

Shear jamming transition in alternating shear rotation for frictional and frictionless suspensions

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Alternating shear rotations in dense suspensions have recently shown the ability to reduce both viscosity and dissipation per strain (at a fixed global shear rate). Here, we study alternating shear rotations, with extensive numerical simulations, at various angles and up to their corresponding jamming points. For increasing shear rotation angles, we find that the jamming point is continuously shifted to higher packing fractions for frictional particles, while it remains constant for frictionless particles. As a consequence, the alternating shear rotation is unable to reduce the dissipation per strain for suspensions composed of frictionless particles. We detail the individual contributions, hydrodynamic or contact, to the shear stress, which are uncharted for this protocol. As the angle of rotation increases, the average contact stress decreases. However, we find that the hydrodynamic stress shows the opposite trend, instead increasing with increasing angle. Hence, hydrodynamic stress will dominate up to much higher packing fractions as the angle of rotation increases. In addition, we report how the microstructure varies and establish a one-to-one mapping between the contact number and its contribution to the total stress for both frictionless and frictional particles.

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I. INTRODUCTION

Suspensions are abundant in nature (e.g., magma), industry (e.g., pill formation), and in the human body (e.g., blood). These complex fluids often exhibit non-Newtonian behavior, where the flow resistance is nonlinear as a function of the flow rate, where shear thinning is characterized by a sublinear relationship and shear thickening by a superlinear relationship. Non-Newtonian or not, upon increasing the solid content of a suspension, i.e., increasing the solid fraction ϕ , the system attains rigidity via a jamming transition when $\phi \rightarrow \phi_c$, where ϕ_c is the packing fraction at (shear)jamming. In general, suspensions show fragility [1] where, although they are shear jammed in a particular flow direction, a change in the shear flow direction, i.e., applying a shear rotation, may allow the system to flow [1]. Studying shear rotation has emerged as a promising research area for the last few years, as it shows significant interesting physics, such as history-dependent transverse stress [2,3] or reducing energy cost [4] while transporting. Further, the shear rotation potentially challenges [4] the idea that random organization [5] is the key

mechanism that reduces dissipation. In addition to reducing flow resistance, a repeated sudden change of flow and vorticity directions also shifts the jamming point toward larger solid fractions and hits the maximum for a rotation of 180° , corresponding to a nonpropagating oscillatory shear protocol. Although the jamming transition [6,7] has been extensively studied for various kinds of non-Brownian systems such as granular materials [8,9], emulsions [9], foams [10,11], etc., the area is largely unexplored for alternating shear rotation (ASR), also known as tacking [4], where one performs alternating, and opposite, shear rotations around the gradient direction between strain sequences. To fill this knowledge gap, we here study how the jamming transition and individual contributions, contact and short-range hydrodynamics, to the shear stress are altered by the angle in the alternating shear rotation for frictionless and frictional particles.

In this work, we show that the contact part of the relative viscosity is well described by the Krieger-Dougherty (K-D) relationship [12] at each fixed shear rotation angle. With increasing angle and for frictional grains, we find that the jamming transition shifted to higher packing fractions in a continuous way, while the prefactor of the K-D relationships decreases. For frictionless grains (and within our precision), the jamming packing fraction remains constant as a function of tacking angle, while the K-D's prefactor decreases. We also demonstrate that while contact contribution to the total stress decreases with the tacking angle, the hydrodynamic contribution increases; therefore, they eventually cross at some point in the volume fraction axis. We show that this crossing point increases with the angle of shear rotation. In addition, we

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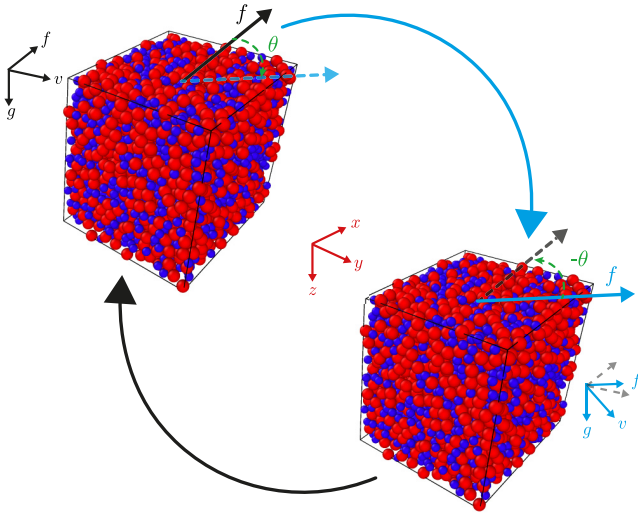


FIG. 1. Sketch of the alternating shear rotation protocol, with shear rotations by an angle θ around the gradient (g) direction. The figure indicates the laboratory frame (xyz) as well as the two flow-vorticity-gradient (fvg) frames. Small and large particles are shown in blue and red, respectively.

report how the microstructure varies with tacking angle and packing fraction in the alternating shear rotation. In the end, we show that the contact contribution of the shear stress versus the number of contacts per particle, across all packing fractions and angles, can be unified into two master curves: one for frictionless grains and the other for frictional grains, once a geometric factor is taken into account.

II. ALTERNATING SHEAR ROTATION

A shear rotation consists of rotating the flow direction about the gradient axis of a sheared fluid (suspension); unlike simple shear flow reversals, which merely invert the flow, it reorients the flow by an angle θ , thereby defining a broader class of unsteady shear protocols. Alternating shear rotations are protocols in which alternating and opposing ($\pm\theta$) shear rotations are repeated after a certain strain. The steady shear and shear reversal can be seen as two extreme cases of shear rotations, with θ being 0° and 180° , respectively. This protocol is similar to the one recently proposed by Blanc *et al.* [2], but differs on two points: First, our strain is much smaller than in Ref. [2], and, hence, our viscosity never relaxes to its steady-shear value. Second, and related to the first point, this is for repeated and opposing shear rotations (see Fig. 1 for a sketch of the shear protocol). Upon rotations, the flow and vorticity directions change, and stresses need to be expressed in these rotated frames.

The stress component in a rotated flow-gradient plane, σ'_{fg} , can be directly obtained from the laboratory (xyz) frame stress tensor σ as the projection of the traction vector on the plane normal to the gradient direction $\hat{z}$ along the rotated flow $\hat{f}$ direction:

$$\sigma'_{fg} = (\sigma \hat{z}) \cdot \hat{f} = \sigma_{xz} \cos \theta_{fx} + \sigma_{yz} \sin \theta_{fx}. \quad (1)$$

Alternatively, this component can be obtained by rotating the full stress tensor $\sigma' = \mathbf{R}^T \sigma \mathbf{R}$, where $\mathbf{R}$ is the rotation matrix

about the gradient direction, and extracting the fg component from σ' , which gives the same result. Relative viscosities, along the flow directions, can then easily be obtained by dividing the shear stress component of the stress tensors by $\eta_f \dot{\gamma}$, where η_f is the fluid viscosity and $\dot{\gamma}$ the shear rate. The latter is kept constant and independent of the angle.

As a suspension is sheared, it develops an anisotropic force and contact network [13], where contact between particles is more frequent along the compression axis, which increases its resistance to flow. When the flow direction is reversed, this network partly collapses [1], as evident from the immediate decrease in the number of contacts per particle (see Fig. S1 in Supplemental Material [14]), resulting in a corresponding dip in the viscosity-strain curve [2,15,16]. Continuing shearing after a reversal causes the contacts to rebuild, eventually reaching a steady shear state in the rotated direction. However, frequent changes in direction, particularly reversing after short strain amplitudes, can prevent reaching this steady shear state.

III. SIMULATION DETAILS

In our simulations, unless stated otherwise, we subject each direction to a 1% strain before changing direction. We perform dynamic particle simulations solving trajectories using LAMMPS [17] under applied shear (strain imposed) at constant volume, with contact forces (including friction) and hydrodynamics included, following previous studies [4,5,18]. We simulate a bidisperse 50:50 mixture of hard particles with a diameter ratio of 1:1.4 at fixed packing fractions ϕ . This mixture avoids crystallization and is well studied in the literature [4,5,15,19]. The particles are immersed in isodense fluids, described as a continuum, with the viscosity η_f . The solid particles are modeled as athermal, noninertial, and spherical. The suspension is confined in a three-dimensional periodic box using Lees-Edwards boundary conditions.

A particle undergoes three types of force and torque: Stokes drag, contact, and lubrication forces. The Stokes drag force and torque are given by

$$\begin{aligned} \vec{F}_i^s &= -6\pi\eta_f a_i (\vec{v}_i - \vec{v}_f), \\ \vec{\tau}_i^s &= -8\pi\eta_f a_i^3 (\vec{\omega}_i - \vec{\omega}_f). \end{aligned} \quad (2)$$

Here, a_i , v_i , and ω_i are the radius, linear velocity, and angular velocity of particle i , while v_f and ω_f are the fluid's linear and angular velocity (vorticity), and η_f is the viscosity of the fluid. We apply linear shear in various directions; therefore, we generally have $\vec{v}_f = \dot{\gamma} z \hat{f}$, where $\hat{f}$ is the unit vector in the flow direction, z the gradient direction coordinate, and $\dot{\gamma}$ the shear rate along the flow direction. In a corresponding way, we have $\vec{\omega}_f = \dot{\gamma} (\hat{f} \times \hat{z})/2$, $\hat{z}$ being the unit vector along the gradient direction. The contact force between particles is described by harmonic springs:

$$\vec{F}_{ij} = k_n \delta_n^{ij} \hat{n}^{ij} + k_t \delta_t^{ij} \hat{t}^{ij}. \quad (3)$$

Here, δ_n^{ij} is the normal overlap between two particles, while δ_t^{ij} is the relative tangential displacement. Particles are counted to be in contact if there is a nonzero overlap between them. The springs' normal and tangential stiffness constants are denoted by k_n and k_t . We choose $k_t = (2/7)k_n$, and k_n and k_t values large enough such that our particles fulfill near-hard sphere

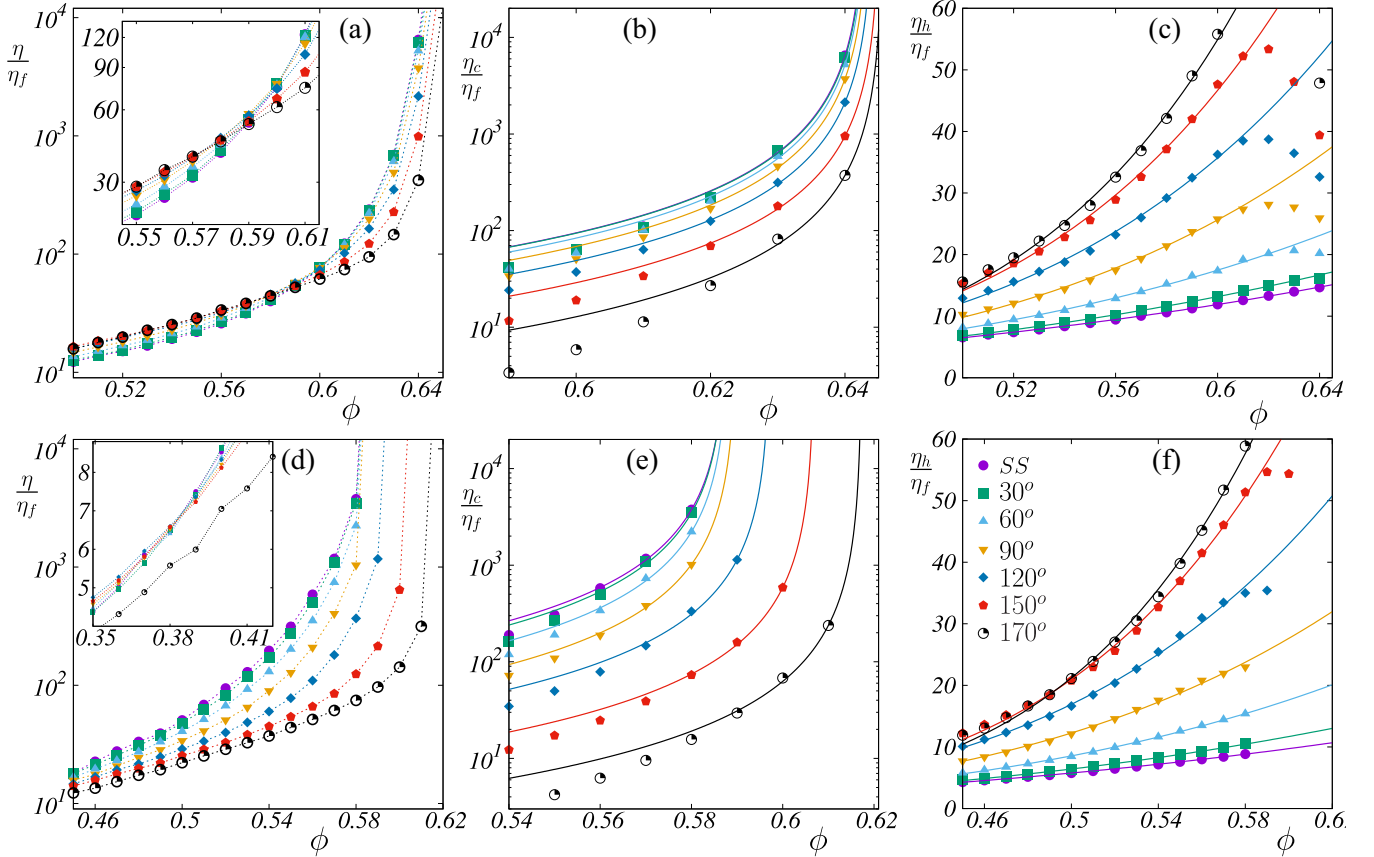


FIG. 2. Relative viscosity η/η_f along with packing fractions ϕ for different shear rotation angles θ for (a) frictionless and (d) frictional particles ($\mu = 1$). (Inset) Enlarged version of the figure to show the change in relative viscosity trend from increasing to decreasing with tacking angles. Contact contribution to relative viscosity η_c/η_f near shear jamming as a function of packing fraction for (b) frictionless and (e) frictional particles. The curves are fitted with the K-D relationship (see Fig. S5 of Supplemental Material [14] for corresponding dependencies on the normal viscosities). Hydrodynamic contribution to relative viscosity η_h/η_f as a function of packing fraction for (c) frictionless and (f) frictional particles. The branches below jamming are all well fitted with power law $\alpha (\phi/\phi_m)^\beta$, where ϕ_m is a reference packing fraction of the fit, i.e., 0.50. Dashed lines in panels (a) and (d) are guides to the eye, while solid lines in panels (b), (c), (e), and (f) are best fits.

conditions (typically $k_n/Pd \sim 10^4$, where P is the pressure and d is the diameter of the smaller particles). We chose parameters such that the Stokes number $\rho \dot{\gamma} \langle a \rangle^2 / \eta_f \ll 1$, where $\langle a \rangle$ denotes the average radius and ρ the density of both solid particles and the fluid. The unit vectors along the normal and the tangential direction of two particles are denoted by $\hat{n}^{ij}$ and $\hat{t}^{ij}$. The tangential forces for each contact are restricted by the Coulomb friction criteria $|\vec{F}_{ij}^t| < |\mu \vec{F}_{ij}^n|$, where μ is the coefficient of friction. When a lubricating film separates two particles, the hydrodynamic interactions among particles rely on the resistance matrix formalism outlined in Refs. [20,21]. The code can be found in Ref. [22].

In LAMMPS, we can only shear along the principal directions of the simulation box. To achieve any given flow direction, we, hence, shear along two of these principal directions simultaneously (but at various rates), using a common gradient direction $\hat{z}$. The flow direction is then given by $\hat{f} = \cos \theta_{fx} \hat{x} + \sin \theta_{fx} \hat{y}$, where $\hat{x}$ and $\hat{y}$ are the two of the simulation box's principal axes and θ_{fx} is the angle between $\hat{f}$ and $\hat{x}$.

For each angle of shear rotation, we collect data over 36 accumulated strains along the flow path (corresponding

to 1800 alternating shear rotation cycles), after removing the first four accumulated strains using the same protocols.

IV. THE JAMMING TRANSITION

We now conduct an analysis on the shear jamming transition for non-Brownian suspensions subjected to alternating shear rotations [4] at various tacking angles θ under applied shear (strain imposed) at constant volume and strain sequences (amplitude) of 1%, using numerical simulations as described above. In Figs. 2(a) and 2(d), we plot the viscosity along the flow directions with a wide range of packing fractions for different alternating shear rotation angles for frictionless and frictional particles with friction coefficient $\mu = 1$, respectively. For each fixed angle, we see the viscosity as a function of the packing fraction increasing, as has been observed before in the steady-state conditions. For the frictionless case, we see that regardless of the angle, the viscosity diverges roughly at the same packing fraction of 0.64. This value is the same as found for the isotropic jamming transition, and is identified as the random close packing (RCP)

in three dimensions [23]. For the frictional case, the steady-state shear jamming is considerably smaller than that in the corresponding frictionless case, 0.59 compared to 0.64 (RCP), due to additional mechanically stabilizing tangential forces [24]. As one increases the alternating shear rotation angle, the shear jamming packing fraction shifts to higher values and approaches 0.62, below RCP. In both cases, the densest packings are geometrically limited by the RCP. Frictionless jamming occurs at or very close to this limit and cannot be increased by changing the shear protocol. In contrast, frictional packings, whose steady-state jamming is below RCP, can achieve higher packing fractions under different protocols. Surprisingly, for frictionless grains, increasing the shear rotation angle at fixed packing fractions has opposite effects on the viscosity depending on the packing fraction. Below a packing fraction of around 0.585, the viscosity is found to increase with increasing angle, while above it decreases. This results in a clear crossing point of the curves at various angles as seen in the inset of Fig. 2(a), with no detectable size dependency (see Fig. S6 of Supplemental Material [14]). We find similar findings (except for the highest angle) for the frictional case, but where the crossing now is shifted to much smaller packing fractions around 0.385, as can be seen in the inset of Fig. 2(d). We now examine the individual contributions of contact forces and hydrodynamics to viscosity, η_c and η_h , with increasing volume fraction at various angles of shear rotations; see Figs. 2(b) and 2(c) for the frictionless case and Figs. 2(e) and 2(f) for the frictional case. As can be seen from the different scales, logarithmic on contact versus linear on hydrodynamics, the contact forces are always orders of magnitude greater than the hydrodynamic ones close to their corresponding shear jamming transition regardless of the angle. However, at lower packing fractions and larger angles, these terms start to be on the same order of magnitude (see Fig. S4 of Supplemental Material [14] for explicit comparisons). We denote the packing fraction where the two contributions, contact and hydrodynamic, have the same magnitude by $\phi_{c/h}$. Interestingly, the two contributions react in opposite ways upon increasing shear rotation angle. The contact contribution always decreases upon increasing angle, while the hydrodynamics increase. This explains why we can have the total viscosity increasing at low packing fractions and decreasing at high ones for an increasing angle, as it depends on which contribution is dominant and most responsive. The increase in hydrodynamic stress could, in principle, arise from larger relative velocities, smaller interparticle gaps, or enhanced alignment along the compression axis. However, neither these effects individually nor their simple combination can account for the observed growth, which instead arises from a coupled three-way correlation among them that is outlined later in this article. Furthermore, the hydrodynamics do not show any divergence approaching shear jamming. Instead, it shows a nonmonotonic behavior as a function of packing fraction close to the shear jamming and at the highest angles. The contact part of the viscosity can be fitted by using a K-D relationship, as $\eta_c/\eta_f = \eta_0/\eta_f(\phi_c - \phi)^{-2.5\phi_c}$, where η_0 and ϕ_c all depend on the rotation angle θ , and are shown in Figs. 2(b) and 2(e) as lines. The hydrodynamic contribution is fitted as $\eta_h/\eta_f \sim \alpha(\phi/\phi_m)^\beta$, where ϕ_m is a reference packing fraction, and we use 0.5 for both frictional

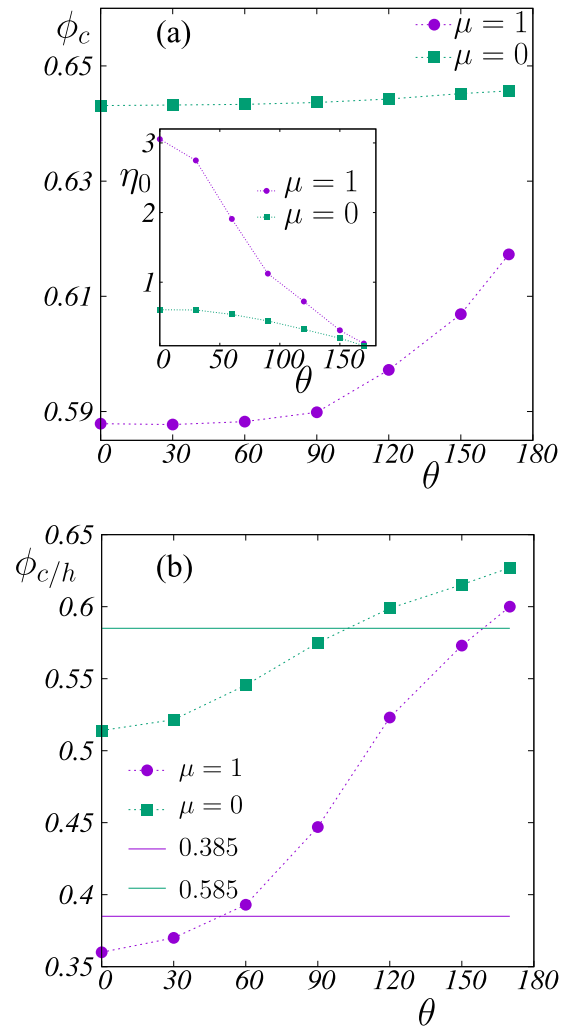


FIG. 3. (a) Change in jamming point with alternating shear rotation angle for both frictionless and frictional particles by fitting the simulation data with the K-D relationship. While for frictional particles a visible increase is noticed, for frictionless particles the system barely changes its jamming point. (Inset) The prefactor in the K-D relationship decreases for both frictionless and frictional particles, indicating that the contact contribution decreases in both cases when a shear rotation is applied. (b) Crossing packing fraction of hydrodynamics and contact contribution for frictionless and frictional particles. Solid lines correspond to the crossing point at which the rheology changes behavior as a function of angle. Dashed lines serve as guides to the eye.

and frictionless cases. It is important to note that the hydrodynamic contribution does not diverge as we regularize the minimum distance between any pair of particles, thereby simulating particle-scale roughness. This clarifies why we use ϕ rather than inverse powers of $(\phi_c - \phi)$ in our fits of the hydrodynamics. The parameters and how they vary with shear rotation angle are illustrated in Fig. 3 and Fig. S2 in Supplemental Material [14]. From Fig. 3(a), we see that shear jamming packing fraction does not show any detectable changes for the frictionless case as the angle is increased, while for the frictional case we see a clear increase when the angle is above 90° and is continuously increasing toward its

maximum at $\theta \rightarrow 180^\circ$, corresponding to alternating shear reversal (or oscillatory shear). Similar trends are observed for the other stress components (see Fig. S5 in Supplemental Material [14]). The K-D prefactor η_0 is seen to decrease with the shear rotation angles for both cases, frictional and frictionless, as seen in the inset of Fig. 3(a). The fact that ϕ_c increases (or stays constant) and η_0 decreases with increasing rotation angle shows that viscosity is lowered with increasing angle for a given fixed packing fraction. One might attempt to collapse the viscosity as a function of ϕ/ϕ_c , accounting for the tacking-angle dependence of ϕ_c . However, this fails in the frictionless case, where ϕ_c is angle independent, and also in the frictional case (see Fig. S7 in Supplemental Material [14]). Hence, viscosity cannot be described solely by ϕ_c and must also depend on geometric factors such as the tacking angle.

The hydrodynamic increases, with increasing angle, can be seen from Figs. 2(c) and 2(f) but further supported by the fitted parameters seen in Fig. S2 of Supplemental Material [14]. Next, we look at $\phi_{c/h}$ as a function of θ , in Fig. 3(b). A compelling observation emerges from our analysis: A tug-of-war scenario unfolds as the contributions from contact forces and lubrication contend with each other. At a certain packing fraction, they intersect, where the contact contribution equals that of lubrication. Noteworthy is the revelation that the crossing point shifts along the packing fraction axis with the angle of shear rotation, as evident in Fig. 3(b). This suggests that as the angle of shear rotation increases, a higher packing fraction is required for the contact contribution to surpass hydrodynamics. Surprisingly, the net effect of these opposing forces leads to an almost fixed crossing point in the packing space for the total viscosity [see the insets of Figs. 2(a) and 2(d)]. To make this a bit clearer, we also plot the packing fraction at which the total viscosity crosses the steady-state, $\theta = 0^\circ$, curve, marked as solid lines in Fig. 3(b).

V. DISSIPATION

Increasing the angle not only leads to lower viscosity but can also lead to the possibility of a reduction in the dissipation (per propagation strain), i.e., a lowering in energy consumption for mass transporting the suspension by shear. As shown previously, this is dictated not only by the viscosity reduction but also by the flow path geometry [4]. In general, the viscosity reduction needs to be substantial enough to compensate for the longer path one needs to take by doing a tacking motion.

In Fig. 4, we plot the dissipation per strain for both frictionless and frictional cases at three different tacking angles. Once the viscosity is known, the relative dissipation per strain is calculated as $W = \frac{2}{(1+\cos\theta)} \frac{\eta}{\eta_{ss}}$ following Ref. [4], where η is the viscosity obtained from the ASR protocol and η_{ss} is the corresponding viscosity under steady shear (i.e., $\theta = 0$) at the same packing fraction. The quantity W is defined as $W \equiv W_{ASR}/W_{SS}$, where W_{ASR} denotes the energy dissipated per unit strain in the ASR protocol and W_{SS} the corresponding dissipation under simple shear. Values of $W < 1$ indicate reduced dissipation, implying an energetic advantage for transporting the suspension compared to steady shear. For frictional particles, a substantial gain (below unity) is observed beyond a certain packing fraction for all three angles, whereas no

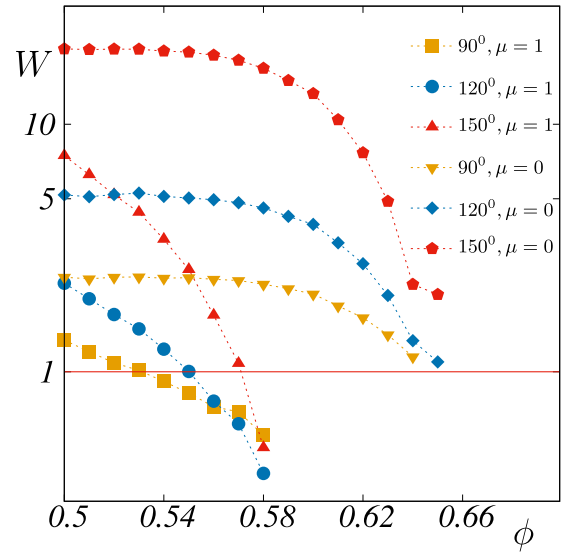


FIG. 4. Dissipation per unit strain for three different angles of shearing for both frictional and frictionless systems. In frictionless systems, the value does not drop below 1, preventing any decrease in dissipation per strain. In contrast, frictional systems exhibit a substantial gain after reaching a certain packing fraction for all three angles.

such gain (remaining above unity) is found for frictionless particles. This difference arises from the significantly smaller viscosity reduction in the frictionless case compared to the frictional one, the primary reason being that the shear jamming packing fraction remains unchanged as the tacking angle is varied, as shown in Fig. 3(a).

VI. INCREASE IN THE HYDRODYNAMIC CONTRIBUTIONS

To rationalize the increase in hydrodynamic stress with tacking angle, we examine the pairwise origin of the lubrication contribution for the frictional case. The relevant stress component can be written as $\sigma_{fg}^h = \frac{1}{2}(f_f^h \Delta g + f_g^h \Delta f)$, where f_f and f_g are the lubrication forces [20,21] in the flow and gradient directions and Δf , Δg the corresponding separations. As shown in Ref. [25], the dominant contribution arises from the normal lubrication force, for which $f_f \simeq X_A^{11} v_{n,f}$ and $f_g \simeq X_A^{11} v_{n,g}$, where X_A^{11} denotes the normal resistance, scaling inversely with the interparticle gap, and $v_{n,f/g}$ are the normal components of the relative velocity. Substituting yields the leading-order expression

$$\sigma_{fg}^h \simeq \frac{1}{2} X_A^{11} (v_{n,f} \Delta g + v_{n,g} \Delta f). \quad (4)$$

Figure 5 shows the ratio of cycle-averaged quantities at tacking angles 120° and 60° . The lubrication resistance and geometric separations vary by only a few percent, while normal relative velocities increase by $\sim 10\%$. Even the lubrication forces grow by only $\sim 50\%$, far too weak to explain the $>100\%$ increase in hydrodynamic stress.

Instead, the data show that the enhancement arises from a strong three-way correlation among (1) normal velocity differences, (2) interparticle gaps, and (3) pair alignment with the instantaneous compression axis. This correlated variation—

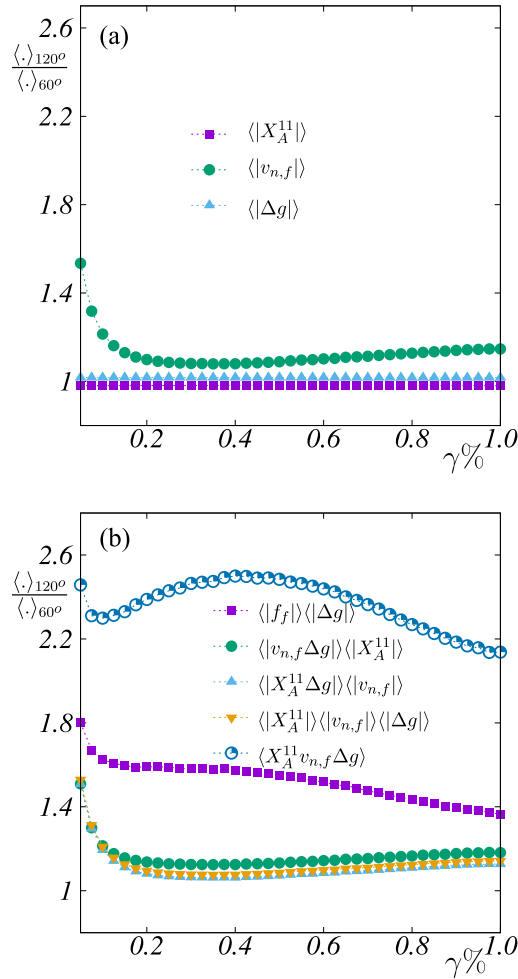


FIG. 5. Ratio of cycle-averaged component-wise contributions to the hydrodynamic stress for tacking angles 120° and 60° at $\phi = 0.5$. Shown are (a) the ratios of lubrication resistance X_A^{11} , normal relative velocity components, and geometric projection terms, along with (b) their combined contributions entering Eq. (4). While individual quantities change only weakly with angle, their correlated variation leads to a strong enhancement of the hydrodynamic stress.

not any single microscopic factor—produces the pronounced increase in hydrodynamic stress at larger tacking angles. This three-way correlation may ultimately be traced back to a small number of dominant hydrodynamic pair interactions and represents an interesting direction for future work.

VII. MICROSTRUCTURE

Next, we have a detailed look at the microstructure, examining how the number of elastic contacts per particle varies at various tacking angles and packing fractions. In Figs. 6(a) and 6(b), we depict the average contact number as the packing fraction increases for different ASR angles for both frictionless and frictional particles. Notably, at each angle, this metric is higher with greater packing fractions, which is expected as a result of the increased likelihood of overlap as packing density increases. The observed decrease in the number of contacts, at a fixed packing fraction, with an increasing shear rotation angle aligns with the trends seen in Figs. 2(a), 2(b), 2(d),

and 2(e), where a wider angle leads to the destruction of more contacts and leads to a lower contact contribution of the viscosity and the overall viscosity for the same packing fraction, as seen in previous studies [19].

Notably, the frictionless data all merge at $z \simeq 6$ as one approaches RCP, with a steeper increase in z close to RCP for higher angles to compensate for lower values further below this point. This gives a physical (contact) interpretation of why the jamming packing fraction remains constant as a function of tacking angle, but with a lower K-D prefactor. For the frictional case, the z versus ϕ curves at fixed angles never approach each other, and the sharp increase in z is shifted to higher packing fractions for higher angles, indicating higher shear jamming packing fractions.

Despite maintaining an exponential relationship akin to previous findings cited in Ref. [19], the scaling factor $\eta/\eta_f \sim 14 \exp(z)$ is not universally applicable, primarily due to variations in the contribution of hydrodynamics across different packing fractions. It is important to note that the total contribution includes hydrodynamics, which differs across packing fractions, necessitating its exclusion, especially when mapping with the number of contacts. To address this, we plot the contact contribution to relative viscosity against the average coordination number for different packing fractions in Fig. 6(c). Strikingly, all curves for $\mu = 0$ and $\mu = 1$ converge separately to two different branches once we rescaled the contact contribution by a factor f , reflecting the θ -dependent efficiency of the contact network to transmit shear stress. We plot the θ dependency of this factor in the inset of Fig. 6(c), together with best fits using power-law functions (expressions given in the inset). We fit these two rescaled coordination number branches with two important facts in mind: First, the stress contribution should diverge at the isostatic point, which we assume to be independent of the shear rotation to respect mechanical stability (Maxwell's criterion). For frictionless conditions, we have $z_{\text{iso}}^{\mu=0} = 2D = 6$, and for infinite friction, $z_{\text{iso}}^{\mu=\infty} = D + 1 = 4$, where D is the dimension (2D or 3D) of the system. Second, the stress contribution from contact should be null at $z = 0$. The frictionless branch is fitted [26] by $\eta_c/\eta_f \sim 25(\frac{z}{6-z})^2 f(\theta)$, where we used the standard value of $z_{\text{iso}}^{\mu=0} = 6$ but with a divergence steeper than predicted by RCP theories [27]. However, in the case of frictional branch, as we have a friction coefficient $\mu = 1$, which is far from infinite friction, the isostatic point is not exactly at $z_{\text{iso}} = 4$, but rather a slightly bigger value [14]. Since the exact z_{iso} is not easy to predict for the frictional case, we approximate $z_{\text{iso}}^{\mu=1} \approx z_{\text{iso}}^{\mu=\infty} = 4$ and use $\eta_c/\eta_f \sim 45(\frac{z}{4-z})^{2.5} f(\theta)$ as our fit for the frictional branch. Both branches are nicely captured with these fit functions.

VIII. DISCUSSION

We have detailed how the viscosity and its various contributions, the number of contacts and contact network, vary after a shear rotation. As shown in Fig. 3(a), the shear jamming point shifts with tacking angle only for frictional particles, approaching the frictionless (near-RCP) value as the angle increases. As a result, the viscosity decreases less for suspensions composed of frictionless particles than frictional

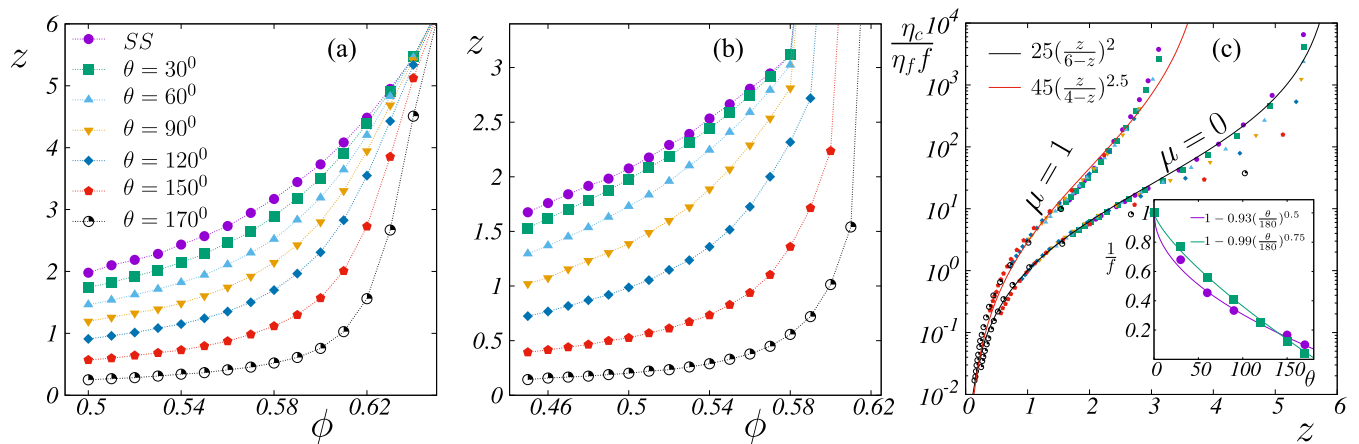


FIG. 6. (a) The number of average contacts z as a function of packing fractions ϕ for various shear rotation angles θ for (a) $\mu = 0$ and (b) $\mu = 1$ (dashed lines as guides to the eye). In panel (a), the curves appear to approach the same jamming point. In contrast, the curves in panel (b) do not show a tendency to reach the same jamming point. (c) Scaled contact contribution to the viscosity with contact number showing two different branches for frictional and frictionless particles (see Fig. S3 of Supplemental Material [14] for unscaled contact contributions). (Inset) Factor by which the contact contribution is scaled. Purple circles: frictionless; green squares: frictional. Dashed lines in panels (a) and (b) are guides to the eye, while solid lines in panel (c), including the inset, are best fits.

particles when increasing the tacking angle and precludes any substantial dissipation gains for the frictionless case using alternative shear rotations, as compared to the frictional case. We, furthermore, find that shear rotations delay the buildup of contacts but do not suppress their final values at the jamming threshold. Moreover, the number of contacts per particle continues to follow two distinct master curves, one for frictionless systems and one for frictional systems, once corrected by an angular geometric factor $f(\theta)$, highlighting the persistent and fundamental distinction between the two. For frictionless particles, we found the existence of a distinct packing fraction (of around 59%) where below the viscosity increases, i.e., thickens, upon increasing angle, while above the opposite happens, i.e., thins. This effect is explained by the contact and hydrodynamic stresses opposing responses to an increasing angle, where the former is caused by contact breakage and the latter by a combined effect of larger relative velocities, smaller interparticle gaps, and enhanced alignment along the compression axis [28].

ASR tuning of the rheology shows strong similarities to previously studied protocols, including pure and rotary oscillatory rheology [29,30], as well as oscillatory or perpendicular shear perturbations [19,31]. In these approaches, the primary

shear flow is perturbed either in amplitude or in direction. Taken together, these similarities point toward the possibility of a unified framework for rheological control.

Our work is important for many flows that involve a change in flow geometry, e.g., extrusion or landslides on varying slopes, and paves the way for more energy-efficient transport protocols exploiting geometry. Having mapped out the ASR response for both frictional and frictionless particles, a natural continuation would be to explore the response of ASR on discontinuous shear-thickening (DST) suspensions [20,32], which might provide further insight into the physics at the DST point [33–35].

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DATA AVAILABILITY

The data that support the findings of this study, including the numerical simulation input files, are publicly available in Zenodo [36].

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