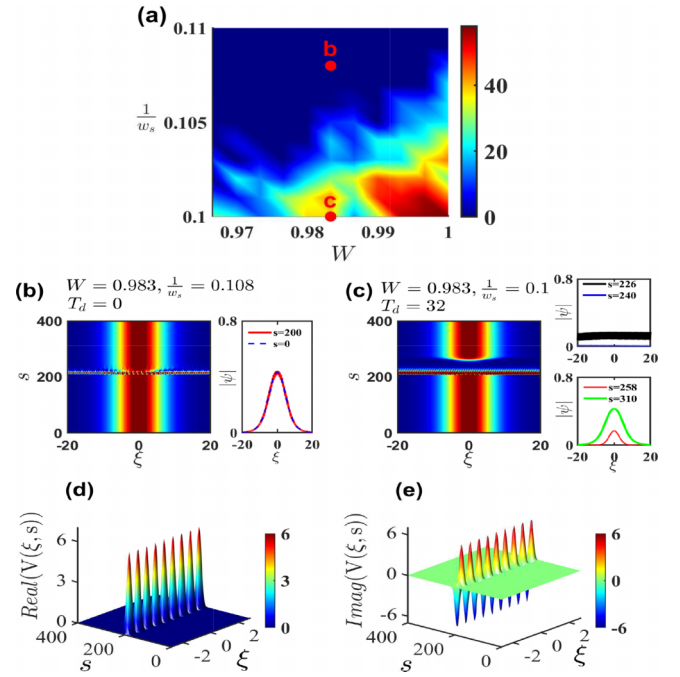


FIG. 9. (a) Contour plot showing the disappearance time of the bright solitonlike state vs the ratio of imaginary to real-part intensities  $W$  and the inverse temporal Gaussian width  $\frac{1}{w_s}$ , where the PT-symmetric barrier strength varies as a Gaussian function in time. (b), (c) The evolution results for  $W = 0.983$  with  $\frac{1}{w_s} = 0.108$  and  $\frac{1}{w_s} = 0.1$ , respectively, corresponding to the red points marked with labels in panel (a). In the right inset of panel (b), the blue dashed line and the red solid line, respectively, represent the profiles when  $s = 0$  and  $s = 200$ . Right insets in panel (c): black, blue, red, and green lines, respectively, represent the profiles when  $s = 226$  ( $S_1$ ),  $s = 240$ ,  $s = 258$ , and  $s = 310$  ( $S_2$ ). (d), (e) The real and imaginary parts of the PT-symmetric periodic lattice potential with the inverse temporal Gaussian width  $\frac{1}{w_s} = 0.1$ .

time  $T_d$  versus the ratio of imaginary to real-part intensities  $W$  and the inverse temporal Gaussian width  $\frac{1}{w_s}$ . The blue region corresponds to areas with smaller numerical values, while the red region represents larger values. It can be observed that, within a certain parameter range, larger values of  $W$  and smaller values of  $\frac{1}{w_s}$  result in a longer disappearance time of the solitonlike state (red region in the lower right). Figures 9(b) and 9(c) show the evolution results for  $W = 0.983$  with  $\frac{1}{w_s} = 0.108$  and  $0.1$ , respectively. By comparing the evolution results of the two cases, it can be observed that when  $\frac{1}{w_s}$  is relatively large, the bright solitonlike state directly transmits through the barrier without exhibiting switching dynamics (i.e.,  $T_d = 0$ ). In contrast, when  $\frac{1}{w_s}$  is relatively small, the bright solitonlike state demonstrates switching dynamics consistent with the behavior described in the main text. In the inset on the right side of Fig. 9(c), the black, blue, red, and green lines represent the profiles when  $s = 226$ ,  $s = 240$ ,  $s = 258$ , and  $s = 310$ , respectively. The disappearance time is determined as  $T_d = 258 (S_2) - 226 (S_1) = 32$  (64 ps). Figures 9(d) and 9(e) illustrate the real and imaginary parts of the PT-symmetric periodic potential under the parameters  $\sigma_9 = 6$ ,  $W = 1$ ,  $k = 5$ , and  $\frac{1}{w_s} = 0.1$ , respectively.

TABLE II. Fixed dimensionless parameters and their values in the figures.

| Symbol     | Physical meaning                             | Value (Figs.)        |
|------------|----------------------------------------------|----------------------|
| $\sigma_2$ | Cross-interaction coefficient                | 0.3 (all)            |
| $\sigma_3$ | Nonlinear gain coefficient                   | 0.20 (1–3, 5–9)      |
| $\sigma_4$ | Linear loss coefficient                      | 0.3423 (1–3, 5–9)    |
| $\sigma_5$ | Reservoir loss rate                          | 1 (2–9)              |
| $\sigma_6$ | Condensation rate                            | 4 (1–3, 5–9)         |
| $\sigma_7$ | Homogeneous pump intensity                   | 1 (2–9)              |
| $w_0$      | Gaussian pump width                          | 1 (2, 4); 5 (3, 5–9) |
| $\sigma_9$ | Real-part strength of PT-symmetric potential | 6 (all)              |

### APPENDIX C: SUMMARY OF DIMENSIONLESS PARAMETER VALUES

In this Appendix, we summarize the dimensionless parameters used throughout the main text to avoid repetitive listings in figure captions and section descriptions. Table II lists the fixed parameters that remain unchanged across all sections, along with their physical meanings and the figures in which they appear. Table III summarizes the variable parameters that are intentionally tuned in different contexts, specifying their values, the corresponding figures, and the physical motivations for their variation.

#### Physical intentions:

$\sigma_1 = -3$  (Fig. 2): To examine Kerr nonlinearity effect on gap solitonlike states.

$\sigma_8$ : To control the amplitude of gap solitonlike states (Fig. 2), to generate stable bright solitonlike states (Figs. 3–9), and to tune the amplitude of single-peak solitonlike states (Fig. 6).

$W$ : To ensure PT-symmetry unbroken (Figs. 1 and 2), to locate the region of bright solitonlike state switching

TABLE III. Variable parameter values in different figures and the physical intentions behind their variations.

| Symbol           | Physical meaning                                                  | Value (Figs.)                                                              |
|------------------|-------------------------------------------------------------------|----------------------------------------------------------------------------|
| $\sigma_1$       | Self-interaction coefficient                                      | −1 (2(a)–2(c2); 3–9); −3 (2(a), 2(d1) and 2(d2))                           |
| $\sigma_8$       | Gaussian pump intensity                                           | 5.2, 10 (2); 2.33 [3–5, and 6(c)–6(f), 7–9]; 2–6.5 [6(b)]                  |
| $W$              | Intensity ratio of imaginary/real parts of PT-symmetric potential | 0.1 (1 and 2); 0–1 [3(a)]; 0.88–1 [5(a) and 5(b)]; 1 (4, 6–8); 0.97–1 (9); |
| $\delta$         | Temporal length of the PT-symmetric barrier                       | 0.1 (3 and 4, 6(b)–6(f)); 0.08 (6(b), 7 and 8); 0.08–0.1 (5)               |
| $\mu$            | Chemical potential                                                | −4 to −1.1 [2(a)]                                                          |
| $a, b$           | Gain/loss scaling                                                 | 0.001–2 [4(a)]                                                             |
| $d$              | Peak distance                                                     | 10, 20, 30 [6(c)]                                                          |
| $N$              | Peak number                                                       | 1, 2, 3, 4, 5 [6(c)]                                                       |
| $\sigma'_8$      | Pulse intensity                                                   | 6–25 [7(c)]; 8, 10, 20, 50 [8(a)–8(d)]                                     |
| $\Delta s$       | Pulse duration                                                    | 1–20 [7(c) and 7(d)]; 5, 10, 20, 30 [8]                                    |
| $s_{\text{int}}$ | Injection time                                                    | 240 (7); 240–320 (8)                                                       |
| $1/w_s$          | Inverse temporal width of Gaussian PT-symmetric barrier           | 0.1–0.11 (9)                                                               |

dynamics [Fig. 3(a)], and to control the disappearance time of bright solitons (Figs. 5 and 9).

$\mu$ : To obtain gap solitonlike states of different branches.

$a, b$ : To verify the switching dynamics originates from nonequilibrium nature.

$d, N$ : To examine effects of peak distance and peak number on disappearance time.

$\sigma'_8, \Delta s, s_{\text{int}}$ : Active control of disappearance time via non-resonant pump pulse to demonstrate real-time tunability.

$1/w_s$ : To control barrier time variation rate for Gaussian-type PT-symmetric barrier.

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