

Mechanism of the nonequilibrium phase transition in self-propelled particles with alignment

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(Received 11 February 2025; accepted 30 June 2025; published 22 July 2025)

Self-propelled particles with alignment, displaying ordered collective motions such as swarming, can be investigated by the well-known Vicsek model. However, challenges still remain regarding the nature of the associated phase transition. Here, we use the landscape-flux approach combined with the coarse-grained mapping method to reveal the underlying mechanism of the order-disorder nonequilibrium phase transition in Vicsek model systems featuring diverse noise characteristics. It is found that the nonequilibrium flux inside the landscape in the density-alignment degree phase space always rotates counterclockwise, and tends to delocalize or destabilize the point attractor states, providing the dynamical driving force for altering the landscape shape and the system state. Furthermore, the variations in the averaged flux and entropy production rate exhibit pronounced differences across various noise types. This not only helps to reveal the dynamical and thermodynamical mechanisms of the order-disorder transition but also offers a useful tool to recognize the continuity of the transition. Our findings present a perspective for exploring nonequilibrium phase transition behaviors and other collective motions in various complex systems.

DOI: [10.1103/fqn3-4qkv](https://doi.org/10.1103/fqn3-4qkv)

I. INTRODUCTION

Self-propelled particles, widely existing as both natural objects and artificial swimmers [1–11], have the ability to execute purposeful systematic movement by taking in and dissipating energy, which causes the system to be far from the equilibrium and to exhibit many novel dynamical behaviors different from those in the equilibrium system. For instance, self-propelled particles can spontaneously organize into chains [1,5], vortices [2,3,12], clusters [4,5,11,13], and notably, exhibit ordered collective motion and swarming behaviors [4,5,14,15]. Among diverse theoretical models of self-propelled particles, the Vicsek model with an alignment term [15] can successfully reproduce the ordered collective motion observed in nature like bird flocks and bacterial

colonies. Notably, the Vicsek model system undergoes a phase transition from an ordered to a disordered state dependent on the decreasing particle density or the increasing noise intensity. Inspired by this, a series of studies [16–27] have delved into the Vicsek model and its various forms, such as introducing density-dependent particle velocity [16], obstacles [17], chirality [18], memory effect [19], delayed effect [20], etc. [21–27], so that many novel collective motions emerge spontaneously. For example, when the Vicsek particle (VP) velocity is correlated with its local density [16], abundant dynamical patterns can be observed, including moving clumps, active lanes, and asters. These findings underscore the practical potential of VP systems across various fields, including robotics [28], drone technology [29], and so on.

For the original Vicsek model [15], the phase transition from the ordered collective motion to the disordered one was considered continuous (second order transition), which is proved by the smooth transition of the average normalized velocity dependence on the noise intensity and their scaling behavior [15]. However, it was found that there exists a coexisting phase consisting of traveling bands and dilute disordered phase, which emerges between the ordered phase and disordered phase, implying the transition

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is a discontinuous or weakly first order transition rather than a second order one [30,31]. More interestingly, when the intrinsic noise (also called the scalar or angular noise) in the original model is replaced by the extrinsic noise (the vectorial noise), not only do the traveling bands in the coexisting phase emerge more easily, but also the variation in the noise-dependent average normalized velocity becomes more violent, both indicating that the order-disorder phase transition changes to a sharper one (first order transition) [30–33]. There exist many theoretical approaches to study the phase transitions (including the order-disorder transition in VP systems and the well-known motility-induced phase separation) in nonequilibrium systems, such as the scaling behavior [15], Binder cumulant behavior [32], the scalar ϕ^4 field theory [34,35], the kinetic model [36–38], and the compression-based entropy (the computable information density) method [31]. For example, the compression-based entropy method gives a good approximation to the thermodynamic entropy during the ordered-disordered phase transition in the VP system under intrinsic or extrinsic noise [31]. However, only position (density) information was used to analyze the phase transition [31], while the velocity information was ignored. Motivated by this, it is very necessary to establish a framework based on both position (density) and velocity information of VPs, which not only can present clearly and visually the order-disorder phase transition behaviors for different noise types, but also allow us to reveal the underlying dynamical and thermodynamical origins of the associated nonequilibrium phase transition.

To address this, we introduce the nonequilibrium landscape and flux approach [39], which is widely used in various fields, including cell fate decision making [40], cancer research [41], neural networks [42], and ecological systems [43], among others [44–47]. In this approach, the driving forces for the nonequilibrium dynamics of the system are determined by both the landscape and rotational flux, analogous to the driving force of charged particle in electric field (potential gradient force) and magnetic field (Lorentz force). Compared to the existing theoretical methods for studying the nonequilibrium phase transitions, this landscape-flux approach combined with the coarse-grained mapping method [48] offers interesting physics and insights. We can establish and visualize the nonequilibrium potential and flux landscape of VP systems in the phase space of the local density and the local alignment degree, which reflects global states of the system as well as changes in dynamical and thermodynamical characteristics. As the noise intensity increases, the landscape changes continuously/discontinuously from one potential basin located at a large local alignment degree (the ordered flocking phase) to two basins (the coexisting phase consisting of traveling bands), and finally to one basin located at a small local alignment degree (the disordered phase) under intrinsic/extrinsic noise, respectively. Furthermore, the nonequilibrium flux inside the potential here always rotates counterclockwise in the $\rho - q$ phase space (increasing ρ as the x axis and increasing q as the y axis), providing the dynamical driving force for the shape changes of the landscape. Moreover, the obtained averaged flux and entropy production rate (EPR) exhibit distinct variation behaviors near the transition threshold, which not only can be utilized to reveal the

dynamical and thermodynamical origins of the phase transition, but also serve as early warning signals of the transition. Notably, the variation behavior near the transition threshold differs in intensity for different noise types, offering an effective tool to recognize the continuity of the order-disorder transition. Additionally, forward and backward optimal transition paths between the two phases in the coexisting phase do not overlap with each other, indicating that the time-reversal symmetry of the VP system is broken.

II. VICSEK MODEL UNDER INTRINSIC AND EXTRINSIC NOISE

We consider a quasi two-dimensional system with size W under periodic boundary conditions, including N VPs. The motion of the i th particle located at $\mathbf{x}_i$ obeys the following equation:

$$\mathbf{x}_i(t + \Delta t) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t. \quad (1)$$

Herein, Δt denotes the simulation time step, and $\mathbf{v}_i(t) \equiv v_0(\cos \theta_i(t), \sin \theta_i(t))$, where v_0 and θ_i denote, respectively, the amplitude and direction of $\mathbf{v}_i$. The evolution rule of θ_i for the VP under intrinsic noise [15] is given as

$$\theta_i(t + \Delta t) = \arg \left[\sum_{r_{ij} < R} \mathbf{v}_j(t) \right] + \eta \xi_i(t). \quad (2)$$

Here, the first term represents the alignment effect of the VP, i.e., θ_i always tends to orient toward the average direction of its neighbors, whose distance (r_{ij}) from the given particle is smaller than the interaction radius R . Here, $\arg(\mathbf{v})$ denotes the average of the orientation of $\mathbf{v}$. The second term denotes the intrinsic noise, with η the noise intensity and ξ_i the white noise distributed uniformly ranging from $-\pi$ to π . For the situation of extrinsic noise, the evolution of θ_i should be replaced by [31]

$$\theta_i(t + \Delta t) = \arg \left[\sum_{r_{ij} < R} \mathbf{v}_j(t) + \eta \xi_i(t) \right], \quad (3)$$

where ξ_i is a unit vector with a random direction, i.e., $\xi_i = (\cos(\zeta_i), \sin(\zeta_i))$ with ζ_i a random number selected from $[0, 2\pi]$. In the rest of this paper, we fix $R = 1$, $v_0 = 0.5$, $W = 100$, $N = 20\,000$, $\Delta t = 1$, and average density $\rho_0 = N/W^2 = 2$. We first run each simulation for 100 000 time steps to ensure the system reaches the steady state. Then, we perform the simulation for another 200 000 time steps in the steady state for sampling and analysis.

III. ORDER-DISORDER PHASE TRANSITION FOR DIFFERENT NOISE

By gradually increasing the noise strength η from 0.05 to 1, we observe transitions of the VP system from an ordered phase to a disordered one. To measure the order degree of the VP system, we first introduce the average normalized velocity $\phi = |\sum_{i=1}^N \mathbf{v}_i|/(Nv_0)$, Fig. 1(a), where $\phi = 1$ represents VPs moving in the same orientation so that the system is in the ordered phase, while ϕ being close to 0 implies VPs moving

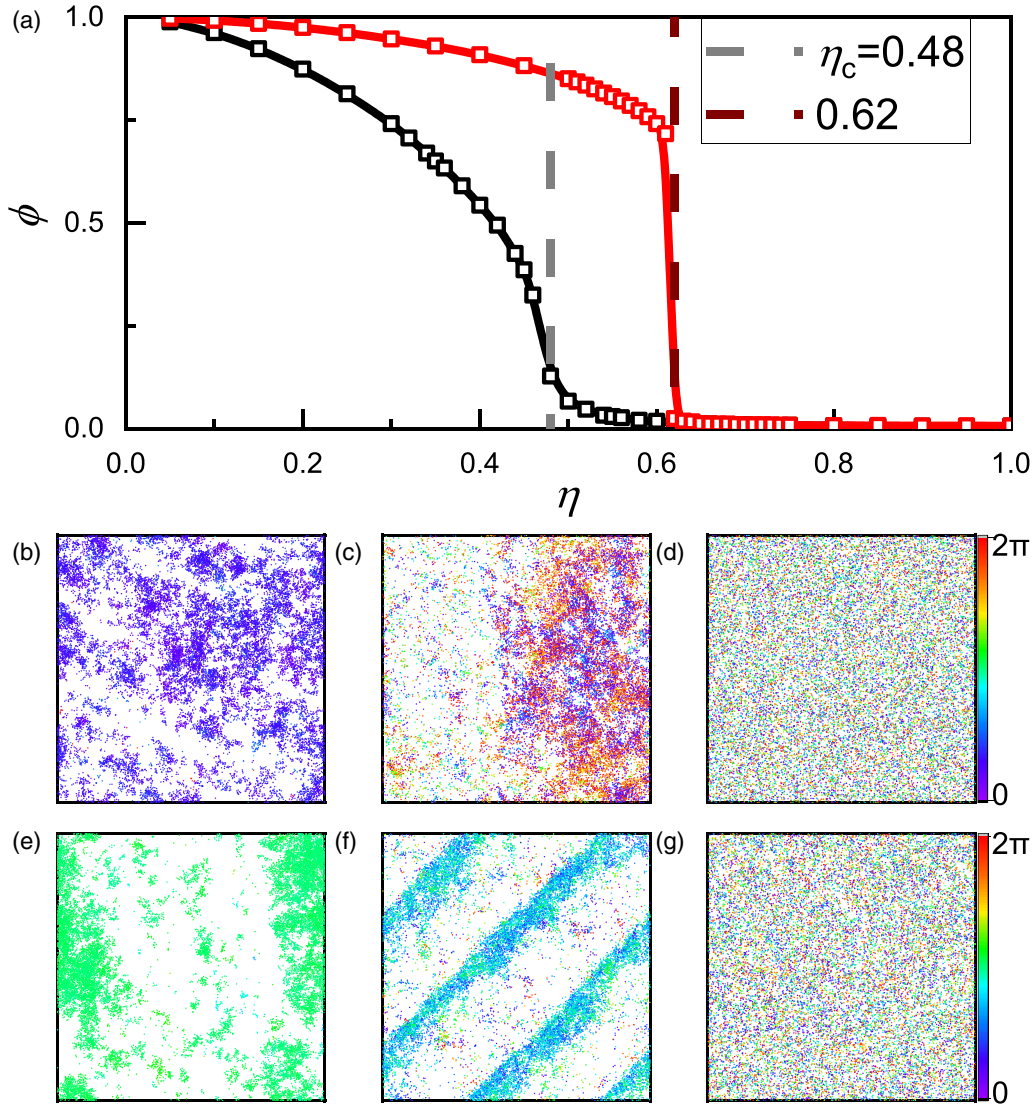


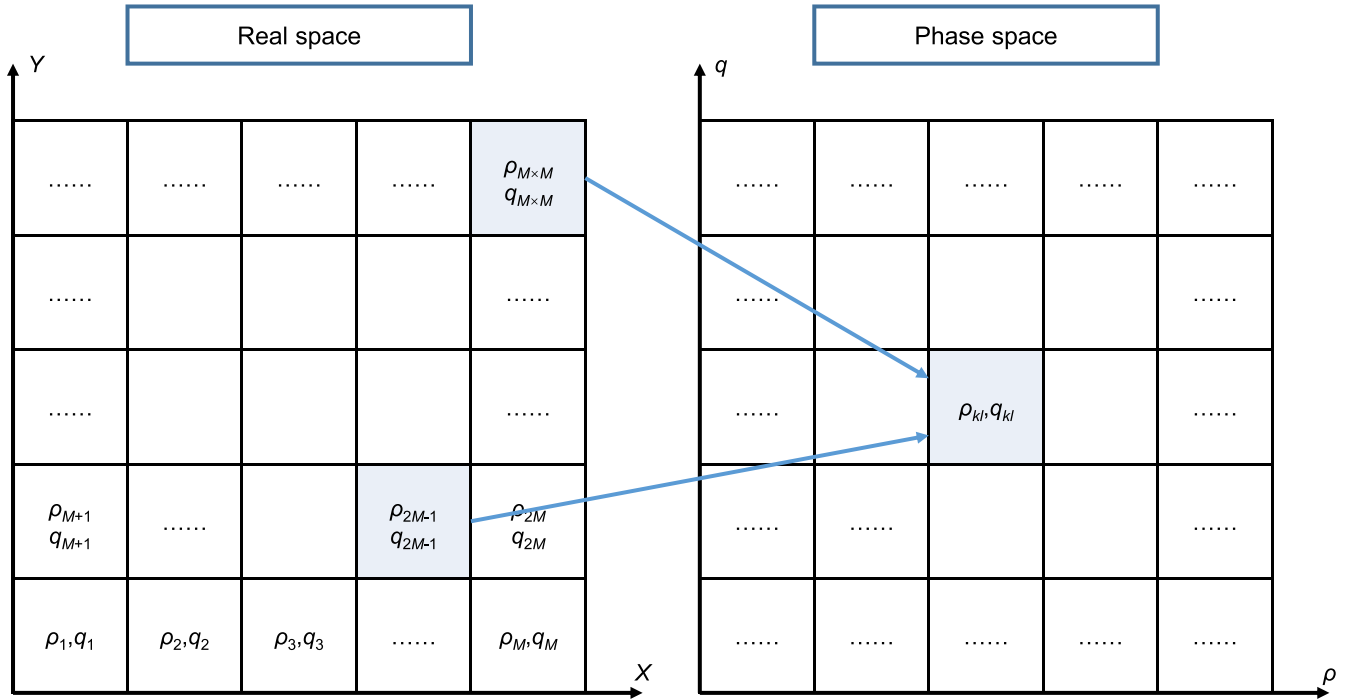
FIG. 1. (a) ϕ vs η for the VP system under intrinsic noise (black symbols and line) and extrinsic noise (red symbols and line), with dashed lines the critical transition points η_c . (b)–(g) Typical snapshots in the steady states for different phases with $\eta = 0.1$ (b), 0.4 (c), and 0.9 (d), respectively, under intrinsic noise as well as with $\eta = 0.1$ (e), 0.6 (f), and 0.8 (g), respectively, under extrinsic noise. (b) and (e) represent the ordered flocking phases. (c) and (f) denote the coexisting phases with traveling bands. (d) and (g) show the disordered phases.

in random directions such that the system is disordered. For intrinsic noise [the black symbols and line in Fig. 1(a)], it is observed that as the noise intensity η increases, ϕ decreases smoothly from almost 1 to 0, indicating that the VP system changes smoothly from the ordered phase to the disordered one. For small noise such as $\eta = 0.1$ as shown in Fig. 1(b), it is found that VPs gather into clusters and move in similar directions, so that the VP system forms the flocking phase with ϕ close to 1. For larger noise such as $\eta = 0.4$, Fig. 1(c), many VPs gather and move collectively in one direction, further forming a traveling band of the dense phase, while the other particles move randomly in the dilute phase. For noise large enough such as $\eta = 0.9$, Fig. 1(d), the motion of VPs becomes almost random, and the system becomes disordered. When the intrinsic noise is replaced by extrinsic noise [red symbols and line in Fig. 1(a)], ϕ still decreases from almost 1 to 0. However, the transition is much sharper than that under

intrinsic noise. For small and large enough noise, the VP system also forms the ordered flocking phase, Fig. 1(e), and disordered phase, Fig. 1(g), respectively. Interestingly, when noise changes from intrinsic to extrinsic, the coexisting phase that appears at moderate noise levels shows a clearer traveling band. This represents that the order-disorder phase transition in the VP system becomes sharper or more violent when the intrinsic noise is replaced by the extrinsic one, consistent with observations in Refs. [30,31].

IV. COARSE-GRAINED MAPPING METHOD AND LANDSCAPE-FLUX THEORY

In order to reveal the origin of the order-disorder phase transition in the VP system under intrinsic or extrinsic noise, we use the coarse-grained mapping method [48] and the landscape-flux approach [39,45].


 FIG. 2. Schematic of the coarse-grained mapping from the evolution of cells in real space to that in the $\rho - q$ phase space.

For the coarse-grained mapping method to transform the VP system from the real space into the phase space, we first divide the real space into square cells with number M for both the x and y dimensions (the left picture in Fig. 2). Next, we calculate the time evolution of the two order parameters (the local density ρ_m and local alignment degree q_m , where m indicates the cell index) for each cell in the real space by collecting simulation data obtained from Eqs. (1)–(3) (more details and definitions can be found in the Supplemental Material [49]). Subsequently, we map them into the $\rho - q$ phase space, which is also divided into cells with $\delta\rho$ along the ρ dimension and δq along the q dimension (the right picture in Fig. 2). As an example shown in Fig. 2, at time t , there are only two cells in the real space (labeled $2M - 1$ and M^2) satisfying with $\rho(t) \in [k\delta\rho, (k+1)\delta\rho]$ and $q(t) \in [l\delta q, (l+1)\delta q]$ (k and l are non-negative integers). These two cells can be mapped into the same cell (k, l) in the $\rho - q$ phase space (blue cells). In this case, the probability $P(\rho, q, t)$ of the cell (k, l) in the $\rho - q$ phase space is $2/M^2$, while the evolution of the cell (k, l) is obtained as follows: $\dot{\rho}(\rho_{kl}, q_{kl}, t) = \{[\rho_{2M-1}(t + \Delta t) - \rho_{2M-1}(t)] + [\rho_{M^2}(t + \Delta t) - \rho_{M^2}(t)]\} / (2\Delta t)$ and $\dot{q}(\rho_{kl}, q_{kl}, t) = \{[q_{2M-1}(t + \Delta t) - q_{2M-1}(t)] + [q_{M^2}(t + \Delta t) - q_{M^2}(t)]\} / (2\Delta t)$.

On this basis, we can get the evolution of all the system cells in the $\rho - q$ phase space, which is well defined on the coarse-grained level (see details in the SM [49]). The stochastic dynamical equation can be written as

$$\dot{\mathbf{X}}(t) = \mathbf{F}(\rho, q) + \boldsymbol{\xi}(\rho, q, t), \quad (4)$$

where $\mathbf{X} = (\rho, q)^T$. $\mathbf{F}$ is the deterministic driving force, which can be described as $\mathbf{F}(\rho, q) = \langle \dot{\mathbf{X}}(\rho, q, t) \rangle$, where $\langle \cdot \rangle$ denotes time averaging. $\boldsymbol{\xi}$ is the stochastic force, which can be calculated according to the following formula: $\boldsymbol{\xi}(\rho, q, t) =$

$\dot{\mathbf{X}}(\rho, q, t) - \langle \dot{\mathbf{X}}(\rho, q, t) \rangle$. Also, the time correlations of $\boldsymbol{\xi}$ should satisfy with $\langle \boldsymbol{\xi}(\rho, q, t) \boldsymbol{\xi}(\rho, q, t') \rangle = 2\mathbf{D}(\rho, q)\delta(t - t')$, where $\mathbf{D}$ is the diffusion coefficient tensor represented by a 2×2 matrix, which can be obtained from the Fourier transform of the time correlation functions (the power spectrum functions) of $\boldsymbol{\xi}$. We plot an example of the power spectrum functions in Fig. S3 of the SM [49] and find that $\boldsymbol{\xi}$ in the $\rho - q$ phase space is white noise, demonstrating that the dynamics of the VP system in such phase space is indeed Markovian. In addition, by calculating the time averaging of $P(t)$ in the steady state for each cell in the $\rho - q$ phase space, the steady-state probability distribution $P(\rho, q)$ in the $\rho - q$ phase space can be obtained.

The probabilistic evolution for the order parameter under the stochastic Langevin dynamics specified above follows a Fokker-Planck diffusion equation as $\frac{\partial P}{\partial t} = -\nabla \cdot \mathbf{J}$ where $\mathbf{J} = \mathbf{F}P - \mathbf{D} \cdot \nabla P$. According to the landscape-flux approach, the nonequilibrium potential U and the nonequilibrium flux $\mathbf{J}$ of the VP system in the $\rho - q$ phase space under the steady state can be quantified as (details can be found in the SM [49])

$$U(\rho, q) = -\ln P(\rho, q), \quad (5)$$

$$\mathbf{J}(\rho, q) = \mathbf{F}(\rho, q)P(\rho, q) - \nabla \cdot [\mathbf{D}(\rho, q)P(\rho, q)]. \quad (6)$$

Unlike the equilibrium system, where $\mathbf{J}$ must vanish, a nonequilibrium system exhibits different behavior. When such a system reaches the steady state (with $\partial P / \partial t = 0$), the condition $\nabla \cdot \mathbf{J} = 0$ merely indicates local conservation rather than requiring $\mathbf{J}$ itself to be zero. In general, the flux can exist in the form of a rotational curl or more precisely recurrent field. Therefore, the driving force for the nonequilibrium dynamics is determined by both the gradient of the potential and the curl flux (see more details in the SM [49]).

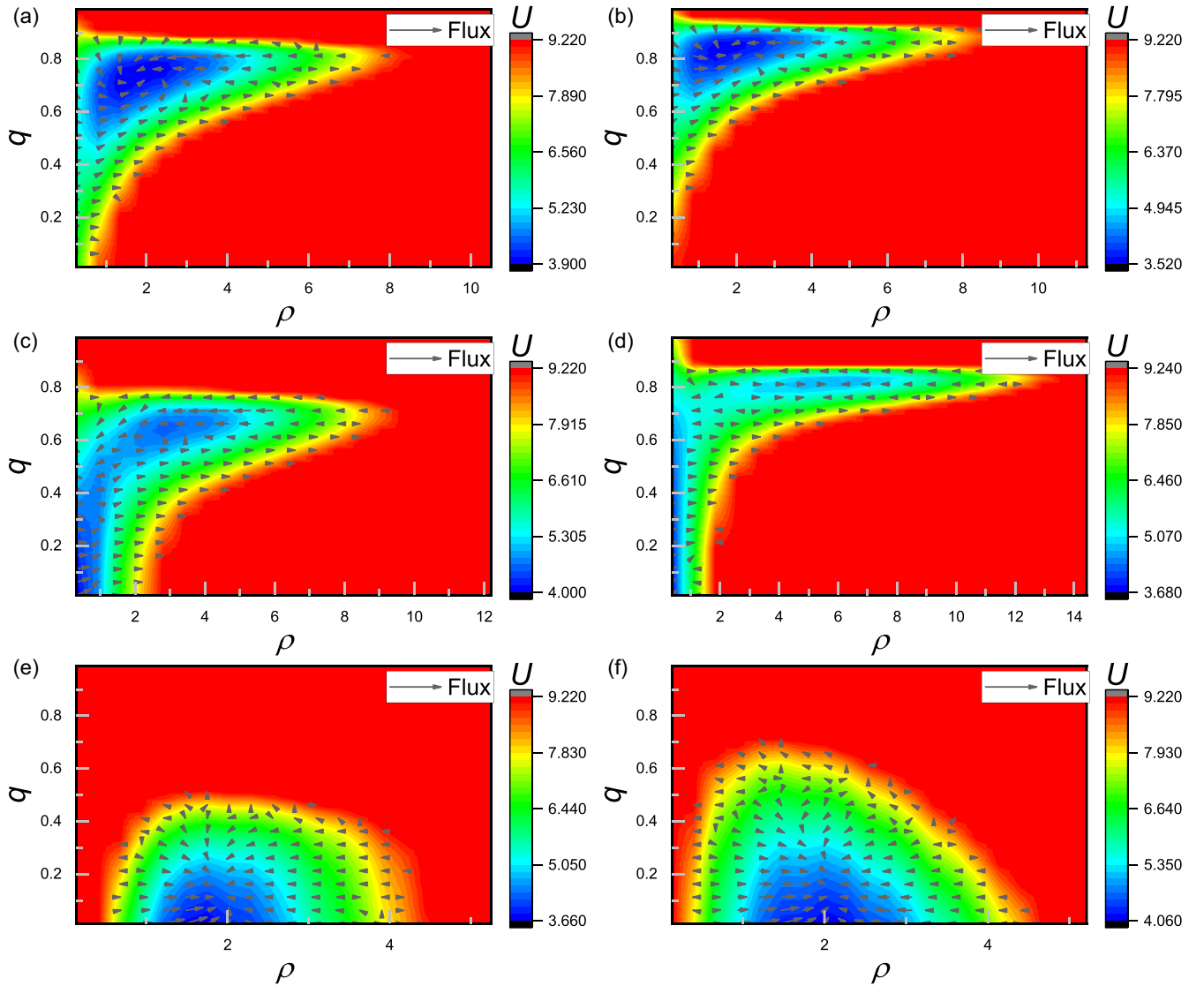


FIG. 3. The nonequilibrium potential and flux landscape in the $\rho - q$ phase space of the VP system under intrinsic (a), (c), (e) and extrinsic (b), (d), (f) noise. The noise intensity is $\eta = 0.3$ (a); 0.4 (c); 0.5 (b); 0.6 (d), (e); and 0.62 (f), respectively. The flux is represented by the black arrows, and the nonequilibrium potential is presented by the colored background.

V. LANDSCAPE AND FLUX IN THE PHASE SPACE

First, we quantify the potential landscape in the $\rho - q$ phase space (see Fig. S4 [49] for typical potential) and observe that the number of potential basins changes from 1 to 2 and finally back to 1 for both intrinsic and extrinsic noise, in agreement with the phase transition behavior exhibited in the VP system (Fig. 1). More interestingly, the landscape changes continuously when the system undergoes intrinsic noise, but it changes discontinuously for extrinsic noise, demonstrating that the order-disorder phase transition is sharper for extrinsic noise.

We then focus on the dynamical nature of the VP system by presenting the nonequilibrium flux inside the landscape in the $\rho - q$ phase space Fig. 3. The nonzero nonequilibrium flux demonstrates that the detailed balance of the VP system is broken. For the ordered flocking phase in the VP system, both

under intrinsic and extrinsic noise, Figs. 3(a) and 3(b), there is a counterclockwise rotating flow around the potential well. Different from the gradient force resulting from the nonequilibrium potential, which always attracts the system down into the potential basin, the curl flux favors global movements so that the system can not be localized in the basin any more, providing the dynamical driving force to create the coexisting phase with the ordered dense phase and the disordered dilute phase. For the coexisting phase Figs. 3(c) and 3(d), it is observed that the flux continues to rotate counterclockwise between the ordered dense phase and the disordered dilute phase, demonstrating that the nonequilibrium flux serves as the dynamical driving force for the order-disorder phase separation. When noise becomes sufficiently large, Figs. 3(e) and 3(f), the rotation of the nonequilibrium flux inside the single potential basin becomes weak and indistinct due to the large noise.

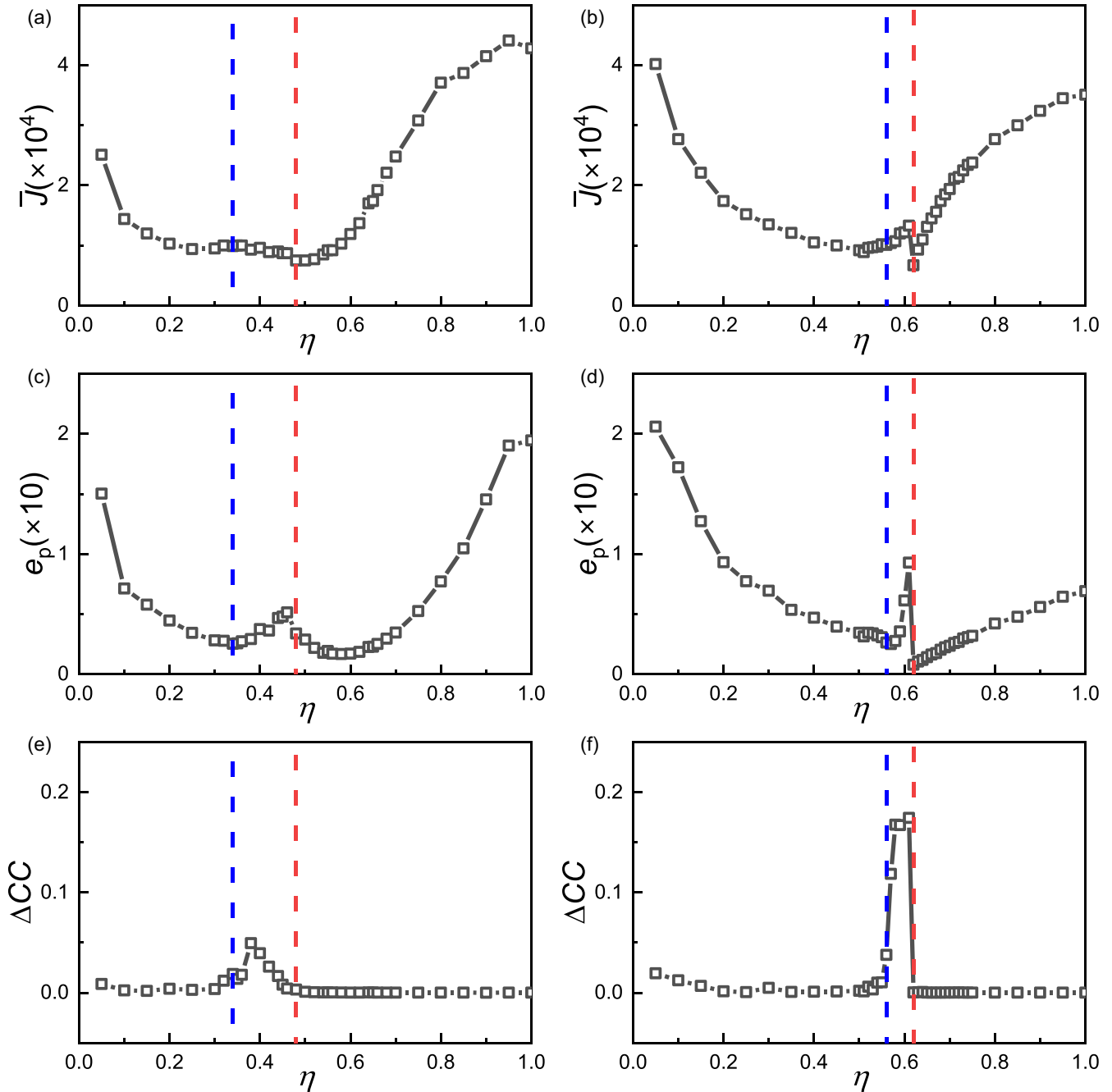


FIG. 4. Dynamical and thermodynamical mechanisms of the phase transition in the VP system. Plots of $\bar{J}$ (a), (b); e_p (c), (d); and ΔCC (e), (f) versus the noise strength η under intrinsic (a), (c), (e) and extrinsic (b), (d), (f) noise. The blue/red dashed lines (η_b/η_c) represent the phase boundaries between the coexisting phase and the ordered phase/the disordered phase.

VI. DYNAMICAL AND THERMODYNAMICAL ORIGINS OF NONEQUILIBRIUM PHASE TRANSITION

We now explore the dynamical and thermodynamical origins of the phase transition. To reveal the dynamical origin, we introduce the averaged flux $\bar{J} = \langle |\mathbf{J}(\rho, q)| \rangle$ because the nonequilibrium flux decomposed from the driving force can determine the nonequilibrium aspect of the dynamics for the system. We quantitatively calculate the averaged flux $\bar{J}$ and plot the dependence of $\bar{J}$ on η as shown in Figs. 4(a) and 4(b). For the VP system under intrinsic noise [Fig. 4(a)], as η

increases, $\bar{J}$ first decreases in the region of the ordered flocking phase, remains relatively unchanged with fluctuations in the coexisting phase, and finally increases in the disordered phase. This means that the slope of $\bar{J}$ changes from 0 to a positive value when η crosses the order-disorder phase transition threshold η_c . In contrast to the case of intrinsic noise, it is observed that $\bar{J}$ initially increases but later decreases in the coexisting phase when noise is extrinsic [Fig. 4(b)], resulting in a peak appearing just before the phase transition threshold. In other words, the slope of $\bar{J}$ changes from a negative value to a positive one as η exceeds η_c . The variations

in the slopes of $\bar{J}$ near the phase transition threshold imply that the nonequilibrium flux is the dynamical origin of the order-disorder phase transition. Due to the rotational nature of the flux as a driving force for nonequilibrium dynamics, the flux tends to be delocalized rather than being localized in the state space (it likes to rotate around spanning the state space). This is in clear contrast to another driving force, the landscape gradient where it always tends to converge to the point attractor in state space. As flux becomes larger, states tend to become more delocalized and the initial point attractor can become less stable. Eventually, further increase of flux relative to landscape gradient can lead to instability and possible bifurcation or dynamical transition from monostable to bistable phase. Thus, the flux serves as the dynamical origin of the bifurcation or nonequilibrium transition. More interestingly, we notice that compared to the situation under intrinsic noise, the variation of $\bar{J}$ is more sudden and sharper under extrinsic noise, suggesting that the order-disorder phase transition is sharper under extrinsic noise.

Now we focus on the thermodynamics of the VP system under different noise types. To quantify the detailed balance breaking and time-reversal symmetry violation in the $\rho - q$ phase space of the VP system, we introduce the entropy production rate e_p of the system [48]:

$$e_p = \iint d\rho dq \frac{\mathbf{J}^T(\rho, q) \mathbf{D}^{-1}(\rho, q) \mathbf{J}(\rho, q)}{P(\rho, q)}. \quad (7)$$

As shown in Figs. 4(c) and 4(d), for both intrinsic and extrinsic noise, e_p decreases in the ordered flocking phase, but increases in the coexisting phase and the disordered phase, further exhibiting a peak at the boundary between these two phases. Since entropy production rate measures the thermodynamic cost of the action of the flux destabilizing the current state and generating a new state, the emergence of this peak near the boundary between the ordered and disordered phases provides an indicator for the thermodynamical origin of the nonequilibrium phase transition in the VP system. Furthermore, it is observed that the peak of e_p in extrinsic noise is sharper than that in intrinsic noise, demonstrating again that the phase transition is sharper or more violent under extrinsic noise compared to that under intrinsic noise. This finding corresponds to that the compression-based entropy (the computable information density) exhibits a higher peak in extrinsic noise than the intrinsic noise [31].

Finally, we introduce the average difference ΔCC between the two-point cross correlations forward and backward in time (more details and definitions can be found in the SM [49]), as plotted in Figs. 4(e) and 4(f). It is observed that ΔCC shows a peak in the phase separation region for both intrinsic and extrinsic noise, illustrating that the time irreversibility of the VP system reaches its maximum during the phase transition process. The sharper peak of ΔCC under extrinsic noise indicates again the sharper nature of the phase transition with a clearer traveling band emergence under extrinsic noise conditions. More interestingly, $\bar{J}$, e_p , and ΔCC all show significant variations prior to reaching the critical transition point η_c . Along with the peak appearance, this provides early warning signals for the order-disorder phase transition in the VP system with both intrinsic and extrinsic noise.

VII. OPTIMAL TRANSITION PATHS IN THE COEXISTING PHASE

Furthermore, to unravel the fundamental mechanisms of the dynamical process between the dense and dilute phases in the coexisting phase, we employ the landscape and flux approach to analyze the optimal transition paths [50]. The optimal transition path represents the highest probability path for the system switching from one phase to another, which can be obtained by minimizing path integrals of the effective Hamiltonian-Jacobian action weights in length space. As depicted in Figs. 5(a) and 5(b), it is evident that for both intrinsic and extrinsic noise, the backward optimal transition paths (black lines) originating from the dense phase initially exhibit a slow decrease in q but a rapid decrease in ρ , followed by a sharp decline in q until reaching the dilute phase. Conversely, the forward optimal transition paths (red lines) switching from the dilute phase to the dense phase involve simultaneous increases in both ρ and q . Notably, the forward and backward optimal transition paths do not overlap with each other, indicating that the time-reversal symmetry is broken. These nonoverlapped paths result from the combined effect of the potential and the curl flux. Next, we focus on the dynamical behaviors of the traveling band in the coexisting phase. As shown in Figs. 5(c) and 5(d), it is found that for both intrinsic and extrinsic noise, compared to the region ahead of the band (red boxes) with VPs originating from the disordered phase, the area behind the band (black boxes) primarily comprises VPs that have just exited the traveling band. Consequently, at similar ρ values, the black boxes exhibit larger q values than the red boxes, which aligns with the observation that the backward optimal transition path [black lines in Figs. 5(a) and 5(b)] consistently lies above the forward path (red lines).

VIII. CONCLUSION AND DISCUSSION

In summary, by using the landscape-flux approach combined with the coarse-grained mapping method in the VP system, we quantified the nonequilibrium potential and flux field in the $\rho - q$ phase space for both intrinsic and extrinsic noise. Compared to the existing theoretical methods, our approach and results not only reflect the global states of the system changing smoothly/sharply from the ordered flocking phase to the disordered phase under intrinsic/extrinsic noise, but also reveal the dynamical and thermodynamical mechanisms of the phase transition.

In the dynamical domain, we have constructed the nonequilibrium potential landscape of the VP system within the coarse-grained $\rho - q$ phase space. This landscape effectively captures global information about the VP system, including phase states and their relative stabilities. Changes in landscape morphology depending on noise intensity allow us to classify transition types under different noise conditions. Additionally, by calculating the nonequilibrium flux (serves as the dynamical driving force), we have analyzed optimal transition paths (most probable paths) between dense (band) and dilute phases in the coexistence region. We observed that the forward and the backward optimal paths do not overlap with each other, indicating the time-reversal symmetry breaking. These nonoverlapping paths reveal the dynamics of VPs either

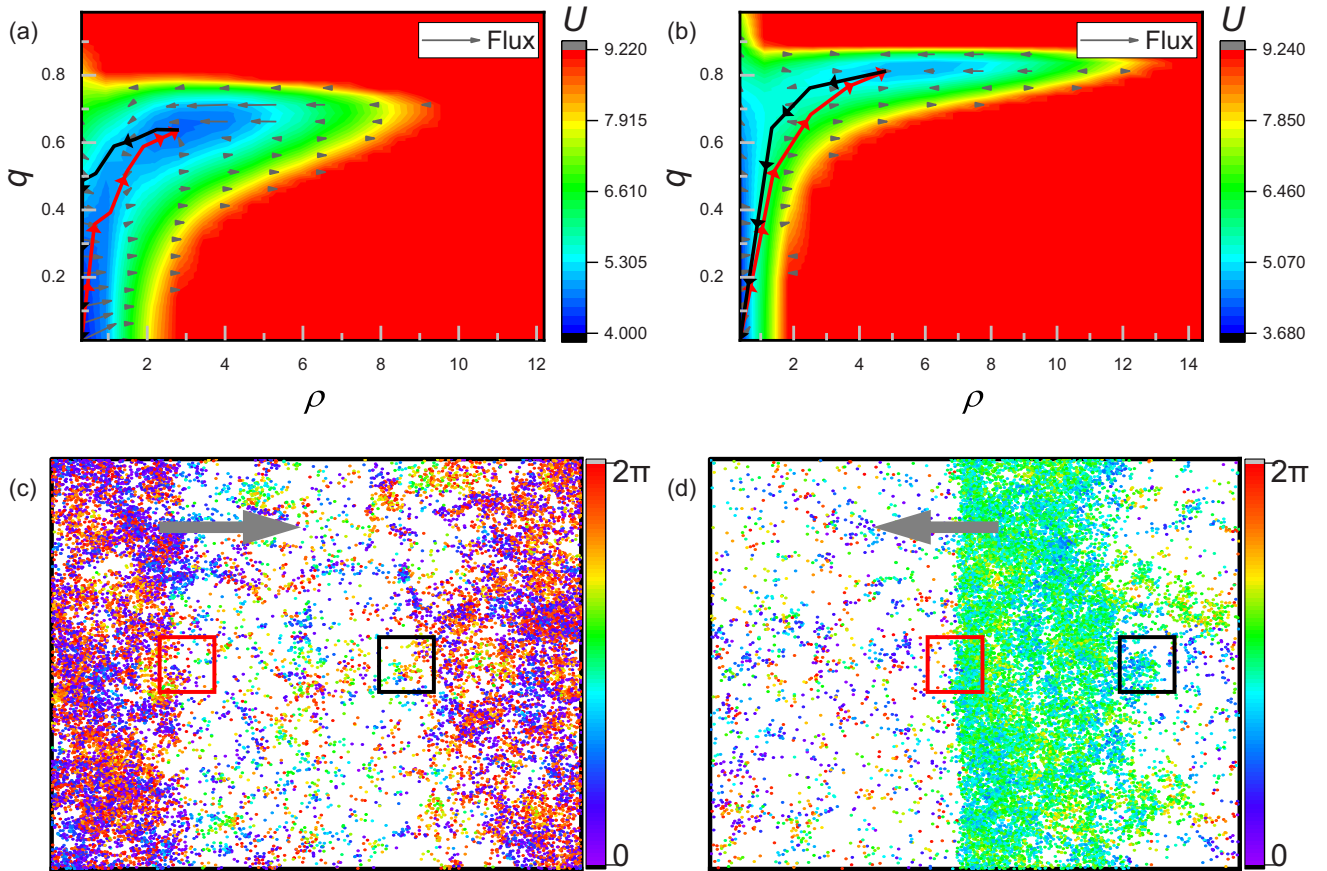


FIG. 5. Optimal transition paths between dense and dilute phases in the coexisting phase of the VP system. (a) and (b) show the backward (black lines) and forward (red lines) optimal transition paths between two phases under intrinsic ($\eta = 0.4$) and extrinsic ($\eta = 0.6$) noise, respectively. (c) and (d) display typical snapshots in the steady states corresponding to (a) and (b), respectively. The arrows represent the moving directions of the traveling bands.

forming or dissolving the traveling bands. Consequently, our results provide physical insights into the visual landscape, the dynamical driving force, and the dynamics of VPs in the coexisting phase with traveling bands.

In the thermodynamic domain, Cavagna *et al.* investigated the compression-based entropy (or computable information density) of the VP system [31], which serves as a reasonable approximation to thermodynamic entropy. Their analysis focused solely on positional (density) information while neglecting velocity data. In contrast, our study introduces a density-alignment degree ($\rho - q$) phase space using the coarse-grained mapping method, where density and alignment degree represent positional and velocity information of the VP system, respectively. This phase space proves more appropriate as both density and alignment degree capture the essential nature of various phases in VPs. Furthermore, we have determined the EPR associated with time-reversal symmetry breaking and apparent dissipation at this coarse-grained scale ($\rho - q$ space) of the VP system. This EPR enables exploration of thermodynamic properties and changes in the VP system during order-disorder phase transitions. Our findings reveal distinct EPR variation patterns near the phase transition threshold for different noise types, demonstrating that EPR acts as the thermodynamic driving force or origin for these transitions. Notably, EPR variations are more abrupt and

pronounced under extrinsic noise, paralleling observations made using the compression-based entropy method [31].

Our findings not only provide a physical and visual way to reveal the underlying mechanism of the phase transition in the VP system, but also offer a new perspective to explore other collective behaviors in more complex nonequilibrium systems.

ACKNOWLEDGMENTS

We thank Wei Wu for thorough discussions on this study. R.Y. and J.S. are supported by WIUCASQD2022012, NSFC12404237, and NSFC12234019, and the Zhejiang Provincial Natural Science Foundation of China under Grant No. LQN25A050004.

J.W. proposed and designed the research. J.S. provided the model and supervised the research. R.Y. performed the simulations and data analysis. All authors discussed the results and co-wrote the manuscript.

The authors declare no competing interests.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, Active particles in complex and crowded environments, *Rev. Mod. Phys.* **88**, 045006 (2016).
- [2] H. Wioland, F. G. Woodhouse, J. Dunkel, J. O. Kessler, and R. E. Goldstein, Confinement stabilizes a bacterial suspension into a spiral vortex, *Phys. Rev. Lett.* **110**, 268102 (2013).
- [3] Y. Sumino, K. H. Nagai, Y. Shitaka, D. Tanaka, K. Yoshikawa, H. Chaté, and K. Oiwa, Large-scale vortex lattice emerging from collectively moving microtubules, *Nature (London)* **483**, 448 (2012).
- [4] H. Karani, G. E. Pradillo, and P. M. Vlahovska, Tuning the random walk of active colloids: From individual run-and-tumble to dynamic clustering, *Phys. Rev. Lett.* **123**, 208002 (2019).
- [5] J. Yan, M. Han, J. Zhang, C. Xu, E. Luijten, and S. Granick, Reconfiguring active particles by electrostatic imbalance, *Nat. Mater.* **15**, 1095 (2016).
- [6] G. Malescio and G. Pellicane, Stripe phases from isotropic repulsive interactions, *Nat. Mater.* **2**, 97 (2003).
- [7] E. A. Jagla, Phase behavior of a system of particles with core collapse, *Phys. Rev. E* **58**, 1478 (1998).
- [8] T. Dotera, T. Oshiro, and P. Ziherl, Mosaic two-lengthscale quasicrystals, *Nature (London)* **506**, 208 (2014).
- [9] Y. Du, H. Jiang, and Z. Hou, Self-assembly of active core corona particles into highly ordered and self-healing structures, *J. Chem. Phys.* **151**, 154904 (2019).
- [10] J. Zhang and S. Granick, Natural selection in the colloid world: active chiral spirals, *Faraday Discuss.* **191**, 35 (2016).
- [11] Y. Gou, H. Jiang, and Z. Hou, Assembled superlattice with dynamic chirality in a mixture of biased-active and passive particles, *Soft Matter* **15**, 9104 (2019).
- [12] A. Bricard, J.-B. Caussin, D. Das, C. Savoie, V. Chikkadi, K. Shitara, O. Chepizhko, F. Peruani, D. Saintillan, and D. Bartolo, Emergent vortices in populations of colloidal rollers, *Nat. Commun.* **6**, 7470 (2015).
- [13] I. Buttinoni, J. Bialké, F. Kümmel, H. Löwen, C. Bechinger, and T. Speck, Dynamical clustering and phase separation in suspensions of self-propelled colloidal particles, *Phys. Rev. Lett.* **110**, 238301 (2013).
- [14] C. W. Reynolds, Flocks, herds and schools: A distributed behavioral model, *SIGGRAPH Comput. Graph.* **21**, 25 (1987).
- [15] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Novel type of phase transition in a system of self-driven particles, *Phys. Rev. Lett.* **75**, 1226 (1995).
- [16] F. D. C. Farrell, M. C. Marchetti, D. Marenduzzo, and J. Tailleur, Pattern formation in self-propelled particles with density-dependent motility, *Phys. Rev. Lett.* **108**, 248101 (2012).
- [17] O. Chepizhko, E. G. Altmann, and F. Peruani, Optimal noise maximizes collective motion in heterogeneous media, *Phys. Rev. Lett.* **110**, 238101 (2013).
- [18] B. Liebchen and D. Levis, Collective behavior of chiral active matter: Pattern formation and enhanced flocking, *Phys. Rev. Lett.* **119**, 058002 (2017).
- [19] K. H. Nagai, Y. Sumino, R. Montagne, I. S. Aranson, and H. Chaté, Collective motion of self-propelled particles with memory, *Phys. Rev. Lett.* **114**, 168001 (2015).
- [20] V. Holubec, D. Geiss, S. A. M. Loos, K. Kroy, and F. Cichos, Finite-size scaling at the edge of disorder in a time-delay Vicsek model, *Phys. Rev. Lett.* **127**, 258001 (2021).
- [21] A. Morin, J.-B. Caussin, C. Eloy, and D. Bartolo, Collective motion with anticipation: Flocking, spinning, and swarming, *Phys. Rev. E* **91**, 012134 (2015).
- [22] A. Costanzo, Milling-induction and milling-destruction in a Vicsek-like binary-mixture model, *Europhys. Lett.* **125**, 20008 (2019).
- [23] D. Gulich, G. Baglietto, and A. F. Rozenfeld, Temporal correlations in the Vicsek model with vectorial noise, *Physica A* **502**, 590 (2018).
- [24] S. Roy, M. J. Shirazi, B. Jantzen, and N. Abaid, Effect of visual and auditory sensing cues on collective behavior in Vicsek models, *Phys. Rev. E* **100**, 062415 (2019).
- [25] Y. Wu, J. Li, D. Chen, X. Li, and H.-T. Zhang, Pattern phase transitions in 3d minimal Vicsek model, *Europhys. Lett.* **134**, 50004 (2021).
- [26] X. Lu, C. Zhang, and B. Qin, An improved Vicsek model of swarm based on remote neighbors strategy, *Physica A* **587**, 126553 (2022).
- [27] F. You, H.-X. Yang, Y. Li, W. Du, and G. Wang, A modified Vicsek model based on the evolutionary game, *Appl. Math. Comput.* **438**, 127565 (2023).
- [28] X. Lei, Y. Xiang, M. Duan, and X. Peng, Exploring the criticality hypothesis using programmable swarm robots with Vicsek-like interactions, *J. R. Soc. Interface.* **20**, 20230176 (2023).
- [29] X. Liu, X. Xiang, Y. Chang, C. Yan, H. Zhou, and D. Tang, Hierarchical weighting Vicsek model for flocking navigation of drones, *Drones* **5**, 74 (2021).
- [30] H. Chaté, F. Ginelli, G. Grégoire, and F. Raynaud, Collective motion of self-propelled particles interacting without cohesion, *Phys. Rev. E* **77**, 046113 (2008).
- [31] A. Cavagna, P. M. Chaikin, D. Levine, S. Martiniani, A. Puglisi, and M. Viale, Vicsek model by time-interlaced compression: A dynamical computable information density, *Phys. Rev. E* **103**, 062141 (2021).
- [32] G. Grégoire and H. Chaté, Onset of collective and cohesive motion, *Phys. Rev. Lett.* **92**, 025702 (2004).
- [33] M. Nagy, I. Daruka, and T. Vicsek, New aspects of the continuous phase transition in the scalar noise model (SNM) of collective motion, *Physica A* **373**, 445 (2007).
- [34] R. Wittkowski, A. Tiribocchi, J. Stenhammar, R. J. Allen, D. Marenduzzo, and M. E. Cates, Scalar φ^4 field theory for active-particle phase separation, *Nat. Commun.* **5**, 4351 (2014).
- [35] E. Tjhung, C. Nardini, and M. E. Cates, Cluster phases and bubbly phase separation in active fluids: Reversal of the Ostwald process, *Phys. Rev. X* **8**, 031080 (2018).
- [36] G. S. Redner, M. F. Hagan, and A. Baskaran, Structure and dynamics of a phase-separating active colloidal fluid, *Phys. Rev. Lett.* **110**, 055701 (2013).
- [37] G. S. Redner, A. Baskaran, and M. F. Hagan, Reentrant phase behavior in active colloids with attraction, *Phys. Rev. E* **88**, 012305 (2013).
- [38] G. S. Redner, C. G. Wagner, A. Baskaran, and M. F. Hagan, Classical nucleation theory description of active colloid assembly, *Phys. Rev. Lett.* **117**, 148002 (2016).
- [39] J. Wang, L. Xu, and E. Wang, Potential landscape and flux framework of nonequilibrium networks: robustness, dissipation, and coherence of biochemical oscillations, *Proc. Natl. Acad. Sci. USA* **105**, 12271 (2008).

- [40] L. Zhu, S. Yang, K. Zhang, H. Wang, X. Fang, and J. Wang, Uncovering underlying physical principles and driving forces of cell differentiation and reprogramming from single-cell transcriptomics, *Proc. Natl. Acad. Sci. USA* **121**, e2401540121 (2024).
- [41] C. Li and J. Wang, Quantifying the landscape for development and cancer from a core cancer stem cell circuit, *Cancer Res.* **75**, 2607 (2015).
- [42] H. Yan, L. Zhao, L. Hu, X. Wang, E. Wang, and J. Wang, Nonequilibrium landscape theory of neural networks, *Proc. Natl. Acad. Sci. USA* **110**, E4185 (2013).
- [43] L. Xu, D. Patterson, A. C. Staver, S. A. Levin, and J. Wang, Unifying deterministic and stochastic ecological dynamics via a landscape-flux approach, *Proc. Natl. Acad. Sci. USA* **118**, e2103779118 (2021).
- [44] C. Li and J. Wang, Landscape and flux reveal a new global view and physical quantification of mammalian cell cycle, *Proc. Natl. Acad. Sci. USA* **111**, 14130 (2014).
- [45] X. Fang, K. Kruse, T. Lu, and J. Wang, Nonequilibrium physics in biology, *Rev. Mod. Phys.* **91**, 045004 (2019).
- [46] J. Wang, Landscape and flux theory of non-equilibrium dynamical systems with application to biology, *Adv. Phys.* **64**, 1 (2015).
- [47] W. Wu and J. Wang, Landscape framework and global stability for stochastic reaction diffusion and general spatially extended systems with intrinsic fluctuations, *J. Phys. Chem. B* **117**, 12908 (2013).
- [48] J. Su, Z. Cao, J. Wang, H. Jiang, and Z. Hou, Dynamical and thermodynamical origins of motility-induced phase separation, *Cell Rep. Phys. Sci.* **5**, 101817 (2024).
- [49] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/fqn3-4qkv> for computational explanations, definitions of parameters, details of methods and approaches employed in this study as well as additional figures.
- [50] J. Wang, K. Zhang, and E. Wang, Kinetic paths, time scale, and underlying landscapes: A path integral framework to study global natures of nonequilibrium systems and networks, *J. Chem. Phys.* **133**, 125103 (2010).