

Correlations of the current density in many-body Landau level states

Daniel Spasic-Mlacak¹ and Nigel R. Cooper¹*TCM Group, Cavendish Laboratory, University of Cambridge, J.J. Thompson Avenue, Cambridge CB3 0US, United Kingdom*

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Motivated by recent advances in quantum gas microscopy, we investigate correlation functions of the current density in many-body Landau level states, such as the Laughlin state of the fractional quantum Hall effect. For states fully in the lowest Landau level, or in a vortexable Aharonov-Casher band, we present an exact relationship which shows that all correlation functions involving the current density are directly related to correlation functions of the number density. We calculate perturbative corrections to this relationship arising from Landau level mixing due to interparticle interactions and show that this provides a method by which to extract the system's interaction energy. Finally, we comment on the applicability of our results to lattice systems.

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Introduction. Recently, significant progress has been made towards the realization of fractional quantum Hall (FQH) and other many-body Landau level (LL) states in quantum simulators. Many ways to generate effective magnetic fields have been proposed for cold-atom and photonic systems [1,2] and have been successfully realized experimentally [3–7], including for strongly interacting bosons in flat Chern bands [8,9]. Recent experiments have also studied the evolution of Bose-Einstein condensates near the lowest Landau level (LLL) of rotating gases [10], and of strongly correlated few-fermion systems [11]. These quantum simulators offer the potential to study the FQH effect in new settings, such as investigating bosons rather than fermions and different types of interactions between particles, as well as to probe and detect properties of the FQH states in novel ways [12].

One of the most powerful methods of extracting information from quantum simulations is through measurements of multipoint correlation functions [8,11,13], as has been applied to other systems such as atomic superfluids [14] and the Fermi-Hubbard model [15,16]. Motivated by recent advances in quantum gas microscopy which allow for the measurement of local *current* operators in optical lattices [17], we investigate current density correlation functions of FQH states (and, more broadly, any generic many-body LL states) to gain insight into how the properties of such states can be probed by measurements of these correlations.

In this paper, we report on an exact relationship that holds in the LLL between correlation functions of the current density and correlation functions of the number density. We show that deviations from this relationship arise from interparticle interactions and hence that measurements of the current density and number density correlators can be used as a way to determine the interaction energy of the many-body system experimentally. Finally, we discuss how our findings can be

related to lattice settings and comment on discretization deviations from the LLL relationship for single-particle states in the Harper-Hofstadter model.

We consider particles moving in two dimensions and subjected to gauge fields, such that they are well described by particles of charge e and mass M acted on by a perpendicular magnetic field B . The single-particle energy levels are the discrete LLs with energy difference $\hbar\omega_c$ between each level, where $\omega_c = eB/M$ is the cyclotron frequency. For the symmetric gauge, with the vector potential $A_\alpha(\mathbf{r}) = \frac{B}{2}\epsilon_{\alpha\beta}r_\beta$, where α, β are coordinate indices and $\epsilon_{\alpha\beta}$ is the two-dimensional Levi-Civita symbol, the single particle states have definite angular momentum $m = 0, 1, 2, \dots$. The position representations of the single particle wave functions in the LLL are

$$\langle \mathbf{r} | m \rangle = \frac{1}{\sqrt{2\pi l_B^2 m!}} \left(\frac{r}{\sqrt{2}l_B} \right)^m \exp\left(-\frac{r^2}{4l_B^2}\right) e^{im\theta}, \quad (1)$$

where $l_B = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length and $\mathbf{r} = (r, \theta)$ is the position in polar coordinates.

We use these single-particle states to form a generic first-quantized N -body state in the LLL as

$$|\psi_{\text{LLL}}\rangle = \sum_{\{m_i\}} c_{m_1 m_2 \dots m_N} |m_1\rangle |m_2\rangle \dots |m_N\rangle, \quad (2)$$

where $|m_i\rangle$ denotes the single particle state for particle i and the notation $\sum_{\{m_i\}}$ indicates N sums over $m_1, \dots, m_N$. The complex coefficients $c_{m_1 m_2 \dots m_N}$ are symmetric (antisymmetric) for bosons (fermions) under particle exchange. We are interested in calculating correlation functions of number density, $\hat{\rho}(\mathbf{r})$, and current density, $\hat{\mathbf{j}}(\mathbf{r})$, defined as

$$\hat{\rho}(\mathbf{r}) = \sum_{i=1}^N \hat{\rho}_i(\mathbf{r}) = \sum_{i=1}^N \delta(\mathbf{r} - \hat{\mathbf{r}}_i), \quad (3)$$

$$\hat{\mathbf{j}}(\mathbf{r}) = \sum_{i=1}^N \hat{\mathbf{j}}_i(\mathbf{r}) = \sum_{i=1}^N \frac{1}{2} \left\{ \frac{\hat{\boldsymbol{\pi}}_i}{M}, \hat{\rho}_i(\mathbf{r}) \right\}, \quad (4)$$

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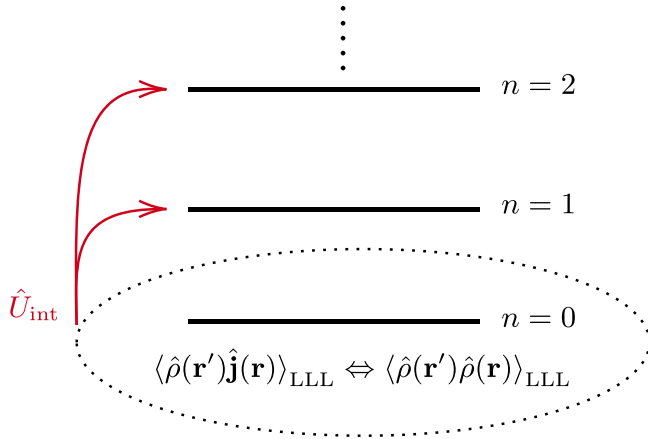


FIG. 1. In the LLL, there is a direct correspondence between current density and number density correlators. When contact interactions $\hat{U}_{\text{int}}$ are turned on, the LLL states are perturbatively mixed with higher LL states, causing a deviation to the correspondence between correlators.

where $\hat{\mathbf{p}}_i$ and $\hat{\mathbf{r}}_i$ are the (canonical) momentum and position of particle i , respectively, from which $\hat{\boldsymbol{\pi}}_i \equiv \hat{\mathbf{p}}_i - e\mathbf{A}(\hat{\mathbf{r}}_i)$ is its kinetic momentum and $\{\hat{C}, \hat{D}\} \equiv \hat{C}\hat{D} + \hat{D}\hat{C}$. The current density operator is defined such that, together with the density operator, it obeys the continuity equation for the system described above [18].

The key observation which underlies our general result below is that, for states defined in (1), the matrix elements of the current density and number density obey the following relation:

$$\langle m_i | \hat{j}_{\alpha,i}(\mathbf{r}) | m'_i \rangle = \frac{\hbar}{2M} \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle m_i | \hat{\rho}_i(\mathbf{r}) | m'_i \rangle. \quad (5)$$

When $m_i = m'_i$, Eq. (5) can be physically interpreted as the magnetization current [18] due to the cyclotron motion, arising from an average magnetization density in the z direction of $M_z = -\frac{\partial E}{\partial B} \langle \hat{\rho} \rangle$ with the LLL energy $E = \hbar\omega_c/2$. By acting on the one-body states with the current density and number density operators (3) and using the observation (5), the following general result for the many-body LLL state can be derived (see Fig. 1):

$$\langle \hat{\rho}(\mathbf{r}') \hat{j}_\alpha(\mathbf{r}) \rangle_{\text{LLL}} = \frac{\hbar}{2M} \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle \hat{\rho}(\mathbf{r}') \hat{\rho}(\mathbf{r}) \rangle_{\text{LLL}}. \quad (6)$$

This result can also be extended to an arbitrary number of current density and number density operator insertions. Since the only requirement for Eq. (6) to hold is that the N -body wave function is fully contained in the LLL, it can be applied to various states describing both the integer and fractional quantum Hall effects. In Fig. 2, we show numerically obtained correlators for the bosonic Laughlin state at half filling [19], which are shown to agree with Eq. (6). Moreover, Eqs. (5) and (6) hold more generally for an Aharonov-Casher band [20], where the single-particle states are given by

$$\langle \mathbf{r} | \tilde{m} \rangle \sim (x + iy)^m e^{-\phi(\mathbf{r})}, \quad (7)$$

where $\nabla^2 \phi(\mathbf{r}) = B(\mathbf{r})$ in the gauge $A_x(\mathbf{r}) = -\partial_y \phi(\mathbf{r})$, $A_y(\mathbf{r}) = \partial_x \phi(\mathbf{r})$ for a nonuniform magnetic field $B(\mathbf{r})$ [21,22]

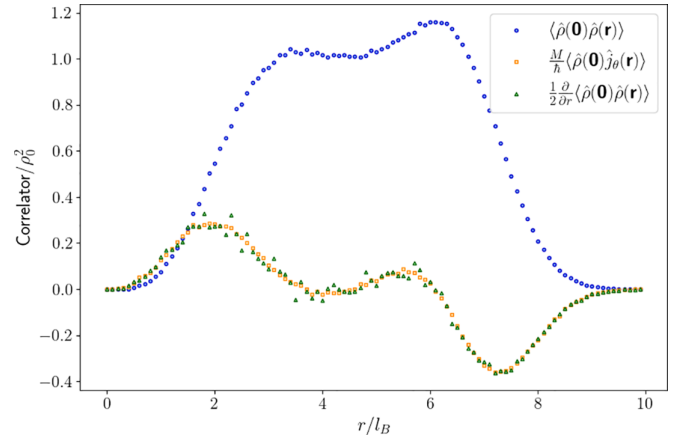


FIG. 2. Plot of current density-number density and number density-number density correlators obtained numerically using the Metropolis Monte Carlo method for the filling fraction $\nu = \frac{1}{2}$ Laughlin state in a disk geometry with particle number $N = 15$, after 500 000 iterations and discarding the first 80 000 “thermalization” runs. The radial derivative of $\langle \hat{\rho}(\mathbf{0}) \hat{\rho}(\mathbf{r}) \rangle$ can be seen to coincide with the angular component of the current density-number density correlator, $\langle \hat{\rho}(\mathbf{0}) \hat{j}_\theta(\mathbf{r}) \rangle$, demonstrating the correspondence in Eq. (6).

(see Supplemental Material for derivations of the statements above [23]).

This relationship shows that one cannot extract further information with current density correlators than with number density correlators for LLL states. For single-particle states in higher LLs, Eq. (5) is no longer valid due to the presence of Laguerre polynomials, which introduce non-holomorphic factors to the wave functions. However, a result of similar form to (5) may be derived for the n th LL by replacing the single-particle number density and current density operators with $\hat{\rho}_i^{(n)}(\mathbf{r}) = \frac{1}{n!} (\hat{a}_i^\dagger)^n \hat{\rho}_i(\mathbf{r}) (\hat{a}_i)^n$, $\hat{j}_{\alpha,i}^{(n)}(\mathbf{r}) = \frac{1}{n!} (\hat{a}_i^\dagger)^n \hat{j}_{\alpha,i}(\mathbf{r}) (\hat{a}_i)^n$, where $\hat{a}_i$ ($\hat{a}_i^\dagger$) is the annihilation (creation) operator which lowers (raises) the LL of the i th single-particle state, given by $\hat{a}_i = (\hat{\pi}_{i,x} + i\hat{\pi}_{i,y})l_B/(\sqrt{2}\hbar)$. Inserting these operators for the many-body correlator reproduces (6), so long as the many-body state is fully contained within a single LL and there is no mixing of LLs present. Note that this is no longer a simple correspondence between number density and current density correlators, due to the nontrivial form of the operators $\hat{a}_i$, highlighting the special property of the LLL within this context.

Effects of Landau level mixing. When mixing of LLs is present due to interactions, we expect a deviation from the LLL result (6) (see Fig. 1). In fact, we will show that this deviation can be used to extract the interaction energy of the system. To explore this connection, we consider a system of bosons with contact interactions:

$$\hat{H} = \frac{1}{2M} \sum_{i=1}^N [\hat{\mathbf{p}}_i - e\mathbf{A}(\hat{\mathbf{r}}_i)]^2 + g \sum_{i < j=1}^N \delta(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j). \quad (8)$$

We assume that the contact interaction is weak, $g \ll \hbar\omega_c l_B^2$, so that we may consider the mixing with higher LLs perturbatively. We impose a length-scale cutoff $\lambda_{\text{X}} \ll l_B$ for the contact potential, which later plays a role in avoiding diver-

gences in the perturbation theory. As Eq. (6) only holds in the LLL, we expect deviations due to perturbation theory in higher LLs of the contact interaction. We define the deviation from the LLL result as

$$\Delta_\alpha(\mathbf{r}) \equiv \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle \hat{\rho}(\mathbf{0}) \hat{\rho}(\mathbf{r}) \rangle - \frac{2M}{\hbar} \langle \hat{\rho}(\mathbf{0}) \hat{j}_\alpha(\mathbf{r}) \rangle. \quad (9)$$

We first consider a system of two particles, where analytical expressions for LLL wave functions are known. Namely, according to Kohn's theorem [24], the two-body Hamiltonian can be separated into relative coordinate $\hat{\mathbf{r}} = \hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2$ and center-of-mass coordinate $\hat{\mathbf{R}} = \frac{1}{2}(\hat{\mathbf{r}}_1 + \hat{\mathbf{r}}_2)$ terms to obtain an effective one-body Hamiltonian, with interaction potential as a function of the relative coordinate only. Perturbation theory can then be performed on LLL wave functions of the form given in (1), with effective magnetic length $\tilde{l}_B = \sqrt{2}l_B$, i.e., an effective charge $\tilde{e} = e/2$, due to the introduction of the relative coordinate [importantly, this will also be the charge associated with the vector potential in Eq. (3)].

Note that the center-of-mass part of the wave function is not affected by the contact potential and is not relevant for correlators in a translationally invariant system. Formally, we consider a large system, with points $\mathbf{r}$ and $\mathbf{r}'$ deep inside the bulk where the system is locally translationally invariant. Then the properties can be derived by considering a uniform system, with periodic boundary conditions for which translational invariance is exact. This allows us to integrate out the center-of-mass contribution in the correlators for a translationally invariant system by using center-of-mass translation operators $\hat{t}(\mathbf{x}) = \prod_i \exp(-i\mathbf{x} \cdot \hat{\mathbf{K}}_i)$, where $\hat{\mathbf{K}}_i = \hat{\pi}_i - \hbar \mathbf{e}_z \times \hat{\mathbf{r}}_i / l_B^2$ is the “pseudomomentum” chosen such that the translation operators commute with the dynamical momentum $\hat{\pi}_i$ [25], and introducing the replacement

$$\langle \psi | \dots | \psi \rangle \rightarrow \int \langle \psi | \hat{t}^\dagger(\mathbf{x}) \dots \hat{t}(\mathbf{x}) | \psi \rangle d^2\mathbf{x} \quad (10)$$

for all correlators, where $\mathbf{x}$ parametrizes the translationally invariant system. For example, for the number density correlator, we would have $\langle \delta(\hat{\mathbf{r}}_1) \delta(\mathbf{r} - \hat{\mathbf{r}}_2) \rangle \rightarrow \langle \delta(\mathbf{r} - (\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2)) \rangle$. From now on, we will carry out this replacement implicitly and refer back to it when discussing the perturbations of the N -body system.

We use the perturbed wave functions to find the first-order deviation (9) (see Supplemental Material [23]):

$$\Delta_\theta(\mathbf{r}) = \frac{g}{4\pi^2 \hbar \omega_c \tilde{l}_B^6} r \exp\left(-\frac{r^2}{2\tilde{l}_B^2}\right) \sum_{n=1}^{n_X} \frac{1}{n} L_{n-1}^1\left(\frac{r^2}{2\tilde{l}_B^2}\right). \quad (11)$$

The cutoff in the sum n_X ensures that there are no divergences and corresponds to a minimum range of interaction λ_X such that $n_X \sim (\frac{l_B}{\lambda_X})^2$. Defining the interaction energy as $E_{\text{int}} \equiv \langle \psi_0 | \hat{U}_{\text{int}} | \psi_0 \rangle$, the orthogonality relation of generalized Laguerre polynomials (see Supplemental Material [23]) can be used to obtain the relation

$$E_{\text{int}} = \frac{1}{2} \hbar \omega_c \int |\mathbf{r} \times \Delta(\mathbf{r})| d^2\mathbf{r}. \quad (12)$$

This result shows that the interaction energy of the two-body system can be found by measuring number density and current density correlators that appear in (9). Analogous results also

hold for more general short-range interactions of the form $\nabla^{2j} \delta(\mathbf{r})$ introduced in the context of the FQH effect [26]. Thus (12) extends also to fermionic states with odd relative angular momentum (see Supplemental Material [23]).

In order to verify the validity of (12) for a many-body state of the Hamiltonian (8), we utilize Schmidt decomposition to factorize the N -body wave function into a system with two particles and $N - 2$ particles, respectively,

$$\Psi^{(0)}(\mathbf{r}_i, \mathbf{r}_j, \{\mathbf{r}_k\}) = \sum_{m,M} \psi_m^{(0)}(\mathbf{r}_{ij}) \psi_M^{(0)}(2\mathbf{R}_{ij}) \Phi_{m,M}^{(0)}(\{\mathbf{r}_k\}), \quad (13)$$

where we have used relative and center-of-mass angular momentum quantum numbers m and M to label the states, $\psi_{m(M)}^{(0)}$ correspond to single-particle LLL wave functions in Eq. (1) with angular momentum m (M) and effective magnetic length $\tilde{l}_B$, $\Phi_{m,M}^{(0)}$ is some (unnormalized) wave function of $N - 2$ particles, and $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{R}_{ij} = (\mathbf{r}_i + \mathbf{r}_j)/2$ are the relative and center-of-mass coordinates respectively for the positions of particles i and j . The contact potential then acts pairwise on the many-body wave function and perturbatively raises the LL of the two-body wave functions in relative coordinate, such that

$$\Psi^{(n)}(\mathbf{r}_i, \mathbf{r}_j, \{\mathbf{r}_k\}) = \sum_{m,M} \psi_m^{(n)}(\mathbf{r}_{ij}) \psi_M^{(0)}(2\mathbf{R}_{ij}) \Phi_{m,M}^{(0)}(\{\mathbf{r}_k\}), \quad (14)$$

where $\psi_m^{(n)}$ is the wave function for the n th LL and $\{\mathbf{r}_k\}$ is a set of $N - 2$ coordinates. By applying (10) to all correlators involving wave functions (13) and (14) to eliminate two-body center-of-mass wave functions $\psi_M^{(0)}$, the perturbation problem for these states can be reduced to that of the two-body case (see Supplemental Material for key simplifications [23]). Therefore, we conclude that Eq. (12) can be used to calculate the interaction energy of a system of N interacting particles using current density and number density correlators.

A useful cross-check of (12) in the N -body context is the Laughlin state of bosons, given by

$$\Psi_L(z_1, \dots, z_N) = \prod_{i < j=1}^N (z_i - z_j)^2 \exp\left(-\frac{1}{4l_B^2} \sum_{k=1}^N |z_k|^2\right), \quad (15)$$

where $z_i = x_i + iy_i$ is the position of the i th particle in complex form. This wave function is the ground state of (8) for filling fraction $\nu = \frac{1}{2}$, with a vanishing interaction energy [27]. This is consistent with (12), as the Laughlin state is fully in the LLL, resulting in a vanishing deviation. We note, however, that other states of (8) with differing filling fractions, or states at nonzero temperature, will have a nonvanishing interaction energy which could, in turn, be probed by current density correlators as described.

We have not included a confining potential in the Hamiltonian (8), as box traps are often used in quantum gas microscopy experiments [28], whose potential does not impact the bulk correlators of the system. However, if the system of interest has a different confining potential, it would be natural to measure connected correlators of current density, where the operator $\hat{j}_\alpha(\mathbf{r})$ is replaced by $\hat{j}_\alpha(\mathbf{r}) - \langle \hat{j}_\alpha(\mathbf{r}) \rangle$. This would remove any one-body contributions to the current arising from the potential [23].

Lattice effects. Finally, we discuss the application of the presented continuum results to lattice systems. In the perturbative calculation, we considered two energy scales: the cyclotron energy $\hbar\omega_c$ and the interaction strength $U \equiv g/l_B^2$. The lattice introduces an additional energy scale corresponding to the bandwidth of the Chern band ϵ . This induces another contribution to the current density related to the dispersion of the band. The correlator of current density with number density can then be decomposed into three contributions at the separate energy scales:

$$\langle \hat{\rho}(\mathbf{0}) \hat{j}_a(\mathbf{r}) \rangle = \frac{l_B}{\hbar} [\hbar\omega_c f(\mathbf{r}) + U g(\mathbf{r}) + \epsilon h(\mathbf{r})]. \quad (16)$$

The first contribution will have functional scaling $f(\mathbf{r})$ related to the number density correlator according to (6), whereas $g(\mathbf{r})$ corresponds to the deviation given in (9) and $h(\mathbf{r})$ is some further deviation dependent on the energy dispersion of the band. To access a regime of strong interactions where strongly correlated, fractional Chern insulator states can appear, one requires that this bandwidth is small compared to the interaction strength (which is typically small compared to the cyclotron energy) [29], i.e., $\epsilon \ll U \ll \hbar\omega_c$. Thus, in such a regime, the additional contribution to the current density described by $h(\mathbf{r})$ is small and our continuum results above still provide an accurate description. Indeed, for the special case of Aharonov-Casher bands [21], which can arise in certain lattices [20,30] and for which $\epsilon = 0$, we have shown that (6) holds exactly.

The applicability of our results to *tight-binding* lattice models can be investigated using the Harper-Hofstadter model. In the low magnetic flux limit, the ground states of the model are given exactly by the continuum LLL wave functions, as the length scale l_B becomes larger than the lattice spacing a . The Harper-Hofstadter Hamiltonian [31,32] is given by

$$\hat{H} = -t \sum_{\mathbf{x}} \hat{c}_{\mathbf{x}+a\mathbf{e}_x}^\dagger \hat{c}_{\mathbf{x}} + e^{i2\pi n_\phi x} \hat{c}_{\mathbf{x}+a\mathbf{e}_y}^\dagger \hat{c}_{\mathbf{x}} + \text{H.c.}, \quad (17)$$

where t is the hopping amplitude, $\mathbf{x}$ are the coordinates of sites on the lattice, $\mathbf{e}_x, \mathbf{e}_y$ are the unit vectors in the x and y directions, respectively, and $n_\phi = p/q$ is the magnetic flux per unit cell (where p and q are integers for a system with periodic boundary conditions). We note that we have now switched to the Landau gauge, $\mathbf{A}(\mathbf{r}) = Bx\mathbf{e}_y$. The ground states of the model can be easily obtained by diagonalizing (17) in momentum space to obtain eigenstates $|\mathbf{k}\rangle$, where the crystal momentum is restricted to the magnetic Brillouin zone $k_x = [0, 2\pi/aq], k_y = [0, 2\pi/a]$.

To investigate the validity of continuum results on the lattice at a single-particle level, we take the curl of both sides of Eq. (5) and then discretize the Laplacian acting on the density matrix elements. Noting that, on the lattice, the current density operator is defined on bonds, we approximate the curl of the current density matrix element by a line integral around a plaquette of the square lattice [see Fig. 3(a)] divided by the area of the enclosed region, giving the following discretization:

$$\langle \mathbf{k} | \hat{T}_{\Delta\mathbf{r}} \sum_{\text{edge}} \hat{j}_{\text{edge}} \hat{T}_{\Delta\mathbf{r}} | \mathbf{k} \rangle = \frac{t}{\hbar} \langle \mathbf{k} | \sum_{\text{vert}} \hat{\rho}_{\text{vert}} - 4\hat{\rho}_{\mathbf{x}_c} | \mathbf{k} \rangle + \delta_{\mathbf{k}}, \quad (18)$$

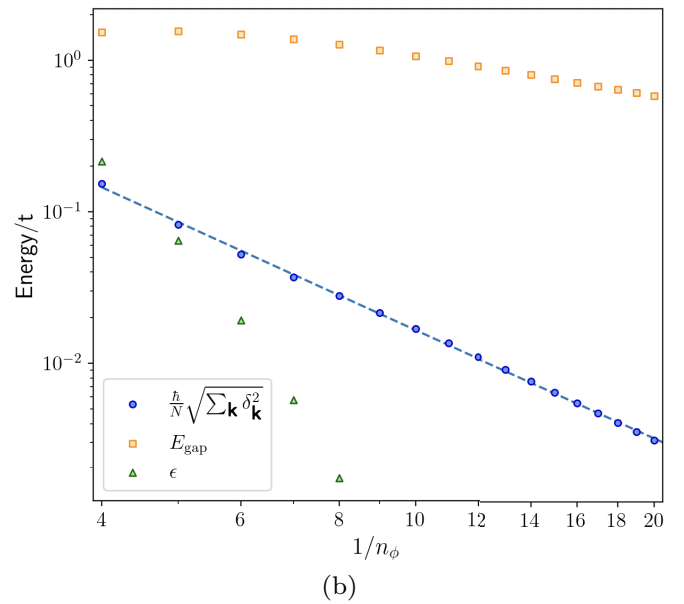
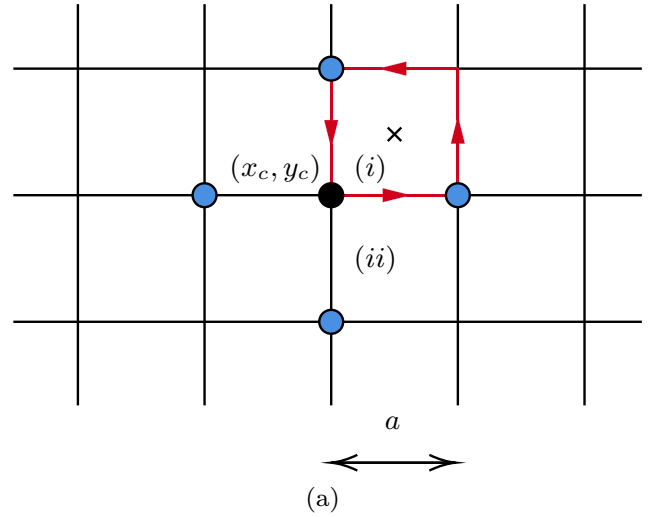


FIG. 3. (a) Schematic diagram of the discrete density and current density operators for the central site at (x_c, y_c) . The blue circles correspond to number density operators defined on vertices, whereas the red arrows correspond to current density operators on the edges. (b) Log-log plot of the root mean square of the lattice deviation $\delta_{\mathbf{k}}$, Eq. (18), compared to the band gap E_{gap} and bandwidth ϵ as a function of n_ϕ . In the limit of small n_ϕ , the root-mean-square $\delta_{\mathbf{k}}$ approaches zero algebraically, with exponent ~ 2.5 , in the continuum limit of the Harper-Hofstadter model.

where $|\mathbf{k}\rangle$ are the Bloch states of (17) and the two sums are over edges and vertices as depicted in Fig. 3(a) and centered at the point $\mathbf{x}_c = (x_c, y_c)$. The operators on the lattice are now given by $\hat{\rho}_{\mathbf{x}} = \hat{c}_{\mathbf{x}}^\dagger \hat{c}_{\mathbf{x}}$, $\hat{j}_{\mathbf{x} \rightarrow \mathbf{x}+a\mathbf{e}_x} = -\frac{it}{\hbar} \hat{c}_{\mathbf{x}}^\dagger \hat{c}_{\mathbf{x}+a\mathbf{e}_x} + \text{H.c.}$, and $\hat{j}_{\mathbf{x} \rightarrow \mathbf{x}+a\mathbf{e}_y} = -\frac{it}{\hbar} e^{i2\pi n_\phi x} \hat{c}_{\mathbf{x}}^\dagger \hat{c}_{\mathbf{x}+a\mathbf{e}_y} + \text{H.c.}$, where $\hat{c}_{\mathbf{x}}$ is the particle annihilation operator on the lattice and $\delta_{\mathbf{k}}$ is the deviation due to lattice and finite difference effects. The operators $\hat{T}_{\Delta\mathbf{r}}$ are included such that the expectation value of the current operators is centered at the same point as the Laplacian of densities, due to the curl of current operators being defined at $(x_c + a/2, y_c + a/2)$ [see Fig. 3(a)]. This shift operator can be

realized by implementing twisted boundary conditions with appropriately chosen values of torsion ($\Delta\varphi_x, \Delta\varphi_y$). (Note that for the states in the chosen Landau gauge, which are spatially extended along y , this amounts to a shift in the y momentum.) We have also performed the replacement $\frac{\hbar}{2Ma^2} \rightarrow \frac{t}{\hbar}$ to obtain the lattice version of the continuum formula (5).

Figure 3(b) shows the comparison between energy associated with the lattice discretization error, as measured by the root mean square of $\delta_{\mathbf{k}}$, compared to the band gap E_{gap} and bandwidth ϵ for different fluxes. We find that the lattice deviation falls algebraically with decreasing flux, approaching zero in the continuum limit $n_\phi \rightarrow 0$. (By contrast, the bandwidth falls exponentially.) The energy associated with the deviation remains small compared to the band gap E_{gap} for all flux values, enabling perturbative calculations in the many-body context when $\frac{\hbar}{N} \sqrt{\sum_{\mathbf{k}} \delta_{\mathbf{k}}^2} \ll U \ll E_{\text{gap}}$. The exact form of perturbative deviations due to interactions on the lattice may differ from those in (11) away from the continuum limit, as the higher bands will generally have a different Chern number and quantum geometry compared to the higher LLs in the continuous. We leave the study of higher Chern band mixing for future work.

Conclusion. To conclude, we have presented a direct relationship between current density and number density correlators for many-body states in the LLL, which does not have a simple correspondence in higher LLs. Furthermore, we have shown that the deviations from this relationship can be used to determine the system's interaction energy for states which are perturbatively lifted out of the LLL via interactions. We have also discussed the applicability of our results to lattice settings. The natural continuation of this work is to investigate the properties of current density correlators for many-body states on lattice models, such as the Harper-Hofstadter model. Furthermore, it would be interesting to explore connections between correlators and analogous quantities in the corresponding field theories (e.g., Chern-Simons theory), where the emergent gauge field couples the number density of particles with the current density [33].

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