

# Self-contained power optimization for wave energy conversion: A step change in generation

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Wave energy holds immense potential, yet to date progress has been hampered by substantial design challenges and perceived high levelized cost of electricity (LCoE). Control strategies could be vital for improving the economic viability of wave energy converters (WECs), yet practical implementations demonstrating performance remain limited. Here, we present, and implement experimentally for the first time, a self-sufficient control system requiring no external measurements. It is physically demonstrated for the complex hinged, Moored, MultiMode, Multifloat WEC, called M4. The controller operates within a linear noncausal framework to optimize generator torque, where excitation forces are estimated and predicted autoregressively. A real-time state-space hydrodynamic model, based on the Cummins-type linear diffraction-radiation formulation, enables operation based solely on motor-generator torque-speed data. In a first-of-its-kind wave basin experiment, the control strategy is shown to over double energy capture (up to a 125% increase) whilst also demonstrating performance in both unidirectional and directionally spread irregular wave conditions, mimicking real ocean complexity. Site-specific assessment across four European wave sites demonstrates substantial implications for system design and performance: The optimal device size is approximately 40% less than that defined by the average peak period of a site, as often used, and with control implemented, the average power per unit mass (approximately proportional to the inverse of LCoE) increased by about 100% across the four sites. This represents a step change toward the commercial viability of wave energy.

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## I. INTRODUCTION

The potential for wave energy is vast [1], with early estimates indicating that the UK alone has between 30 and 100 GW of available mean power [2,3]. More recent studies suggest that the global wave energy resource exceeds 2 TW [4], making it comparable to offshore wind. Wave energy also has the advantage of greater availability at about 90% compared with 40% for offshore wind [5]. There is therefore high complementarity between offshore wind and wave power generation systems, and substantial collocation potential, which has the potential to greatly reduce the required total installed capacity to match demand [6].

To extract the vast available wave resource, there have been many designs of wave energy converters (WECs), dating back to the 1800s [7]. Several pioneering WEC concepts were developed in the 1970s (e.g., Refs. [8,9]) during the oil crisis [10]. Despite these, and subsequent important WEC

developments (e.g., Refs. [11,12]), wave energy remains an almost completely untapped resource. This is largely due to remaining substantial design challenges, particularly around survivability in extremes, power take-off and mooring design, along with the perceived high but currently unknown levelized cost of electricity (LCoE). Recent concepts demonstrate promise at overcoming these challenges, including the Moored, MultiMode, Multifloat (M4) wave energy converter, designed and developed at the University of Manchester. The M4 system offers  $\mathcal{O}(\text{MW})$  scale power production [13] with strong survivability characteristics [14] and significant recent developments in reducing mooring line loads [15,16] as well as realistic power take-off (PTO) system developments [17]. The system performance data are widely and openly available and an 1:4 scale version has recently been deployed at sea under open-access data principles [18,19]. As such, the M4 system is used in the current work, representing a complex yet highly promising, active, and relevant system, where control improvements have the potential to substantially increase power performance and reduce LCoE.

WECs must be designed for the realistic ocean environment in which they must operate, which is highly variable [20], complex, and at times extremely hostile. Realistic wave conditions are irregular and directionally spread also known as short-crested (often multimodal [21]) particularly in deeper waters where the wave energy is greatest. These variable “sea states” are typically characterized by the statistical properties of significant waveheight  $H_{m0}$  and peak period  $T_p$  (or alternative measures of wave period) along with parameters describing the frequency and directional bandwidth of the spectrum. Scaled experimental testing in wave basins plays

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an important role in the design, testing, and de-risking of wave energy converter designs in realistic environments, enabling assessment of performance and response in a controlled setting for a fraction of the cost of sea trials. These test campaigns use the principle of Froude similitude, using appropriately scaled WEC models and wave conditions, and will typically aim to cover the equivalent range of  $H_{m0}$ - $T_p$  combinations expected at full scale. For the M4 system, tank testing has been used to validate numerical models, iterate, improve (and prove) designs, reviewed in Ref. [22], and more recently focus on the reduction of mooring line loads in realistic conditions including directional wave conditions [15,16,23].

In addition to device design, control strategies are widely acknowledged for their potential to increase the power conversion efficiency of WECs [24]. Initially, control was introduced to maximize wave energy extraction by tuning mechanical components so that the WECs' resonant frequency matches the frequency of the incoming waves [9], which is analogous to the impedance principle used in the electrical circuit analysis. Early control strategies are essentially based on this intuitive concept, e.g., complex-conjugate control [25], latching/declutching control [26], phase control [27], reactive control [28], etc., and have been extended to arrays of WECs [29]. However, since the frequency of ocean waves is constantly varying, achieving control optimality relies on accurate real-time prediction to meet the noncausal optimality condition [30]. In recent years, advanced strategies are being extensively studied for WEC control, which are formulated as energy maximization problems subject to persistent wave excitation forces and constraints for safe operation [31,32]. The theoretical efficacies of some advanced control methods have been demonstrated by numerical simulations. Linear noncausal optimal control (LNOC) [33] has been applied to the multifloat M4 WEC both assuming knowledge of future wave conditions [34] and using autoregression (AR) for prediction of excitation forces [35,36] successfully improving energy capture by up to 100%. The accuracy of a linear hydrodynamic model is implicit in such tests and, while this has been shown to be successful without control for M4 [13,37], control increases motion thus increasing nonlinearity [38]; wave conditions are also assumed to be linear. It is thus essential to test control strategies in laboratory wave basins representing real-world nonlinearity and complexity before time consuming and costly sea trials are undertaken. LNOC has already been demonstrated for the Mocean attenuator-type device using measured wave data [39–41]. However, this approach has the obvious disadvantage over AR in requiring extra wave measurement hardware (expensive in ocean conditions), and wave prediction accuracy is particularly limited in realistic complex sea states with spread and crossing waves [42,43]. There is thus a gap in the development and experimental validation of standalone WEC control systems for realistic conditions.

We describe major progress on WEC control through the following key aspects: (1) Using the Hydrodynamics Laboratory at the University of Manchester, we successfully validated significant improvement in control performance of a standalone control algorithm. To achieve this, the wave force prediction must be estimated solely from the generator torque-speed measurements. In this work, excitation

wave force estimation is based on a Kalman filter with a harmonic oscillator (KFHO) model [44] with a lumped state-space representation, and future excitation is predicted using autoregression [45]. This combination, termed LNOC-AR, represents an approach by the authors that will be shown to be effective. (2) The experiments were conducted with waves representing realistic sea conditions involving irregular and directionally spread waves not previously tested with control. (3) This is applied to the 1-2-2 (1 bow float, 2 mid floats, and 2 stern floats) M4 system, which represents a challenging WEC controller design and real-time control experiment with two power take-offs. Application to this problem demonstrates broad applicability for multifloat and multi-PTO systems. (4) The experimental results are then scaled up according to Froude similitude for four typical sea sites across the Europe, showing the significant impact of the advanced optimal control on reducing the LCoE, enhancing potential of wave energy commercialization.

The rest of the paper is laid out as follows. Section II introduces the M4 WEC's control-oriented modeling and the theoretical formulation of the LNOC-AR algorithm. Section III presents the experimental setup in the University of Manchester Hydrodynamics Laboratory. Experimental results assessing the controller performance against an optimized linear damper are reported in Sec. IV. Section V assesses the expected results across four different European wave climates to assess full-scale implications of control on performance and design. Section VI discusses some limitations of this study, leading to a few promising future directions. Conclusions are made in Sec. VII.

## II. WEC MODEL AND CONTROL ALGORITHMS

### A. Hydrodynamic modeling

Figure 1 shows a schematic diagram of the M4 WEC considered in this work. There are, in total, five floats, one bow float connected rigidly with two middle floats and two stern floats connected separately on the other side. When moored, from a single point mooring, the platform heads naturally into wave direction due to wave drift forces. As waves propagate through the device, motions of the bow, mid, and stern floats are out of phase to varying degrees, which causes the device to rotate about the hinges. PTOs are placed on top of middle floats to generate electricity from the rotational motion.

The LNOC-AR algorithm is a model-based control method, so a dynamic model of the M4 WEC is required. In this work, a time domain linear diffraction model is developed for the M4 WEC, based on the Fortran code OREGEN\_X [37], and represented in state-space form as required for control. The modeling procedure is briefly reported here. Readers are referred to Ref. [34] for more details. There are, in total, eight degrees of freedom (DOFs) for the 1-2-2 M4 WEC due to the hinge constraint. They are the surge, sway, heave, roll, and yaw motions of the whole body and pitch motions of the bow-mid float frame as well as those of two independent stern floats. However, wave power is mainly captured by motions in surge, heave, and pitch. Thus, the problem can be simplified and only DOFs related to power capture are considered. The motion vector can be chosen as  $q = [x_0 \ z_0 \ \theta_1 \ \theta_2]^T$ , where  $x_0$

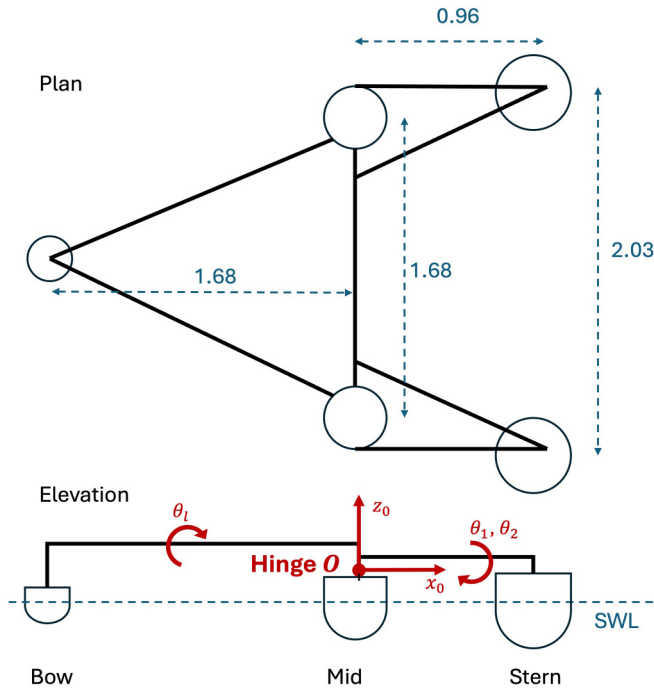


FIG. 1. A diagrammatic representation of 1:40 scale M4 1-2-2 WEC system in both plan and elevation views. SWL: still water line. Units are in meters.

and  $z_0$  are the surge and heave motions of the reference point,  $\theta_l$  is the pitch of the bow-mid float frame, and  $\theta_1$  and  $\theta_2$  are the pitch motions of the two stern floats, as shown in Fig. 1.

Based on Newton's second law of motion, the equation of motion for the M4 WEC in the time domain is

$$(M + m_\infty)\ddot{q}(t) = f_{e,q}(t) + f_{rd,q}(t) + f_{rs,q}(t) + f_{pto,q}(t), \quad (1)$$

where  $t$  is time,  $M$  is the  $5 \times 5$  mass and inertia matrix,  $m_\infty$  is the  $5 \times 5$  added mass matrix derived at infinite frequency within Cummins's formulation for radiation damping (described below),  $f_{e,q}(t)$  is the wave excitation force,  $f_{rd,q}(t)$  is the radiation damping force,  $f_{rs,q}(t)$  is the hydrostatic restoring force, and  $f_{pto,q}(t)$  is the PTO torque. Note that mooring force is not modeled for simplification but a small stiffness term is added to the surge to prevent model drift.

For the model to be run in real time, we are required to assume linear wave theory. It is to be noted that application of linear hydrodynamics in control has been criticized [38] since this often increases response and therefore nonlinearity. The response of the M4, however, has been shown to essentially linear even in steep waves [13,14,37], and we will demonstrate in this paper the effectiveness of this assumption. Under these assumptions of linear theory, the wave excitation force is independent of the WEC motion  $q$ . It can be calculated using

$$f_{e,q}(t) = \text{Re} \left\{ \sum_{n=1}^N H(\omega_n) F_{ex}(\omega_n) e^{i\omega_n t} \right\}, \quad (2)$$

where  $H(\omega_n)$  is the complex wave spectrum and  $F_{ex}(\omega_n)$  is the frequency-dependent excitation force per unit wave amplitude.  $N$  is the number of total frequencies considered and in this case  $N = 200$ . This equation is used in numerical

simulations to generate the wave excitation force but is not used for real-time control in experiments. In experiments, the excitation forces are obtained by an observer that will be discussed in Sec. IIC.

The radiation force in the time domain is modeled using Cummins's method [46] with

$$f_{rd,q}(t) = - \int_{-\infty}^t F_{rd}(t - \tau) \dot{q}(\tau) d\tau, \quad (3)$$

where  $F_{rd}$  is a  $5 \times 5$  matrix with an impulse response function (IRF) in each entry. The length of the IRFs is set to 8 s thus capturing at least four peak periods for all conditions (see Table I), while modeling has shown little effect of reducing this to two. To derive a state-space representation, these convolutions are replaced by a state-space model. The radiation damping state-space representation can be written as

$$\begin{aligned} \dot{z}_s &= A_s z_s + B_s \dot{q}(t), \\ f_{rd,q}(t) &= C_s z_s + D_s \dot{q}(t), \end{aligned} \quad (4)$$

where  $z_s$  is the auxiliary state variable of the identified state-space subsystem, which has no physical meaning. The size of the state matrices  $A_s$ ,  $B_s$ ,  $C_s$ , and  $D_s$  can be chosen manually. Note that the frequency-dependent excitation force, added mass, and radiation damping coefficients were computed from the boundary element method solver OREGEN\_BEM [47], checked against the commonly used software WAMIT [48].

The hydrostatic restoring force can be calculated using

$$f_{rs,q}(t) = -Kq(t), \quad (5)$$

where  $K$  is the  $5 \times 5$  hydrostatic restoring matrix.

Equation (1) can now be rewritten as

$$\begin{aligned} (M + m_\infty)\ddot{q}(t) - f_{rd,q}(t) + Kq(t) &= f_{e,q}(t) + f_{pto,q}(t), \\ \dot{z}_s &= A_s z_s + B_s \dot{q}(t), \\ f_{rd,q}(t) &= C_s z_s + D_s \dot{q}(t). \end{aligned} \quad (6)$$

As a benchmark for LNOC-AR, the PTO torque  $f_{pto,q}$  is modeled as a linear damper,

$$f_{pto,q}(t) = \begin{bmatrix} 0 \\ 0 \\ -\tau_{pto1}(t) - \tau_{pto2}(t) \\ \tau_{pto1}(t) \\ \tau_{pto2}(t) \end{bmatrix}, \quad (7)$$

with torque  $\tau_{pto1} = -B_{mech}(\dot{\theta}_l - \dot{\theta}_1)$  and  $\tau_{pto2} = -B_{mech}(\dot{\theta}_l - \dot{\theta}_2)$ , where  $B_{mech}$  is a constant damping gain identical for each PTO and  $(\dot{\theta}_l - \dot{\theta}_1)$ ,  $(\dot{\theta}_l - \dot{\theta}_2)$  are the angular velocities of the relative pitch motions. The value of  $B_{mech}$  is optimally tuned for benchmark comparison. A torque limit  $\tau_{max} = 10$  Nm is implemented as in Eq. (8) to protect the mechanical couplings on the hinge. This applies to both controllers.

$$\tau_{pto} = \begin{cases} -\tau_{max}, & \text{for } \tau_{pto} < -\tau_{max}, \\ \tau_{pto}, & \text{for } -\tau_{max} \leq \tau_{pto} \leq \tau_{max}, \\ \tau_{max}, & \text{for } \tau_{pto} > \tau_{max}. \end{cases} \quad (8)$$

With active control,  $\tau_{pto1}$  and  $\tau_{pto2}$  are calculated using the LNOC-AR algorithm. By defining a state vector  $x := [q, \dot{q}, z_s]^T$ , the final state-space representation of the M4 WEC

TABLE I. Irregular sea states used in the experiment.

| Sea states | $T_p$ (s) | Target $H_{m0}$ (m) | $\frac{H_{m0}k_p}{2}$ | $s$ ( $\sigma_\theta$ ) |
|------------|-----------|---------------------|-----------------------|-------------------------|
| U101–U111  | 0.8–1.8   | 0.03                | 0.094–0.022           | $\infty$ ( $0^\circ$ )  |
| U201–U211  | 0.8–1.8   | 0.05                | 0.16–0.037            | $\infty$ ( $0^\circ$ )  |
| U301–U311  | 0.8–1.8   | 0.07                | 0.22–0.052            | $\infty$ ( $0^\circ$ )  |
| S001       | 1.3       | 0.05                | 0.062                 | 100 ( $8.06^\circ$ )    |
| S002       | 1.3       | 0.05                | 0.062                 | 25 ( $15.9^\circ$ )     |
| S003       | 1.5       | 0.05                | 0.049                 | 100 ( $8.06^\circ$ )    |
| S004       | 1.5       | 0.05                | 0.049                 | 25 ( $15.9^\circ$ )     |

can be written as

$$\dot{x} = A_c x + B_{w,c} f_{e,q} + B_{u,c} f_{pto,q}, \quad y = C_c x, \quad (9)$$

where the system matrices are

$$A_c = \begin{bmatrix} 0_{5 \times 5} & I_{5 \times 5} & 0_{5 \times n} \\ -(M + m_\infty)^{-1} K & -(M + m_\infty)^{-1} D_s & -(M + m_\infty)^{-1} C_s \\ 0_{n \times 5} & B_s & A_s \end{bmatrix}, \quad (10)$$

$$B_{w,c} = \begin{bmatrix} 0_{5 \times 5} \\ (M + m_\infty)^{-1} \\ 0_{n \times 5} \end{bmatrix}, \quad (11)$$

$$B_{u,c} = \begin{bmatrix} 0_{5 \times 1} \\ (M + m_\infty)^{-1} [0, 0, -1, 1, 1]^\top \\ 0_{n \times 1} \end{bmatrix}, \quad (12)$$

$$C_c = \begin{bmatrix} 0_{1 \times 5} & [0 \ 0 \ 1 \ -1 \ 0] & 0_{1 \times n} \\ 0_{1 \times 5} & [0 \ 0 \ 1 \ 0 \ -1] & 0_{1 \times n} \end{bmatrix}, \quad (13)$$

where  $I$  is the identity matrix and  $0$  is matrix with zeros in all entries. The size of  $A_c$  depends on the selected model order  $n$  for the radiation subsystem, and in this case,  $A_c \in \mathbb{R}^{20 \times 20}$ .

### B. The LNOC controller

The LNOC controller is designed based on the state-space model derived in the last subsection. This is briefly discussed in this section and readers can refer to Ref. [34] for more details.

The LNOC algorithm is derived by solving the optimal control problem with a cost function

$$\min_{u_0, \dots, u_N} \sum_{k=0}^N \{y_k^\top u_k + \frac{1}{2} x_k^\top Q x_k + \frac{1}{2} u_k^\top R u_k\} \quad (14)$$

subject to the discrete-time version of the system [Eq. (9)]:

$$x_{k+1} = A x_k + B_w w_k + B_u u_k, \quad y_k = C x_k, \quad (15)$$

where  $w_k$  is the  $5 \times 1$  excitation force at discrete-time instant  $k$  and  $u_k$  is the PTO torque (control input). Note that here  $y_k$  is the relative pitch velocity:

$$y_k = \begin{bmatrix} \dot{\theta}_{l,k} - \dot{\theta}_{1,k} \\ \dot{\theta}_{l,k} - \dot{\theta}_{2,k} \end{bmatrix}. \quad (16)$$

The mechanical power absorbed by the WEC is  $p_k = -y_k^\top u_k$ . The first term in the cost function [Eq. (14)] is used to maximize the absorbed energy. The second and the third terms are used to penalize the state and input of the control system.

The weighting matrices  $Q$  and  $R$  are tunable parameters that have a substantial impact on the performance of the controller. The parameter  $R$  can be used to handle the soft constraint of the control input. The tuning methodology of  $Q$  and  $R$  was described previously in Ref. [34].

The control policy obtained from solving the above optimal control problem has the form of

$$u_k = K_x x_k + K_d w_{k,n_p}. \quad (17)$$

Here,  $w_{k,n_p}$  is the incoming excitation force sequence  $w_{k,n_p} := [w_k, w_{k+1}, \dots, w_{k+n_p-1}]^\top$  and  $n_p$  is the length of the prediction horizon.  $K_x$  and  $K_d$  are constant control gain matrices and the formulas for calculating them are

$$K_x = -(R + B_u^\top V B_u)^{-1} (C + B_u^\top V A), \quad (18)$$

$$K_d = -(R + B_u^\top V B_u)^{-1} B_u^\top \Psi, \quad (19)$$

and

$$V = Q + A^\top V A - (C + B_u^\top V A)^\top (R + B_u^\top V B_u)^{-1} (C + B_u^\top V A), \quad (20)$$

where  $V$  is the solution of the algebraic Riccati equation [Eq. (20)]. Note that the control policy described in Eq. (17) consists of two parts, which are the feedback (causal) part  $K_x x_k$  and the feed-forward (noncausal) part  $K_d w_{k,n_p}$ .

### C. The KFHO state observer

To realize the control law in Eq. (17), the system state  $x_k$  and the incoming excitation  $w_{k,n_p}$  are needed. A typical way of doing this is to design an observer to observe  $x_k$  and rely on external wave prediction algorithms to estimate  $w_{k,n_p}$ . In this work, the state estimator is used to observe  $x_k$  and  $w_k$  simultaneously, and then  $w_{k,n_p}$  is predicted based on the observed historical  $w_k$  by the AR algorithm that will be introduced later. The combination of the state estimator and the AR predictor enables the standalone property of the LNOC-AR algorithm, thus requiring no additional hardware, and effectively making the performance gains that will be reported at almost zero cost.

The task of the wave excitation force estimator is thus to estimate  $x_k$  and  $w_k$  based on the real-time measurement of  $y_k$  and  $u_k$ . To this end, a mathematical model is required to perform the time transition of  $w_k$ . A harmonic oscillator model is adopted here. Assuming the excitation force follows a perturbation model (harmonic oscillator):

$$w = a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t) + \dots + a_N \sin(\omega_N t), \quad (21)$$

where  $a_1$  to  $a_N$  are amplitude coefficients to be estimated,  $\omega_1$  to  $\omega_N$  are the predefined set of frequencies, and  $N$  is the number of harmonic components. Then, immediately we have

$$\dot{w} = \omega_1 a_1 \cos(\omega_1 t) + \omega_2 a_2 \cos(\omega_2 t) + \dots + \omega_N a_N \cos(\omega_N t) \quad (22)$$

and

$$\ddot{w} = -\omega_1^2 a_1 \sin(\omega_1 t) - \omega_2^2 a_2 \sin(\omega_2 t) - \dots - \omega_N^2 a_N \sin(\omega_N t) \quad (23)$$

Let  $H_e = [a_1 \sin(\omega_1 t) \ a_2 \sin(\omega_2 t) \ \dots \ a_N \sin(\omega_N t)]^\top$ , and thus  $w = [1 \ 1 \ \dots \ 1] H_e$ . Then,

$$\begin{bmatrix} \dot{H}_e \\ \ddot{H}_e \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Omega & 0 \end{bmatrix} \begin{bmatrix} H_e \\ \dot{H}_e \end{bmatrix}, \quad (24)$$

with

$$\Omega = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_N^2 \end{bmatrix}. \quad (25)$$

Define an augmented state  $\eta = [x, H_e, \dot{H}_e]^\top$ .

$$\begin{aligned} \dot{\eta} &= A_{a,c} \eta + B_{a,c} f u, \\ y &= C_{a,c} \eta, \end{aligned} \quad (26)$$

with

$$\begin{aligned} A_{a,c} &= \begin{bmatrix} A_c & B_{w,c} H_{2f} & 0 \\ 0 & 0 & I \\ 0 & -\Omega & 0 \end{bmatrix}, \quad B_{a,c} = \begin{bmatrix} B_{u,c} \\ 0 \end{bmatrix}, \\ C_{a,c} &= [C_c \ 0], \end{aligned} \quad (27)$$

and

$$H_{2f} = \begin{bmatrix} [1 \ 1 \ \dots \ 1] & & \\ & \ddots & \\ & & [1 \ 1 \ \dots \ 1] \end{bmatrix}. \quad (28)$$

The discrete-time version of the lumped state-space model in Eq. (26) can be written as

$$\eta_{k+1} = A_a \eta + B_a u_k, \quad y_k = C_a \eta. \quad (29)$$

Now, the wave excitation force estimation problem becomes a state estimation problem, which can be tackled by a standard Kalman Filter. It takes the control input  $u_k$  and output  $y_k$  from measurement as the estimator's inputs, and an estimated state information  $\hat{\eta}_k$  as the output. This standard Kalman filter algorithm operates as follows. Denote the former input of the noncausal controller with observer  $\hat{u}_{k-1}$ , and the estimated state  $\hat{\eta}_{k-1}$ . First, *a priori* estimation with the former state information is calculated,

$$\hat{\eta}_k^- = A_a \hat{\eta}_{k-1} + B_a \hat{u}_{k-1}, \quad (30)$$

and then the error covariance  $P_k^-$  of this priori estimation is calculated with the predefined model error covariance  $Q_{\text{kal}}$

$$P_k^- = A_a P_{k-1} A_a^\top + Q_{\text{kal}}. \quad (31)$$

The Kalman gain is

$$K_{\text{kal}} = P_k^- C_a^\top (C_a P_k^- C_a^\top + R_{\text{kal}})^{-1}, \quad (32)$$

with predefined noise covariance matrix  $R_{\text{kal}}$ . The estimated state information can be calculated as

$$\hat{\eta}_k = \hat{\eta}_k^- + K_{\text{kal}} (y_k - C_a \hat{\eta}_k^-). \quad (33)$$

The last step in a Kalman algorithm loop is to update the error covariance matrix  $P_k$  for the next time step:

$$P_k = (I - K_{\text{kal}} C_a) P_k^-. \quad (34)$$

The noncausal optimal control policy with the states estimated by the KFHO estimator can now be rewritten using the estimated state  $\hat{x}_k$  as

$$\hat{u}_k = K_x \hat{x}_k + K_d w_{k,n_p}, \quad (35)$$

where  $\hat{x}_k$  is taken from the estimated augmented state  $\hat{\eta}_k$ . The estimated wave excitation force  $w_k$  can also be derived from the  $H_e$  component in  $\hat{\eta}_k$ .

### D. The AR excitation predictor

In Eq. (35), the wave excitation prediction term  $w_{k,n_p}$  represents the prediction of the wave excitation force at time  $k$  into the future of  $n_p$  steps provided by the wave predictor. This can be predicted based on the observed  $w_k$  by the AR algorithm. The fundamental assumption of an AR model is that the value  $w_k$  depends linearly on its previous values  $w_{k-p}, \dots, w_{k-1}$ , through a set of parameters  $\Phi := [\phi_1, \dots, \phi_p]$ . This assumption can be written in the form of

$$\hat{w}_k = \sum_{i=1}^p \phi_i w_{k-i}, \quad (36)$$

where notation  $\hat{w}_k$  represents the predicted value of  $w_k$ . Here,  $p$  is the order of the AR model, with an AR model of order  $p$  being denoted as AR( $p$ ).

To resolve the set of parameters  $\Phi$ , a set of training data with length  $L$  is used to train the model.  $\Phi$  can be obtained by minimizing the sum of prediction errors over the training horizon  $L$ ,

$$J = \sum_{k=p+1}^L (\eta_k - \hat{\eta}_k)^2, \quad (37)$$

which leads to a linear least-square (LLS) problem. The total number of data needed for one training process is  $L + p$ . With these  $L + p$  data, the AR( $p$ ) model training process can be written as

$$\begin{bmatrix} w_{p+1} \\ w_{p+2} \\ \vdots \\ w_{p+L} \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & \cdots & w_p \\ w_2 & w_3 & \cdots & w_{p+1} \\ \vdots & \vdots & \ddots & \vdots \\ w_L & w_{L+1} & \cdots & w_{p+L} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{bmatrix}, \quad (38)$$

or in matrix form

$$Y = X\Phi. \quad (39)$$

Solving the LLS problem gives the AR( $p$ ) coefficient set as

$$\Phi = (X^\top X)^{-1} X^\top Y. \quad (40)$$

The incoming wave excitation force term  $w_{k,n_p}$  can then be formed by using this AR( $p$ ) model repeatedly with the past  $p$  estimated wave excitation forces,

$$w_{k,n_p} = [\hat{w}_k, \hat{w}_{k+1}, \dots, \hat{w}_{k+n_p-1}]^\top, \quad (41)$$

which feeds into Eq. (35), providing the predicted excitation required for the LNOC algorithm to maximize power.

### III. EXPERIMENTAL SETUP AND CONTROL IMPLEMENTATION

#### A. The wave tank and wave conditions

The experiments were conducted in the Wide Flume in the University of Manchester Hydrodynamics Laboratory. The flume is 20-m long, 5-m wide with a variable uniform depth up to a maximum of 0.8 m that is used for the experiments. Eight force-feedback piston-type wavemakers are used to generate irregular wave conditions.

The M4 and LNOC-AR controller are tested under both unidirectional and directionally spread irregular wave conditions. JONSWAP [49] frequency spectra  $S(f)$  are used for all conditions with varying significant waveheight ( $H_{m0}$ ) and peak period ( $T_p$ ) values, with a peak enhancement factor,  $\gamma = 3.3$ , used throughout. For  $H_{m0} = 0.05$  and  $T_p$  values of 1.3 and 1.5 s, we also assess performance in moderately directionally spread conditions. For generating directional waves, the directional spreading function  $D(\theta)$  is based on a cos-2s distribution as defined in Eq. (42) with  $\theta$  the direction and  $\bar{\theta}$  the mean direction. The directional energy spectrum  $E(f, \theta) = S(f)D(f, \theta)$ , and we use a frequency independent directional spreading function, i.e.,  $D(f, \theta) = D(\theta)$ . The  $s$  parameter controls the directional spreading and  $\alpha$  is set to ensure that  $\int_{-\pi}^{\pi} D(\theta) d\theta = 1$ . The random directional method [50] is used to define the angles associated with individual frequency components, similar to Ref. [51]. Due to the relatively narrow tank width, we also employ the Dalrymple method [52] to obtain the desired directional distribution at a chosen  $x$  position (chosen to be the bow float) at the location of the WEC, noting that away from this location the directional distribution begins to deviate from that specified. Table I summarizes all the sea states tested in the experiment, where we also include the spectral steepness range  $\frac{H_{m0}k_p}{2}$  (with  $k_p$  the peak wave number), and the standard deviation of the directional distribution, approximated using  $\sigma_\theta = \sqrt{\frac{2}{s+1}}$ . The

spectral steepness is included to highlight that comparable performance is achieved in nonlinear wave conditions including breaking cases:

$$D(\theta) = \alpha \left( \cos \left( \frac{\theta - \bar{\theta}}{2} \right) \right)^{2s}. \quad (42)$$

Measurements of the incident spectrum are estimated at the WEC's location for all the sea states in Table I, without the WEC installed. This was conducted using the frequency-domain reflection analysis approach outlined in Ref. [53] with wave gauge spacings based on a Golomb ruler to optimize the coarray separations (as in Ref. [54]). The estimated incident spectrum is used to quantify the measured  $H_{m0}$  and incident power of each sea state, with  $H_{m0}$  values generally within 5% of the target. Sea state characterization of the incident  $H_{m0}$  values is presented in Appendix A (see Fig. 9).

#### B. The M4 1-2-2 physical model

As shown in Fig. 2, the 1-2-2 M4 has, from right to left, one bow float, two mid floats, and two stern floats. Waves propagate from bow to stern float through the device. Phase differences of the floats induce rotation of the device about the hinges, which is the principle method of power extraction. The WEC is moored using a single point mooring, and for these experiments the mooring buoy is attached to a mass on the bed using a single inelastic cable. Table V in Appendix B summarizes the technical specifications of the M4 physical model.

#### C. Control implementation

In this section, the real-time implementation of the torque control is introduced as depicted in Fig. 3. Two servo motors (Kollmorgen AKM 43E-ANCNC-00) are mounted on top of the mid floats representing the power take-off, and acting as actuators to provide the required torque for control. These motors are powered by two servo drives (Kollmorgen AKD-P01206-NBCC-E000), which also outputs real-time encoder data for speed calculation and current for torque estimation. A dSPACE MicroLabBox is used for implementing control algorithms as well as data collection. The LNOC-AR and the benchmark linear damping controller algorithms are programmed in matlab/Simulink and compiled directly to the dSPACE. Controllers and data collection both run at 50 Hz that is proved to be sufficient. Estimates of extracted power are calculated from measurements of torque and speed.

In the experiment, an AR model with order  $p = 100$  is used with training data length  $L = 1500$  (30 s with the sampling rate of 50 Hz). The AR model is retrained every 4 s with the latest estimated excitation forces. The KFHO observer uses a frequency band  $[\omega_1, \omega_2, \dots, \omega_N] = [0.5, 0.6, \dots, 1.2]$  Hz. The LNOC control gain  $Q$  and  $R$  as well as the Kalman Filter covariance matrices  $Q_{kal}$  and  $R_{kal}$  are carefully tuned for the best performance. All the key tuning parameters and their values are shown in Table II. These tunings are consistent for all the sea states.

Note that the tested sea states are 300-s long and have 256 s repeat time. As a result, mean power values for each sea state are calculated using the last 256 s in each 300 s

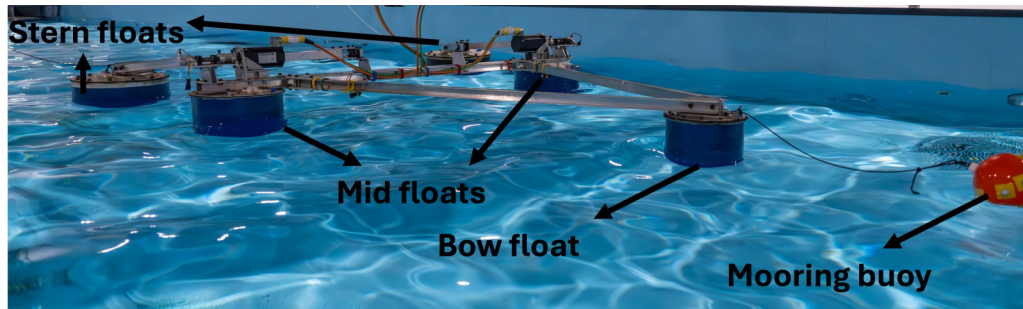


FIG. 2. The M4 wave energy converter (1:40 scale) installed in the Wide Flume in the University of Manchester Hydrodynamics Laboratory.

window, eliminating the ramping up period and maximizing stationarity of the conditions. A trigger signal is shared between the wave paddles and the control unit (dSPACE) for data synchronization.

Tuning of the damping ratio  $B_{\text{mech}}$  for the linear damping controller is performed, which is necessary to ensure the best benchmark. Previous study has identified that the optimal damping ratio has little sensitivity to the change of sea states [22]. This is done here with sea state  $H_{m0} = 0.05$  m and  $T_p = 1.3$  s, which is the peak period for maximum power for the M4 WEC. The optimal damping ratio is 8 Nms/rad, giving a mean power of 0.85 W. This is used for all the sea states.

## IV. EXPERIMENTAL RESULTS

Experimental results are presented in this section. First, example time-series results are presented for a unidirectional and directionally spread condition, before key statistics are presented over all conditions to demonstrate broader performance. Figure 4 shows comparison of the LNOC-AR against the linear damping control in one specific sea state (U209), with  $H_{m0} = 0.05$  m and  $T_p = 1.5$  s with no directional

spreading. This corresponds to a moderate sea state in full scale. In Fig. 4, subplot (a) shows the accumulated energy of both PTOs, comparing the two control methods. Subplots (b)–(d) are short example time series showing the total instantaneous power, the torque, and the speed of a motor on one side, respectively. In this sea state, the mean power for the linear damper is 0.58 W and for LNOC-AR is 0.93 W demonstrating an improvement of 59%. Bidirectional power flow is required with the LNOC-AR, which is evident in subplot (b). By drawing some power from the PTO, LNOC-AR enables a more significant power generation (positive) compared with linear damping. The RMS value of torque is 2.39 Nm with the linear damper and 4.59 Nm with LNOC-AR. Similarly, the RMS value of speed is 0.30 rad/s with the damper and 0.34 rad/s with LNOC-AR. These statistics indicate that increasing torque is associated with increased power capture. Clearly, however, the LNOC-AR improvement is not only due to increasing torque amplitude, but also due providing torque at the correct time, thanks to the AR predicted information. As presented in Fig. 6, the maximum improvement is found at a smaller sea state,  $H_{m0} = 0.03$  m and  $T_p = 1.6$  s. In this case, the mean power for the linear damper is 0.12 W and for LNOC-AR is 0.27 W providing an improvement of 125%.

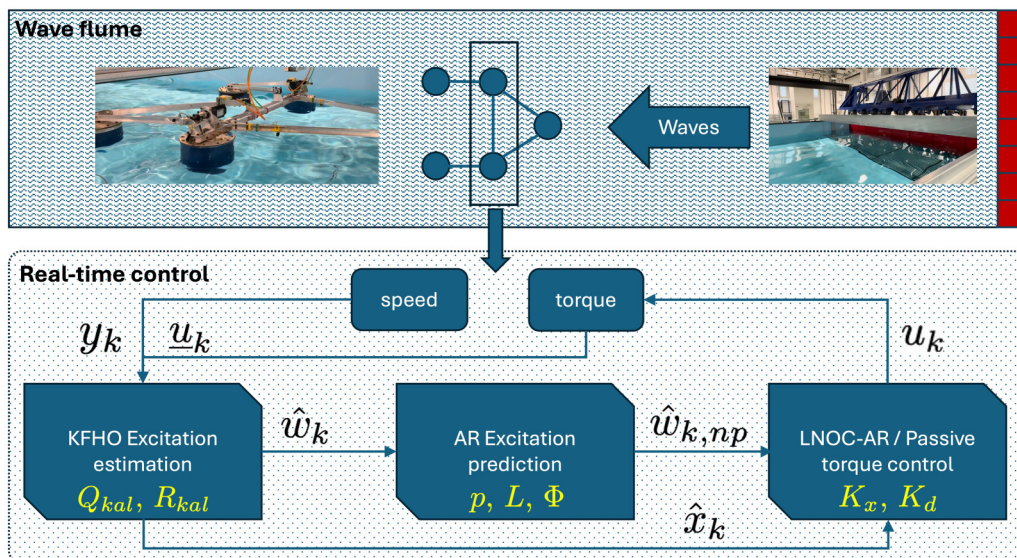


FIG. 3. Illustrative diagram for the experiment setup and control implementation. An underline is added to distinguish the measured torque from the control signal. The hat symbol represents estimated values and tuning parameters are in yellow.

TABLE II. Tunings of all the LNOC-AR parameters. Identity matrix of size  $n$  is represented with  $I_n$ .

| Parameter | KFHO                                                  |               | LNOC |             | AR  |              |
|-----------|-------------------------------------------------------|---------------|------|-------------|-----|--------------|
|           | $Q_{kal}$                                             | $R_{kal}$     | $Q$  | $R$         | $p$ | $L$          |
| Value     | $\begin{bmatrix} I_{20} \\ 10^4 I_{80} \end{bmatrix}$ | $10^4 I_{10}$ | 0    | $0.012 I_2$ | 100 | 30 s         |
|           |                                                       |               |      |             |     | Retrain time |
|           |                                                       |               |      |             |     | 4 s          |

Real ocean waves are all directionally spread to some degree. It is thus important to find out how the LNOC-AR method performs in directionally spread wave conditions. Figure 5 presents the data collected for a case with directionally spreading  $s = 25$ , and with  $T_p = 1.5$  s and  $H_{m0} = 0.05$  m (S002 in Table I), in comparison to those from the sea state in unidirectional conditions with  $T_p = 1.5$  s and  $H_{m0} = 0.05$  m (U208 in Table I). One can observe that in spreading wave condition, the instantaneous power of PTO 1 and PTO 2 are frequently out of phase as in Fig. 5(a), while in unidirectional wave condition they are effectively in phase, as in Fig. 5(b). Figure 5(c) shows the comparisons of cumulative energy. It can be noticed that while both controllers provide slightly less power due to spreading effect, the improvement of LNOC-AR over linear damper is still substantial in directionally spreading wave conditions, demonstrating the robustness of the LNOC-AR. Figure 6 summarizes all the results including both unidirectional (solid and dashed lines) and directionally spread (circles and triangles) wave conditions. Results are presented in subplots split by the target  $H_{m0}$  values, with the mean power increase across all conditions being 40%. It is important to point out again that these improvements are mechanical power rather than electrical. It should be noted

that the M4 system has been shown to be remarkably linear through experimental analysis with no damping [14] and with damping [13] comparing with hydrodynamic modeling. The power results collapse to the nondimensional capture width ratio. This is also the case here when a constant torque loss, estimated to be close to 1 Nm, is added to the measured torque. The values with the largest  $H_{m0}$  (and largest torques) are most representative of mechanical power but the prediction of full-scale values will be underestimated to some degree. Evidently, the LNOC-AR controller improves performance across all tested conditions (noting that the tuning is consistent across all cases) as expected from modeling results [34,35], and performance gains are more significant for longer peak periods away from the resonant period. This makes it possible to have a smaller scale device to optimize LCoE in full-scale sea conditions, which will be demonstrated in Sec. V.

Assessing Fig. 6(b), it is evident that the performance increase in directionally spread conditions is comparable to those in unidirectional conditions, albeit with generally lower

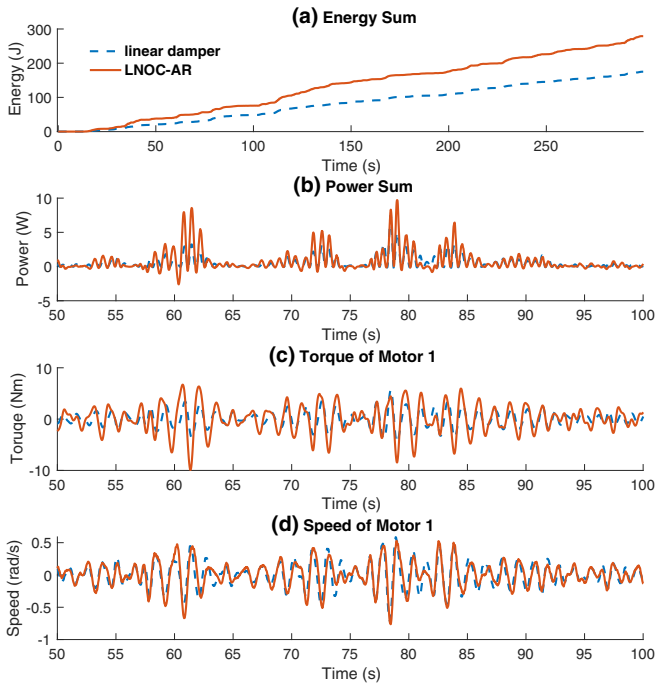


FIG. 4. Experiment data, with sea state U209:  $H_{m0} = 0.05$  m,  $T_p = 1.5$  s, and  $s = \infty$ . The cumulative energy (a) and instantaneous power (b) summed over both motors are presented, along with time series of torque (c) and speed (d) for motor 1.

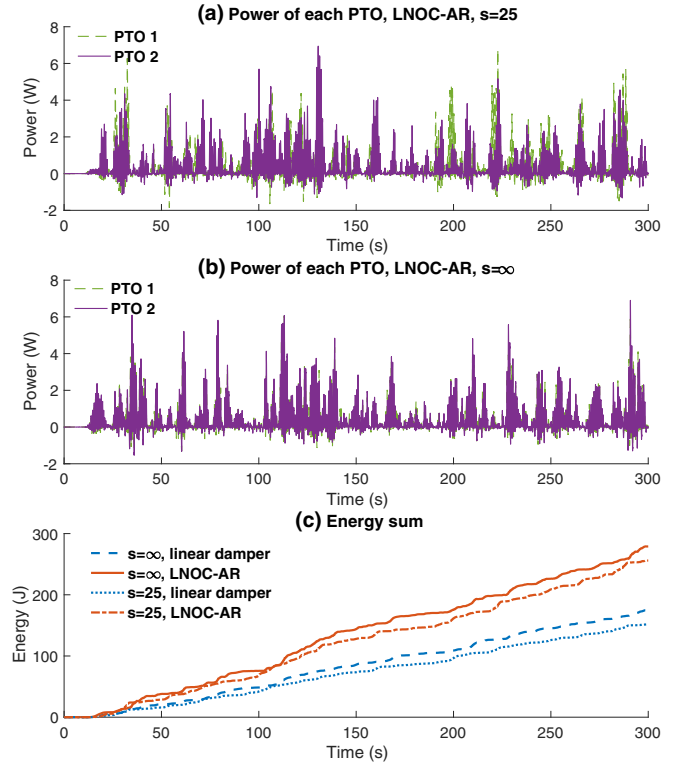


FIG. 5. Experiment data, with sea states:  $H_{m0} = 0.05$  m and  $T_p = 1.5$  s. Panel (a) depicts the instantaneous power of both PTOs in directionally spreading waves with  $s = 25$ . Panel (b) depicts the instantaneous power of both PTOs in unidirectional waves with  $s = \infty$ . Panel (c) presents the cumulative energy summed over both motors.

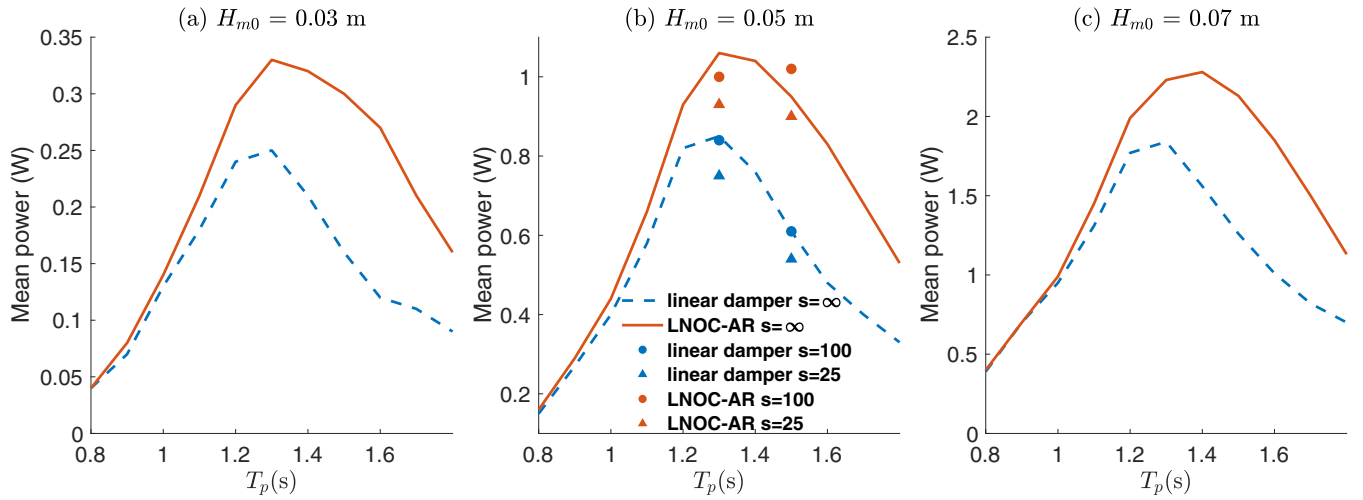


FIG. 6. Summary of LNOC-AR improvement, in both unidirectional and directional spread wave conditions. Panels (a)–(c) present performance with increasing values of  $H_{m0}$  over the same range of  $T_p$ .

mean power values. With  $s = 100$  and LNOC-AR, the mean power is actually slightly higher than  $s = \infty$ . This is likely a combined result of the very small directional spreading and potential for constructive interference at locations away from the Dalrymple method correction location (bow float). However, with  $s = 25$ , there is a noticeable decrease of mean power for both controllers, as expected. In these directionally spread wave conditions, the LNOC-AR still outperforms the

traditional linear damping controller significantly, which is essential for its application in real seas.

From these results, it is clear that the control strategy presented can enable a step change in wave power generation. Here, we make some comments on model efficacy. The underlying hydrodynamic model is for unidirectional waves in an infinite domain. Previous model predictions for response and power have been generally performed well when a slightly

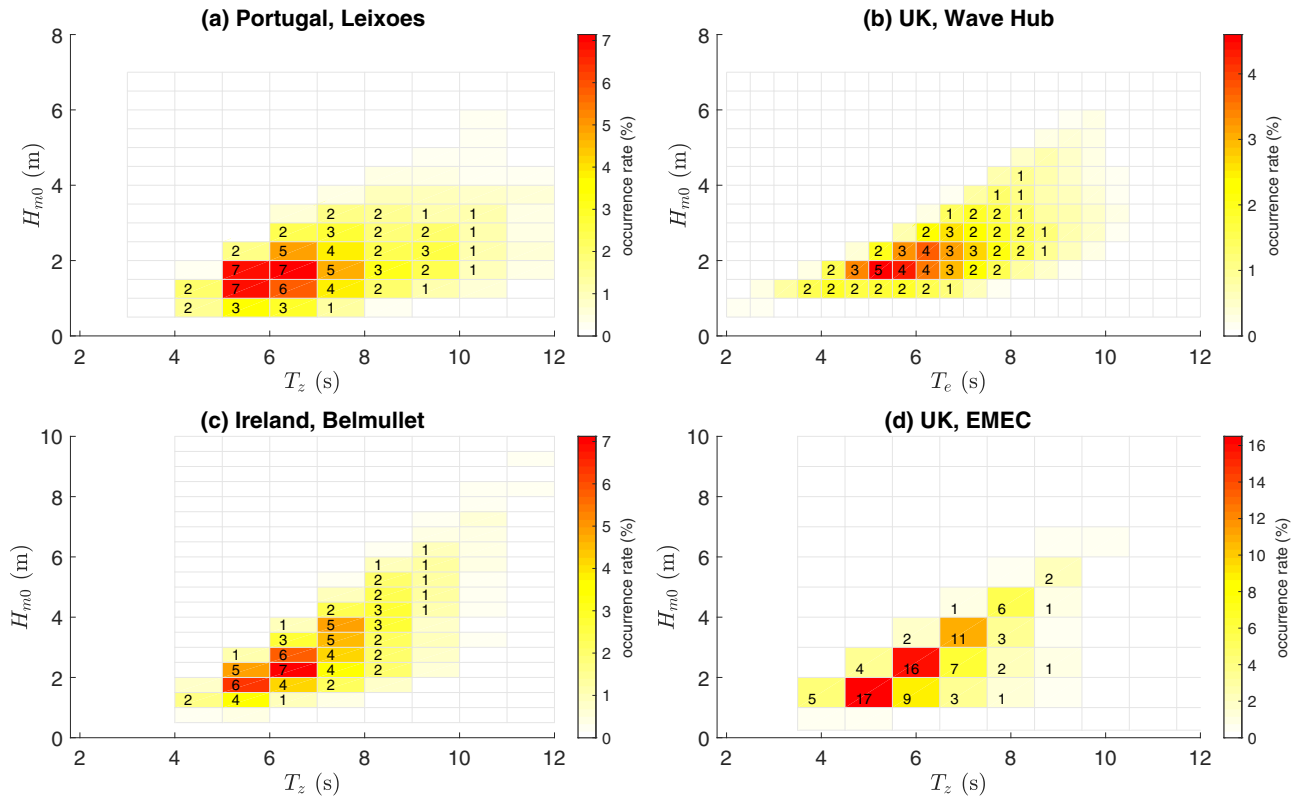


FIG. 7. Scatter diagrams of occurrence rate for four sites. Numbers in bins are occurrence rates over 1 in percentage, rounded to integer. It is assumed that  $T_p = 1.4T_z$  and  $T_p = T_e/0.85$ . Data source: Panels (a) and (c) from Ref. [55], panel (b) from Ref. [56], and panel (d) from Ref. [57].

TABLE III. Mean wave statistics ( $H_{m0}$  and  $T_p$ ) of each site.

| Site name         | $\bar{H}_{m0}$ (m) | $\bar{T}_p$ (s) | $\bar{T}_z$ or $\bar{T}_e$ (s) in Fig. 7 |
|-------------------|--------------------|-----------------|------------------------------------------|
| Portugal Leixoes  | 1.8                | 9.8             | 7.0 ( $T_z$ )                            |
| UK Wave Hub       | 2.2                | 7.7             | 6.5 ( $T_z$ )                            |
| Ireland Belmullet | 3.0                | 9.7             | 7.0 ( $T_z$ )                            |
| UK EMEC           | 2.2                | 8.4             | 6.0 ( $T_z$ )                            |

smaller size model with a passive damper was compared with experiments in a relatively wide basin (15-m wide) (e.g., Refs. [13,37]). Here, the device length scale is 20% bigger and the basin is only 5-m wide. The measured power with passive damping control was about 20% smaller than the model predictions with an effective infinite width basin, consistent with the finite width effect, with torque loss considered. Nevertheless, this model has still enabled control to improve measured power absorption by over 100% for some conditions, with performance being demonstrably maintained in more complex directional waves. This is extremely promising for the general robustness of the model in complex realistic sea states.

## V. SITE-SPECIFIC PERFORMANCE ASSESSMENT

In this section, a study is carried out to understand the impact of proposed scaled control test results in a full-scale scenario over a real range of site conditions. Four typical wave energy sites in Europe are considered, which are located at Leixoes, Portugal [55], Wave Hub, UK [56], Belmullet, Ireland [55], and EMEC, UK [57]. Figure 7 shows sea state scatter diagrams of each site with occurrence data from the

above literature. Only sea states with occurrences over 1% are displayed in scatter diagram. The Mean  $H_{m0}$  and  $T_p$  values for each site are presented in Table III, calculated as a weighted sum of each sea states  $H_{m0}$  and  $T_p$  values, with estimates of  $T_p$  from available mean period ( $T_z$ ) or energy period ( $T_e$ ) values.

Froude scaling is applied to the experiment results to calculate the mean power, power per unit mass ( $\propto \text{LCoE}^{-1}$ ) and power variability at different scales for these four sites. Note that we assume unidirectional JONSWAP spectra with  $\gamma = 3.3$  for all sites, which is an approximation to give qualitative evaluation. Real ocean waves are directionally spread, so the absolute values of power are likely overpredicted, but relative performance gains will be approximately equivalent, at least for modest values of directional spreading, as demonstrated in Fig. 6.

The mean power at a given site is calculated as

$$P_{\text{site}} = \sum_{i=1}^N P_i \mathbb{P}_i, \quad (43)$$

where  $P_i$  is the mean power for a specific sea state  $i$  at the site and  $\mathbb{P}_i$  is the associated probability of occurrence, with  $N$  the total number of sea states and  $\sum_{i=1}^N \mathbb{P}_i = 1$ . For a given sea state defined by a  $H_{m0}-T_p$  combination on the scatter diagram,  $P_i$  can be calculated. With scale  $l$ , the peak period in experiment is  $T_{p,\text{lab}} = T_{p,i}/\sqrt{l}$ . We use  $H_{m0,\text{lab}} = 0.05$  m as reference noting that this does not account for torque loss and therefore gives a conservative estimate of mechanical power and  $P_{\text{lab}}$  is the mean power in the associated experiment. Since power is proportional to  $H_{m0}^2$ , at full scale

$$P_i = P_{\text{lab}} l^{1.5} (H_{m0}/H_{m0,\text{lab}})^2. \quad (44)$$

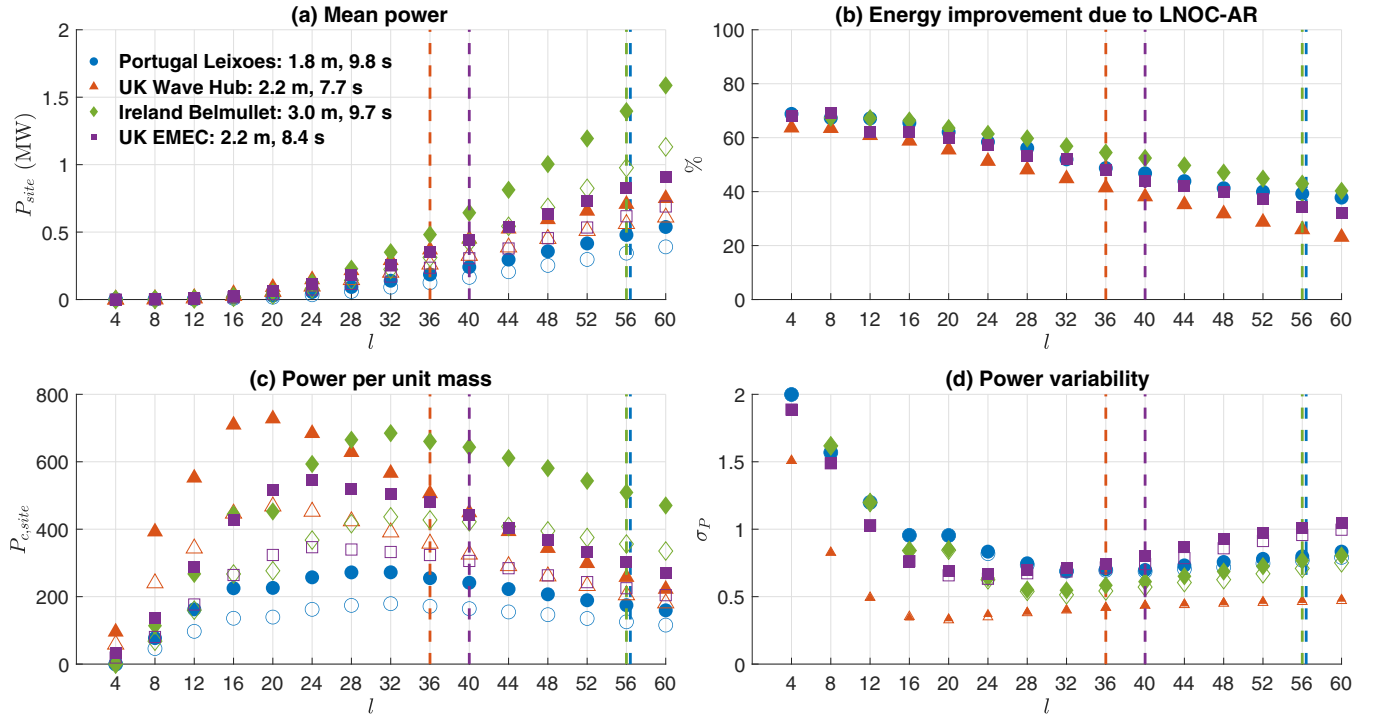


FIG. 8. Site-specific assessment. In panels (a), (c), and (d), empty markers represent linear damper and filled markers represent LNOC-AR. In panel (b), energy improvement is differences of energy output between LNOC-AR and linear damper over that of linear damper in percentage.

TABLE IV. Optimal scale of M4 for each site and overall benefit from LNOC-AR.

| Site name         | $l_{\text{ref}}$ | $l_{\text{opt}}$ for max $P_c$ | $l_{\text{opt}}/l_{\text{ref}}$ | Optimal $\sigma_P$ at $l_{\text{opt}}$ | Net increase in $P_c$ |
|-------------------|------------------|--------------------------------|---------------------------------|----------------------------------------|-----------------------|
| Portugal Leixoes  | 56               | 32                             | 0.57                            | 0.69                                   | 118%                  |
| UK Wave Hub       | 36               | 20                             | 0.56                            | 0.34                                   | 104%                  |
| Ireland Belmullet | 56               | 32                             | 0.57                            | 0.54                                   | 92%                   |
| UK EMEC           | 40               | 24                             | 0.60                            | 0.67                                   | 79%                   |

If the  $H_{m0}$  ratio was  $l$ , then power would scale as  $l^{3.5}$  as often specified.

A caveat here is in the experiment, the  $T_{p,\text{lab}}$  range is from 0.8 to 1.8 s, which with a scaling factor  $\sqrt{l}$  may not cover all the peak periods in the site scatter diagrams. In these cases,  $P_{\text{lab}}$  is extrapolated based on simulation data of the linear hydrodynamic model. Additionally, high sea states will cause  $P_i$  above a practical limit. Hence, a power limit is applied to  $P_i$ , which is based on the maximum laboratory value that occurs with  $T_{p,\text{lab}} = 1.3$  s. The mean  $H_{m0}$  with  $T_p$  for maximum power thus defines a reference power and the power limit is 10 times this as the typical peak to mean value for a sea state [17,58].

To determine the scale for the optimum power per unit cost for a given site, a relative power per unit mass is calculated by

$$P_{c,\text{site}} = P_{\text{site}}/l^3, \quad (45)$$

assuming mass proportional to  $l^3$  and total cost proportional to mass cost, making the metric  $P_{c,\text{site}}$  approximately inversely proportional to LCoE. Note that  $P_{\text{site}}$  is a function of  $l$  as discussed.

The power variability is calculated by

$$\sigma_P = \frac{\sum_{i=1}^N (\sqrt{(P_i - P_{\text{site}})^2} \mathbb{P}_i)}{P_{\text{site}}}. \quad (46)$$

Results are presented in Fig. 8. Note that a range of  $l$  from 4 to 60 is chosen defining the experiment scale as  $l = 1$ . Normally, the typical scale  $l_{\text{ref}}$  of a WEC is determined by the mean  $T_p$  of a given site  $\hat{T}_p$  and its own resonant frequency  $\hat{T}_p$  (the most energetic one if multiple resonant frequencies exist) as  $l_{\text{ref}} = (\hat{T}_p/\hat{T}_p)^2$ . This  $l_{\text{ref}}$  helps match the device's resonant period to the site's mean  $T_p$ , which is considered the "optimal" scale of a WEC for a site. It will be shown later that  $l_{\text{ref}}$  is actually not optimal and it reaffirms the same conclusion drawn in the literature, e.g., Refs. [59,60]. Note that in experiment,  $\hat{T}_p = 1.3$  s as demonstrated in Fig. 6. The mean  $\bar{H}_{m0}$  and  $\hat{T}_p$  of each site are shown in Fig. 8's legend for the reader's convenience. The typical expected scale  $l_{\text{ref}}$  is plotted as a vertical dashed line with horizontal position  $l = l_{\text{ref}}$  to help understand these results.

It is evident from Fig. 8(a) that, as expected,  $P_{\text{site}}$  increases as the device scale  $l$  increases. However, in contrast, the improvement from linear damping control (empty markers) to LNOC-AR (filled markers) decreases as demonstrated in Fig. 8(b). Thus, the LNOC-AR controller is of greater benefit for smaller scale devices as opposed to larger devices. The improvement in mean power using LNOC-AR compared to the linear damper is greater than 40% for all sites for  $l < 36$  and continues to decrease for larger  $l$ . This is because, as

presented in Fig. 6, LNOC-AR improves power mainly for longer wave peak periods.

Figure 8(c) shows the power per unit mass for each site,  $P_{c,\text{site}}$ , versus  $l$ . As mentioned, this metric is approximately inversely proportional to the LCoE, which is the most critical metric of any energy generation system including wave energy. Taking the UK EMEC site as an example,  $l_{\text{ref}}$  is 40 (purple vertical dashed line) with conventional scale selection. It is found here with either LNOC-AR or linear damper, the optimal scale shifts to 24, which means a device is 60% of the value that was thought to provide the optimal  $P_c$ . The maximum  $P_c$  in this case increases from 305 to 346 with the linear damper and from 440 to 545 with LNOC-AR. The overall benefit is from 305 with linear damper to 545 with LNOC-AR, a 79% improvement of power per unit mass. Table IV presents the optimal scale, scale ratio, power variability at optimal scale, and overall increase in  $P_{c,\text{site}}$  through using LNOC-AR. The net increase in  $P_{c,\text{site}}$  is around 100% across all four sites, demonstrating the potential of the presented approach for optimizing both the scale of the device as well as the control. Detailed site assessment results are presented in Appendix C (see Figs. 10–13).

Power variability is another key aspect of renewable energy as it affects power storage requirements. Lower power variability means the WEC's energy output is more uniform throughout the whole year. It can be seen in Fig. 8(d) that Wave Hub has the lowest  $\sigma_P$  of 0.34, Belmullet is 0.54, EMEC is 0.67, and Leixoes is 0.69. Optimal scales for lowest power variability at all the four sites coincide with optimal scales for highest  $P_c$ , reaffirming the advantage of smaller scale devices. It is also found that LNOC-AR control does not increase power variability. Therefore, the implementation of LNOC-AR improves both the power per unit cost and the power variability with substantial implications for wave energy development.

## VI. LIMITATIONS

In this study, we consider mechanical power rather than electrical power due to practical limitations, when comparing performance of different controllers. The level of mechanical power absorbed by the WEC in the tank testing environment is around 1 W, making it almost impossible to integrate a generator and assess PTO losses. At much larger scale, the source of losses may be analyzed objectively as undertaken by our research team in Ref. [17]. Note that the control formulation would have to be altered to simultaneously maximize wave energy capture and minimize energy losses from PTO, power transmission components, e.g., gear box, and power electronics components. This requires a full wave-to-wire model to describe the hydro-mechanical-electrical dynamics of the WEC that can be highly nonlinear. In these scenarios, linear control

formulation may lose its applicability and more computationally demanding nonlinear model predictive control, e.g., Ref. [61], can be considered. However, these losses from mechanical and electrical components are a small percentage of the total energy captured by the floaters in operational conditions, and thus setting energy capture maximization as the major control objective is reasonable for WEC control system design and is commonly used in most literature in this field [31].

Constraint handling is another important aspect of WECs when it comes to reliability and survivability. In this study, PTO constraints are handled by imposing a torque limit on the control torque for protection of the mechanical couplings and tuning of the  $R$  value that links directly to the magnitude of control input [34]. More sophisticated control algorithms can handle constraints optimally, such as model predictive control previously studied by the authors in Ref. [62]. The validation of these controllers on the M4 WEC is one of our future research directions, which can follow the completion of this study.

Finally, the power improvements provided by the LNOC-AR, although significant, do not translate linearly into the reduction of the LCoE of wave energy due to variable costs of many components. Nevertheless assuming cost proportional to overall mass is indicative of relative cost.

## VII. CONCLUSIONS

Increasing power generation from wave energy converters through standalone control has been demonstrated in wave basin tests for the first time. This has been undertaken for the hinged multifloat attenuator M4 with two PTOs using linear noncausal optimal control with a Kalman-type state estimator and an autoregressive predictor (LNOC-AR). Numerical experiments have shown the theoretical efficacy previously but an exact hydrodynamic model was inherent and wave conditions were known. With physical reality, the hydrodynamic

models can only be approximate and indeed are simplified to state-space form to speed up for real-time control. The state observer and prediction are also approximate. Nevertheless it is demonstrated that, for a fixed scale device, energy capture can be increased by up to 125% and an average of 40% across all tested irregular sea states. This increase was also achieved in directionally spread waves with the same unidirectional hydrodynamic model, indicating the robustness of the control system. With autoregressive wave prediction, the system is standalone without any external measurement of wave surface elevations. The controller is found to be more effective for larger wave periods, and it is demonstrated that smaller devices are hence more cost effective than larger ones. Across four European wave sites, an average increase in mean power per unit mass of around 100% is achieved when considering the combined effect of power performance gains and reduction for optimal size, whilst power variability is also reduced. These represent substantial improvements that have the potential to advance the wave energy sector, and were hitherto undemonstrated.

## ACKNOWLEDGMENTS

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

## APPENDIX A: SEA STATE CHARACTERIZATION

Sea state characterization of the incident wave height is given in Fig. 9.

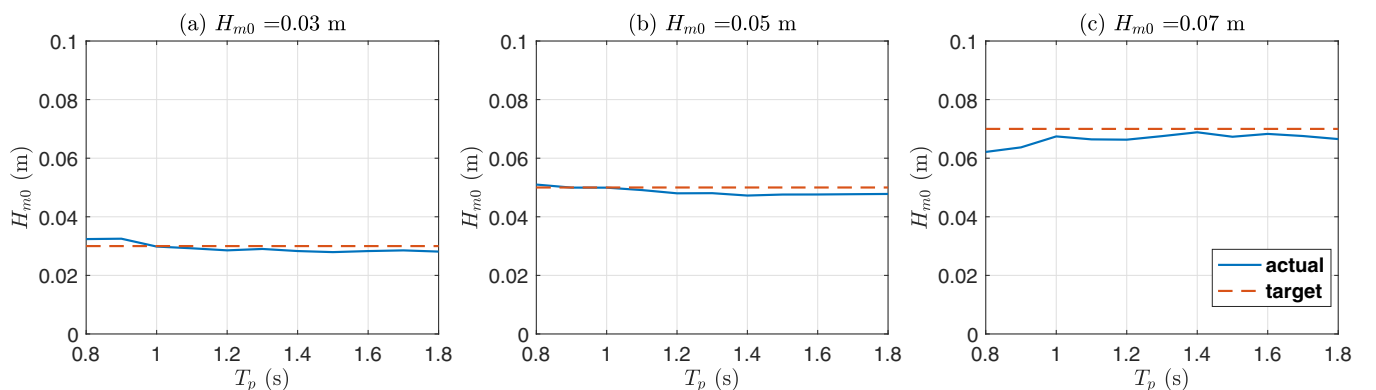


FIG. 9. Comparison between the target significant waveheight and the measured incident significant waveheight.

TABLE V. The M4 physical model specifications.

| Part                | Mass (kg) | $h$ (m) | $t$ (m) | $v$ (m) | Radius (m) | Draft (m) |
|---------------------|-----------|---------|---------|---------|------------|-----------|
| Float 1             | 2.28      | -1.68   | 0       | -0.19   | 0.125      | 0.12      |
| Ballast 1           | 1.5       | -1.68   | 0       | -0.40   |            |           |
| Frame 123           | 10.9      | -0.19   | 0       | -0.11   |            |           |
| Float 2 and ballast | 4.55      | 0       | -0.84   | -0.37   | 0.175      | 0.21      |
| Motor 1             | 4.35      | 0       | -0.73   | 0       |            |           |
| Float 3 and ballast | 4.55      | 0       | 0.84    | -0.37   | 0.175      | 0.21      |
| Motor 2             | 4.35      | 0       | 0.73    | 0       |            |           |
| Float 4             | 21.2      | 0.96    | -1.015  | -0.27   | 0.21       | 0.276     |
| Ballast 4           | 9.0       | 0.96    | -1.015  | -0.51   |            |           |
| Beam 4              | 1.55      | 0.48    | -0.92   | -0.13   |            |           |
| Float 5             | 21.2      | 0.96    | 1.015   | -0.27   | 0.21       | 0.276     |
| Ballast 5           | 9.0       | 0.96    | 1.015   | -0.51   |            |           |
| Beam 5              | 1.55      | 0.48    | 0.92    | -0.13   |            |           |

## APPENDIX B: THE M4 PHYSICAL MODEL SPECIFICATIONS

The physical model specifications are detailed in Table V. Note that center of mass is relative to the WEC hinge.  $h$  (horizontal) is for the  $x$  axis in surge;  $t$  (transversal) is for the  $y$  axis in sway;  $v$  (vertical) is for  $z$  axis in heave. Frame 123 connects the bow and mid floats. The presented radius is of the circular cross section of floats.

## APPENDIX C: ESTIMATED MEAN POWER PLOTTED ON SCATTER DIAGRAM

Detailed assessment results for the four considered sea sites are given in Figs. 10–13, respectively.

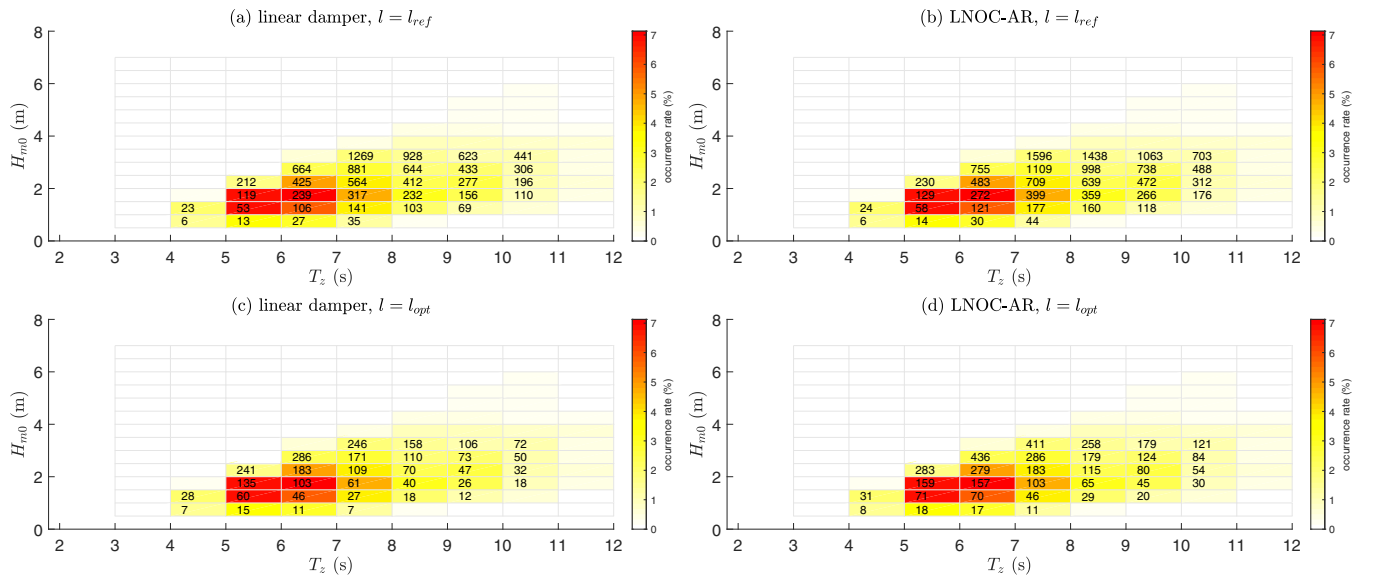


FIG. 10. Portugal Lexioes: mean power  $P_i$  [unit (KW)] in scatter diagram, (a) with linear damper,  $l = l_{ref}$ , (b) with LNOC-AR,  $l = l_{ref}$ , (c) with linear damper,  $l = l_{opt}$ , and (d) with LNOC-AR,  $l = l_{opt}$ .  $P_i$  of sea states with occurrence rate less than 1% are omitted.

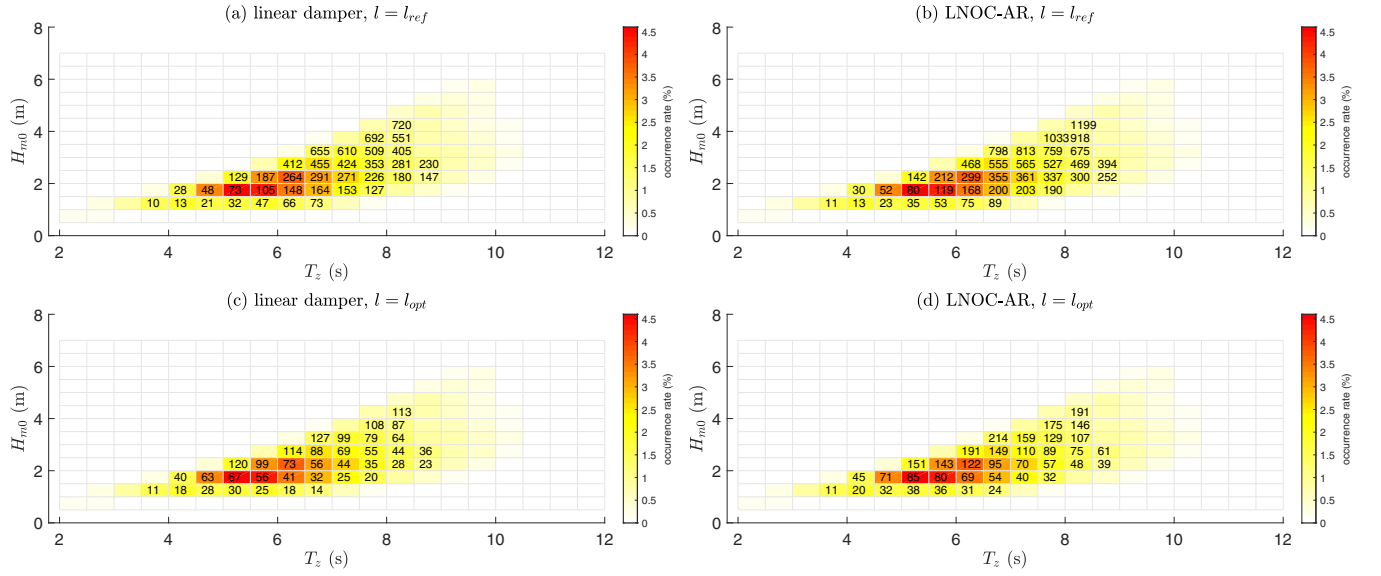


FIG. 11. UK Wave Hub: mean power  $P_i$  [unit (KW)] in scatter diagram, (a) with linear damper,  $l = l_{ref}$ , (b) with LNOC-AR,  $l = l_{ref}$ , (c) with linear damper,  $l = l_{opt}$ , and (d) with LNOC-AR,  $l = l_{opt}$ .  $P_i$  of sea states with occurrence rate less than 1% are omitted.

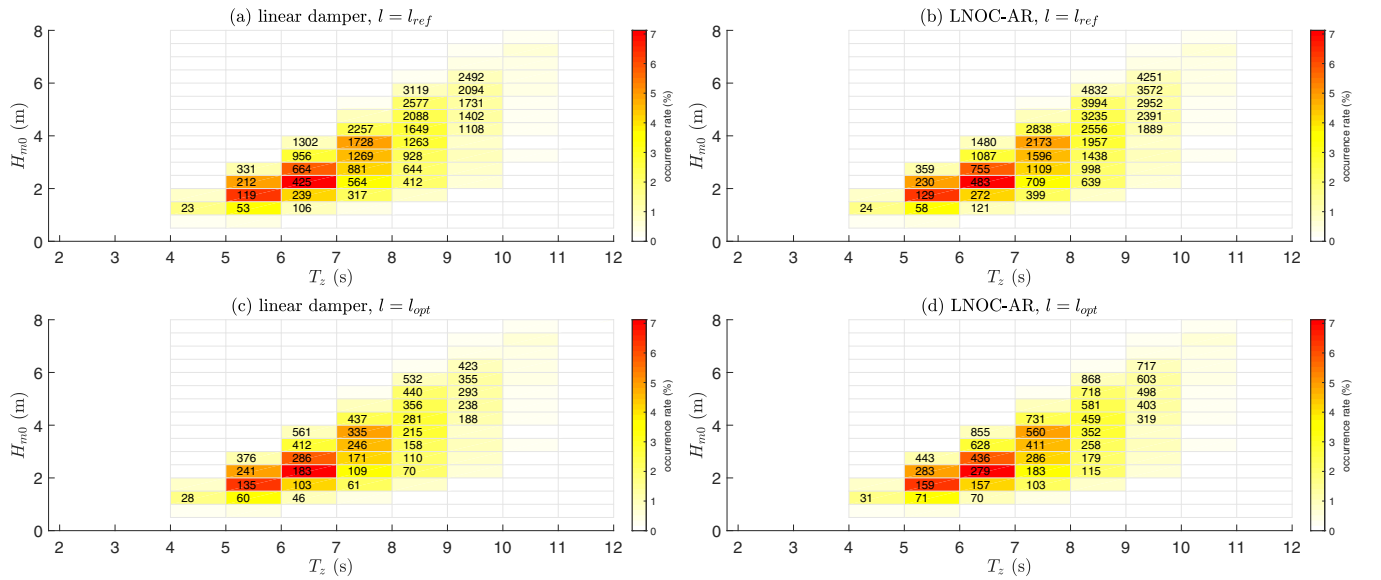


FIG. 12. Ireland Bellmullet: mean power  $P_i$  [unit (KW)] in scatter diagram, (a) with linear damper,  $l = l_{ref}$ , (b) with LNOC-AR,  $l = l_{ref}$ , (c) with linear damper,  $l = l_{opt}$ , and (d) with LNOC-AR,  $l = l_{opt}$ .  $P_i$  of sea states with occurrence rate less than 1% are omitted.

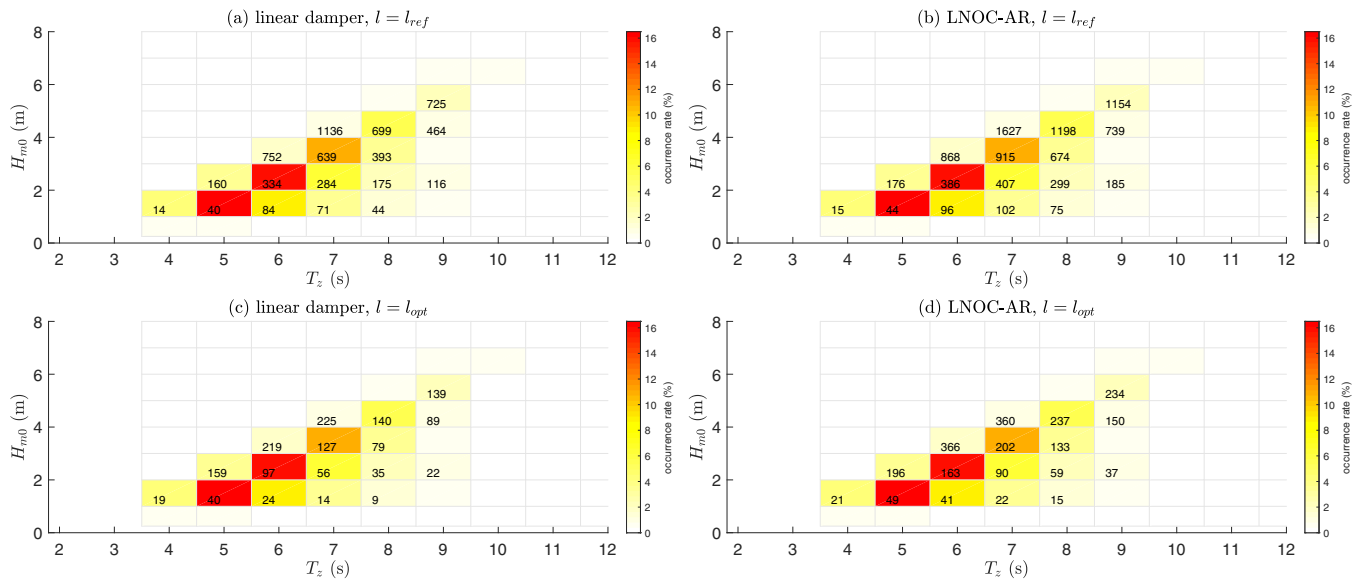


FIG. 13. UK EMEC: mean power  $P_i$  [unit (KW)] in scatter diagram, (a) with linear damper,  $l = l_{ref}$ , (b) with LNOC-AR,  $l = l_{ref}$ , (c) with linear damper,  $l = l_{opt}$ , and (d) with LNOC-AR,  $l = l_{opt}$ .  $P_i$  of sea states with occurrence rate less than 1% are omitted.

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