

Symmetry-controlled hybrid chiral-omega bianisotropy in knot-particle metasurface

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Hybrid chiral-omega bianisotropic responses are challenging to achieve within a single metamaterial layer due to their distinct symmetry requirements. Here, we demonstrate that such hybrid response naturally emerges from mirror symmetry breaking in a single-layer trefoil knot-particle metasurface. A single geometric parameter, the offset of vertical interconnects between top and bottom faces, continuously tunes the response from pure chiral to hybrid chiral-omega behavior. Using dipole moment decomposition, we reveal that symmetry breaking activates the hybrid response by allowing coupling of current modes to orthogonal dipole moments, enabling simultaneous control over transmission and reflection asymmetries. Experimental measurements of complex-valued scattering coefficients validate the predicted hybrid response through direct susceptibility extraction. These results establish geometric symmetry engineering as a design principle for controlling electromagnetic coupling pathways in bianisotropic metamaterials.

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I. INTRODUCTION

Bianisotropic metasurfaces provide a rich platform for controlling and manipulating electromagnetic waves. Their magnetoelectric coupling enables electric fields to induce magnetic currents and vice versa. These additional degrees of freedom allow for intricate control of the phase, amplitude, and polarization of electromagnetic waves scattered from the surface [1,2]. This unique control capability fundamentally stems from the underlying symmetry properties of the metasurface unit-cell (or meta-atom) and lattice structures. Symmetry principles govern electromagnetic responses in metamaterials and metasurfaces through the geometric structure and spatial configuration of meta-atoms [3–6]. In bianisotropic systems, mirror symmetry, governed by the parity operator ($\mathcal{P}$), plays an essential role as it directly determines the coupling pathways between electric and magnetic responses [7].

When exploring reciprocal metasurfaces, their magnetoelectric coupling is typically classified into *chiral* and *omega* types. Chiral response arises from broken mirror symmetry, generating parallel electric-magnetic dipole coupling ($\mathbf{p} \parallel \mathbf{m}$) that enables polarization rotation and circular dichroism [8,9]. Omega-type bianisotropy, conversely, emerges from orthogonal dipole coupling ($\mathbf{p} \perp \mathbf{m}$), enabling asymmetric reflection for unidirectional absorption and anomalous refraction [10–13]. These distinct coupling mechanisms demand fundamentally different structural symmetries, creating a

design challenge when both responses are desired within a single metasurface element. Previous studies have explored this symmetry-response connection through various geometric configurations, demonstrating different response types by designing elements with specific symmetry properties or by changing excitation conditions relative to the element's symmetry axes [14–16]. Previously [17], we demonstrated that toroidal knot particles provide a promising platform for chiral bianisotropy, with simple design rules arising from their topological features. In this work, we leverage symmetry breaking as a practical design tool. We demonstrate that a hybrid chiral-omega response emerges directly from mirror-symmetry breaking in a flat trefoil-knot structure, tunable through a single geometric parameter. By offsetting layer interconnects in a quasiplanar configuration, we break mirror symmetry, activating omega-type coupling alongside the chiral response. This combination provides simultaneous tailoring of transmission and reflection asymmetries within a single-layer architecture [9]. The physical mechanism is studied by examining how even and odd current modes excite the in-plane dipole moments, establishing an intuitive connection between structural symmetry, current harmonics, and the resulting bianisotropic response type.

II. FORMULATION AND ANALYSIS

A (p, q) -torus knot has the parametric form [18]

$$\mathbf{r}(s) = \begin{pmatrix} (a + b \cos qs) \cos ps \\ (a + b \cos qs) \sin ps \\ b \sin qs \end{pmatrix}, \quad (1)$$

where $s \in [0, 2\pi]$, a and b are the major and minor radii of the host torus, and p and q are coprime integers defining the knot winding numbers. The trefoil knot, corresponding to $(p, q) = (2, 3)$, is the simplest nontrivial knot and is shown in Fig. 1(a).

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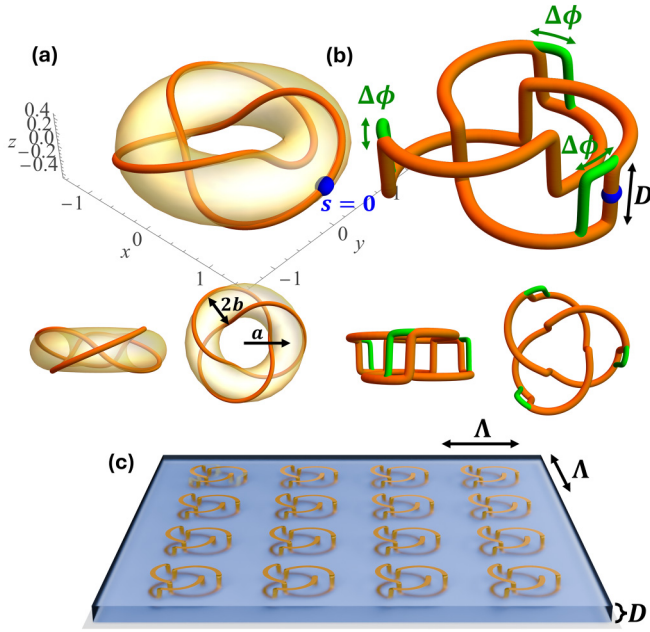


FIG. 1. Trefoil knot-particle geometry and metasurface implementation. (a) Three-dimensional (3D) trefoil knot with major radius a and minor radius b , where the blue dot indicates the parameter origin $s = 0$. (b) Quasiplanar implementation: Orange traces show symmetric configuration and green indicates via offset $\Delta\phi$, breaking mirror symmetry between the top and bottom layers. (c) Schematic of periodic knot-particle metasurface array on substrate. D is the substrate thickness, and Λ is the lattice periodicity.

While the general 3D structure is easier to describe and study due to its simple analytic representation in Eq. (1), its implementation as a metasurface with metallic wire knot-particle unit cells is both challenging and limited in terms of available materials. For practical implementation, a flattened structure is preferred; therefore, we transition to a quasiplanar geometry where the knot's 3D structure is projected onto a substrate. The new knot curve has (x, y) components from Eq. (1) with $(p, q) = (2, 3)$, and a modified z component

$$z(s) = \frac{D}{2} \text{sgn}[\sin(3s)], \quad (2)$$

where D is the substrate thickness and $\text{sgn}(\cdot)$ is the sign function. This geometry leads to two flat curves at $z = \pm D/2$, which are *mirror symmetric* with respect to three planes passing through the central axis and oriented at 120° intervals [orange, Fig. 1(b)]. The electric connection between the top and bottom curves is achieved by vertical vias positioned at $s_{\text{via}} = n\pi/3$ [$z(s_{\text{via}}) = 0$, six transitions]. It is important to note that while this flattened structure has a different geometry, it is topologically equivalent to the trefoil and therefore exhibits similar electromagnetic response [17]. To this flat setup, we can add degrees of freedom without increasing the difficulty of either modeling or fabrication; these can be used to further tailor the symmetry properties and obtain a richer playground for the metasurface response. One such degree of freedom is obtained by offsetting alternate vertical connections (three of the six vias) by angle $\Delta\phi$ in the s parameter space, as seen in Fig. 1(b). This preserves

the fundamental trefoil topology while breaking the mirror symmetry between top and bottom faces. The via positions become $s_{\text{via}} = [0, 2\pi/3, 4\pi/3]$ for the preserved locations and $s_{\text{via}} = [\pi/3, \pi, 5\pi/3] + \Delta\phi$ for the offset ones [green, Fig. 1(b)]. Applying the same offset $\Delta\phi$ to all three vias in an alternating way maintains the threefold rotational symmetry of the structure. When these quasiplanar knot particles are arranged in a two-dimensional square periodic array to form a metasurface [shown in Fig. 1(c), where Λ is the period], the physical nature of their collective electromagnetic behavior is described, approximately, by their macroscopic surface susceptibility tensors in the generalized sheet transition conditions (GSTCs) [19]

$$\hat{z} \times \Delta \mathbf{E} = -j\omega\mu_0 \overline{\overline{\chi}}^{\text{mm}} \cdot \mathbf{H}_{\text{t,av}} - j\omega\sqrt{\epsilon_0\mu_0} \overline{\overline{\chi}}^{\text{me}} \cdot \mathbf{E}_{\text{t,av}}, \quad (3)$$

$$\hat{z} \times \Delta \mathbf{H} = j\omega\epsilon_0 \overline{\overline{\chi}}^{\text{ee}} \cdot \mathbf{E}_{\text{t,av}} + j\omega\sqrt{\epsilon_0\mu_0} \overline{\overline{\chi}}^{\text{em}} \cdot \mathbf{H}_{\text{t,av}}, \quad (4)$$

where $\hat{z}$ is the unit normal, ω is the angular frequency, ϵ_0 and μ_0 are the free-space permittivity and permeability, $\Delta \mathbf{E}$ and $\Delta \mathbf{H}$ are field discontinuities across the surface, and $\mathbf{E}_{\text{t,av}}$ and $\mathbf{H}_{\text{t,av}}$ are average tangential fields on the metasurface. For reciprocal media, these are constrained by $\overline{\overline{\chi}}^{\text{ee}} = (\overline{\overline{\chi}}^{\text{ee}})^T$, $\overline{\overline{\chi}}^{\text{mm}} = (\overline{\overline{\chi}}^{\text{mm}})^T$, and $\overline{\overline{\chi}}^{\text{em}} = -(\overline{\overline{\chi}}^{\text{me}})^T$ [20].

To connect these macroscopic susceptibility tensors to the excitation of the knot particle, we analyze the electromagnetic response using the excited multipoles in each knot. The multipole moments for arbitrary-sized knot particles are rigorously given in Refs. [21,22]. However, to capture the main physical mechanism while keeping calculations analytically tractable, we focus on cases where $k|\mathbf{r}(s)| \ll 1$. Knot particles have many different resonance frequencies corresponding to different dominant current modes, and the subwavelength approximation is suitable when operating around the first resonance frequency, where the total wire length is roughly one wavelength [18] and dipole moments dominate the electromagnetic response [17]. Under this approximation, the electric and magnetic dipole moments can be expressed as

$$p_\gamma \approx \frac{1}{j\omega} \int_0^{2\pi} I_s(s) \mathbf{t}(s)_\gamma ds, \quad (5)$$

$$m_\gamma \approx \frac{1}{2} \int_0^{2\pi} I_s(s) [\mathbf{r}(s) \times \mathbf{t}(s)]_\gamma ds, \quad (6)$$

where $\gamma \in \{x, y, z\}$, $I_s(s)$ is the current distribution along the wire, and $\mathbf{t}(s) = d\mathbf{r}/ds$ is the tangent vector along the wire. In turn, these dipole moments directly connect to the macroscopic susceptibility tensors through [19]

$$\frac{1}{\Lambda^2} \begin{pmatrix} \mathbf{p} \\ \mathbf{m}/\mu_0 \end{pmatrix} = \begin{pmatrix} \epsilon_0 \overline{\overline{\chi}}^{\text{ee}} & \overline{\overline{\chi}}^{\text{em}}/c_0 \\ \overline{\overline{\chi}}^{\text{me}}/\eta_0 & \overline{\overline{\chi}}^{\text{mm}} \end{pmatrix} \begin{pmatrix} \mathbf{E}_{\text{t,av}} \\ \mathbf{H}_{\text{t,av}} \end{pmatrix}, \quad (7)$$

where Λ^2 is the unit cell area, $\mathbf{p}$ and $\mathbf{m}$ are the electric and magnetic dipole moment vectors, $c_0 = 1/\sqrt{\epsilon_0\mu_0}$ is the speed of light in vacuum, and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the free-space impedance. To evaluate the dipole moments and understand how geometric symmetry breaking controls the bianisotropic response, we analyze the current distribution along the knot particle. The current in any wire element can be written as $I(s) = \sum_n [\alpha_n \sin(ns) + \beta_n \cos(ns)]$, where α_n and β_n represent the antisymmetric (odd) and symmetric (even) complex

TABLE I. Dipole coupling mechanisms for different bianisotropic responses.

Response	Dipole coupling
Chiral	$p_{x,\alpha} \leftrightarrow m_{x,\alpha}, p_{y,\beta} \leftrightarrow m_{y,\beta}$
Omega	$p_{x,\alpha} \leftrightarrow m_{y,\alpha}, p_{y,\beta} \leftrightarrow m_{x,\beta}$
Hybrid	Both of the above

amplitudes of the n th harmonic, respectively. As demonstrated in Ref. [17], the dominant harmonic contribution around the first resonance of a trefoil occurs at $n = 1$ (for detailed current analysis and simulation, see the Supplemental Material [23]), allowing us to approximate the current distribution as

$$I(s) \approx \alpha \sin(s) + \beta \cos(s). \quad (8)$$

Substituting this current into Eqs. (5) and (6), and using the geometry defined by Eqs. (1) and (2) with the via offset incorporated, yields the dipole moments

$$p_x = \frac{3\alpha b\pi}{2j\omega}, \quad p_y = \frac{3\beta b\pi}{2j\omega}, \quad (9)$$

$$m_x = \frac{3\pi D}{16} [\alpha F_1(\Delta\phi) + \beta F_2(\Delta\phi)], \quad (10)$$

$$m_y = \frac{3\pi D}{16} [\beta F_1(\Delta\phi) - \alpha F_2(\Delta\phi)], \quad (11)$$

where the functions $F_1(\Delta\phi) = 16a \cos^2(3\Delta\phi/2) - 10b \sin^2(3\Delta\phi)$ and $F_2(\Delta\phi) = 8a \sin(3\Delta\phi) - 6b\Delta\phi + 5b \sin(6\Delta\phi)$ encode the geometric dependence on the via offset parameter $\Delta\phi$. This calculation assumes an electrically thin substrate by neglecting the contribution of the vias themselves to the dipole moments, and the variation of the current along the via.

To understand the fundamental coupling mechanisms, we decompose the magnetic dipole moments according to their current mode origins: $m_{x,\alpha} = m_x|_{\beta=0}$, $m_{x,\beta} = m_x|_{\alpha=0}$, and similarly for the y components. The electric dipoles p_x and p_y are driven exclusively by the α (odd) and β (even) current modes, respectively, while each magnetic dipole component receives contributions from both modes with relative strength controlled by $F_1(\Delta\phi)$ and $F_2(\Delta\phi)$. The key to understanding symmetry control lies in the behavior of these geometric functions. For symmetric vias ($\Delta\phi = 0$), $F_2(0) = 0$ eliminates cross-coupling, leaving only parallel dipole coupling corresponding to pure chiral response. For offset vias ($\Delta\phi \neq 0$), $F_2(\Delta\phi) \neq 0$ enables orthogonal cross-coupling alongside the persistent parallel coupling, resulting in hybrid chiral-omega response. These coupling mechanisms are summarized in Table I.

These distinct coupling pathways manifest directly in the scattering coefficients [9]. Chiral coupling produces nonvanishing cross-polarized transmission $t_{ij} \neq 0$ ($i, j \in \{x, y\}$, $i \neq j$), while omega coupling enables direction-dependent copolarized reflection $r_{ii}^+ \neq r_{ii}^-$, where the superscript $\pm$ denotes illumination from the positive or negative z direction. The hybrid response combines both mechanisms, providing simultaneous control over transmission and reflection asymmetries within a single-layer structure (see detailed scattering analysis

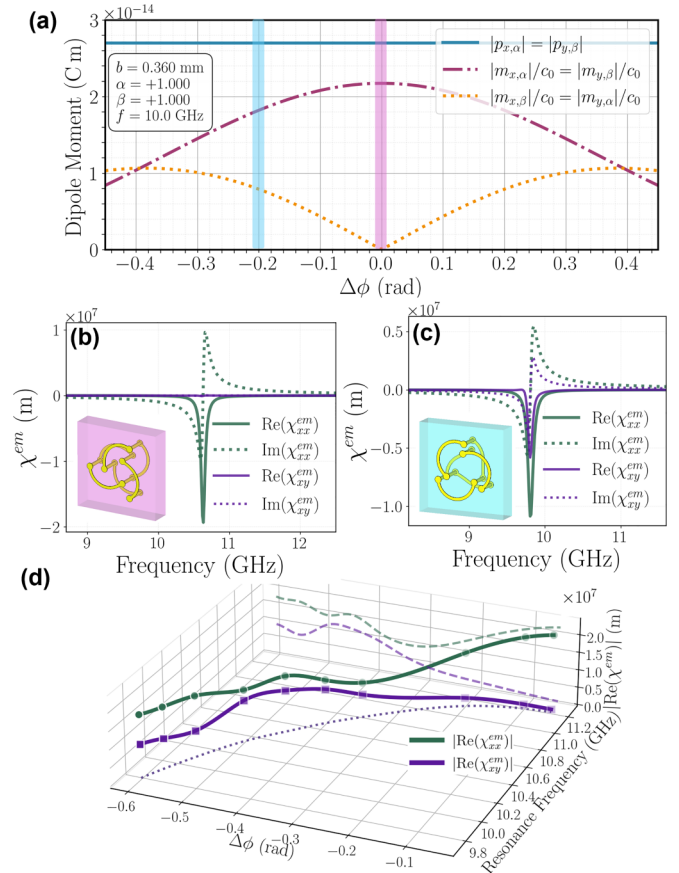


FIG. 2. Symmetry breaking in quasiplanar trefoil knot particles. (a) Dipole moment components vs via offset $\Delta\phi$ ($\Lambda = 3.9$ mm, $a = 0.96$ mm, $b = 0.36$ mm). Transparent magenta and blue lines indicate $\Delta\phi = 0$ and $\Delta\phi = -0.6$ rad, respectively. (b) Electromagnetic susceptibility for symmetric design ($\Delta\phi = 0$, thickness = 0.91 mm) showing pure chiral response. (c) Susceptibility for offset design ($\Delta\phi = -0.6$ rad, thickness = 0.82 mm) exhibiting hybrid chiral-omega behavior. Both designs use metallic trace width = 0.1 mm on Isola Astra MT77 substrate ($\epsilon_r = 3$, $\tan\delta = 0.0017$) with silver surface finish. (d) Chiral ($|\text{Re}(\chi_{xx}^{em})|$, green) and omega ($|\text{Re}(\chi_{xy}^{em})|$, purple) susceptibilities evaluated at resonance ($\text{Im}(\chi_{xx}^{em}) = \text{Im}(\chi_{xy}^{em}) = 0$) vs $\Delta\phi$ and resonance frequency. Dotted lines project onto the $\Delta\phi$ /frequency plane showing resonance drift, while dashed lines project onto the $\Delta\phi/|\text{Re}(\chi^{em})|$ plane showing amplitude trends. Square and circular markers indicate discrete simulated configurations. Continuous curves between markers are obtained by interpolation [23]. The $\Delta\phi = -0.6$ rad configuration corresponds to the fabricated design.

in the Supplemental Material [23]). Figure 2 demonstrates this coupling mechanism control. Panel (a) shows the dipole moment components versus $\Delta\phi$, evaluated at a fixed resonance frequency (10 GHz) and assuming $|\alpha| = |\beta| = 1$ to isolate the geometric influence of via offset on the multipole coupling, showing that regardless of the actual values of α, β , the coupling required for omega-type response emerges only upon symmetry breaking. In practice, they are usually excited with similar amplitudes [23]. As discussed, while all dipole components (p_x, p_y, m_x, m_y) exist throughout the $\Delta\phi$ range, the macroscopic bianisotropic response depends

on which coupling pathways are active. At $\Delta\phi = 0$ (transparent vertical magenta line), only parallel coupling channels are active, while at $\Delta\phi = -0.2$ rad (transparent vertical blue line), both parallel and orthogonal coupling pathways are enabled, resulting in a hybrid chiral-omega response. This fixed-frequency analysis captures the qualitative behavior well for small offsets; quantitative deviations at larger $|\Delta\phi|$ arise from resonance drift and α , β variations [as presented in the realistic scenario in panel (d)]. Importantly, this symmetry-based coupling mechanism remains valid when considering additional current harmonics in Eq. (8). It is worth highlighting that the magnetic dipole moments in Eqs. (10) and (11) are proportional to substrate thickness D , which must have a non-negligible value to facilitate a significant magnetic and bianisotropic response. Additionally, as a basic building block, the analysis above describes the shift $\Delta\phi$ for via connections on one side of the substrate. In practice, our simulated and fabricated models implement the same shift on both sides, which results in an even simpler form for $\Delta\phi$ dependence— $F_1 = 8a \cos(3\Delta\phi)$ and $F_2 = 8a \sin(3\Delta\phi)$ with the same dipole coupling mechanism. This simplified form nicely manifests the threefold rotation symmetry of this case in its $2\pi/3$ periodicity.

III. RESULTS AND DISCUSSION

To validate this analysis on a realistic model that can be fabricated, we use full-wave simulations in CST Microwave Studio [24] with periodic boundary conditions. The unit cell parameters are $\Lambda = 3.9$ mm, $a = 0.96$ mm, and $b = 0.36$ mm. The knot is implemented as metallic traces with width = 0.1 mm, and the substrate is Isola Astra MT77 ($\epsilon_r = 3$, $\tan \delta = 0.0017$) with silver immersion surface finish. Although the nominal substrate thickness is 0.88 mm, fabrication tolerances resulted in variations between samples. The final measured thicknesses were 0.91 mm for the symmetric design and 0.82 mm for the symmetry-broken design. These measured values were used in the corresponding simulations to ensure accurate modeling. It is worth noting that both thickness and via offset affect the resonance frequencies in simulations and measurements. While the thinner substrate in the symmetry-broken design would nominally shift resonance to higher frequencies (owing to the reduced effective wire length), the observed lower resonance frequency is dominated by increased capacitive loading from via offset. The simulated unit cells are shown in the insets of Figs. 2(b) and 2(c). Since we aim to conduct experiments utilizing printed circuit board (PCB) fabrication, we employ a practical model that includes via pads. We extract the complex reflection and transmission coefficients for bidirectional excitation and map them to surface susceptibilities through generalized sheet transition conditions [2,25,26]. As shown in panel (b), the symmetric configuration with $\Delta\phi = 0$ exhibits strong chiral response (green) $|\chi_{xx}^{\text{em}}|$ reaching approximately 1.8×10^7 m at resonance (10.67 GHz), while the omega response (purple) $|\chi_{xy}^{\text{em}}|$ remains negligible (below 0.1×10^7 m, originating in small numerical inaccuracies), confirming pure chiral behavior. At $\Delta\phi = -0.6$ rad [Fig. 2(c)], the symmetry-breaking mechanism discussed above activates orthogonal coupling alongside the existing chiral response. The responses of chiral $|\chi_{xx}^{\text{em}}|$

(green) and omega $|\chi_{xy}^{\text{em}}|$ (purple) are significant at the shifted resonance frequency of ~ 9.8 GHz, reaching $|\chi_{xx}^{\text{em}}| \approx 1.0 \times 10^7$ m and $|\chi_{xy}^{\text{em}}| \approx 0.5 \times 10^7$ m, confirming the predicted hybrid chiral-omega behavior.

To complete the picture of the via offset mechanism, we extract the chiral ($|\text{Re}(\chi_{xx}^{\text{em}})|$) and omega ($|\text{Re}(\chi_{xy}^{\text{em}})|$) susceptibilities at resonance (where $\text{Im}(\chi_{xx}^{\text{em}}) = 0$ and $\text{Im}(\chi_{xy}^{\text{em}}) = 0$) for various offset angles. Figure 2(d) presents a 3D visualization showing both responses versus $\Delta\phi$ and resonance frequency. The chiral response exhibits modest variation across the offset range, maintaining values around 1.0 – 2.0×10^7 m, while the omega response emerges from zero at $\Delta\phi = 0$, reaches a maximum around $\Delta\phi \approx -0.4$ rad, and then decreases at larger offsets. The dotted projection onto the $\Delta\phi$ /frequency plane shows resonance drift due to capacitive loading, shifting between 9.8 and 11.2 GHz. Dashed projections onto the $\Delta\phi/|\text{Re}(\chi^{\text{em}})|$ plane show the amplitude for both responses, with a matching trend to the approximate model shown in Fig. 2(a) for small values of $\Delta\phi$. While the analytical model in Fig. 2(a) uses the first current harmonic, the symmetry-based coupling mechanism persists at higher harmonics since via offset enables cross-coupling between orthogonal dipole components regardless of harmonic content (see discussion in the Supplemental Material [23]). These results provide quantitative design guidelines for selecting offset angles based on desired response characteristics and frequency range.

To experimentally validate the symmetry-controlled bianisotropic response mechanism, we fabricated symmetric and symmetry-broken PCB metasurfaces using the same materials and dimensions as those in the simulations (fabrication details are provided in the caption of Fig. 3). The fabricated surfaces measure $18 \text{ cm} \times 18 \text{ cm}$, corresponding to $\sim 6\lambda \times 6\lambda$ at the operational frequencies. The complex transmission and reflection coefficients (amplitude and phase) were measured using a Compass focused beam system [27] with automated positioning [Fig. 3(a)] (for detailed measurement setup specifications, see Supplemental Material [23] and Refs. [28,29] therein), and mapped to surface susceptibilities using the same GSTC approach as the simulations. Figures 3(b)–3(c) summarize the experimental validation of symmetry-controlled chiral and hybrid chiral-omega responses. Chiral response is shown in blue (simulation) and gold (measurement), and omega response in black (simulation) and light orange (measurement), with solid lines for real parts and dashed lines for imaginary parts. For the symmetric design [$\Delta\phi = 0$, panel (b)], the chiral response $|\chi_{xx}^{\text{em}}|$ reaches approximately 2.5×10^7 m at ~ 10.6 GHz, while the omega response $|\chi_{xy}^{\text{em}}|$ remains negligible, confirming pure chiral behavior. The symmetry-broken design [$\Delta\phi = -0.6$ rad, panel (c)] demonstrates comparable magnitudes: chiral $|\chi_{xx}^{\text{em}}| \approx 1.0 \times 10^7$ m and omega $|\chi_{xy}^{\text{em}}| \approx 0.8 \times 10^7$ m at ~ 9.8 GHz, validating hybrid chiral-omega behavior. Minor discrepancies arise from fabrication tolerances, including substrate thickness variations (discussed above), pad and wire loading effects, and material dispersion not fully captured in the idealized simulations.

Our results reveal that bianisotropic response type is not fixed by topology but can be continuously tuned through symmetry engineering. The trefoil knot structure provides a

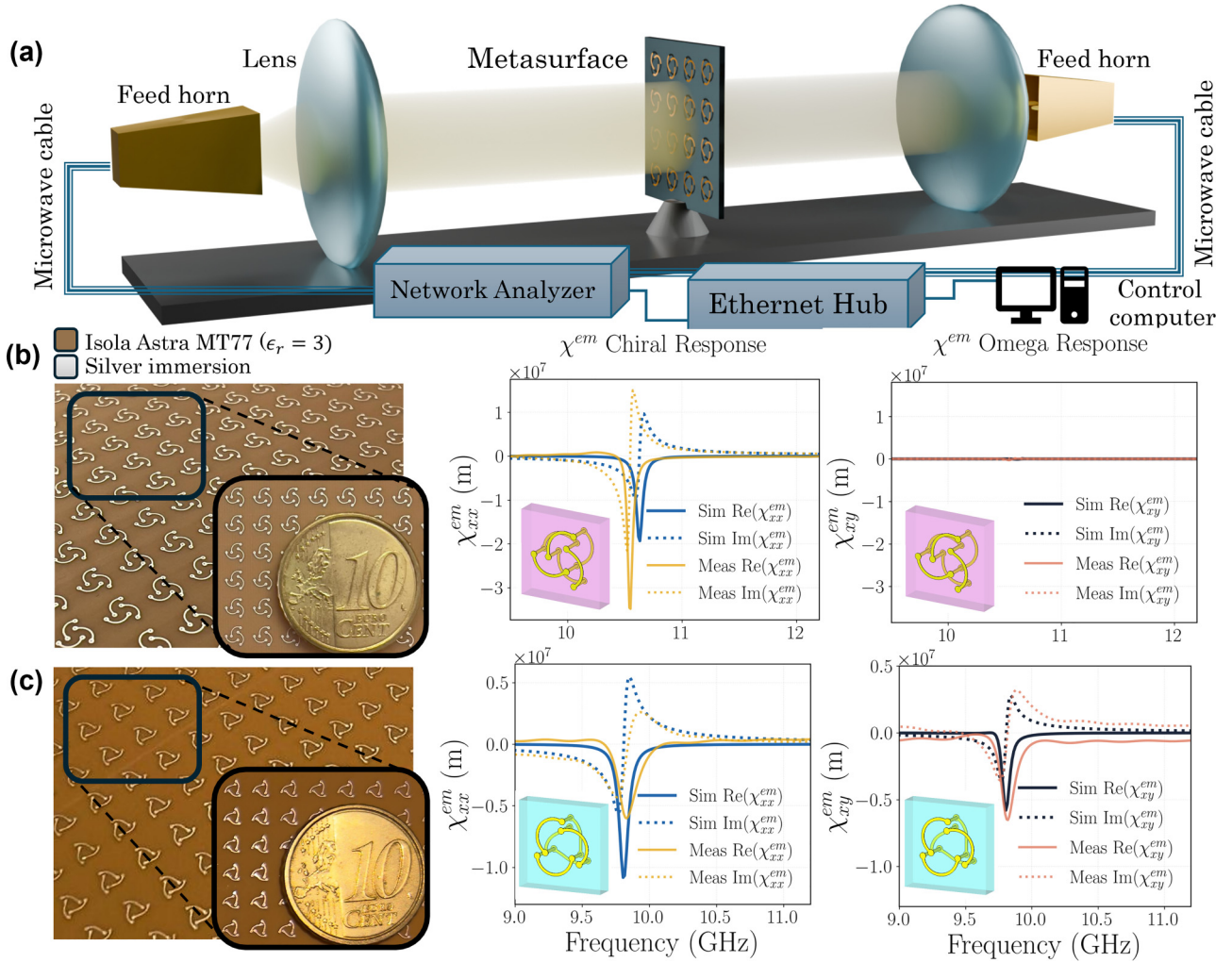


FIG. 3. Experimental validation of symmetry-controlled bianisotropic response. (a) Compass focused beam measurement setup. (b), (c) Symmetric ($\Delta\phi = 0$, thickness = 0.91 mm) and symmetry-broken ($\Delta\phi = -0.6$ rad, thickness = 0.82 mm) designs showing fabricated samples (left), chiral response (middle), and omega response (right). Simulation and measurement are shown with solid and dashed lines for real and imaginary parts, respectively. PCB fabrication on Isola Astra MT77 ($\epsilon_r = 3$, $\tan \delta = 0.0017$) with silver immersion finish and 0.1 mm trace width. Scale: 10 euro cent coin.

natural platform where small geometric perturbations control electromagnetic coupling mechanisms, representing a practical symmetry-breaking approach that eliminates the need for multilayer architectures or material engineering. In addition, it enables intuitive modeling that relates current distribution properties to the expected electromagnetic susceptibility. The hybrid response demonstrated here provides simultaneous control over both cross-polarized transmission and copolarized reflection asymmetries, expanding the design space for single-layer bianisotropic devices. Applications include polarization converters with tunable asymmetric transmission, broadband absorbers leveraging combined bianisotropic mechanisms [30], and radar cross-section engineering where independent control over scattering channels enables advanced signature design [31]. Furthermore, reconfigurable metasurfaces with mechanical or electrical control of via positions could dynamically switch between response types. The design principles established here, utilizing geometric symmetry to control electromagnetic coupling pathways, may

be extended beyond knot particles to general bianisotropic metamaterial design. The same principles used in this work can also be applied to the design of metasurfaces at higher frequencies, such as terahertz, infrared, and optical frequencies. This would require finding and properly modeling the required materials and using more advanced fabrication techniques. Yet from a theoretical perspective, the same principles apply.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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