

Toward extracting scattering phase shift from integrated correlation functions on quantum computers

Peng Guo^{*}

*College of Arts and Sciences, Dakota State University, Madison, South Dakota 57042, USA
and Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*



(Received 22 April 2025; accepted 22 October 2025; published 12 November 2025)

Based on an established relation that relates the integrated correlation functions for a trapped system to the infinite volume scattering phase shifts through a weighted integral, we propose to extract the infinite volume scattering phase shifts through quantum simulation of the integrated correlation functions of trapped two-particle systems on quantum computers. The integrated correlation function can be computed by an ancilla-based algorithm. The proposal is demonstrated with a simple contact interaction fermion model.

DOI: [10.1103/PhysRevResearch.7.043164](https://doi.org/10.1103/PhysRevResearch.7.043164)

I. INTRODUCTION

Few-body interaction and scattering processes play a key role in a large variety of physics phenomena ranging from nuclear reactions in nuclear and astrophysics to electron-phonon scattering in material sciences. The determination of few quantum particles' interaction and scattering amplitudes is a fundamental problem in many areas of modern physics, including nuclear, atomic, and condensed matter physics. Numerical simulations of scattering processes of strongly coupled quantum systems, such as strong interaction between quarks and gluons in nuclear physics and strongly correlated electron system in condensed matter physics, remain challenging problems. Classical Monte Carlo simulations suffer a number of fundamental issues, including lack of access to real-time dynamics, critical slowing down [1], infamous sign problems in fermion systems [2,3], and signal-to-noise ratio issues [4,5] in lattice quantum chromodynamics (QCD) calculations of few-nucleon systems. Despite challenges, a lot of progress has been made in recent years on determining and extracting few-hadron interactions in nuclear and hadron physics. Typically, computations based on stochastic evaluation of the path integral are performed in an artificial trap, such as a harmonic oscillator trap in nuclear physics or a periodic finite box in lattice QCD calculation. The discrete low-lying energy spectra are extracted by evaluating the exponential decay of few-body correlation functions in Euclidean space-time. Eigenenergies are then converted into scattering phase shifts by applying the Lüscher formula [6] in periodic boxes or Busch-Englert-Rzażewski-Wilkens (BERW) formula [7] in harmonic oscillator traps (see also extension of such formula in inelastic and three-body channels, e.g., Refs. [8–47]).

Quantum computers have emerged as a promising alternative to simulate quantum systems more efficiently and offer the prospect to overcome some of the limitations in classical simulations. Number of quantum algorithms toward computing two-body scattering on quantum computers has been proposed recently, such as (1) the Lüscher formula-like approach in Ref. [48,49]: extracting eigenenergies by variational quantum eigensolver in quantum computing and then converting them into scattering phase shifts by Lüscher- or BERW-like formula; (2) measuring phase shift by wave packet time delay [50]; and (3) reconstructing scattering amplitudes through the coupling of the particle with an ancillary spin-1/2 [51].

In the present work, we propose the direct computation of the integrated correlation functions of trapped particle systems on quantum computers; then, the scattering phase shifts in infinite volume can be established by the difference of integrated correlation functions between interacting and noninteracting particle systems in the trap through a weighted integral [52–54]. This article is organized as follows: (1) we first summarize the formalism that relates the integrated correlation functions for a particle system in a trap to the infinite volume scattering phase shifts; (2) next, we show how the Hamiltonian of trapped system and correlation functions can be mapped onto quantum circuits that can be evaluated on quantum computers; and (3) finally, the feasibility of the proposed approach is demonstrated with a simple contact interaction fermion model followed by the discussion and summary.

II. TWO-PARTICLE SCATTERING AND INTEGRATED CORRELATION FUNCTION FORMALISM

In this section, we briefly outline and summarize the fundamentals of the integrated correlation function formalism that offers an alternative approach for extracting the infinite volume scattering phase shifts from few-body systems in a trap. The complete technical details can be found in Refs. [52–54].

A simple $1+1$ dimensional nonrelativistic field theory model of spin-1/2 fermion interaction via a contact interaction

^{*}Contact author: peng.guo@dsu.edu

in a trap is used in this work. The Hamiltonian operator of the trapped fermion system is

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \int dx \hat{\psi}_{\sigma}^{\dagger}(x) \left[-\frac{1}{2m} \frac{d^2}{dx^2} + U(x) \right] \hat{\psi}_{\sigma}(x) + V_0 \int dx \hat{\psi}_{\uparrow}^{\dagger}(x) \hat{\psi}_{\downarrow}^{\dagger}(x) \hat{\psi}_{\downarrow}(x) \hat{\psi}_{\uparrow}(x), \quad (1)$$

where $\sigma = \uparrow, \downarrow$ and m refer to the fermion polarizations and mass, respectively, and $\hat{\psi}_{\sigma}(x)$ stands for the fermion field operator. $U(x)$ represents the trap potential and V_0 is the strength of the contact interaction potential between two fermions with opposite polarizations. The commonly used artificial traps include a periodic finite box in lattice QCD and a harmonic oscillator trap in the nuclear physics community.

As illustrated in Refs. [52–54], the infinite volume two-particle elastic scattering phase shift, $\delta(\epsilon)$, can be related to the integrated correlation functions for two particles in a trap through a weighted integral

$$C(t) - C_0(t) \xrightarrow{\text{trap} \rightarrow \infty} \frac{it}{\pi} \int_0^{\infty} d\epsilon \delta(\epsilon) e^{-i\epsilon t}, \quad (2)$$

where $C(t)$ and $C_0(t)$ are integrated correlation functions for two interacting and noninteracting particles in the trap. The difference of integrated trapped two-particle correlation functions converges rapidly to its infinite volume limit that is given by a weighted integral over scattering phase shift at short time.

The integrated forward time propagating two-particle correlation function for nonrelativistic systems is defined through summing over all the modes of two-particle correlation functions along the diagonal,

$$C(t) = \int dr \langle 0 | \hat{\mathcal{O}}(r) e^{-i\hat{H}t} \hat{\mathcal{O}}^{\dagger}(r) | 0 \rangle. \quad (3)$$

Here, $\hat{\mathcal{O}}^{\dagger}(r)$ and $\hat{\mathcal{O}}(r)$ denote creation and annihilation operators to create two particles with relative coordinate of r at time 0 at source, and then annihilate them with relative coordinate of r at later time t at sink, respectively. For instance, the total spin-singlet two spin-1/2 fermion operator after projecting out the center-of-mass motion in a periodic box trap is given by

$$\hat{\mathcal{O}}^{\dagger}(r) = \frac{1}{\sqrt{L}} \int_0^L dx_2 \frac{\psi_{\uparrow}^{\dagger}(r+x_2) \psi_{\downarrow}^{\dagger}(x_2) - \psi_{\downarrow}^{\dagger}(r+x_2) \psi_{\uparrow}^{\dagger}(x_2)}{2}, \quad (4)$$

where L stands for the length of a periodic box and $\hat{\psi}_{\sigma}(x)$ is required to satisfy the periodic boundary condition: $\hat{\psi}_{\sigma}(x+L) = \hat{\psi}_{\sigma}(x)$.

We have also shown in Refs. [52–54] that the energy spectral representation of integrated correlation function is

$$C(t) = \text{Tr}[e^{-i\hat{H}t}] = \sum_{\epsilon} e^{-i\epsilon t}, \quad (5)$$

where the energy spectrum of trapped interacting systems, ϵ , can be determined by Lüscher formulalike quantization conditions.

III. SECOND QUANTIZATION REPRESENTATION OF TRAPPED SYSTEM AND FERMION-TO-QUBIT MAPPING

One of the major advantages of putting particles in a trap is that the Hamiltonian operator of interacting particle systems can be written in the trap basis with discrete energy spectra, which is convenient and suitable for the further mapping onto quantum gate operations in quantum computing.

By expanding field operators in the trap basis,

$$\hat{\psi}_{\sigma}(x) = \sum_n \varphi_n(x) a_{n,\sigma}, \quad \hat{\psi}_{\sigma}^{\dagger}(x) = \sum_n \varphi_n^*(x) a_{n,\sigma}^{\dagger}, \quad (6)$$

where $a_{n,\sigma}$ and $a_{n,\sigma}^{\dagger}$ are the annihilation and creation operators for a noninteracting fermion state with eigenenergy of $\epsilon_n^{(0)}$ in a trap, and $\varphi_n(x)$ are the eigensolutions of Hartree-Fock-like equation,

$$\left[-\frac{1}{2m} \frac{d^2}{dx^2} + U(x) \right] \varphi_n(x) = \epsilon_n^{(0)} \varphi_n(x), \quad (7)$$

the second quantization representation of Hamiltonian of trapped fermion system is thus given by

$$\hat{H} = \sum_{\sigma,n} \epsilon_n^{(0)} a_{\sigma,n}^{\dagger} a_{\sigma,n} + \sum_{n_1,n_2,n'_1,n'_2} V_{n'_1,n'_2;n_1,n_2} a_{\uparrow,n_1}^{\dagger} a_{\downarrow,n_2}^{\dagger} a_{\downarrow,n'_2} a_{\uparrow,n'_1}, \quad (8)$$

where

$$V_{n'_1,n'_2;n_1,n_2} = V_0 \int dx \varphi_{n'_1}^*(x) \varphi_{n'_2}^*(x) \varphi_{n_1}(x) \varphi_{n_2}(x). \quad (9)$$

The second quantization representation of Hamiltonian operator as in Eq. (8) can then be mapped onto qubit quantum registers by applying the Jordan-Wigner transformation (see, e.g., Refs. [55–57]).

Similarly, the two-particle creation and annihilation operators can be expanded in terms of trap basis and then subsequently mapped onto qubit quantum registers by applying the Jordan-Wigner transformation. For instance, with a periodic box trap, the two-particle creation operator in Eq. (4) now has the form:

$$\hat{\mathcal{O}}^{\dagger}(r) = \frac{1}{\sqrt{L}} \sum_{k=\frac{2\pi n}{L}, n \in \mathbb{Z}} e^{-ikr} \frac{a_{k,\uparrow}^{\dagger} a_{-k,\downarrow}^{\dagger} - a_{k,\downarrow}^{\dagger} a_{-k,\uparrow}^{\dagger}}{2}. \quad (10)$$

The integrated correlation functions can be computed on quantum computers by an ancilla-based algorithm proposed in Ref. [55] (also see a nice review article in Ref. [58]). The demonstration of quantum circuit for computing integrated two interacting particles' correlation function $C(t)$ is given in Fig. 1. The quantum circuit for computing integrated two noninteracting particles' correlation function $C_0(t)$ can be constructed in a similar way by turning the interaction off. The scattering phase shifts can then be extracted from simulation results of $C(t) - C_0(t)$ by using the relation in Eq. (2).

IV. PROOF OF PRINCIPLES

In this section, the proposed approach is demonstrated with fermion interaction in a periodic box of size L as a specific

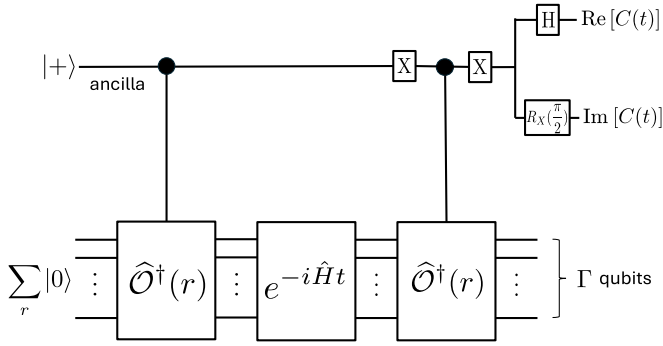


FIG. 1. Demo of quantum circuit for computing the integrated two-particle correlation function $C(t)$.

example. Instead of mapping the trapped system onto quantum registers by Jordan-Wigner transformation, we will show that the mapping and formalism can be simplified drastically by using eigenvector basis of trapped system. With a contact interaction, two-fermion scattering is exactly solvable, and the two-fermion correlation function is given in a simple form that can be checked numerically rather straightforwardly.

A. Quantum circuit in trap basis representation

For a periodic box trap, the trap wave function that satisfies the periodic boundary condition is simply given by plane waves: $\varphi_k(x) = \frac{1}{\sqrt{L}}e^{ikx}$, where $k = \frac{2\pi n}{L}$ and $n \in \mathbb{Z}$ are discrete particle momenta. Two-particle state in center-of-mass frame can thus be defined by

$$|k\rangle = \frac{a_{k,\uparrow}^\dagger a_{-k,\downarrow}^\dagger - a_{k,\downarrow}^\dagger a_{-k,\uparrow}^\dagger}{2} |0\rangle, \quad k = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}, \quad (11)$$

and the Hamiltonian matrix in $|k\rangle$ basis is therefore given by a simple form,

$$\hat{H} = \sum_k 2\epsilon_k^{(0)} |k\rangle\langle k| + \frac{V_0}{L} \sum_{k,k'} |k'\rangle\langle k|, \quad (12)$$

where $\epsilon_k^{(0)} = \frac{k^2}{2m}$. The integrated correlation function in Eq. (3) is now reduced to

$$C(t) = \sum_k \langle k | e^{-i\hat{H}t} | k \rangle. \quad (13)$$

The Hamiltonian matrix $\hat{H}$ can be decomposed by a diagonalizable matrix $\hat{E}$ and a unitary matrix $\hat{S}$:

$$\hat{H} = \hat{S} \hat{E} \hat{S}^\dagger, \quad (14)$$

and using identity $e^{-i\hat{H}t} = \hat{S} e^{-i\hat{E}t} \hat{S}^\dagger$, Eq. (5) is thus recovered.

The infinite volume scattering phase shift for a contact interaction is given by (see, e.g., Ref. [52])

$$\delta(\epsilon) = \cot^{-1} \left(-\frac{\sqrt{2\mu\epsilon}}{\mu V_0} \right), \quad (15)$$

where $\mu = \frac{m}{2}$ is the reduced mass of two-fermion system. Hence, the analytic expression of the right-hand side of

Eq. (2) can be obtained,

$$C(t) - C_0(t) \xrightarrow{L \rightarrow \infty} \frac{1}{2} \operatorname{erfc} \left(\mu V_0 \sqrt{\frac{it}{2\mu}} \right) e^{(\mu V_0)^2 \frac{it}{2\mu}} - \frac{1}{2}. \quad (16)$$

Instead of using Jordan-Wigner transformation, the discrete two-particle state vector $|k\rangle$'s can be mapped directly onto Γ quantum registers. The integrated correlation function in Eq. (13) can be rewritten as

$$C(t) = \sum_{k=\frac{2\pi n}{L}}^{n \in [-N, N]} \langle 0 | \hat{P}_k e^{-i\hat{H}t} \hat{P}_k^\dagger | 0 \rangle, \quad (17)$$

where $N = \frac{2\Gamma-1}{2}$ is the cutoff of state vector $|k\rangle$'s and the projection operator $\hat{P}_k^\dagger$ is defined by

$$\hat{P}_k^\dagger = |k\rangle\langle 0|. \quad (18)$$

Equation (17) can thus be computed by using quantum circuit in Fig. 1. The unitary time evolution of the kinematic term in the Hamiltonian matrix in Eq. (12) is simply given by a diagonal matrix,

$$\langle k' | e^{-i\hat{H}_0 t} | k \rangle = \delta_{k,k'} e^{-2i\epsilon_k^{(0)} t}. \quad (19)$$

The second potential term of the Hamiltonian matrix in Eq. (12), $\hat{H}_V$, is a constant matrix; hence, we find

$$\langle k' | e^{-i\hat{H}_V t} | k \rangle = \begin{cases} \frac{e^{-i(2N+1)\frac{V_0}{L}t} + 2N}{2N+1}, & \text{if } k = k', \\ \frac{e^{-i(2N+1)\frac{V_0}{L}t} - 1}{2N+1}, & \text{otherwise.} \end{cases} \quad (20)$$

With Γ quantum registers, the quantum circuit of $e^{-i\hat{H}_V t}$ is given by

$$\begin{aligned} e^{-i\hat{H}_V t} &= \frac{e^{-i(2N+1)\frac{V_0}{L}t} + 2N}{2N+1} I_{\Gamma-1} \otimes \cdots \otimes I_1 \otimes I_0 \\ &+ \frac{e^{-i(2N+1)\frac{V_0}{L}t} - 1}{2N+1} I_{\Gamma-1} \otimes \cdots \otimes I_1 \otimes X_0 \\ &+ \cdots + \frac{e^{-i(2N+1)\frac{V_0}{L}t} - 1}{2N+1} X_{\Gamma-1} \otimes \cdots \otimes I_1 \otimes I_0 \\ &+ \frac{e^{-i(2N+1)\frac{V_0}{L}t} - 1}{2N+1} I_{\Gamma-1} \otimes \cdots \otimes X_1 \otimes X_0 + \cdots \\ &+ \frac{e^{-i(2N+1)\frac{V_0}{L}t} - 1}{2N+1} X_{\Gamma-1} \otimes \cdots \otimes X_1 \otimes X_0. \end{aligned} \quad (21)$$

The sum includes all the possible single X -gate insertions, two X -gate insertions, etc., up to Γ X -gate insertions. The unitary time evolution of the total Hamiltonian matrix can then be computed via Trotterization approximation [59,60]; e.g., the lowest order of Trotterization is given by

$$e^{-i\hat{H}\delta t} \xrightarrow{\delta t \rightarrow 0} e^{-i\hat{H}_0 \delta t} e^{-i\hat{H}_V \delta t}. \quad (22)$$

B. Data processing in real-time simulations

The difference of integrated correlation functions, $\Delta C(t) = C(t) - C_0(t)$, in the real-time simulation is a fast oscillating function around its infinite volume limit (see

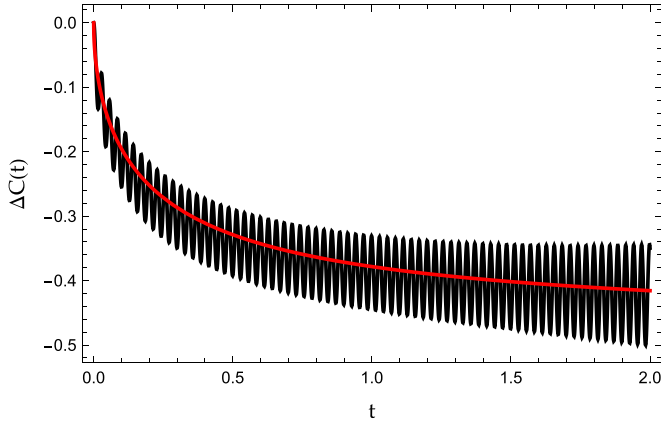


FIG. 2. Demo plots of real part of $\Delta C(t)$ (black) vs its infinite volume limit given by the right-hand side of Eq. (16) (red) with parameters $V_0 = 2.5$, $\mu = 1$, $L = 90$, and $N = 300$.

demo plot in Fig. 2). The amplitude of the fast oscillating function of $\Delta C(t)$ decreases slightly as momenta cutoff N is increased. Meanwhile, the frequency of oscillating motion increases as N is increased, where the frequency of oscillation motion is proportional to momentum cutoff $\frac{2\pi N}{L}$. We remark that the oscillatory behavior of the correlation function for a finite volume system with periodic boundary condition is in fact a quite well-known effect in nuclear physics, especially in finite volume scattering problems. For instance, the finite volume Green's function is typically periodic oscillating function compared with a smooth function of its infinite volume counterpart (see, e.g., discussion in Ref. [33]). The infinite limit of some finite volume quantities, such as Green's function and correlation function, can only be achieved with some careful mathematical manipulations, such as analytical continuation of finite volume Green's function into complex energy plane (see, e.g., Ref. [33]). The oscillating behavior can also be smoothed out via further data processing by averaging out $\Delta C(t)$ over a short period of time that is larger than the oscillation period (see, e.g., averaged tunneling time in a finite system as the size of the system is increased in Ref. [61]). Assuming that the time of $\Delta C(t)$ is measured up to t_0 , let us split the range of measurement up to N_t segment of size of $\Delta t = \frac{t_0}{N_t}$, where $\Delta t \gg \frac{L}{2\pi N}$, the average $\Delta C(t)$ in i th segment can be computed by

$$\Delta C_{\text{avg}}(t_i) = \frac{1}{\Delta t} \int_{t_{i-1}}^{t_i} [C(t) - C_0(t)] dt, \quad (23)$$

where $i \in [1, N_t]$ and $t_i = (i - \frac{1}{2})\Delta t$ sits at the center of i th segment. The demo of $\Delta C_{\text{avg}}(t_i)$ versus its infinite volume limit is shown in Fig. 3.

To extract the scattering phase shifts, the scattering phase shift must first be modeled and parametrized in terms of a few parameters and kinematic factors based on either chiral

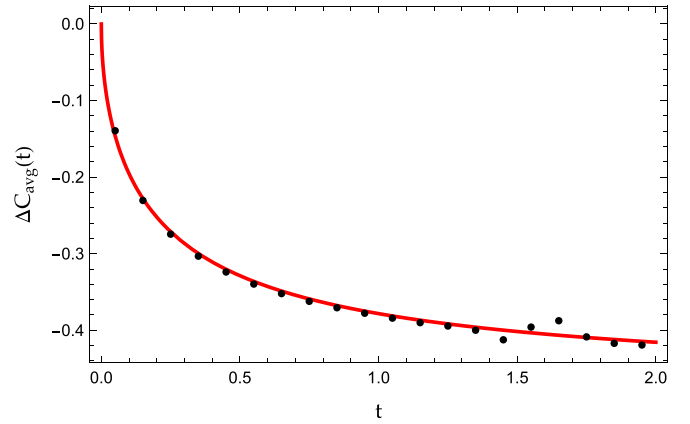


FIG. 3. Demo plots of real part of $\Delta C_{\text{avg}}(t)$ (black) vs its infinite volume limit given by the right-hand side of Eq. (16) (red) with parameters $V_0 = 2.5$, $\mu = 1$, $L = 90$, $N = 1000$, $t_0 = 2$, and $N_t = 20$.

perturbation theory or K -matrix formalism, etc. The phase shift can then be obtained by fitting quantum simulation data of integrated correlation functions with these free parameters through an integral.

V. SUMMARY AND OUTLOOK

In summary, we propose to extract and determine the infinite volume scattering phase shifts through quantum simulation of the integrated correlation functions of few-particle systems in a trap on quantum computers. The formalism is based on the relation in Eq. (2) that relates the integrated correlation functions for a trapped system to the infinite volume scattering phase shifts through a weighted integral in Refs. [52–54]. The Hamiltonian and correlation functions of a two-particle system in the trap basis can be easily mapped onto quantum registers and are suitable and convenient for quantum simulation. The integrated correlation function can then be computed by an ancilla-based algorithm proposed in Ref. [55] (see also Fig. 1). The proposal is demonstrated with a simple contact interaction fermion model that yields the analytic solutions in finite volume, and the further data processing to smooth out the fast oscillating behavior in real-time simulation is discussed.

ACKNOWLEDGMENTS

This research is supported by the U.S. National Science Foundation under Grant No. PHY-2418937 and in part by National Science Foundation under Grant No. PHY-2309135 to the Kavli Institute for Theoretical Physics (KITP).

DATA AVAILABILITY

No data were created or analyzed in this study.

[1] S. Schaefer, R. Sommer, and F. Virotta (ALPHA Collaboration), Critical slowing down and error analysis in lattice QCD simulations, *Nucl. Phys. B* **845**, 93 (2011).

[2] E. Y. Loh, J. E. Gubernatis, R. T. Scalettar, S. R. White, D. J. Scalapino, and R. L. Sugar, Sign problem in the numerical simulation of many-electron systems, *Phys. Rev. B* **41**, 9301 (1990).

- [3] P. de Forcrand, Simulating QCD at finite density, in *Proceedings of the XXVII International Symposium on Lattice Field Theory (LAT2009) Beijing, P.R. China, July 26-31, 2009* (PoS LAT2009, Beijing, 2009).
- [4] G. Peter Lepage, The analysis of algorithms for lattice field theory, in *Proceedings of the Boulder Advanced Study Institute (ASI)* (CERN, Boulder, CO, USA, 1989), pp. 97–120.
- [5] C. Drischler, W. Haxton, K. McElvain, E. Mereghetti, A. Nicholson, P. Vranas, and A. Walker-Loud, Towards grounding nuclear physics in QCD, *Prog. Part. Nucl. Phys.* **121**, 103888 (2021).
- [6] M. Lüscher, Two particle states on a torus and their relation to the scattering matrix, *Nucl. Phys. B* **354**, 531 (1991).
- [7] T. Busch, B.-G. Englert, K. Rzazewski, and M. Wilkens, Two cold atoms in a harmonic trap, *Found. Phys.* **28**, 549–559 (1998).
- [8] K. Rummukainen and S. A. Gottlieb, Resonance scattering phase shifts on a nonrest frame lattice, *Nucl. Phys. B* **450**, 397 (1995).
- [9] N. H. Christ, C. Kim, and T. Yamazaki, Finite volume corrections to the two-particle decay of states with non-zero momentum, *Phys. Rev. D* **72**, 114506 (2005).
- [10] V. Bernard, M. Lage, U.-G. Meißner, and A. Rusetsky, Resonance properties from the finite-volume energy spectrum, *J. High Energy Phys.* **08** (2008) 024.
- [11] S. He, X. Feng, and C. Liu, Two particle states and the s-matrix elements in multi-channel scattering, *J. High Energy Phys.* **07** (2005) 011.
- [12] M. Lage, U.-G. Meißner, and A. Rusetsky, A method to measure the antikaon-nucleon scattering length in lattice QCD, *Phys. Lett. B* **681**, 439 (2009).
- [13] M. Döring, U.-G. Meißner, E. Oset, and A. Rusetsky, Unitarized chiral perturbation theory in a finite volume: Scalar meson sector, *Eur. Phys. J. A* **47**, 139 (2011).
- [14] P. Guo, J. Dudek, R. Edwards, and A. P. Szczepaniak, Coupled-channel scattering on a torus, *Phys. Rev. D* **88**, 014501 (2013).
- [15] P. Guo, Coupled-channel scattering in 1 + 1 dimensional lattice model, *Phys. Rev. D* **88**, 014507 (2013).
- [16] S. Kreuzer and H. W. Hammer, Efimov physics in a finite volume, *Phys. Lett. B* **673**, 260 (2009).
- [17] K. Polejaeva and A. Rusetsky, Three particles in a finite volume, *Eur. Phys. J. A* **48**, 67 (2012).
- [18] M. T. Hansen and S. R. Sharpe, Relativistic, model-independent, three-particle quantization condition, *Phys. Rev. D* **90**, 116003 (2014).
- [19] M. Mai and M. Döring, Three-body unitarity in the finite volume, *Eur. Phys. J. A* **53**, 240 (2017).
- [20] M. Mai and M. Döring, Finite-volume spectrum of $\pi^+\pi^+$ and $\pi^+\pi^+\pi^+$ systems, *Phys. Rev. Lett.* **122**, 062503 (2019).
- [21] M. Döring, H. W. Hammer, M. Mai, J. Y. Pang, A. Rusetsky, and J. Wu, Three-body spectrum in a finite volume: The role of cubic symmetry, *Phys. Rev. D* **97**, 114508 (2018).
- [22] P. Guo, One spatial dimensional finite volume three-body interaction for a short-range potential, *Phys. Rev. D* **95**, 054508 (2017).
- [23] P. Guo and V. Gasparian, A solvable three-body model in finite volume, *Phys. Lett. B* **774**, 441 (2017).
- [24] P. Guo and V. Gasparian, Numerical approach for finite volume three-body interaction, *Phys. Rev. D* **97**, 014504 (2018).
- [25] P. Guo and T. Morris, Multiple-particle interaction in (1 + 1)-dimensional lattice model, *Phys. Rev. D* **99**, 014501 (2019).
- [26] M. Mai, M. Döring, C. Culver, and A. Alexandru, Three-body unitarity versus finite-volume $\pi^+\pi^+\pi^+$ spectrum from lattice QCD, *Phys. Rev. D* **101**, 054510 (2020).
- [27] P. Guo, M. Döring, and A. P. Szczepaniak, Variational approach to N -body interactions in finite volume, *Phys. Rev. D* **98**, 094502 (2018).
- [28] P. Guo, Propagation of particles on a torus, *Phys. Lett. B* **804**, 135370 (2020).
- [29] P. Guo and M. Döring, Lattice model of heavy-light three-body system, *Phys. Rev. D* **101**, 034501 (2020).
- [30] P. Guo, Threshold expansion formula of N bosons in a finite volume from a variational approach, *Phys. Rev. D* **101**, 054512 (2020).
- [31] P. Guo and B. Long, Multi- π^+ systems in a finite volume, *Phys. Rev. D* **101**, 094510 (2020).
- [32] P. Guo, Myth of scattering in finite volume, [arXiv:2007.04473](https://arxiv.org/abs/2007.04473) [hep-lat].
- [33] P. Guo and B. Long, Visualizing resonances in finite volume, *Phys. Rev. D* **102**, 074508 (2020).
- [34] P. Guo, Modeling few-body resonances in finite volume, *Phys. Rev. D* **102**, 054514 (2020).
- [35] P. Guo and V. Gasparian, Charged particles interaction in both a finite volume and a uniform magnetic field, *Phys. Rev. D* **103**, 094520 (2021).
- [36] P. Guo and B. Long, Nuclear reactions in artificial traps, *J. Phys. G: Nucl. Part. Phys.* **49**, 055104 (2022).
- [37] P. Guo, Coulomb corrections to two-particle interactions in artificial traps, *Phys. Rev. C* **103**, 064611 (2021).
- [38] P. Guo and V. Gasparian, Charged particles interaction in both a finite volume and a uniform magnetic field II: Topological and analytic properties of a magnetic system, *J. Phys. A* **55**, 265201 (2022).
- [39] I. Stetcu, B. R. Barrett, U. van Kolck, and J. P. Vary, Effective theory for trapped few-fermion systems, *Phys. Rev. A* **76**, 063613 (2007).
- [40] I. Stetcu, J. Rotureau, B. R. Barrett, and U. van Kolck, An effective field theory approach to two trapped particles, *Ann. Phys.* **325**, 1644 (2010).
- [41] J. Rotureau, I. Stetcu, B. R. Barrett, M. C. Birse, and U. van Kolck, Three and four harmonically trapped particles in an effective field theory framework, *Phys. Rev. A* **82**, 032711 (2010).
- [42] J. Rotureau, I. Stetcu, B. R. Barrett, and U. van Kolck, Two and three nucleons in a trap and the continuum limit, *Phys. Rev. C* **85**, 034003 (2012).
- [43] T. Luu, M. J. Savage, A. Schwenk, and J. P. Vary, Nucleon-nucleon scattering in a harmonic potential, *Phys. Rev. C* **82**, 034003 (2010).
- [44] C.-J. Yang, Chiral potential renormalized in harmonic-oscillator space, *Phys. Rev. C* **94**, 064004 (2016).
- [45] C. W. Johnson *et al.*, From bound states to the continuum, *J. Phys. G: Nucl. Part. Phys.* **47**, 123001 (2020).
- [46] X. Zhang, Extracting free-space observables from trapped interacting clusters, *Phys. Rev. C* **101**, 051602 (2020).
- [47] X. Zhang, S. R. Stroberg, P. Navrátil, C. Gwak, J. A. Melendez, R. J. Furnstahl, and J. D. Holt, *Ab initio* calculations of low-energy nuclear scattering using a generalized Lüscher method, *Phys. Rev. Lett.* **125**, 112503 (2020).

- [48] P. Wang, W. Du, W. Zuo, and J. P. Vary, Nuclear scattering via quantum computing, *Phys. Rev. C* **109**, 064623 (2024).
- [49] F. Turro, K. A. Wendt, S. Quaglioni, F. Pederiva, and A. Roggero, Evaluation of phase shifts for nonrelativistic elastic scattering using quantum computers, *Phys. Rev. C* **110**, 054604 (2024).
- [50] E. Gustafson, Y. Zhu, P. Dreher, N. M. Linke, and Y. Meurice, Real-time quantum calculations of phase shifts using wave packet time delays, *Phys. Rev. D* **104**, 054507 (2021).
- [51] G. Mussardo, A. Stampiggi, and A. Trombettoni, Reflection and transmission amplitudes in a digital quantum simulation, *EPJ Quantum Technol.* **11**, 65 (2024).
- [52] P. Guo and V. Gasparian, Toward extracting the scattering phase shift from integrated correlation functions, *Phys. Rev. D* **108**, 074504 (2023).
- [53] P. Guo, Toward extracting the scattering phase shift from integrated correlation functions. II. A relativistic lattice field theory model, *Phys. Rev. D* **110**, 014504 (2024).
- [54] P. Guo and F. X. Lee, Toward extracting scattering phase shift from integrated correlation functions. III. Coupled channels, *Phys. Rev. D* **111**, 054506 (2025).
- [55] G. Ortiz, J. E. Gubernatis, E. Knill, and R. Laflamme, Quantum algorithms for fermionic simulations, *Phys. Rev. A* **64**, 022319 (2001).
- [56] R. Somma, G. Ortiz, J. E. Gubernatis, E. Knill, and R. Laflamme, Simulating physical phenomena by quantum networks, *Phys. Rev. A* **65**, 042323 (2002).
- [57] P. Jordan and E. Wigner, Über das paulische äquivalenzverbot, *Z. Phys.* **47**, 631 (1928).
- [58] F. Tacchino, A. Chiesa, S. Carretta, and D. Gerace, Quantum computers as universal quantum simulators: State-of-the-art and perspectives, *Adv. Quantum Technol.* **3**, 1900052 (2020).
- [59] H. F. Trotter, On the product of semi-groups of operators, *Proc. Am. Math. Soc.* **10**, 545 (1959).
- [60] N. Hatano and M. Suzuki, Finding exponential product formulas of higher orders, *Lect. Notes Phys.* **679**, 37 (2005).
- [61] P. Guo, V. Gasparian, E. Jódar, and C. Wischart, Tunneling time in PT-symmetric systems, *Phys. Rev. A* **107**, 032210 (2023).