

Ising model with power law resetting

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We investigate the nonequilibrium dynamics of the nearest-neighbour Ising model subjected to stochastic resetting, where the system is intermittently returned to an initial configuration with magnetization m_0 , with the inter-reset times drawn from the power law distribution $\alpha\tau_0^\alpha/\tau^{\alpha+1}$. The heavy-tailed resets generate magnetization distributions that differ significantly from both equilibrium dynamics and the previously studied Ising model with exponentially distributed reset times. In two dimensions, for $T > T_C$, we find a quasiferro state for all α , marked by a double-peaked distribution that diverges at $m = 0$ and $m = m_0$; no steady state exists for $\alpha < 1$, while a stationary state emerges for $\alpha > 1$. For $T < T_C$, power law resetting produces two distinct regimes separated by a crossover exponent $\alpha^* = 1 - c$: a single-peaked ferromagnetic phase localized at m_{eq} for $\alpha < \alpha^*$, and a dual-peaked ferromagnetic phase with divergences at m_{eq} and m_0 for $\alpha > \alpha^*$. Analytic results in one and two dimensions, supported by simulations, yield a rich phase diagram in the (T, α) plane and reveal how heavy-tailed resetting generates nonequilibrium phases very different from those seen in the case of exponential resetting.

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I. INTRODUCTION

Stochastic resetting is a process in which evolving dynamical system is suddenly brought back to a predecided state at random times and allowed to evolve via its natural dynamics in between two such resets. This seemingly simple mechanism is found to have intriguing effects in system dynamics, which often lead to nontrivial steady-state behavior. This concept has emerged as a versatile tool for modeling and analyzing different phenomena since it was originally introduced in the context of a diffusion search problem [1]. In that particular work, it was shown that this mechanism leads to a more efficient searching process, and the first passage time can be minimized by tuning the resetting rate. It was also found that this process breaks the detailed balance and leads to the emergence of a nonequilibrium steady state. Since this pioneering work, stochastic resetting has attracted significant research interest. Early works examined diffusion with space-dependent or randomly distributed resets [2–4] and the optimization of first-passage times, including scenarios with partial absorption [5,6]. Subsequent studies extended the concept to discrete-time jumps [7], memory-dependent resetting [8], fractional Brownian motion [9], and resets to extremal positions [10]. Noninstantaneous or finite-speed resets have also been considered to account for spatiotemporal constraints [11–14], along with complex geometries such as bounded

domains [15–17], comb structures [18], and Sisyphus random walks [19]. Beyond classical diffusion, stochastic resetting has been explored in quantum dynamics [20], run-and-tumble particles [21], and telegrapher's processes [22]. Experimental confirmations for diffusion with resetting have been reported using optically trapped colloidal particles [23,24]. Applications extend across disciplines: in biology for transcriptional pauses, protein-DNA target search, and active intracellular transport [25–27], in ecology for animal relocation and movement patterns [28,29], in computer science for randomized search algorithms [30,31], in psychology for visual search and pattern recognition [32,33], and in finance for reset options and warrant valuations [34,35]. The wide-ranging effects of stochastic resetting on search efficiency, first-passage behavior, and nonequilibrium dynamics make it a rich and rapidly expanding area of research.

While most studies of stochastic resetting have focused on noninteracting systems, its effects in interacting systems are particularly important, as they reflect more realistic dynamics [36]. To understand the range of behaviors induced by resetting, researchers have explored diverse interacting systems, from simple two-particle setups to complex many-particle models. In the simplest case of two-particle systems, stochastic resetting has been investigated in predator-prey models [37,38], interacting Brownian motion [39], and diffusion with birth-death dynamics [40–42]. Studies on many-particle systems have examined how stochastic resetting affects models such as the symmetric simple exclusion process (SSEP) [43,44], the totally asymmetric simple exclusion process (TASEP) [45], fluctuating interfaces [46,47], coagulation-diffusion processes [48], zero-range processes [49], and the Ising model [50]. Further, in such systems, it is possible to have dynamics involving the reset of a part of the system instead of the whole system. The effects of such local resetting

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have also been explored in binary aggregation [51], SSEP [52], and TASEP [53]. In this work, we focus on the nearest-neighbour Ising model with stochastic resetting, which is an archetypal model to study phase transition, making it an ideal platform to investigate how resetting modifies dynamics and steady-state behavior.

In a previous work, Magoni *et al.* [50] studied the Ising model undergoing stochastic resetting at a constant rate. They considered a nearest-neighbour Ising model, which evolves according to Glauber dynamics in the absence of reset. The system, which is trying to evolve to its equilibrium state, is repeatedly brought back to a configuration with magnetization m_0 after random intervals of time that are distributed exponentially ($re^{-r\tau}$). The quantity under observation was the distribution of the magnetization of the system at long times. They found that the introduction of resetting leads to a nontrivial phase behavior. The system can exist in a “ferro” phase ($T < T_c$), where the stationary magnetization distribution $P_{\text{stat}}(m)$ develops a clear gap at $m = 0$ and peaks at a finite value of m , or a “para” phase ($T > T_c$, $r(T) < r^*(T)$), where $P_{\text{stat}}(m)$ instead exhibits a divergent peak at $m = 0$ with no gap. Apart from these two phases, which are broadly similar to the usual ferromagnetic and paramagnetic phases, their work also shows the emergence of a “pseudoferro” phase for $T > T_c$ and $r(T) > r^*(T)$, where $P_{\text{stat}}(m)$ remains gapless but now vanishes at $m = 0$ and peaks at a finite value of m ($m = m_0$). The introduction of resetting thus leads to a richer and more interesting behavior than the usual Ising dynamics.

In this work, we study the Ising model with stochastic resetting such that the time intervals between the resets are distributed according to a power law ($\alpha\tau_0^\alpha/\tau^{\alpha+1}$). The paradigmatic nature of the Ising model makes it an ideal system to study the effects of new dynamical features, while the choice of power law distributed time intervals is guided by their ubiquity in natural as well as human-made systems. Such waiting time distributions commonly appear in complex systems and lead to events that occur in bursts with long pauses in between. This type of behavior is observed in diverse systems like earthquakes [54], neuron firing [55], stock market fluctuations [56–58], and human activity patterns [59–61]. Power law resetting introduces heavy-tailed intervals, allowing rare but arbitrarily long times between resets, which leads to a richer interplay between the system’s intrinsic relaxation and the resetting mechanism. Previous studies have shown that power law resetting can significantly alter system dynamics compared to exponential resets. In the context of diffusion, power law resetting produces a wide range of long-time behaviors: Depending on the tail exponent α , the motion may remain unbounded or become stationary [62], whereas exponential resetting always drives the system to a stationary state [1]. Similarly, in TASEP, exponential resetting leads to a steady state with monotonic density evolution, while power law resetting can generate long-lived, time-dependent scaling and nonmonotonic density profiles when the tail exponent is small [45]. These examples highlight how the choice of reset distribution qualitatively affects long-time dynamics, thus motivating this study.

Our framework is the same as in Ref. [50]. We consider a nearest-neighbor Ising model with initial magnetization m_0 .

The system evolves according to Glauber dynamics [63] toward its equilibrium state with magnetization m_{eq} . In the absence of resetting, for dimensions (D) greater than one, the system reaches a ferromagnetic phase ($m_{\text{eq}} > 0$) for $T < T_c$ and a paramagnetic phase ($m_{\text{eq}} = 0$) for $T \geq T_c$ at long times. The distribution of magnetization in equilibrium is a delta function centered at m_{eq} corresponding to the temperature chosen. Now we introduce a resetting protocol. The system is brought back to a configuration with magnetization m_0 (initial magnetization) at random intervals of time that are distributed according to a power law, $\alpha\tau_0^\alpha/\tau^{\alpha+1}$. In between the resets, the system evolves via Glauber dynamics. This resetting process can be physically realized by a “rapid quench” protocol as described in Ref. [50] by replacing the exponential time interval distribution with a power law distribution. We observe the magnetization $m(t) = \frac{1}{N} \sum_i \langle s_i(t) \rangle$ at time t and analyze how it is distributed.

In our work, we see that the resetting at power law times leads to nontrivial magnetization distributions that are distinctly different from those seen in the Ising model with exponentially distributed reset times. In two-dimensions (2D), the Ising model without reset exists in the paramagnetic ($T > T_c$) or ferromagnetic ($T < T_c$) phase. The introduction of power law resetting leads to the following behavior: For $T > T_c$, we see a “quasiferro” (QF) state in which for all values of α , the probability distribution takes a double-peaked structure. For $\alpha < 1$, the system does not reach a steady state. The probability density function (PDF) at large times diverges at both the origin ($m = 0$) and $m = m_0$. On the other hand, for $\alpha > 1$, the system goes to a stationary state at long times, and the probability function continues to have the double-peaked structure with divergences at $m = 0$ and $m = m_0$ as before. We thus see that the behavior is very different from the paramagnetic state of the Ising model, as well as the “pseudoferro” phase seen in Ref. [50]. For $T < T_c$, we see that depending on the value of α , there is an emergence of two different phases. For $\alpha < 1$, again the system does not go to a steady state in the large time limit. But for $\alpha < \alpha^*$, we see a single-peaked magnetization distribution at large times. The probability peaks only at $m = m_{\text{eq}}$ in this “single-peaked ferro state” (SPF). On the other hand, for $\alpha > \alpha^*$, we again see a double-peaked probability function that diverges at $m = m_{\text{eq}}$ as well as at $m = m_0$ at long times. For $\alpha > 1$, the system possesses a steady state with the probability function again peaking at m_0 and m_{eq} . So, in effect, for $\alpha > \alpha^*$, we have a “double-peaked ferro state” (DPF), which is different from the case of $\alpha < \alpha^*$. Figure 1 shows a plot on the (T, α) plane indicating the various phases along with the behavior of magnetization distribution in each case. For $T < T_c$, we see a crossover line at $\alpha = \alpha^*$ (where $\alpha^* = 1 - c$, $0 < c < 1$, where c is a dynamical parameter corresponding to Ising-Glauber dynamics for $T < T_c$) that separates the two distinct phases we discussed above. For the one-dimensional (1D) Ising model, the original Ising system exists in a paramagnetic state at all finite temperatures. Correspondingly, the introduction of the power law resetting leads to a QF state with a time-independent steady-state distribution existing only for $\alpha > 1$. We establish these results using analytical solutions for the cases of 1D and 2D Ising models and validating them with numerical simulations.

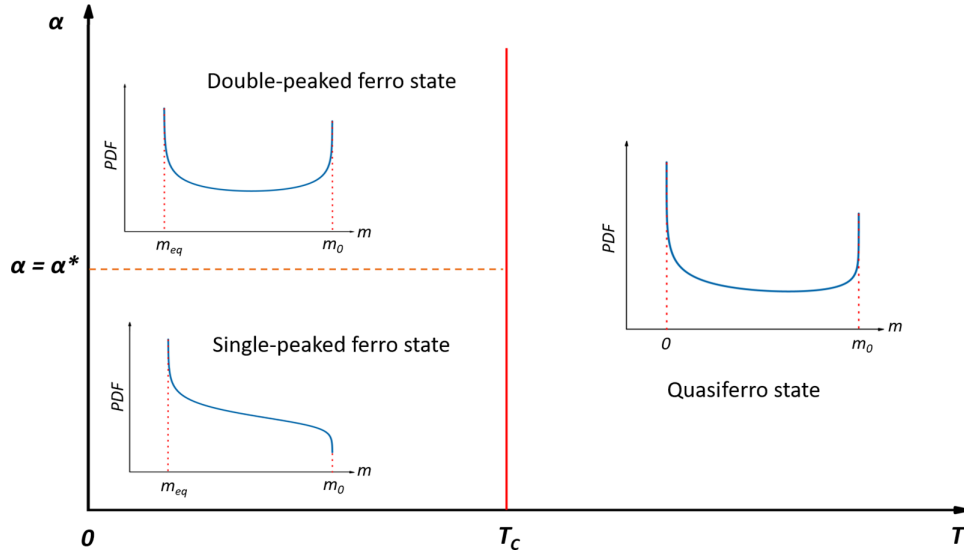


FIG. 1. Phase diagram in the (T, α) plane for the 2D Ising model with power law stochastic resetting. The schematic shows the qualitative forms of the magnetization distribution in the different regimes. For $T > T_c$, the system exhibits a quasiferromagnetic state for all α , characterized by a double-peaked distribution with divergences at $m = 0$ and $m = m_0$. For $T < T_c$, power law resetting gives rise to two distinct ferromagnetic phases separated by the crossover line $\alpha = \alpha^*$ (dashed orange), where $\alpha^* = 1 - c$ and c is the Glauber dynamical exponent. For $\alpha < \alpha^*$, the distribution is single peaked, diverging only at the equilibrium magnetization m_{eq} . For $\alpha > \alpha^*$, a double-peaked ferromagnetic state emerges, with divergences at both m_{eq} and m_0 .

The paper is organized as follows: In Sec. II, we introduce the Ising model with power law resetting. We begin by briefly recalling the behavior of the model without resetting and then present the renewal-based analytical framework used to incorporate stochastic resetting. Section III focuses on the 1D Ising model, where we derive the magnetization distribution under resetting and compare our analytical predictions with numerical simulations. In Sec. IV, we extend the analysis to the 2D Ising model and examine how resetting affects the magnetization distribution across different temperature regimes, again validating our results through simulations. Finally, we summarize our findings and outline possible directions for future work in Sec. V.

II. ISING MODEL WITH POWER LAW RESETTING

We consider a d -dimensional nearest-neighbour Ising model with N sites and periodic boundary conditions. The Hamiltonian is given by $H = -J \sum_{\langle i,j \rangle} s_i s_j$. The initial state is chosen such that the magnetization of the configuration equals m_0 . For this, we generate a configuration by independently choosing the spins to take values ± 1 with probability $(1 \pm m_0)/2$, respectively (where $m_0 \in [0, 1]$). This is also the state to which the system resets repeatedly. In the absence of reset, the system follows Glauber dynamics, which consists of flipping a single spin with rate $w(s_i \rightarrow -s_i) = \frac{1}{1 + e^{\beta \Delta E}}$, where $\beta = (1/k_B T)$ and $\Delta E = 2J s_i \sum_{j \in n.n.} s_j$ is the change in energy when the spin s_i is flipped [63]. In one dimension, this rate takes the form $w(s_i \rightarrow -s_i) = (1/2)(1 - \gamma s_i(s_{i-1} + s_{i+1})/2)$ where, $\gamma = \tanh(2\beta J)$, [63].

Now, we introduce a resetting protocol in which the system is returned, at random times, to the initial state of magnetization m_0 , where the inter-reset times are drawn from a power

law distribution

$$\rho(\tau) = \frac{\alpha \tau_0^\alpha}{\tau^{\alpha+1}}, \quad \tau \in [\tau_0, \infty), \alpha > 0. \quad (1)$$

In between resets, the system relaxes toward its equilibrium state corresponding to the temperature T via Glauber dynamics. Our aim is to determine the probability distribution of the magnetization, $m(t) = \frac{1}{N} \sum_i \langle s_i(t) \rangle$, at the observation time t . To do this, we employ a renewal approach: Each reset restarts the dynamics, so the magnetization distribution at time t is solely determined by the evolution that follows the last reset prior to t . The corresponding renewal equation is

$$P_r(m, t) = \int_0^t d\tau f(t, t - \tau) P_0(m, \tau), \quad (2)$$

where $f(t, t - \tau)$ is the probability density at time $t > 0$ for the last reset to have happened at the time instant $t - \tau$, with $\tau \leq t$. $P_0(m, \tau)$ is the probability distribution of the magnetization in the absence of resetting. The spatially averaged quantity $m(t)$ has a deterministic evolution and therefore this distribution is simply given by $P_0(m, \tau) = \delta(m - m(\tau))$. For power law resets, the behavior of $f(t, t - \tau)$ depends on the exponent α [62]. For $\alpha < 1$,

$$f_{\alpha < 1}(t, t - \tau) = \frac{\sin(\pi\alpha)}{\pi} \tau^{-\alpha} (t - \tau)^{\alpha-1} \quad (3)$$

and thus,

$$P_r(m, t) = \frac{\sin(\pi\alpha)}{\pi} \int_0^t d\tau \tau^{-\alpha} (t - \tau)^{\alpha-1} P_0(m, \tau). \quad (4)$$

When $\alpha > 1$, for $\tau \geq \tau_0$, we have

$$f_{\alpha > 1, \tau \geq \tau_0}(t, t - \tau) = \frac{1}{\tau_0} \left(\frac{\alpha - 1}{\alpha} \right) \left(\frac{\tau}{\tau_0} \right)^{-\alpha}. \quad (5)$$

In addition, the probability density of resetting in the small interval $[0, \tau_0]$ follows from normalization of $f_{\alpha>1}(t, t - \tau)$: $\int_0^{\tau_0} d\tau f_{\alpha>1, \tau < \tau_0}(t, t - \tau) = 1 - \int_{\tau_0}^t d\tau f_{\alpha>1, \tau \geq \tau_0}(t, t - \tau)$. By combining these contributions, we obtain

$$P_r(m, t) = \frac{1}{\tau_0} \left(\frac{\alpha - 1}{\alpha} \right) \int_{\tau_0}^t d\tau \left(\frac{\tau}{\tau_0} \right)^{-\alpha} P_0(m, \tau) + \left[1 - \frac{1}{\alpha} \left(1 - \left(\frac{\tau_0}{t} \right)^{\alpha-1} \right) \right] P_0(m, \tau_0). \quad (6)$$

III. RESETTING IN THE 1D ISING MODEL

For the 1D Ising model, $T_C = 0$, and the system shows a paramagnetic behavior at all finite temperatures. We consider a one-dimensional lattice with periodic boundary conditions. In the absence of resets, the magnetization $m(t)$ evolves deterministically via Glauber dynamics [63] as

$$m(t) = m_0 e^{-(1-\gamma)t}, \quad \gamma = \tanh(2\beta J). \quad (7)$$

Using this relaxation trajectory, the effect of stochastic resetting on the probability distribution of the magnetization can be calculated using Eqs. (4) (for $\alpha < 1$) and (6) (for $\alpha > 1$). For $\alpha < 1$, we have

$$P_r(m, t) = \frac{\sin(\pi\alpha)}{\pi} \frac{1}{m} \left[\ln\left(\frac{m_0}{m}\right) \right]^{-\alpha} \left[t(1-\gamma) - \ln\left(\frac{m_0}{m}\right) \right]^{\alpha-1}. \quad (8)$$

We can see that the system does not reach a stationary state. The distribution at any finite time exhibits a characteristic double-peaked structure with divergences at both $m = 0$ and $m = m_0$. As $m \rightarrow 0$, the divergence of the power law dominates over the decaying logarithm, and hence we get a peak at $m = 0$. At $m = m_0$, we again have a divergent behavior via the term $(\ln \frac{m_0}{m})^{-\alpha}$.

When $\alpha > 1$, using Eqs. (7) and (6), we get

$$P_r(m, t) = \frac{1}{[\tau_0(1-\gamma)]^{1-\alpha}} \left(\frac{\alpha-1}{\alpha} \right) \frac{1}{m} \left[\ln\left(\frac{m_0}{m}\right) \right]^{-\alpha} + \left[1 - \frac{1}{\alpha} \left(1 - \left(\frac{\tau_0}{t} \right)^{\alpha-1} \right) \right] \delta(m - m_{\tau_0}). \quad (9)$$

At large times, the system goes to a stationary state with the distribution

$$P_r^{\text{stat}}(m) = \left(\frac{\alpha-1}{\alpha} \right) \left(\frac{1}{[\tau_0(1-\gamma)]^{1-\alpha}} \frac{1}{m} \left[\ln\left(\frac{m_0}{m}\right) \right]^{-\alpha} + \delta(m - m_{\tau_0}) \right). \quad (10)$$

Here again we see a divergence at $m = 0$ and $m = m_0$. For both the cases, $\alpha < 1$ and $\alpha > 1$, we get double-peaked structures that diverge at $m = 0$ and $m = m_0$. Physically, this reflects the competition between two processes: The short bursts of resetting favour the initial state m_0 , while the long tail of the power law supports the relaxation toward zero magnetization.

We can calculate the average of this magnetization distribution as $\langle m \rangle = \int m P_r(m) dm$. For $\alpha < 1$, at a fixed time t ,

$$\langle m \rangle = m_0 {}_1F_1(1-\alpha; 1, -(1-\gamma)t), \quad (11)$$

where ${}_1F_1(1-\alpha; 1, -(1-\gamma)t)$ is Kummer's (confluent hypergeometric) function of the first kind. For $\alpha > 1$,

$$\langle m \rangle = \left(\frac{\alpha-1}{\alpha} \right) ([\tau_0(1-\gamma)]^{\alpha-1} m_0 \Gamma(1-\alpha, \varepsilon) + m_{\tau_0}), \quad (12)$$

where $\Gamma(1-\alpha, \varepsilon)$ is upper incomplete gamma function and $\varepsilon = (1-\gamma)\tau_0$. So, we see that there exists an average nonzero magnetization for any $m_0 \neq 0$. We therefore call this phase a “quasiferro” state, where we have a divergence at $m = 0$ as expected for a paramagnetic state, but the other divergence at $m = m_0$ leads to finite magnetization in the system for all $T > 0$.

The analytical predictions are in a very good agreement with the numerical simulations, as shown in Fig. 2. Figure 2(a) corresponds to $\alpha < 1$, where the system does not reach a steady state and the distribution at a finite time exhibits the expected double-peaked profile. Figure 2(b) corresponds to $\alpha > 1$, where the system reaches a stationary distribution featuring similar divergences at $m = 0$ and near $m = m_0$ as in the $\alpha < 1$ case. The numerical curve again closely follows the theoretical forms. A broadening of the simulation peaks (especially near $m = 0$) arises from finite size effects and the statistical fluctuations inherent in sampling extremely low-probability events. For $\alpha > 1$, a larger number of realizations is required to capture rare magnetization values, which is why in simulations we have used a high number of iterations to obtain a smooth stationary curve. Overall, the simulations validate the analytical structure of the distributions across both regimes.

IV. RESETTING IN THE 2D ISING MODEL

The Ising model exhibits a phase transition in two dimensions as a function of temperature, characterized by a ferromagnetic state for temperatures below the critical temperature T_C and a paramagnetic phase for $T > T_C$. The introduction of exponential resetting [50] with a resetting rate r generates a pseudoferro phase, in addition to the ferro and para phases, where the behavior is similar to the original system without reset. Here, we explore how these behaviors change when the inter-reset distribution follows a power law.

In the 2D Ising model, Glauber dynamics is not exactly solvable as in 1D. For performing the calculations, we use the well-established late-time approximations [64], as used in Ref. [50]. In the absence of resetting, the time dependence of the magnetization is known to follow: (1) for $T > T_C$, at large times $m(t) \approx a_1 e^{-\lambda t}$ and $a_1 \approx m_0$, (2) for $T < T_C$, $m(t) \approx m_{\text{eq}} \pm a e^{-bt^c}$, where a , b , and c are constants that can be numerically determined, and (3) for $T = T_C$, at large times $m(t) \approx b_c t^{-\phi}$, where $\phi = \beta/\nu z$. Here, ν is the Ising critical exponent related to the correlation length, β is the correlation length associated with the order parameter, and z is the dynamical critical exponent associated with the Ising-Glauber dynamics at $T = T_C$. This approximation does not hold for smaller values of time. Therefore, for early times, guided by numerics, we approximate the magnetization as $m(t) \approx m_0 + At^\zeta$. The parameters A and ζ can be fixed by matching to the known long-time behavior.

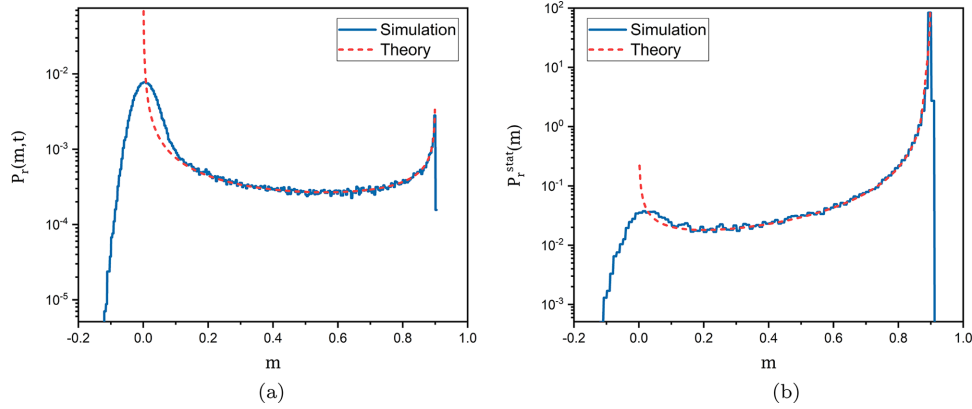


FIG. 2. Plot of the PDF of the magnetization in the 1D Ising model with power law resetting for different values of α : (a) Finite-time distribution at $t = 12$ for $\alpha = 0.5$ ($\alpha < 1$), lattice size $L = 2000$, $m_0 = 0.9$ and $\gamma = 0.6$, averaged over 10^5 realizations. (b) Steady-state distribution for $\alpha = 1.5$ ($\alpha > 1$), obtained with $L = 1000$, $m_0 = 0.9$, and $\gamma = 0.6$, averaged over 10^6 realizations. The red dashed lines correspond to the analytical predictions in Eqs. (8) and (10), and the blue curves correspond to the numerical simulations.

(1) We first consider the case $T > T_C$, where the steady state of the system without resetting is the paramagnetic state $m = 0$. Starting from the initial state with magnetization m_0 , the system is expected to decay toward equilibrium exponentially, and at large times $m(t) \approx a_1 e^{-\lambda t}$. We assume that this form is true at all times and take $a_1 = m_0$. The calculations here are along the lines of those for one dimension. For $\alpha < 1$, the probability density at time t is

$$P_r(m, t) = \frac{\sin(\pi\alpha)}{\pi} \frac{1}{m} \left[\ln\left(\frac{m_0}{m}\right) \right]^{-\alpha} \left[t\lambda - \ln\left(\frac{m_0}{m}\right) \right]^{\alpha-1}. \quad (13)$$

As in the 1D model, no steady state exists in this regime. The distribution diverges both at $m = 0$ and initial magnetization m_0 , reflecting the dominance of very short and very long intervals between resets. When $\alpha > 1$, we have

$$P_r^{\text{stat}}(m) = \left(\frac{\alpha-1}{\alpha} \right) \left(\frac{1}{[\tau_0\lambda]^{1-\alpha}} \frac{1}{m} \left[\ln\left(\frac{m_0}{m}\right) \right]^{-\alpha} + \delta(m - m_{\tau_0}) \right). \quad (14)$$

The above expressions are similar to those seen in the one-dimensional case discussed above, and we refer to this phase with coexistence of near-zero and near-initial magnetization as a QF state.

To generate the theoretical curves shown in Fig. 3, we first simulated the 2D Ising model without resetting for $T > T_C$ at $T = 3.5$ on a 256×256 lattice starting from the initial magnetizations $m_0 = 0.5$ and $m_0 = 0.99$. The relaxation of the magnetization was fitted to the exponential form $m(t) = a_1 e^{-\lambda t}$, yielding $\lambda \approx 0.113$, and we took $a_1 = m_0$. These fitted parameters were then used in the analytical expressions for the power law resetting dynamics. Figure 3 compares the theoretical predictions Eqs. (13) and (14) with numerical simulations at $T = 3.5$. For $\alpha < 1$, the finite-time distribution at $t = 40$ shows the expected divergences at $m = 0$ and $m = m_0$. For $\alpha > 1$, the system reaches a stationary distribution exhibiting the same divergences, corresponding to the QF state discussed earlier. In both cases, the agreement between theory and simulations is excellent. As before, we see the broadening of the $m = 0$ peak.

(2) For $T < T_C$, if there is no resetting, the equilibrium state of the system is a ferromagnetic state with a finite

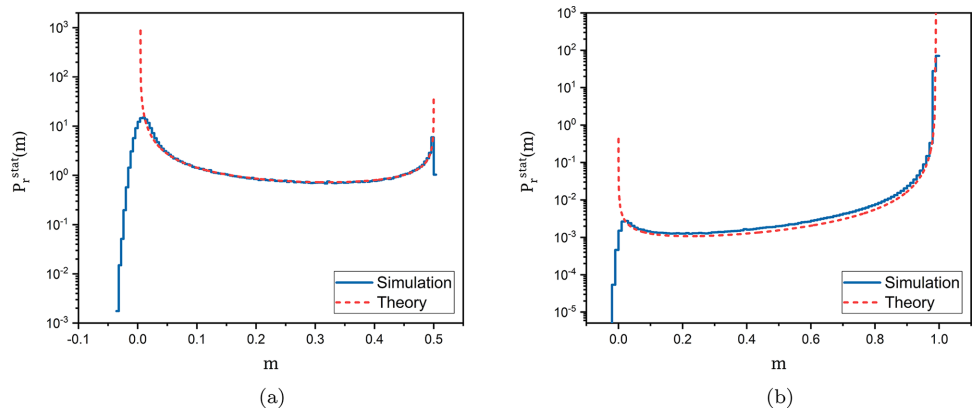


FIG. 3. Plot of the PDF of the average magnetization in the 2D Ising model of lattice size 256×256 with power law resetting at a temperature $T > T_C$ ($T = 3.5$) (a) Finite-time distribution at $t = 40$ for $\alpha = 0.5$ ($\alpha < 1$) and $m_0 = 0.5$, averaged over 10^6 independent realizations. (b) Stationary distribution for $\alpha = 1.5$ ($\alpha > 1$) and $m_0 = 0.99$, averaged over 2×10^8 realizations.

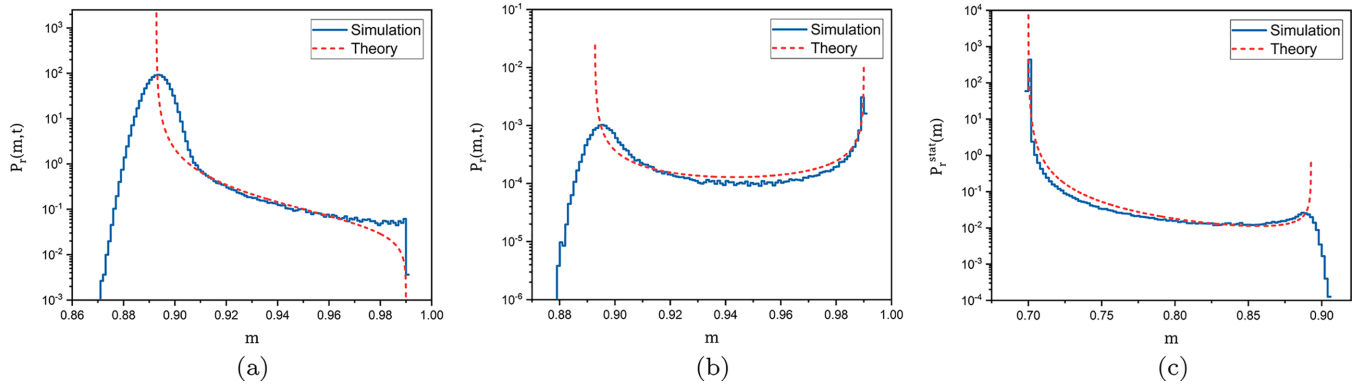


FIG. 4. Probability density function of the average magnetization in the two-dimensional Ising model with power law stochastic resetting for temperatures below the critical point ($T < T_c$). System size $L = 256 \times 256$. (a) Distribution at $t = 120$ for $\alpha = 0.1 < \alpha^*$, $T = 2.05$, and $m_0 = 0.99$, averaged over 3×10^6 realizations, with equilibrium magnetization $m_{eq} = 0.8927$. (b) Distribution at $t = 120$ for $\alpha = 0.8 > \alpha^*$, $m_0 = 0.99$, and $m_{eq} = 0.8927$, averaged over 10^5 realizations. (c) Steady-state distribution for $\alpha = 1.5$, obtained with $m_0 = 0.7$ and averaged over 2×10^7 realizations.

magnetization m_{eq} . A system initially having a different magnetization from m_{eq} will approach this equilibrium value at long times. The approach takes a stretched exponential form at large times $m(t) \approx m_{eq} \pm ae^{-bt^c}$. The values of a , b , and c ($0 < c < 1$) are not known analytically but can be determined

from simulations. In the calculation below, we will consider this stretched exponential form to be valid at all times [and therefore $a = (m_0 \pm m_{eq})$, so that $m(t) = m_0$ at $t = 0$]. When resetting is introduced, for $\alpha < 1$, we get

$$P_r(m, t) = \frac{\sin(\pi\alpha)}{\pi bc} \frac{1}{\pm(m - m_{eq})} \left[\frac{1}{b} \ln \left(\frac{m_0 - m_{eq}}{m - m_{eq}} \right) \right]^{\frac{1-c-\alpha}{c}} \left[t - \left(\frac{1}{b} \ln \left(\frac{m_0 - m_{eq}}{m - m_{eq}} \right) \right)^{\frac{1}{c}} \right]^{\alpha-1}. \quad (15)$$

There is a divergence at $m = m_{eq}$ as evident from the $\frac{1}{m - m_{eq}}$ term (see Fig. 4). Another possibility for a peak arises at $m = m_0$. This will happen when $1 - \alpha - c < 0$. Thus, we see a change in the behavior in our system as the value of α changes. The behavior changes from a single peak at $m = m_{eq}$ as in Fig. 4(a) to a double-peaked structure as seen in Fig. 4(b) as α crosses the value $\alpha^* = 1 - c$. When $\alpha > 1$, in the limit of large time, we get the steady-state expression

$$P_r^{\text{stat}}(m) = \left(\frac{\alpha - 1}{\alpha} \right) \left(\frac{1}{bc\tau_0^{1-\alpha}} \frac{1}{\pm(m - m_{eq})} \left[\frac{1}{b} \ln \left(\frac{m_0 - m_{eq}}{m - m_{eq}} \right) \right]^{\frac{1-c-\alpha}{c}} + \delta(m - m_{\tau_0}) \right), \quad (16)$$

where we have used $a = (m_0 \pm m_{eq})$. One sees that there is a divergence here at $m = m_{eq}$ and $m = m_0$ as can be seen in Fig. 4(c). We thus have a distribution with a double peak again.

The above results show that the system undergoes a change in behavior at $\alpha = \alpha^*$ ($\alpha^* = 1 - c$). When $\alpha < \alpha^*$, the system exhibits a single peak at m_{eq} and we refer to this state as a “single-peaked ferro state”, whereas if $\alpha > \alpha^*$, the system shows two peaks at m_0 and m_{eq} and the system is thus labeled as the “double-peaked ferro state”. The DPF can be further split into two parts: For $\alpha < 1$, there is no steady state, but for $\alpha > 1$, we do have a steady state. The feature of dual peaks in the distribution of magnetization persists for all values of α .

Figure 4 illustrates the behavior of the PDF of the average magnetization in the two-dimensional Ising model with power law stochastic resetting for $T < T_c$. We first simulated the system without resetting at $T = 2.05$ on a 256×256 lattice, for which the equilibrium magnetization is $m_{eq} = 0.8927$. Using an initial magnetization $m_0 = 0.99 > m_{eq}$, we fitted the relaxation curve to the stretched-exponential form $m(t) = m_{eq} + ae^{-bt^c}$ and obtained $b = 0.432$ and $c = 0.591$, giving

the threshold value $\alpha^* = 1 - c = 0.409$. We used these values for the calculation of the PDF for the $\alpha < 1$ case [Eq. (15)]. A second fit was performed using a smaller initial magnetization $m_0 = 0.7 < m_{eq}$ (here, $m(t) = m_{eq} - ae^{-bt^c}$), yielding $b = 0.414$ and $c = 0.718$; these parameters were used in the analysis of the $\alpha > 1$ regime [Eq. (16)]. Figure 4(a) shows the distribution at time $t = 120$ for $\alpha = 0.1 < \alpha^*$, obtained using $m_0 = 0.99$ and averaged over 3×10^6 realizations; the distribution exhibits a single pronounced peak at m_{eq} . Figure 4(b) displays the distribution at the same time for $\alpha = 0.8 > \alpha^*$, again with $m_0 = 0.99$, averaged over 10^5 realizations; here, a second peak emerges near m_0 , signaling the onset of the double-peaked ferro regime. Finally, Fig. 4(c) presents the steady-state distribution for $\alpha = 1.5$, obtained using $m_0 = 0.7$ and averaged over 2×10^7 realizations, where both m_0 and m_{eq} appear as stable long-time magnetization values. The simulations validate the theoretical predictions, showing good correspondence throughout.

(3) The system without reset undergoes a continuous phase transition at $T = T_c$ and a power law dependence is seen for

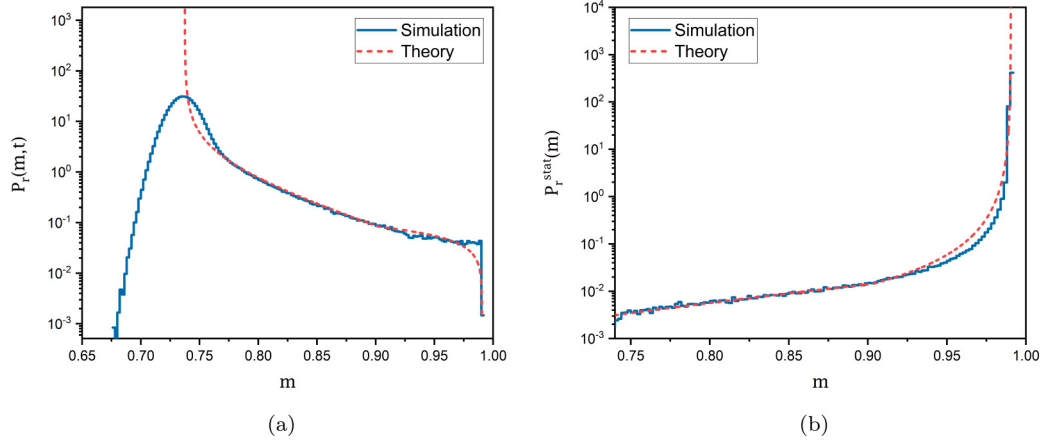


FIG. 5. Probability density function of the average magnetization in the two-dimensional Ising model with power law stochastic resetting at the critical temperature $T = T_c = 2.269$. System size $L = 256 \times 256$. (a) Distribution at $t = 100$ for $\alpha = 0.1$, $m_0 = 0.99$, averaged over 2.4×10^6 realizations. (b) Time-independent distribution for $\alpha = 1.5$, obtained with $m_0 = 0.99$ and averaged over 2×10^7 realizations.

various quantities, with associated critical exponents. We are interested in the behavior of magnetization as a function of time, which in the limit of late times, shows the behavior $m(t) \approx b_c t^{-\phi}$, with $\phi = \beta/\nu z$. For smaller times, we use the approximation, $m(t) \approx m_0 + At^\zeta$. Once we have the values of b_c and ϕ , the parameters A and ζ can be calculated by

equating $m(t)$ and $P_r(m, t)$ at a small time t' (here, t' is chosen as the earliest time at which the large-time approximation of $m(t)$ begins to accurately describe the numerical data). We use these approximate expressions in our calculations below. With resetting and $\alpha < 1$, we have

$$P_r(m, t) = \begin{cases} \frac{\sin(\pi\alpha)}{-\pi\zeta A} \left(\frac{\pm(m-m_0)}{A} \right)^{\frac{1-\zeta-\alpha}{\zeta}} \left[t - \left(\frac{\pm(m-m_0)}{A} \right)^{\frac{1}{\zeta}} \right]^{\alpha-1}, & \text{if } m_0 \geq m \geq m(t'), \\ \frac{\sin(\pi\alpha)}{\pi\phi b_c} \left(\frac{m}{b_c} \right)^{\frac{\alpha-1-\phi}{\phi}} \left[t - \left(\frac{m}{b_c} \right)^{\frac{1}{\phi}} \right]^{\alpha-1}, & \text{if } m < m(t'). \end{cases} \quad (17)$$

For magnetization values close to the initial condition m_0 , the distribution is governed by the early-time behavior $m(t) \approx m_0 + At^\zeta$, leading to a power law dependence with exponent $(1 - \zeta - \alpha)/\zeta$. For smaller magnetization values, the system enters the late-time critical regime $m(t) \approx b_c t^{-\phi}$, which produces a different power law form with exponent $(\alpha - 1 - \phi)/\phi$. In both regions, one sees that the distribution exhibits a power law decay at large times, with the exponents determined jointly by the Ising dynamics (ϕ, ζ) and the resetting dynamics (α). For $\alpha > 1$, we have the following expression in the limit of steady state:

$$[P_r^{\text{stat}}(m)] = \begin{cases} \left(\frac{\alpha-1}{\alpha} \right) \left[\frac{-1}{A\zeta\tau_0^{1-\alpha}} \left(\frac{\pm(m-m_0)}{A} \right)^{\frac{1-\zeta-\alpha}{\zeta}} + \delta(m - m_{\tau_0}) \right], & \text{if } m_0 \geq m \geq m(t'), \\ \left(\frac{\alpha-1}{\alpha} \right) \left[\frac{1}{b_c\phi\tau_0^{1-\alpha}} \left(\frac{m}{b_c} \right)^{\frac{\alpha-1-\phi}{\phi}} + \delta(m - m_{\tau_0}) \right], & \text{if } m < m(t'). \end{cases} \quad (18)$$

We see here an interesting crossover from a power law decay when $\alpha - \phi - 1 < 0$, to a power law increase when $\alpha - \phi - 1 > 0$, with a flat distribution at $\alpha = \phi + 1$.

To obtain the input parameters required for the analytical predictions, we first simulated the 2D Ising model without resetting at $T = T_c$ ($T_c = 2.269$ with $J = k_B = 1$) on a 256×256 lattice, starting from $m_0 = 0.99$. The long-time relaxation of the magnetization follows the critical decay form $m(t) \approx b_c t^{-\phi}$, yielding $b_c = 0.9624$ and $\phi = 0.0578$. This approximation becomes accurate only beyond $t' = 3$; for short times, the data fits well to the $m(t) \approx m_0 + At^\zeta$. Matching the two regimes at $t = t'$ gave $A = -0.0452$ and $\zeta = 0.5981$, which were subsequently used to construct the analytical PDFs. Figure 5(a) shows the magnetization distribution at time $t = 100$ for a heavy-tailed resetting distribution with exponent $\alpha = 0.1$, averaged over 2.4×10^6 realizations. Figure 5(b) presents the time-independent steady-state distribution obtained for $\alpha = 1.5$, using the same initial condition $m_0 = 0.99$ and

averaging over 2×10^7 realizations. These figures illustrate the close agreement between simulations and the analytically constructed PDFs.

V. CONCLUSION

In summary, we have shown that introducing power law stochastic resetting to the Ising-Glauber dynamics leads to qualitatively different nonequilibrium behavior that cannot arise either without resetting or under exponential-reset protocols. By analyzing the relaxation forms of the magnetization and incorporating them into the renewal framework, we obtained explicit expressions for the magnetization distribution in both one and two dimensions. These theoretical predictions were validated through simulations, and together they reveal a rich structure of phases characterized by nontrivial divergences and the coexistence of peaks at the reset value and the equilibrium magnetization.

In the 2D Ising model, for $T > T_C$, the system exhibits a state with a persistent double-peaked magnetization distribution for all values of α . While the distribution is time dependent for $\alpha < 1$ and stationary for $\alpha > 1$, this change does not affect the double-peaked structure. We therefore classify this entire regime as a single phase, the QF state. This state is very different from the usual paramagnetic state expected in the absence of resetting, and also from the para and pseudoferro states seen in the presence of exponentially distributed inter-reset times. For $T < T_C$, varying α leads to a crossover at $\alpha = \alpha^*$ from an SPF state with a magnetization peak at m_{eq} , to a DPF state with two peaks in the magnetization distribution at $m = m_{eq}$ and $m = m_0$. Notice again that this behavior is very different from the simple ferromagnetic behavior seen in the Ising model without reset and also from the ferro state seen in the previous work with exponential resetting, where there is no such crossover. While the existence of a steady state emerges only for $\alpha > 1$, the structure of the magnetization distribution is governed by α^* , rather than by steady-state considerations alone, and therefore we continue with the DPF label for all $\alpha > \alpha^*$. At the critical temperature $T = T_C$, the magnetization distribution displays distinct power law regimes arising from early-time and late-time critical relaxation. For $\alpha < 1$, the distribution remains time-dependent, while for $\alpha > 1$ it reaches a stationary form. In the stationary regime, the exponent $\alpha - \phi - 1$ controls a crossover from a decaying to a growing power law magnetization distribution, with a flat distribution at $\alpha = \phi + 1$. For

the 1D Ising model, where the system remains paramagnetic at all finite temperatures, power law resetting similarly generates a QF state, with a steady-state distribution emerging for $\alpha > 1$.

Thus, we see that the behavior reported here is remarkably rich. We expect that the general features seen here will be relevant beyond the Ising model. Our results highlight the sensitivity of interacting many-body systems to the statistics of resetting events and suggest several natural extensions. These include investigating other spin systems or higher dimensions, and analyzing spatial correlations and domain growth under power law resets. More generally, our findings suggest that non-Poissonian resetting can be used as a general tool in statistical physics to engineer and control nonequilibrium states.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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