

Dispersive topological bound states in the continuum for controllable quantum-state transport in three-state discrete-time quantum walks

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We show that a three-state discrete-time quantum walk (DTQW) on a square lattice, with an $SU(3)$ coin, realizes stroboscopic Floquet phases that host dispersive topological bound states in the continuum (TBICs). Time-independent step operations act as the periodic drive and map the one-step unitary to an effective Floquet Hamiltonian. Chern-number calculations resolve three quasienergy bands and their phase boundaries as the rotation angles are varied, and bulk-boundary correspondence in semi-infinite and fully finite geometries accounts for the edge spectra. At interfaces between media with the same bandwise Chern numbers but different coin parameters, nonchiral edge modes become embedded yet localized TBICs with tunable group velocity and selective excitation, while numerics indicate robustness to moderate coin disorder. Together with feasible routes in trapped-ion, cold-atom, and photonic platforms and an explicit decoherence model, these results identify DTQWs as a controllable setting for protected quantum-state transport and storage, relevant to quantum information and communication technologies.

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I. INTRODUCTION

Bound states in the continuum (BICs) are localized states that exist within the bulk of electronic or photonic bands, enabling radiative-loss-free transport and propagation. This unique property has driven advancements in applications such as lasing [1–5], wave front manipulation, and nonlinear processes like frequency conversion and mixing [6–12]. Expanding on this concept, topological bound states in the continuum (TBICs) harness key features of topological systems—such as bulk-boundary correspondence and edge-state protection—to enable backscatter-free transport and directional propagation [9,13–18]. Previous studies have largely focused on TBICs in higher-order or nonlinear topological systems, where their formation is often attributed to symmetry manipulation in the Hamiltonian [14,15,18] or non-Abelian couplings between lattice sites [19]. Notably, dispersive TBICs have also been observed in systems comprising two distinct valley topological insulators [17]. In this work, we demonstrate that lower-order dispersive TBICs naturally emerge in periodically driven Floquet systems realized as colored quantum walks (cQWs) on discrete lattice structures. Unlike conventional two-state systems [20–22], the cQW framework introduces three internal degrees of freedom, termed “colors,” which provide unprecedented control over Floquet states and TBICs [23–33]. By leveraging the $SU(3)$

group to describe the quantum walker’s spatial evolution, the cQW offers an intrinsic route to realizing Floquet systems, for which fixed external operations at each time step of quantum walk would act as periodic drives. Through tunable rotation parameters, we uncover unconventional edge-state properties, including localized states within topological band gaps and dispersive TBICs that emerge as edge states embedded within the bulk of other topological bands, features absent in analogous systems with spin-1/2 particle and $SU(2)$ symmetry. These TBICs exhibit tunable group velocity and precise excitation control, arising as intrinsic features of the cQW’s dynamics.

Conceptually, our approach extends previous studies that use additional internal degrees of freedom to induce the formation of TBICs. However, it introduces a distinct mechanism: by leveraging $SU(3)$ symmetry and the color structure of the discrete-time quantum walk (DTQW) to generate TBICs as localized edge rather than localized corner states. This first-order realization of TBICs links the gap between higher-order topological states and lower-order systems, providing a controllable platform for investigating the interplay between topology and Floquet dynamics. Moreover, our results establish discrete-time cQWs as a versatile framework for robust quantum Floquet systems, opening avenues for applications in quantum computation and communication technologies [34–40].

To realize TBIC edge state, we consider the motion of a periodically driven DTQW, the quantum analog of a classical random walk. In this system, the dynamics are governed by the internal quantum states of the particle and a coin operator [23–25]. However, unlike classical random walks or conventional DTQWs with two internal degrees of freedom (e.g., spin or light polarization), we introduce three internal states. The coin operator, modeled as a rotation acting on these

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internal states, followed by a conditional shift, together generating dynamics described by a Floquet Hamiltonian. This methodology, based on the SU(3) group, uniquely enables the formation of TBICs on a square lattice, distinguishing our approach from previous models. Our cQW model is experimentally feasible using different platforms such as atoms (trapped ions and optical lattices), light, and synthetic dimensions [41–44]. Specific to ions [45–47] or cold atoms, colors can be three different states of atoms, while for light, colors can be three different orbital-angular momentum states. For atoms and light, coin operations are equivalent to sequential applications of coherent laser pulses, and mode engineering of their angular momentum states using spatial modulators (or q plates) [48–50], respectively. Furthermore, the shift operation in atoms is realizable with a state-dependent optical dipole force, while for light, beam splitters enable spatial motion control. Despite this well-defined motion, challenges in implementation would then be related to the various decoherence mechanisms in atoms (such as spontaneous emission and dephasing) or unwanted light scatterings, which can be reduced using metastable atomic transitions or by minimizing light scattering.

II. MODEL AND THEORY

A DTQW consists of a periodic sequence of unitary operators (U) acting on a Hilbert space. Here, U represents a single step of the walk. The Hilbert space of the walker, $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_c$, is composed of two subspaces: $\mathcal{H}_p$, which is spanned by the particle's position states, and $\mathcal{H}_c$, which is the coin space spanned by the quantum states of the coin operator. In our framework, hopping operations define the particle's movement across a square lattice. The walker is characterized by three mutually orthogonal internal states, termed as red, green, and blue, collectively forming the color basis. The coin operation is represented by rotations within the SU(3) group space. The walker's dynamics are driven by an alternating sequence of rotation and hopping operations, with adjustable parameters and directions. These dynamics are expressed by the unitary operator $U(\theta_1, \theta_2) = T_y R(\theta_2) T_x R(\theta_1)$, where $R(\theta) = e^{iS\theta/2}$, $T_{x/y}$ are the hopping operators that control movement along the x and y directions, respectively, and $R(\theta)$ is the coin rotation operator, defined using operators S from the SU(3) symmetry group. The rotation operator S acts on the internal states of the walker, creating superpositions that influence its translational motion in the xy plane. The translation dynamics, depicted in Fig. 1, incorporate movements to the left and right as well as stationary probabilities and are described as follows [51,52]:

$$T_i = |i, \mathfrak{R}\rangle\langle i, \mathfrak{R}| + |i-1, \mathfrak{G}\rangle\langle i, \mathfrak{G}| + |i+1, \mathfrak{B}\rangle\langle i, \mathfrak{B}|. \quad (1)$$

Here, $|r_i, \text{color}\rangle = |r_i\rangle \otimes |\text{color}\rangle$, where r_i represents position coordinates (x or y), and color denotes the coin space or internal basis states, whose basis are chosen in accordance with SU(3) group, and its nomenclature borrowed from particle physics. If the walker is in the red state ($\mathfrak{R}$), it remains stationary at position r , while the green state ($\mathfrak{G}$) moves it one unit to the left, and the blue state ($\mathfrak{B}$) shifts it one unit to the right along the x or y axis. The internal orthogonal color states are represented as $|\text{color}\rangle = (\mathfrak{R}, \mathfrak{G}, \mathfrak{B})^T$, where

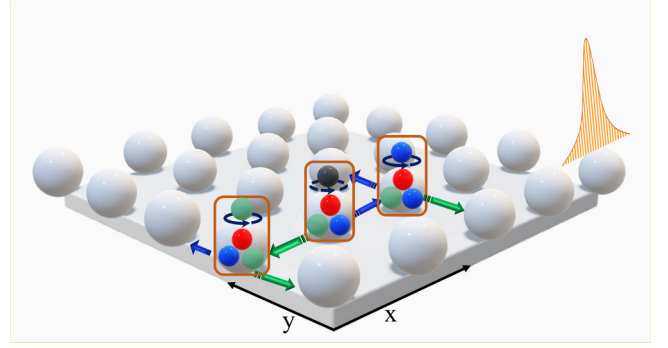


FIG. 1. Schematic of a discrete-time cQW on a square lattice showing Floquet dynamics and topological states. The walker has three internal “colors”: blue/green drive motion along $\pm x$ or $\pm y$, while red corresponds to no shift. Sequential rotations $R(\theta_1)$ and $R(\theta_2)$ mix these colors, controlling the walker's movement. On the right, a topologically localized edge state, fundamental to the realization of dispersive TBICs and Floquet states, is shown at the boundary.

the superscript T denotes the transpose. Rotations within this group are described using the Gell-Mann matrices, λ_i [53–55]. To preserve the structure and dynamics of the DTQW, the rotation operator S is chosen to be unitary, satisfying $S = S^\dagger$:

$$S = \frac{\sqrt{2}}{3}(\lambda_1 + \lambda_4 + \lambda_6) + \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3} \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & -2 \end{bmatrix}. \quad (2)$$

Taking into account translational invariance along the x and y axes, the shift operators in momentum space are $T_l(k) = \text{diag}[1, \exp(ik_l a), \exp(-ik_l a)]$ for $l \in \{x, y\}$, where $k = (k_x, k_y)$ represents the momentum vector within the first Brillouin zone and a is the lattice constant. Accordingly, the unitary operator in momentum space is defined as

$$U_k(\theta_1, \theta_2) = T(k_y)R(\theta_2)T(k_x)R(\theta_1). \quad (3)$$

Following Floquet band theory [28–30], the effective static Hamiltonian of the quantum walk, denoted as $H(\theta_1, \theta_2)$ can be derived through

$$e^{-iH(\theta_1, \theta_2)t/\hbar} = U_k(\theta_1, \theta_2), \quad (4)$$

where we adopt units $t/\hbar = 1$, corresponding to the time duration of a unitary operator. The quasienergy bands of the bulk Hamiltonian can be obtained from Eq. (4) using the unitary operator defined in Eq. (3). Floquet theory predicts that the energy spectrum exhibits periodicity along the energy axis, similar to the periodic structure of momentum space within the first Brillouin zone. As a result, an analogous “energy” Brillouin zone emerges within the energy range $-\pi < E(k_x, k_y) < \pi$.

By varying the parameters (θ_1, θ_2) , the system exhibits a wide range of band structures, demonstrating remarkable flexibility and control over the quasienergy spectrum. This tunability enables the exploration of various phenomena, including band crossings, topological phase transitions, and the emergence of spectral gaps, directly influencing the walker's temporal dynamics [56]. The interplay between these

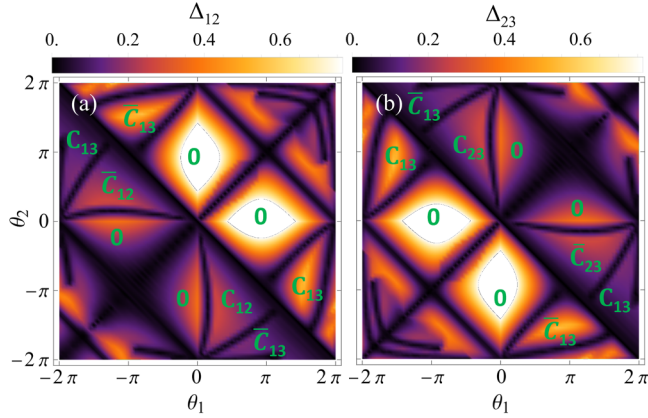


FIG. 2. Energy gap and topological phase diagram in rotation plane (θ_1, θ_2) . Panel (a) shows $\Delta_{12} = |E_1 - E_2|$ and panel (b) shows $\Delta_{23} = |E_2 - E_3|$ (color bar indicating the energy gap size); bright (dark) regions correspond to larger gaps (gap closing resembling topological phase boundaries), respectively. Each region is labeled by the Chern numbers of bands i, j as $C_{ij} = (C_i = 1, C_j = -1) = -C_{ji}$. $C_{ij} = 0$ indicates that the Chern number of at least one band is zero.

parameters provides valuable insight into the system's underlying physics and facilitates the design of tailored energy band structures.

To further investigate the topological properties of the system, we numerically compute the bulk Hamiltonian using Eq. (4). Our analysis reveals that the eigenenergies do not satisfy the conditions $E_\alpha(\mathbf{k}) = E_\alpha(-\mathbf{k})$ or $E_\beta(\mathbf{k}) = -E_\alpha(-\mathbf{k})$ ($\alpha \neq \beta$), confirming the absence of time-reversal and particle-hole symmetries for any nonzero rotation parameters (θ_1, θ_2) . This symmetry breaking originates from the coin operator, which introduces complex phases via rotation. Interestingly, the eigenenergies exhibit a residual symmetry, satisfying $E(k_x, k_y) = E(k_y, k_x)$, which reflects an interchange symmetry in momentum space. To further characterize the system's topology, we compute the Chern number for each energy band by numerically evaluating the band curvature, following the approach in Ref. [61].

$$B_n(\mathbf{k}) = i \sum_{x,y} \epsilon_{xy} \langle \partial_x \psi_n(\mathbf{k}) | \partial_y \psi_n(\mathbf{k}) \rangle, \quad \epsilon_{xy} = -\epsilon_{yx} = 1, \\ C_n = -\frac{1}{2\pi} \int_{BZ} d^2k B_n(\mathbf{k}), \quad (5)$$

where $\psi_n(\mathbf{k})$ represents the eigenstate of the n th quasienergy band of the bulk Hamiltonian. The calculated Chern numbers for each energy band as a function of the rotation parameters (θ_1, θ_2) are shown in Fig. 2, marked by green numbers in the center of each gapped region. Figure 2(a) illustrates the energy gap Δ_{12} between the first and second bands ($\Delta_{ij} = |E_i - E_j|$), while Fig. 2(b) shows the gap Δ_{23} between the second and third bands. The color gradient indicates the magnitude of the energy gap, with darker shades corresponding to smaller gaps and brighter shades indicating larger gaps. Darker regions, where the gap closes, delineate the boundaries between regions with distinct topological phases. The Chern number of topological bands i, j within each region is expressed as

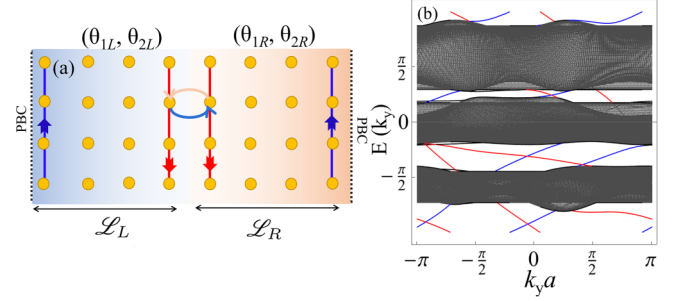


FIG. 3. Semifinite structure of a DTQW with finite size along the x axis. Panel (a) shows slab configuration of two media with periodic boundary conditions applied to the external (left and right) boundaries. The light orange and blue arrows indicate the transition of rotation parameters across the middle boundary, while the rotation parameters at each boundary are determined by their initial position parameters. Panel (b) presents the energy spectrum of two topologically trivial media ($C = 0$), highlighting extended states in black and chiral edge states in blue/red, whose spatial localization corresponds to the external/middle boundaries shown by blue/red big arrows in panel (a). Left and right media consist of $\mathcal{L}_L = \mathcal{L}_R = 60$ sites. The rotation parameters for the left medium are $(\theta_{1L}, \theta_{2L}) = (\pi, \pi/4)$ and for the right medium are $(\theta_{1R}, \theta_{2R}) = (0, \pi)$.

$C_{ij} = (C_i, C_j) = (1, -1)$, with representative values such as $(1, -1)$ or their opposites ($\bar{C}_{ij} = -C_{ij}$). The regions are classified as topologically trivial when the Chern numbers in both panels are zero. Conversely, if one panel displays a zero Chern number while the other does not, the topological nature of the bands is determined by the nonzero panel.

III. RESULTS

To further explore the distinct topological phases, we analyze the bulk-boundary correspondence using a semifinite slab structure. In this setup, we assume translational invariance along the y axis, while the two media consist of a finite number of lattice sites along the x axis. Periodic boundary conditions are applied to the external boundaries, as depicted in Fig. 3(a). The topological invariants of the left and right media govern the number of edge states that appear at both the internal and external boundaries of the system.

We first consider two topologically trivial phases characterized by zero Chern numbers, but with distinct sets of rotation parameters, as shown in Fig. 3(b). This configuration includes one interface at the middle and another at the external boundaries due to the periodic boundary conditions. At the boundaries, each medium retains the rotation parameters corresponding to its initial position. For example, as shown in Fig. 3(a), θ_{1L} determines the transitions from the left (L) to the right (R) medium (light blue arrow), while θ_{1R} determines the reverse transition (light red arrow). The energy spectrum in Fig. 3(b) reveals chiral anomalous edge states localized at both boundaries. In agreement with the bulk-boundary correspondence in Floquet systems [30], the difference in the number of edge states above and below each bandwidth determines the topological invariant associated with that band. In this case, an equal number of edge states on either side of each band means that the bands are topologically trivial. The localized

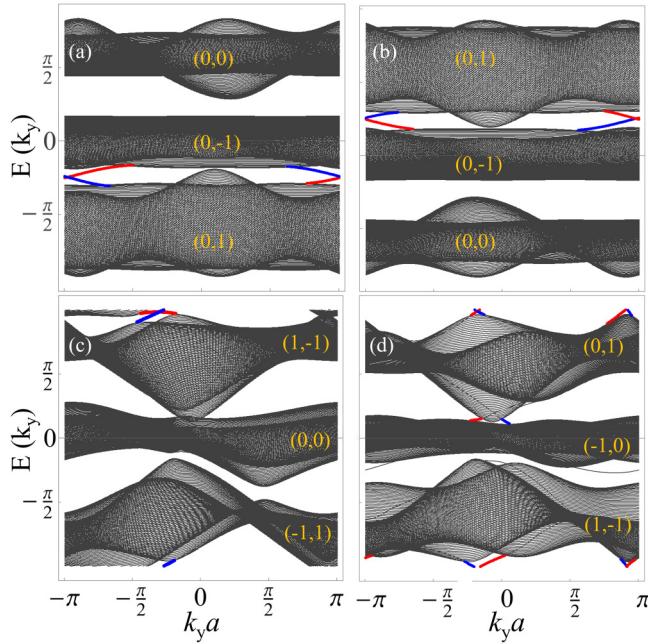


FIG. 4. Energy spectrum of a semifinite structure with two distinct topological phases. Top panels: left medium trivial; bottom row, both media have distinct nonzero topological invariants. Shown in yellow within the bandwidth are the band Chern numbers (C_L, C_R) for the left (L) and right (R) media. The color coding for localized edge states is the same as Fig. 3. The rotation parameters for the left and right media, from top left to bottom right, are $(\theta_{1L}, \theta_{2L}, \theta_{1R}, \theta_{2R}) = (-\pi/32, -\pi, \pi/3, -\pi)$, $(\pi, \pi/32, \pi, -\pi/3)$, $(4, -2.5, 5.3, -2.5)$, and $(\pi/3, -\pi, 4, -2.5)$.

edge states at the middle (external) boundary with negative (positive) chirality are depicted in red (blue). Importantly, swapping the left and right media changes only the chirality of these localized edge states.

We now explore scenarios involving media with distinct topological characteristics, ensuring that at least one medium exhibits nontrivial topology. To describe these configurations, we assign Chern numbers to each band as (C_L, C_R) . Initially, we consider cases where one medium is topologically trivial, while the other possesses nontrivial topology. In particular, we examine two distinct nontrivial topological media with Chern numbers $(C_{1R}, C_{2R}, C_{3R}) = (1, -1, 0)$ and $(0, -1, 1)$, each adjacent to a topologically trivial medium. These configurations are depicted in Figs. 4(a) and 4(b), respectively. The presence of localized edge states between the first and second bands in Fig. 4(a) implies that these bands have opposing Chern numbers. Conversely, the absence of edge states around the third band's bandwidth indicates its topologically trivial nature. In Fig. 4(b), however, localized edge states emerge between the second and third bands with reversed chirality compared to panel (a), reflecting the differing topological numbers. This observation allows us to define the net Chern number for each band as $\tilde{C}_i = C_{iR} - C_{iL}$. We further investigate configurations involving two nontrivial topological media, as depicted in Figs. 4(c) and 4(d). The quasienergy spectrum depicted in Fig. 4(c) shows that the gap between the first and second bands is open, while the gap between the first and third bands is closed. This suggests that, effectively, only one band is

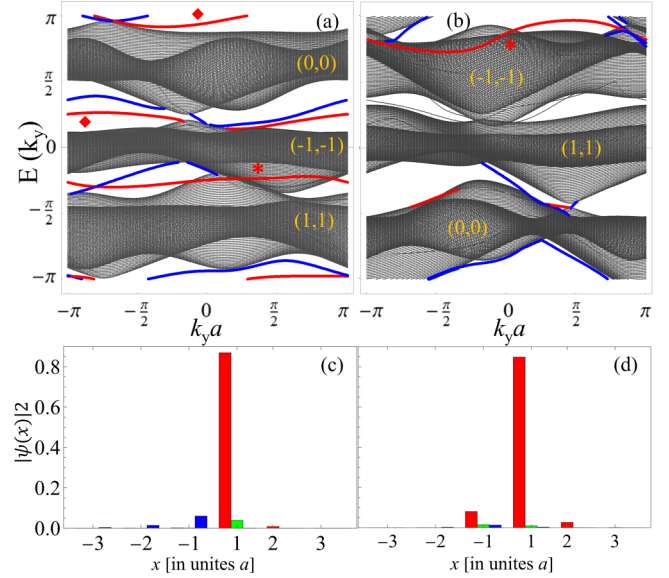


FIG. 5. Energy spectrum and spatial distribution of semiinfinite structures exhibiting identical nontrivial topological properties but differing rotation parameters in the left and right media. In each panel, the Chern numbers for each band are indicated within the bandwidth in yellow as (C_L, C_R) . Localized edge states are color-coded consistently with Fig. 3. In panels (a) and (c), the rotation parameters are $(\theta_{1L}, \theta_{2L}, \theta_{1R}, \theta_{2R}) = (\pi/3, -\pi, -2\pi, 3\pi)$, corresponding to the topological invariant $(C_L, C_R) = (1, -1, 0)$. In panels (b) and (d), the parameters are $(\theta_{1L}, \theta_{2L}, \theta_{1R}, \theta_{2R}) = (-1.3, \pi, 6, -2.5)$, with $(C_L, C_R) = (0, 1, -1)$. The spatial distribution of TBICs, $|\psi|^2$, at momentum $k_y a = 0$ is shown in panels (c) and (d). Here, the dispersive TBICs and midgap states are indicated with red stars and diamonds, respectively.

topologically trivial. Nonetheless, the topological edge states between the third and first bands remain localized within the bands where the gap has closed. In contrast, Fig. 4(d) presents a fully gapped band structure with net Chern numbers calculated as $\tilde{C} = C_R - C_L = (-2, 1, 1)$. This net Chern number indicates the presence of localized edge states around each band gap. For instance, the lowest energy band has a total Chern number of $\tilde{C}_1 = -2$, which corresponds to two pairs of localized edge states.

After examining various topological configurations of the DTQW, we now focus on scenarios where both media share the same topological phase, including potential phase transition points. In Figs. 5(a) and 5(b), both media exhibit Chern numbers $C = (1, -1, 0)$, $C = (0, 1, -1)$, respectively. The results shown in Fig. 5 reveal two notable differences compared to the previous configurations. First, the chiral edge states at the external boundary (blue lines) form pairs with opposite chirality originated from the same topological Chern number of each band at both media, whereas in earlier examples, paired chiral edge states shared the same chirality. Second, since both media possess nontrivial topological characteristics, the localized edge states at the middle boundary lack chiral behavior as they do not connect energy bands. We refer to these localized modes as midgap states when they appear within an open band gap and identify them with red diamond.

These properties make the edge states distinct from those of trivial bands in Fig. 3(b).

In Fig. 5, two strongly localized states, marked by red stars, coexist with the gapless bands yet remain unhybridized with the bulk extended states (shown in gray), forming a realization of TBICs. These states are representative examples of TBICs formed within the continuum. While the TBIC excitation requires the same topological properties with intervening a phase transition, their manifestation originates in including the additional (coin) state in the SU(3) rotation symmetry, as its wave function is dominated by this red-color component. Additionally, these states appear around the quasienergy bandwidth (confined to a finite momentum range $k_y a$) with negative Chern number and exhibiting nonzero momentum-dependent group velocity, as shown in panels (a) and (b). To illustrate the localization of these states at the middle boundary, we present the spatial distribution of the TBICs in panels (c) and (d).

These states can be actively controlled by varying the rotation parameters, which modifies the quasienergy dispersion and manipulates TBICs propagation [56]. Unlike conventional bound states, our dispersive TBICs are topologically robust against disorder [56], yielding tunable states with controlled group velocity, a feature absent in flatband [62–66] or higher-order counterparts [14,15]. Protected by both topology and the BIC mechanism, these states unite disorder immunity with long lifetime, enabling the preservation of information against various loss channels and providing a robust platform for experimental implementation, which is of fundamental importance for understanding the internal quantum states of the quantum walk and has an inevitable role in characterizing light-light and light-matter interactions. Moreover, tunable group velocity of TBICs enables controllable long-range transport, offering a pathway toward robust waveguiding, reconfigurable coupling, and dynamic signal processing in photonic and quantum information [2,4,7,67]. To further investigate the physical characteristics of the BICs, we examine

a fully finite system in the xy plane, considering a finite configuration of two media, where one medium is enclosed by the other [56]. Chiral edge states (shown in blue) remain confined along the system boundaries, whereas nonchiral modes, including midgap states and TBICs, tend to localize near the corners. We emphasize that these corner-localized states are not higher-order topological modes but rather arise from the hybridization of nonchiral edge states in the finite lattice.

IV. CONCLUSION

In summary, we have shown that a three-state DTQW with an SU(3) coin realizes tunable Floquet topology and dispersive TBICs. Chern-number analysis identifies three quasienergy bands and the associated phase transitions, while bulk-boundary correspondence explains the edge spectra in semi-infinite and finite geometries. When topologically equivalent media with different coin parameters are interfaced, localized edge modes become embedded yet localized TBICs with tunable velocity and selective excitation, and they remain numerically robust to moderate disorder. These controllable, protected modes make discrete-time quantum walks a compact platform for engineered quantum-state transport and storage, with direct relevance to quantum information and communication technologies.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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