

# Demonstration of hardware efficient photonic variational quantum algorithm

Iris Agresti<sup>1,\*</sup>, Koushik Paul<sup>2,3,\*</sup>, Peter Schiansky<sup>1,4,9,\*</sup>, Simon Steiner<sup>1,4</sup>, Zhenghao Yin<sup>1,4</sup>, Ciro Pentangelo<sup>5</sup>,  
Simone Piacentini<sup>5</sup>, Andrea Crespi<sup>5,6</sup>, Yue Ban<sup>7</sup>, Francesco Ceccarelli<sup>5</sup>, Roberto Osellame<sup>5</sup>,  
Xi Chen<sup>7,†</sup> and Philip Walther<sup>1,8,9,‡</sup>

<sup>1</sup>*University of Vienna, Faculty of Physics, Vienna Center for Quantum Science and Technology (VCQ), Boltzmannngasse 5, 1090 Vienna, Austria*

<sup>2</sup>*Department of Physical Chemistry, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain*

<sup>3</sup>*EHU Quantum Center, University of the Basque Country UPV/EHU, Barrio Sarriena, s/n, 48940 Leioa, Spain*

<sup>4</sup>*University of Vienna, Faculty of Physics, Vienna Doctoral School of Physics (VDSP), Boltzmannngasse 5, 1090 Vienna, Austria*

<sup>5</sup>*Istituto di Fotonica e Nanotecnologie, Consiglio Nazionale delle Ricerche (IFN-CNR), piazza L. Da Vinci 32, 20133 Milano, Italy*

<sup>6</sup>*Dipartimento di Fisica, Politecnico di Milano, piazza L. Da Vinci 32, 20133 Milano, Italy*

<sup>7</sup>*Instituto de Ciencia de Materiales de Madrid (CSIC), Cantoblanco, E-28049 Madrid, Spain*

<sup>8</sup>*Institute for Quantum Optics and Quantum Information Sciences (IQOQI), Austrian Academy of Sciences, Boltzmannngasse 3, Vienna 1090, Austria*

<sup>9</sup>*QUBO Technology GmbH, 1090 Vienna, Austria*



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Quantum computing has changed the paradigm of computer science, where quantum technologies have promised to outperform classical ones. Such an advantage was only demonstrated for tasks with no application or out of reach for the state-of-the-art quantum technologies. In this context, a promising strategy to find practical use of quantum computers exploits hybrid models, where a quantum device estimates a hard-to-compute quantity, while a classical optimizer trains the parameters. In this work, we demonstrate that single photons and linear optical networks are sufficient for implementing variational quantum algorithms, when the problem specification (ansatz) is tailored to this specific platform. We show this by a proof-of-principle demonstration to tackle an instance of a factorization task, whose solution is encoded in the ground state of a suitable Hamiltonian. This work, which combines Variational Quantum Algorithms with hardware efficient ansätze for linear optics, showcases a promising pathway toward practical applications for photonic quantum platforms.

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## I. INTRODUCTION

In the past decade, quantum computing has risen to the forefront of current technological discussions, fundamentally transforming the field of computation. Indeed, quantum technologies promise to address computational problems exponentially faster and more efficiently, when being compared to their classical counterparts, both for research scopes, e.g., simulating complex physical systems [1,2], and for tackling real-world problems [3,4]. In this context, two specific quantum computing paradigms that have garnered popularity in recent years are *adiabatic quantum computing* (AQC) and *variational quantum algorithms* (VQAs) [5,6]. The first draws inspiration from the concept of adiabatic evolution and it maps

the problem of interest onto finding the ground state of a given Hamiltonian ( $H_f$ ). Since such a state can be experimentally very complex to achieve, the algorithm starts from the implementation of another state (easier to prepare), which corresponds to the ground state of an initial Hamiltonian  $H_i$ . Then, the latter is slowly (and in a controlled way) modified, so that the physical system is maintained in the ground state. In the end, when  $H_i$  converges to  $H_f$ , it is possible to retrieve the solution of the considered computational problem. Despite its robust nature, obtaining fruitful outcomes using AQC on noisy intermediate-scale quantum (NISQ) devices is a challenging task, as adiabatically modifying the initial Hamiltonian requires long sequences of quantum gates [7]. Given the limitations of state-of-the-art quantum computers (e.g., low number of qubits and noisy processes that limit circuit depth), a more feasible alternative to AQC algorithms is constituted by VQAs. These rely on classical optimizers to train parametrized quantum circuits to find the solution to computational problems [9,10]. For instance, such algorithms have been employed in simulating intricate quantum systems to solve quantum chemistry problems [6,11], optimizing financial portfolios [12–15], and tackling combinatorial optimization challenges [16–18], showcasing their effectiveness in addressing real-world problems. In this context, one

\*These authors contributed equally to this work.

†Contact author: [xi.chen@csic.es](mailto:xi.chen@csic.es)

‡Contact author: [philip.walther@univie.ac.at](mailto:philip.walther@univie.ac.at)

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of the most investigated questions was whether this kind of approach could be adopted for the factorization of large numbers, as alternative to Shor's [19–24]. Indeed, despite being efficient, the implementation of such algorithm is highly challenging. This question was answered in the affirmative, as it was possible to design an AQC algorithm to tackle this task, which requires a lower number of qubits [21]. In this work, we want to leverage the lower requirements of this algorithm and design a VQA able to solve the same task while benefiting from the higher experimental feasibility. From the experimental point of view, VQAs have been implemented on gate-based quantum computers, across platforms such as superconducting and trapped ion quantum computers [25]. However, an alternative to these platforms is constituted by photonics, which allows for remarkable performances when exploiting intrinsic properties of photons. A prominent example is the ultimate demonstration of a quantum advantage [26,27] by exploiting the propagation of many photons in complex interferometric networks. This demonstrates that for special-purpose applications that do not require two-qubit gates, which are challenging for photonic systems, such a hardware is very efficient due to its stability, versatility, and ability to operate at ambient temperatures. VQAs have been already implemented on photonic platforms, tackling specific scientific problems, related to quantum simulations of small molecules or sensing applications, with a very recent implementation tackling a multiparameter case through machine learning [28–30]. For further developments, a strategy that has been considered aims at tailoring VQAs to photonic architectures via a so-called hardware efficient ansatz [31–33]. This is done by considering the available quantum hardware and optimal resource usage for designing an algorithm, albeit at the expense of some problem-specific expressiveness.

Here, we implement a VQA, which is custom tailored to photonic platforms, to address a proof-of-principle instance of the factorization problem. Regarding the adopted platform, we employ four input/output ports of a universal photonic integrated circuit (PIC) consisting of a mesh of programmable interferometers that can implement arbitrary unitary transformations on Fock input states. This kind of platform is a natural candidate for adaptive computational algorithms, benefiting from their ease of encoding and manipulating information with high fidelity.

Here, we present such a protocol where we map our problem to the search of the ground state of a suitable Hamiltonian and implement a gradient-based optimization on our photonic platform to obtain the experimental parameters yielding the correct solution. A schematic of this process is depicted in Fig. 1. Albeit choosing it as a benchmark, our approach's scope goes beyond that task. Yet, our method for factorization has a better scalability, in terms of the number of required qubits, when being compared to Shor's algorithm and, in the special case of a number whose factors are successive prime numbers, also compared to the adiabatic quantum factoring proposed in Ref. [21] (see Sec. III of the Supplemental Material [34]). As such, our method is, in particular, applicable on noisy small- and medium-sized quantum hardware, which are within the reach of current technologies.

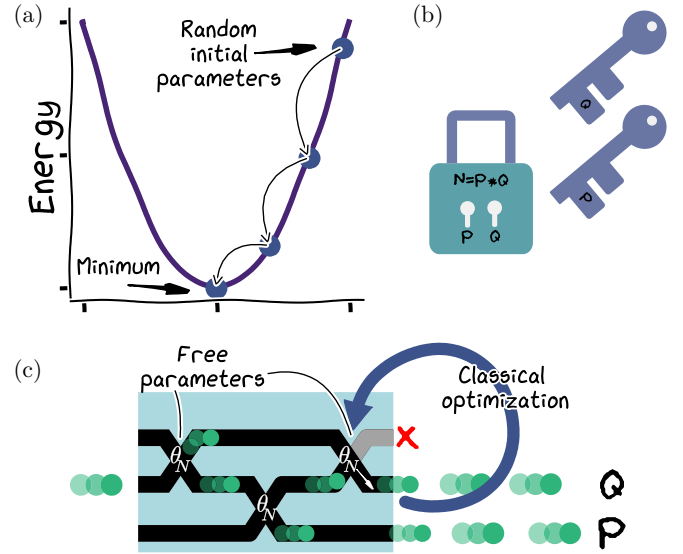


FIG. 1. Variational quantum algorithm to solve a factorization task. (a) In the NISQ era, before fully fault-tolerant large-scale quantum computers are available, VQAs provide a fruitful toolset to tackle a broad variety of problems, whose solution rely on quantum hardware to estimate quantities that are hard to compute and a classical optimization process to find the optimal parameters for the model. (b) The prime factorization problem constitutes one of the cornerstones of modern cryptography. Though quantum algorithms are known to factor prime numbers exponentially faster than classically possible, their implementation requires quantum devices of currently infeasible scale and quality. (c) We apply a variational method to the factorization problem, tailoring the algorithm to a quantum photonic platform.

## II. RESULTS

The implementation of the factorization problem on the PIC using VQA involves three key steps (a scheme of the approach is depicted in Fig. 2). First, through classical pre-processing, we encode the problem into an Ising spin glass Hamiltonian that yields the solution in its ground state. Subsequently, we devise a suitable ansatz by employing a parameterized unitary for the PIC to generate a trial ground state along with a corresponding function to minimize (i.e., loss function). Such a function is designed such that its minimum corresponds to the optimal parameters that solve the task. Finally, we minimize it using a gradient descent optimization to determine the optimal parameters, ultimately providing the ground state. For the first step, the prime factor decomposition of a positive integer  $N$  into two prime numbers  $p$  and  $q$ , where  $p, q \geq 3$ , can be mapped into the following optimization problem [21]:

$$\arg \min_{x, y \in \mathbb{Z}^+} f(x, y), \quad \text{where } f(x, y) = (N - xy)^2, \quad (1)$$

such that, for optimal values of  $x$  and  $y$ , the function  $f(x, y)$  goes to 0 for the values  $x_c$  and  $y_c$  (see Sec. II of the Supplemental Material [34]) that correspond to the solution  $p$  and  $q$  of the factorization problem. The values of  $N$ ,  $p$ , and  $q$  are real integers and can be considered as odd numbers without loss of generality. The lengths of the bit strings corresponding to the

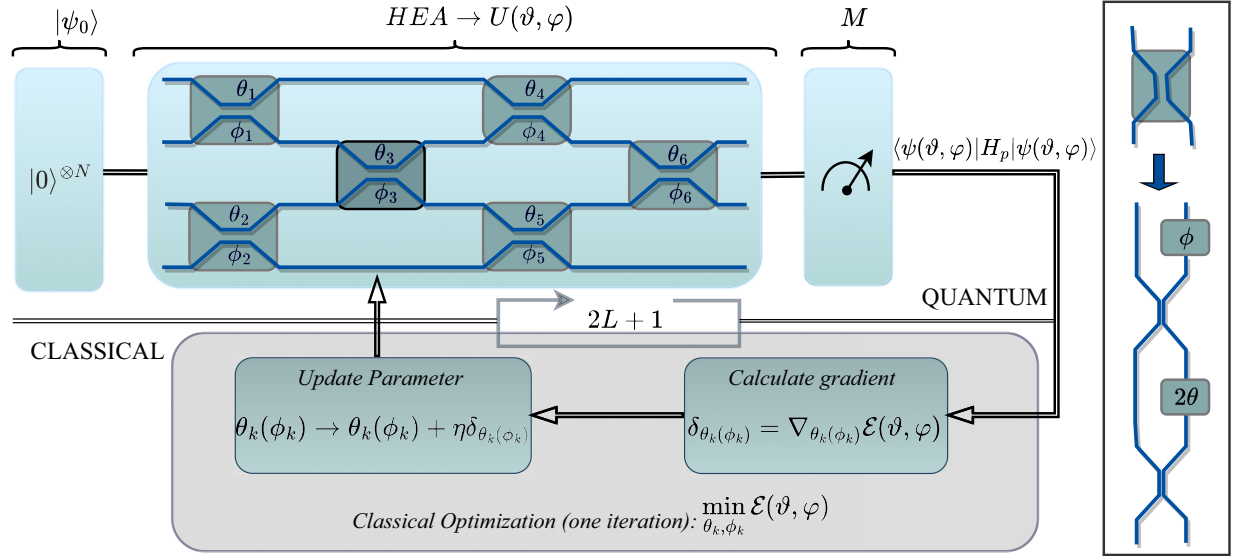


FIG. 2. Conceptual scheme of the proposed single-photon linear-optics-based factorization scheme. The process is hybrid quantum (above) and classical (below) architecture. The quantum part consists of a tunable photonic circuit (PIC) with four input/output modes [8]. This device contains tunable Mach-Zehnder interferometers ( $L = 6$ ), with two tunable parameters,  $\theta$  and  $\phi$ , for enabling arbitrary unitary evolutions. The circuit rotates the input state  $|\psi_0\rangle$  to  $U(\vartheta, \varphi)|\psi_0\rangle$ , which is then measured in the computational basis. The classical part covers the calculation of a cost function  $\mathcal{E}(\vartheta, \varphi)$  related to the expectation value of the Hamiltonian whose ground state provides the solution to a computational problem (here, prime factorization of  $N = 35$ ). Based on the measurement outcome of  $U(\vartheta, \varphi)|\psi_0\rangle$ , a classical gradient descent algorithm is used to adapt  $U(\vartheta, \varphi)$ , minimizing the cost function  $\mathcal{E}(\vartheta, \varphi)$ . The gradients are computed by the forward difference method, which involves  $2L + 1$  times of function evaluations for each iteration of the optimization process.

binary representation of  $x_c$  and  $y_c$  are *a priori* unknown but can be estimated as follows (see Sec. I of the Supplemental Material [34]):

$$n_{x,y} = m(\mathcal{F} \mp \delta N), \quad (2)$$

where  $\mathcal{F} = \lfloor \sqrt{N} \rfloor$  (with  $\lfloor a \rfloor$  indicating the largest integer smaller than or equal to  $a$ ) and  $m(b)$  denotes the smallest number of bits required for representing  $b$ . The offset  $\delta N$ , instead, is required because the gap between two successive primer numbers is not constant and does not grow linearly with  $N$  [35] and it amounts to  $\sqrt{\mathcal{F}^2 - N}$  (see Sec. I of the Supplemental Material [34]). This implies that the solution can be expressed in the following as a binary number with the maximal bit-string length upper bounded by  $n_x$ , for the first factor, and  $n_y$ , for the second:

$$x_c = x_{n_x-1}, \dots, x_1 x_0, \quad y_c = y_{n_y-1}, \dots, y_1 y_0. \quad (3)$$

However, when the number  $N$  to factorize is given by the product of two prime numbers, the number of unknown bits is lower, if we consider that each odd binary number ends with 1. Therefore,  $n_x$  ( $n_y$ ) is reduced to  $n_x - 1$  ( $n_y - 1$ ) and, if we want to encode  $x_c$  and  $y_c$  in a quantum state, the required Hilbert space dimension is  $2^{n_x+n_y-2}$ . Also in this case, these bit lengths are an overestimation of the real length of the factors.

In our implementation, anyway, for practical purposes we can reduce the number of unknown bits even further. Indeed, if we consider the factorization of the number  $35 = 5 \times 7$ , which is equal to the largest number factorized through a simplified version of Shor's algorithm [36], which exploited seven qubits and succeeded roughly 14% of the cases, the length of the bit strings  $x$  and  $y$  constituting the prime factors

must amount to  $n_x = n_y = 3$  (as detailed in Ref. [21] and as reported in Sec. I of the Supplemental Material [34]). This implies that also the most significant bit of the factors must be 1, otherwise a lower number of bits would be sufficient. So, in this specific case, the total number of unknown bits is further reduced, because our factors  $x_c$  and  $y_c$  will have the form  $1x_11$  and  $1y_11$ , respectively. Let us note that this restriction holds only in our case, but the protocol can handle more general cases. Indeed, it is possible to evaluate the maximum number of required bits (as reported in Sec. I of the Supplemental Material [34]), which will be the number of bits outputted by the method. Hence, if a lower amount of bits is sufficient, the most significant ones will be 0. In the present scenario, the Hamiltonian that encodes the problem can be written in an Ising-like form as follows:

$$H_P = [NI - (4I + 2\Pi_z^1 + I)_1] \otimes (4I + 2\Pi_z^1 + I)_2]^2, \quad (4)$$

where  $N$  is the number to factorize,  $I$  is the identity operator,  $\Pi_z^1$  refers to the projector onto the eigenvector of the  $\sigma_z$  operator corresponding to outcome 1, and the subscript indicates the qubit on which it acts (see Sec. II of the Supplemental Material for further details [34]). Then, the state that will minimize Eq. (4) will be represented by a two-qubit state  $|x_1 y_1\rangle$ , where the two unknown bits will be  $x_1$  and  $y_1$ . This is due to the form of the Hamiltonian, because the two factors  $x$ ,  $y$  are expressed, in binary form, as follows:  $4 + 2x_1 + 1$  and  $4 + 2y_1 + 1$ . So, the ground state of  $H_P$  will be the one for which the expected values of the projector onto the eigenstate  $|1\rangle$  of the operator  $\sigma_z$  amount to  $x_1$  and  $y_1$ .

At this point, we need to design a suitable ansatz, so we consider a fixed two-qubit initial state  $|\psi_0\rangle$  that is rotated

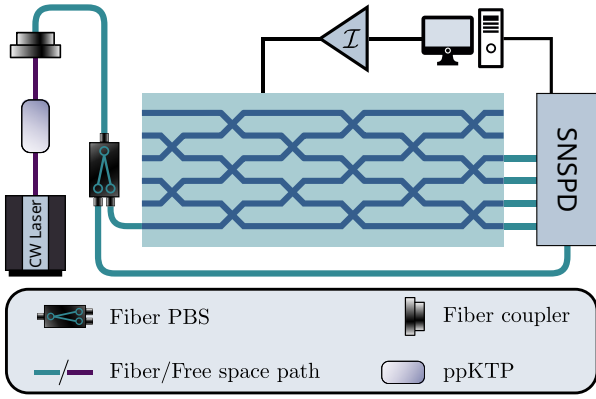


FIG. 3. Experimental setup. Orthogonally polarized single-photon pairs generated by a type-II SPDC process are separated using a fiber polarizing beam splitter (PBS), where one output port is used to herald a pair creation event. The remaining photon is injected into the PIC in a fixed spatial mode representing the input state  $|\psi_0\rangle = |00\rangle$ . The PIC implements a unitary  $U(\vartheta, \varphi)$  on the photon path degree of freedom, creating the trial state  $|\psi(\vartheta, \varphi)\rangle = U(\vartheta, \varphi)|\psi_0\rangle$ . This state is measured in the two-qubit computational basis, where the basis states  $|00\rangle$  ( $|01\rangle$ ) are identified by a photon being present in the first (second) output mode, and so on. From the acquired output statistics, the trial state energy  $\mathcal{E}(\vartheta, \varphi) = \langle \psi(\vartheta, \varphi) | H_p | \psi(\vartheta, \varphi) \rangle$  with respect to the Hamiltonian  $H_p$  of Eq. (4) can be computed. A classical gradient descent algorithm is employed to minimize  $\mathcal{E}(\vartheta, \varphi)$  by varying the internal and external phases  $\vartheta, \varphi$ .

by some parameterized unitary  $U(\vartheta, \varphi)$  to a trial state  $|\psi(\vartheta, \varphi)\rangle = U(\vartheta, \varphi)|\psi_0\rangle$ . This state is then projected on the Z basis and the achieved statistics is employed for computing the expectation value of the energy  $\mathcal{E}(\vartheta, \varphi)$ , as follows:

$$\mathcal{E}(\vartheta, \varphi) = \langle \psi_0 | U^\dagger(\vartheta, \varphi) H_p U(\vartheta, \varphi) | \psi_0 \rangle. \quad (5)$$

According to the variational principle, such an energy value must be greater than or equal to the ground-state energy  $E_0$ . Hence,  $\mathcal{E}(\vartheta, \varphi)$  constitutes the cost function to be minimized, by adaptively changing the parameters of the implemented unitary. Given these premises, the solution boils down to finding the state that corresponds to the ground state of the Hamiltonian  $H_p$ . This implies that our goal is to find the unitary  $U$  such that  $U(\vartheta, \varphi)|\psi_0\rangle = |x_1, y_1\rangle$ . Due to the commutativity property of multiplication, it follows that  $N = p \times q = q \times p$ . This implies that there are two unitaries encoding the correct solution of our problem, as the solution can be given by  $U(\vartheta_1, \varphi_1)|\psi_0\rangle = |01\rangle$  and  $U(\vartheta_2, \varphi_2)|\psi_0\rangle = |10\rangle$ . Hence, for the solution  $|01\rangle$ , the two factors will be  $x_c = 101 = 5$  and  $y_c = 111 = 7$ , while for  $|10\rangle$ , they will amount to  $x_c = 111 = 7$  and  $y_c = 101 = 5$  (see Fig. 2).

From an experimental point of view, we prepare the state  $|\psi_0\rangle$  by harnessing the path degree of freedom in four spatial modes of a PIC, following the encoding  $|\psi_0\rangle = |1, 0, 0, 0\rangle = |00\rangle$ ,  $|\psi_1\rangle = |0, 1, 0, 0\rangle = |01\rangle$ ,  $|\psi_2\rangle = |0, 0, 1, 0\rangle = |10\rangle$  and  $|\psi_3\rangle = |0, 0, 0, 1\rangle = |11\rangle$ , where  $|1, 0, 0, 0\rangle$  denotes a photon populating the first mode of the PIC, and so on. Such a single-photon input is generated by type-II spontaneous parametric down-conversion (SPDC) in a periodically poled  $\text{KTiOPO}_4$  crystal, with a poling period of  $46.2 \mu\text{m}$ , in a single-pass configuration as depicted in Fig. 3, generating

wavelength degenerate photons at  $\lambda_s = \lambda_i = 1550 \text{ nm}$  out of a  $775\text{-nm}$  continuous wave pump. Then, one photon is directly sent to a detection unit consisting of superconducting nanowire single-photon detectors with a detection efficiency of  $\sim 95\%$  and a dark count rate of  $\sim 300 \text{ Hz}$ . It thereby heralds the presence of its twin, which is routed to a tunable optical circuit. This is the core of our experimental apparatus and it is constituted by a six-mode universal PIC, featuring only linear optical elements, i.e., phase shifters and beam splitters, whose architecture follows the optimal design proposed in Ref. [8]. In detail, the circuit consists of a grid of variable beam splitters, i.e., Mach-Zehnder interferometers (see Fig. 2), each characterized by a transformation matrix denoted as  $T_{jk}(\theta, \phi)$ , where  $\theta$  determines the reflectivity and  $\phi$  is an external phase. With a specific ordered sequence  $S$  of these transformation matrices, it is possible to create a programmable unitary operator described as

$$U(\vartheta, \varphi) = D \prod_{j,k \in S, l} T_{jk}(\theta_l, \phi_l), \quad l = \{1, 2, \dots, L\}, \quad (6)$$

where  $D$  is some arbitrary diagonal matrix. It is worth noting that the sets of internal phases  $\vartheta = \{\theta_1, \theta_2, \dots, \theta_L\}$  and external ones  $\varphi = \{\phi_1, \phi_2, \dots, \phi_L\}$  amount to  $2L$  parameters, where  $L$  is the number of Mach-Zehnder interferometers. This structure allows us to perform arbitrary unitary operations on six-dimensional input Fock states, by properly choosing the values of the phase shifts.

The PIC is manufactured in a borosilicate glass substrate through femtosecond laser waveguide writing [37]. The tunability is achieved by fabricating thermo-optic phase shifters (i.e., resistive heaters) in a thin film of gold deposited on the chip surface. The device exhibits an overall transmission of  $54\%$  and can implement arbitrary unitary operations on six modes with an average amplitude fidelity of  $0.9970 \pm 0.0017$  [38]. While the employed PIC has a negligible polarization extinction ratio (PER), the path encoding scheme solely relies on a homogeneous PER along the PIC width to maintain interference.

For our experiment, we need to use only four input and output modes, which are tunable by acting on 12 phase shifters ( $L = 6$ ), while the surrounding actuators are set in reflection ( $\theta = \pi/2$ ) to isolate these modes from the others available in the PIC. Our ansatz is custom tailored to these structures (for further details on the operation of the chip, see Sec. IV of the Supplemental Material [34]).

Hence, to find these two sets of optimal parameters  $(\vartheta_1, \varphi_1)$  and  $(\vartheta_2, \varphi_2)$ , we perform 117 repetitions of the experiment, starting with random initial parameters. For each repetition, we initialize our ansatz by applying 12 random phase shifts ( $L = 6$ ) and injecting the input state as  $|\psi_0\rangle = |00\rangle$ , namely, a single photon in the first mode. Then, we collect the photon statistics at each output, corresponding to a projection of the rotated input onto the computational basis, comprising the states  $|00\rangle$ ,  $|01\rangle$ ,  $|10\rangle$ , and  $|11\rangle$ . Following this, we postprocess the data to evaluate the energy of  $|\psi_0\rangle$  with respect to  $H_p$  on a classical computer, using Eq. (5). Note that, for  $H_p$ , which only involves expected values in the  $\sigma_z$  basis, a full tomography on the output state is not required to achieve

the correct value of Eq. (5) (see Sec. V of the Supplemental Material [34]).

For each of the 117 repetitions of the experiment, we carry out an optimization, through a gradient descent algorithm, made of several iterations. For each step of the optimization, we derive a new set of phases, which is subsequently applied to the circuit through a classical feedback loop. The gradients for each of the  $2L$  parameters are obtained using a forward finite difference method. This requires a total number  $2L + 1$  of circuit evaluations to be performed in each feedback, to update all the phases. Every iteration  $i$  yields a new energy value  $\mathcal{E}(\vartheta_i, \varphi_i)$  and the process stops when it converges to a solution. This implies that the variation between the energies of two consecutive steps falls below a specified threshold  $\epsilon = 10^{-4}$ . At this point, the obtained photon statistics of the last iteration constitutes an approximation of the ground-state energy and therefore represents the solution.

On average, one optimization step is performed every 174 s, of which 42 s are spent acquiring detection events. The remaining time can be attributed to the operation of the PIC. It is worthwhile to point out that the classical optimization algorithm, in principle, requires no knowledge about  $U(\vartheta, \varphi)$  or even the possibly nonlinear mapping between the phases and their control currents (for further details on the classical optimization, see Sec. VI of the Supplemental Material [34]). In our case, though, we exploited a precharacterization of the device to ensure that no light would go to the undetected modes of the six-input/six-output circuit.

The convergence of the energy value with respect to the function evaluations is depicted in Fig. 4, where different curves correspond to different initial phase configurations and the dashed line represents the theoretical limit. It can be clearly seen that the optimization works smoothly and is able to reach the ground state in every instance with an accuracy ranging from 99% to 99.99%. Furthermore, let us point out that the ground state of  $H_p$  is twofold degenerate and each instance of the experiment converges to one particular ground state corresponding to one optimal unitary, i.e., either  $U(\vartheta_1, \varphi_1)$  or  $U(\vartheta_2, \varphi_2)$ . This is visible throughout the 117 repetitions of the experiment, whose averaged probabilities are represented in Fig. 5(a) (purple bars), where we also compare them to the theoretical predictions (in blue). We obtained the solutions  $|01\rangle$  and  $|10\rangle$  with 50.7% and 44.6%, respectively. This implies, as previously mentioned, that the unknown bits for our two factors are  $|x_1 y_1\rangle = |10\rangle$  or, equivalently,  $|01\rangle$ .

This is also reconfirmed in Fig. 5(b), where we plot the fidelities

$$F(\rho, \sigma) = \text{Tr}(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}) \quad (7)$$

between the state  $\sigma$ , which amounts to  $\sqrt{|\alpha|}|0\rangle + \sqrt{(1-|\alpha|)}|1\rangle$ , at each experimental repetition for the last optimization step, and the target state  $\rho$ . Note that the form of state  $\sigma$  is obtained by measuring in the Pauli  $\sigma_z$  basis. Across the 117 experimental repetitions, the measured state emerges as  $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$  with a final fidelity close to unity (depicted in purple). Accordingly, the fidelities to the individual solution states  $|01\rangle$ ,  $|10\rangle$  are close to  $1/\sqrt{2}$  (depicted in blue and green).

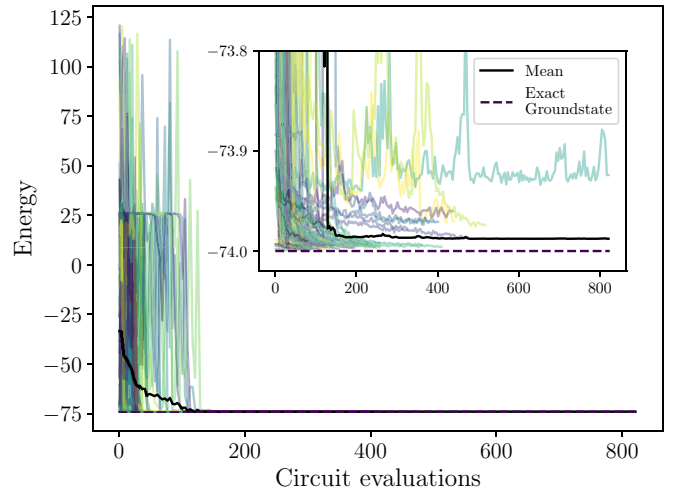


FIG. 4. Convergence of the estimated ground-state energy for different initial configurations. Calculated energy  $\langle \Psi | H_p | \Psi \rangle$  [Eq. (5)] at each optimization step, for 117 different initial configurations of the experiment (colored lines), as they approach the theoretically predicted ground-state energy  $E_g$  (dashed line). The configuration at each step consists in the set of internal and external phases  $(\vartheta, \varphi)$  applied to the integrated circuit, which amounts to a unitary operation  $U(\vartheta, \varphi)$ . Accordingly, at the  $i$ th step, the output state will be  $|\Psi_i\rangle = U(\vartheta_i, \varphi_i)|\psi_0\rangle$ , where  $|\psi_0\rangle$  is the initial input. To evaluate the obtained energy, due to the form of our Hamiltonian  $H_p$  [see Eq. (4)], it is sufficient to postprocess the statistics obtained by projecting the output state onto the computational basis. The black line shows the mean experimental energy at every step, where experiments with a final energy  $\mathcal{E} \geq 0$  have been removed (i.e., excluding 5 out of 117 repetitions). A magnified region close to  $E_g$  is depicted in the inset. The theoretically predicted ground-state energy is reached by the optimization procedure after 27, 28, and 115 steps within a relative uncertainty of 1%, 0.1%, and 0.01%, respectively. Let us note that the achieved energy values are lower than 0, because we are removing a constant term from Eq. (4).

### III. DISCUSSION

This work reports on the implementation of a VQA on an integrated photonic circuit, exploiting an ansatz that is tailored to our platform. In particular, we present a proof-of-principle demonstration for the factorization of  $35 = 5 \times 7$ , executed on a four-input/output mode of a universal PIC. Our strategy involves the use of a qudit with four levels, corresponding to the four modes of the photonic circuit. This is then interpreted as two qubits, encoding the solution in the ground state of an Ising Hamiltonian. Subsequently, we determine its energy through a hybrid quantum-classical optimization problem, utilizing gradient descent. In further detail, we design an ansatz specifically for state-of-the-art PICs that allow high-quality processing of single photons by using only passive optical elements [8]. Our encoding of the two-qubit computational basis involves the binary representation of the mode from which one single photon enters an optical circuit, where the first mode corresponds to  $|00\rangle$ , the second to  $|01\rangle$ , and so forth. At each step of the optimization, we collect the statistics obtained by projecting the evolved input state onto the computational basis and postprocess it to retrieve the energy value. Then, we

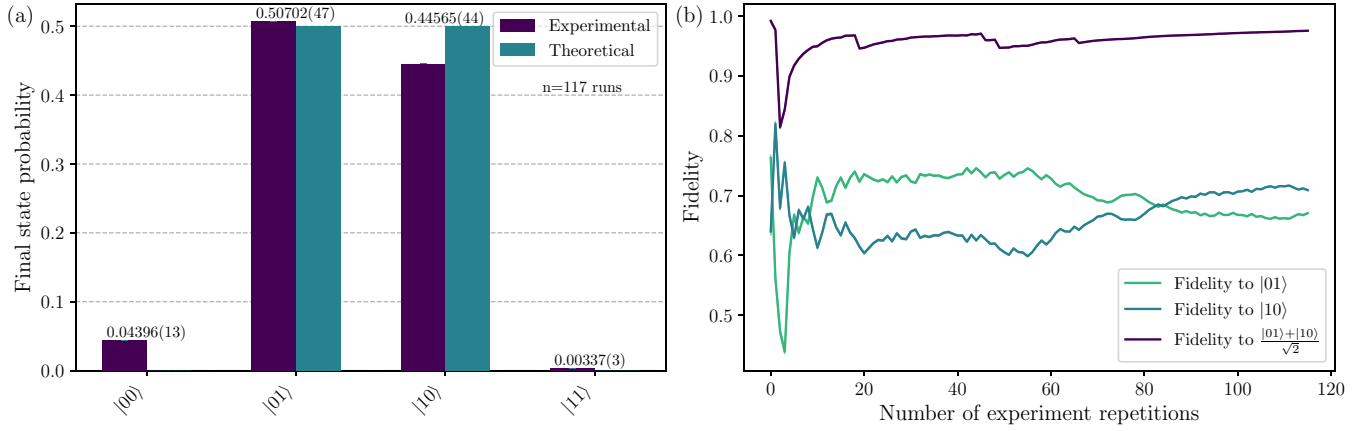


FIG. 5. Experimental factorization results. (a) The bars correspond to the experimental frequencies with which a photon is detected at each of the four outputs of our circuit. The reported values are obtained by averaging over the results of 117 different experimental optimizations, starting from different configurations. For our encoding, injecting one photon in the  $i$ th mode constitutes the  $i$ th element of a two-qubit computational basis. Therefore, the bars representing the highest frequencies indicate the ground state  $|x_1 y_1\rangle$  of the Hamiltonian outlined in Eq. (4), which is an almost equal superposition of  $|01\rangle$  and  $|10\rangle$ . This effectively resolves the factorization problem, allowing us to construct the factors as  $|x_1| = 101 = 5$  and  $|y_1| = 111 = 7$ , or vice versa. (b) As all states of the form  $\sqrt{\alpha}|01\rangle + \sqrt{1-\alpha}|10\rangle$  exhibit the same energy with respect to  $H_p$  (see Sec. IV of the Supplemental Material [34]), every individual optimization converges to one arbitrary of those possible outcomes. The average output of independent repetitions thus approaches the equal superposition  $\alpha = 0.5$ , as can be seen on the fidelity of 0.975 46(5) toward this state after 117 repetitions (violet), while the fidelities to both  $|01\rangle$  (green) and  $|10\rangle$  (teal) concordantly revolve around 0.7. The fidelity [Eq. (7)] is evaluated by interpreting the output distribution measured in the computational basis as normalized state vector. For both plots, the uncertainties (shaded regions for two standard deviations) are evaluated considering an underlying Poissonian statistics using a Monte Carlo simulation and are too small to be visible.

identify a new configuration of our circuit and we iterate the procedure, until it converges to the minimum energy value, corresponding to the ground state of our Hamiltonian. This allows us to reach the correct solution for our factorization problem with a success probability exceeding 98%, in fewer than 150 iterations and for 117 different starting conditions, highlighting its reduced sensitivity to initial parameters. Let us point out that our hybrid classical-quantum VQA takes inspiration from the conventional AQC that aims to prepare the ground state of a problem Hamiltonian. This approach involves a continuous quantum evolution and therefore is limited by the minimum spectral gap dictating the required adiabatic evolution time. More specifically, there are only some theoretical results (see Ref. [39]) proposing conditions to identify problem classes where AQC may be beneficial, and, for factorization, only numerical evidence suggests an increasing spectral gap in certain cases (see Ref. [40]). VQAs circumvent this issue by using classical optimization over parametrized circuits, while still facing challenges such as noise, barren plateaus, and ansatz expressivity. Although no general quantum advantage has been demonstrated also in this case, our approach is well suited for medium-scale devices and provides practical benefits on current hardware. Indeed, this study marks the first instance of factorization on an optical computing device and shows the utility of the photonic devices in variational quantum algorithms, showcasing advantages in terms of resource requirements and resilience against inherent noises. This is reflected by the fact that, with the integrated platforms currently available on the market, our implementation could already factorize a semiprime number up to 93, while considering the product of two consecutive primes, this limit goes up to 899. These numbers are well

beyond what has current been achieved with Shor's algorithm [36] and this limit is likely to increase with recent developments in integrated photonics. Indeed, until now, the largest number factorized through a simplified version of Shor's algorithm is 35, by exploiting seven qubits and obtained a success rate of roughly 14% of the cases [36]. The number of required qubits was then lowered down in a more recent work, where 21 was factored using five qubits [41]. Moreover, although our demonstration is dedicated to the factorization problem, we want to emphasize that our approach can readily be employed to address other challenges, given that our results indicate that universal integrated circuits are a suitable platform for the implementation of VQAs. Let us note that the integrated photonic platforms that are currently on the market are already sufficient to encode up to five qubits using path-encoded single-photon architectures. This places our hardware efficient variational framework well within the range of many established benchmark problems in near-term quantum computing. For instance, ground-state estimation of simple molecular systems such as  $H_2$  using VQE requires only few qubits [31,42], and combinatorial problems like MaxCut on three to four node graphs can also be encoded on systems smaller than five qubits [43]. These examples are representative of diverse problem classes and underscore that our method is not inherently limited to factorization tasks, but is broadly applicable within the capabilities of current photonic hardware. Moreover, the versatility, high achievable fidelity in the implementation of arbitrary unitary transformations, and scalability of photonics platforms make them particularly apt for adaptive protocols, e.g., the training of machine learning algorithms. On the other hand, the limitations of this platform are related to the difficulty of implementing two-qubit gates and hence nonlinear

operations. Indeed, the fact that photons tend not to interact with the environment implies that two-qubit gates can only be implemented probabilistically and heralded through ancillary photons or postselection [44]. These issues were circumvented in our case by encoding  $n$  qubits in  $2^n$  modes, offering a platform that is exactly equivalent to the quantum circuit formalism. While this approach offers the advantage of the number of input photons staying constant—specifically one—-independent of problem size, it exhibits the drawback of the number of necessary PIC modes scaling exponentially in the length of the encoded bit strings. Consequently, the number of parameters to optimize also increases exponentially. Furthermore, the reliance on a single photon relaxes requirements on PIC transmittivity and detection efficiency. As these quantities affect the integration time exponentially in the number of photons, they only impact our protocol in a constant manner. However, the drawbacks can potentially be eased by future developments, as suggested by several recent studies investigating the possibility of implementing nonlinear operations through encoding, feedback loops, and enhancing the expressivity of the circuit through quantum interference [45–47]. Moreover, recent studies have highlighted that optical computational models could grant lower energy consumption, with respect to standard models [48].

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P.S. and I.A. designed and conducted the experiment. K.P. developed the theory, discussing with Y.B. S.S. built the single-photon source. S.S. and Z.Y. participated in the experiment. C.P., S.P., A.C., F.C., and R.O. conducted the design, fabrication, and calibration of the integrated photonic processor. X.C. and P.W. supervised the project. All authors discussed the results and reviewed the manuscript.

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#### DATA AVAILABILITY

The data that support the findings of this study are openly available in Zenodo at [50].

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