

Pushing-induced arrest across lattices and dimensions

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Tracer-media interactions can give rise to transport phenomena beyond classical models; e.g., obstacle pushing can eliminate percolation. We demonstrate that the existing “snowplow” mechanism proposed to explain this effect fails in three dimensions. We show that confinement is governed by emergent trapping—rare “door-closing” events that occur with an approximately constant probability per step at low obstacle densities, thus yielding exponential survival. This allows prediction of the time-dependent mean-squared displacement from short-time estimates of the diffusion constant and trapping probability, providing a minimal description of pushing-induced arrest across lattices and dimensions.

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Introduction. From advancing a chess pawn toward promotion to navigating busy crowds, pushing is synonymous with forward motion. Despite the ubiquity of this heuristic in daily life, classical models describing the motion of tracers through disordered environments typically overlook pushing dynamics. A canonical example of this ruling paradigm is the “ant in a labyrinth” (AIL) model [1–3]. There, an “ant” is depicted as a random walker, surrounded by immovable obstacles as illustrated in Fig. 1(a). This disparity between the static assumptions of traditional percolation models [4–7] and the pushy nature of motion in the real world raises fundamental questions: How does pushing affect transport in disordered media? And does it always translate to forward motion?

Recently, these questions have been studied in controlled experiments. Biswas *et al.* examined the stochastic transport of a self-propelled camphor boat navigating a field of passive paper disks floating on water [8]. The mean first passage time, defined as the average travel time from the center to the boundary of the arena, increased monotonically with the fraction of area occupied by the disks. In contrast, fluctuations in travel time displayed nonmonotonic behavior, reaching lower values at intermediate area fractions.

Although obstacles hinder transport over short timescales, the retention of memory from previous trajectories can facilitate transport in the long term. Altshuler *et al.* investigated the movement of a self-propelled bristle robot within a field of movable obstacles. By directly modifying the positions of obstacles in its path, the tracer creates trails [9]. These trails

give rise to a form of environmental memory that expedites search by increasing the effective diffusion coefficient. Similar environmental memory was observed by Dias *et al.* on a microscopic scale [10]. By employing light-activated Janus particles mixed with passive silica colloids, they demonstrated that environmental memory can arise in a system of “clueless” individuals, promoting group formation. Collectively, these findings indicate that the dynamics of active tracers are fundamentally different when obstacles can be pushed.

Motivated by these findings, Bonomo and Reuveni introduced the Sokoban random walk, a minimal model aimed at capturing the effect of obstacle pushing [11]. In this model, a random walker on a lattice is allowed to push at most one obstacle at a time [see Fig. 1(b)]. While the ability to push obstacles was expected to allow the Sokoban to venture

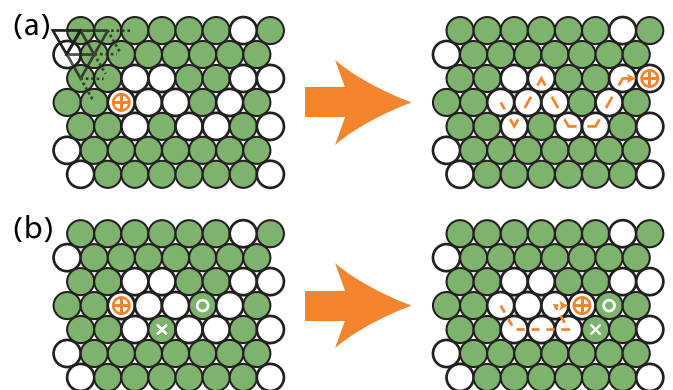


FIG. 1. AIL vs Sokoban random walk. Examples of (a) an AIL and (b) a Sokoban random walk on the Triangular lattice. Green nodes are occupied with obstacles, and white nodes are free. The orange tracers wander around obeying their respective laws of motion. While the AIL escapes, the Sokoban tracer happens to permanently confine itself. Two obstacles are marked with a circle and a cross, for traceability. They are moved by the Sokoban tracer during the walk, but are identical to all other obstacles.

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further away, the model was used to demonstrate that this is not always true. In fact, it was found that the Sokoban always ends up confined. The region visited prior to confinement has a characteristic size, which is set by the initial obstacle density. These features contrast with the AIL model, where a sharp transition between restricted motion and free diffusion occurs at a specific critical density $0 < \rho_c < 1$ [2].

The surprising loss of the percolation transition due to obstacle pushing raises the question of whether this effect is geometry specific or persists more generally. This question was subsequently addressed, revealing that on the infinite-dimensional Bethe lattice, the Sokoban undergoes a percolation transition, occurring at a critical density higher than its nonpushy counterpart, as intuition dictates [12]. This critical density was also found by Vandewalle *et al.* who considered a fluid version of the model in the context of epidemics [13].

Tracer-media interactions can thus qualitatively and quantitatively reshape transport in a topology- and dimension-dependent manner. This calls for a unifying description that remains valid across geometries. Here, we investigate the Sokoban model across lattices and dimensions and show that confinement is governed by emergent trapping—rare, irreversible “door-closing” events—which yields a minimal coarse-grained description of pushing-induced arrest.

The model. We consider the Sokoban model, in which a tracer with limited pushing ability performs a random walk that, among other things, reconfigures its environment. The medium is represented as a lattice, where each node is randomly occupied by an obstacle with probability ρ . A single tracer is initialized on the lattice and proceeds via a sequence of feasible moves. A feasible move for the tracer is one of two types: (1) stepping into a vacant neighboring node or (2) pushing a single obstacle in its direction of motion, provided that the next site is unoccupied. Pushing more than a single obstacle at a time is forbidden. At each time step, the tracer selects a move at random from the current set of feasible moves. For an illustration of such a walk, see Fig. 1(b).

Previous work explored the Sokoban model on the two-dimensional (2D) square lattice [11]. It was found that the mean-squared displacement (MSD) of the walk saturates to a finite value (see Sec. SI of the Supplemental Material [14]):

$$\text{MSD}_\infty = \left(\frac{1 - \rho}{C\rho} \right)^{\frac{2}{\gamma}}, \quad (1)$$

after many steps. Equation (1) relates MSD_∞ , the saturation MSD value, and the obstacle density ρ via the positive constants C and γ . These constants are set by the fractal nature of the walk. Specifically, the authors defined (1) $\mathcal{A}$, the number of distinct nodes the tracer visits during the walk, and (2) $\mathcal{P}$, the number of unvisited nodes, which are within two steps of a visited node, reflecting the region of interest when considering feasible moves. It was shown that the averages of these quantities obey a fractal scaling:

$$\langle \mathcal{A} \rangle \simeq A\sqrt{\text{MSD}_\infty^\alpha}, \quad \langle \mathcal{P} \rangle \simeq B\sqrt{\text{MSD}_\infty^\beta}. \quad (2)$$

Following these definitions, the constants in Eq. (1) are given by $\gamma = \alpha - \beta$ and $C = f\frac{A}{B}$, where f is a factor relating to

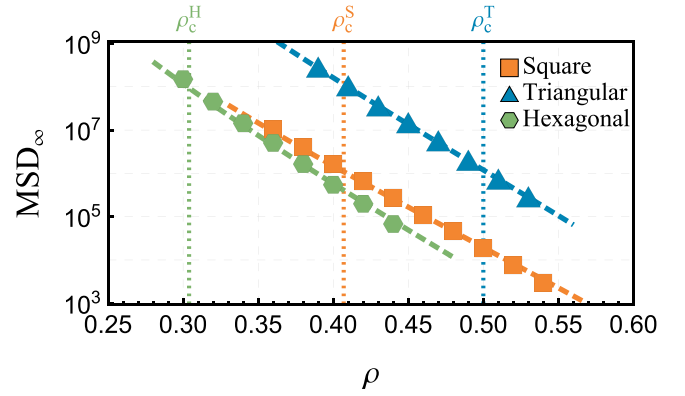


FIG. 2. Existence of “snowplow effect” for the Sokoban in different 2D lattices. The terminal MSD, MSD_∞ , vs the obstacle density ρ for the square, triangular, and hexagonal lattices. Predictions coming from Eq. (1) (dashed lines) are in excellent agreement with Monte Carlo simulations (markers). The dotted vertical lines represent the percolation thresholds for the AIL (no obstacle pushing) in the respective lattices.

occupied nodes in $\mathcal{A}$ and unoccupied nodes in $\mathcal{P}$ at the end of the walk (see Sec. SI of the Supplemental Material [14]).

The “snowplow effect.” To elucidate the physical mechanism behind the emergent confinement in the 2D square lattice, picture plowing snow when clearing your yard. As the snow is pushed aside, it accumulates at the perimeter of the cleared area. Since the cleared area grows faster than its perimeter, the displaced snow will pile up at the rims. Ultimately, the periphery becomes filled with heavy snow piles that are difficult to move. The Sokoban walk displays a similar behavior, which we henceforth refer to as the “snowplow effect.” As the tracer ventures around, it clears obstacles from visited nodes. These obstacles do not vanish, but are rather pushed together into the perimeter of the walk. For the Sokoban walk on the two dimensional square lattice, γ was found to be positive [11], indicating that the area grows faster than the perimeter. Therefore, pushed obstacles inevitably form an impassable, double-layered, wall that confines the walk.

On the 2D square lattice, Eq. (1), which was formulated based on the “snowplow effect,” successfully predicts the empirical MSD_∞ [11]. However, it is unclear whether this is true for other lattices. In fact, little is known about the Sokoban random walk outside the 2D square lattice. Does it always confine itself? And, if so, is the “snowplow effect” the mechanism behind confinement? We now address these questions by studying the Sokoban on the triangular and hexagonal (honeycomb) lattices [2], testing the predictions of Eq. (1). Throughout this work, we omit high obstacle densities, and instead focus on densities where the tracer is able to explore far beyond its immediate surroundings before confinement occurs, i.e., $\sqrt{\text{MSD}_\infty} \gg 1$, allowing us to probe emergent large-scale phenomena rather than trivial local exploration.

In Fig. 2, we compare MSD_∞ values coming from Monte Carlo simulations (symbols) to predictions coming from Eq. (1) (dashed lines). To obtain the latter, we first estimated C and γ via Eq. (2) (see Sec. SI of the Supplemental Material [14] for simulation details and results). We performed

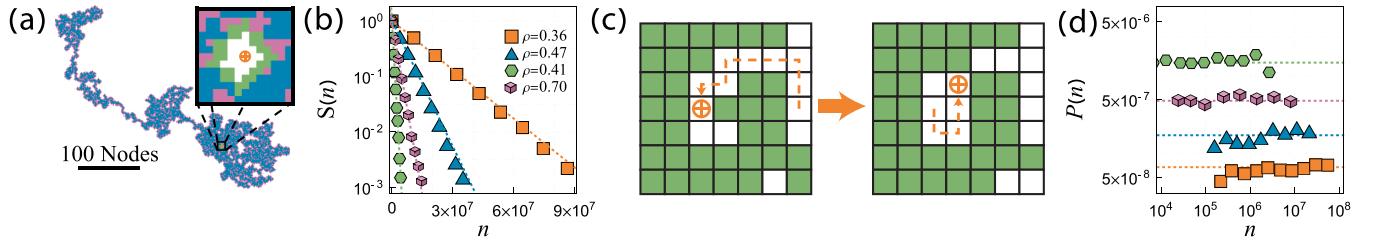


FIG. 3. Trapping of the Sokoban random walk. (a) The Sokoban random walk ends its life in a pocket (inset) that can be orders of magnitude smaller than the size of the walk. Here, the area $\mathcal{A}$ and perimeter $\mathcal{P}$ that were defined below Eq. (1) are colored blue and pink, respectively. Obstacles confining the terminal pocket are colored green. (b) The survival probability $S(n)$ of a Sokoban tracer for various lattices. Dashed lines are exponential fits. Additional densities are presented in Sec. SIV of the Supplemental Material [14]. (c) An example of confinement via trapping. On the left, the tracer enters a pocket of obstacles and then “closes the door,” permanently confining itself to the six nodes inside the pocket. (d) Comparison of the directly measured trapping probability (markers) and the exponential decay rate (dashed lines) extracted from the fits in panel (b).

simulations for the three lattices, with obstacle densities both above and below their respective percolation thresholds.

Below the percolation threshold, the MSD of the AIL diverges as the number of steps grows. Conversely, for the Sokoban we find a finite MSD_∞ . Furthermore, we see that the predictions of Eq. (1) are in excellent agreement with simulations. At very low obstacle densities, computing limitations prevent simulations. There, Eq. (1) predicts that for $\gamma > 0$ and $\rho > 0$, a finite MSD_∞ is obtained. This suggests that pushing eliminates percolation completely at any obstacle density, by means of the “snowplow effect.”

Surprisingly, confinement by the “snowplow effect” does not extend to all lattices. For example, in the Bethe lattice, pushing facilitates percolation [12] and the “snowplow effect” is notably absent. As the Bethe lattice is effectively infinite dimensional [2], the absence of a “snowplow effect” there hints that its emergence is dimension dependent. To probe into this matter, we cranked up the dimension knob and simulated the Sokoban random walk on the three-dimensional (3D) simple cubic (SC) lattice. We measured the fractal dimensions of the volume and surface area, which are the 3D equivalents of $\mathcal{A}$ and $\mathcal{P}$ used in Eq. (2). Finally, we used these to generate a prediction of the MSD_∞ based on Eq. (1). This procedure, which provided accurate estimations in 2D, fails for the 3D SC lattice. While we still observe condensation of obstacles along the perimeter of the walk (see Sec. SII of the Supplemental Material [14]), the tracer is confined well before reaching the predicted MSD_∞ . In fact, the predicted MSD_∞ is orders of magnitude larger than that obtained from simulations (see Sec. SIII of the Supplemental Material [14]).

This mismatch indicates that in 3D, confinement is not controlled by the eventual macroscopic buildup of a snowplowed wall. Rather, there appears to be a competing mechanism that arrests the walk earlier. To characterize the kinetics of this arrest, we must look beyond MSD_∞ , which gives a characteristic size for the walk. To do so, we shift our attention to the time it takes the walk to become confined.

Trapping approach. When examining the behavior of the Sokoban random walk over time, two distinct phases are identified. Initially, the walk explores the lattice, randomly pushing obstacles. At some point, the walk stops discovering new nodes. When this happens, it is confined to a permanent set of nodes. Surprisingly, this terminal cage is orders of

magnitude smaller than the total area visited by the walk [see Fig. 3(a)]. This has also been observed in Refs. [11,15].

Next, we examine this unexplained phenomenon. We define the survival probability $S(n)$ that the walk has not entered its terminal cage for the last time by step n . We estimate this probability using simulations on the square, triangular, hexagonal, and cubic lattices (simulation details in Sec. SIV of the Supplemental Material [14]). Results in Fig. 3(b) and Sec. SIV of the Supplemental Material [14] show that the survival decays exponentially (to a good approximation) with the number of steps, n , across lattices and obstacle densities. This exponential decay can be explained using a trapping approach.

First, we elucidate the trapping event. How does the walk transition from free exploration to the terminal state? Consider the left panel of Fig. 3(c). The path depicted there is completely reversible. This means that the tracer can enter and leave the terminal cage multiple times before confinement occurs. However, shortly after the tracer enters the terminal pocket for the *last* time, an irreversible obstacle pushing event occurs. The tracer enters the pocket and “closes the door” behind itself [see right panel of Fig. 3(c)]. In practice, we set the trapping step as the last time the tracer entered the terminal pocket.

Now that we have established how trapping occurs, we can try and explain the behavior of $S(n)$. We denote the probability of the tracer to confine itself at step n as P_n . The survival is then given by $S(n) = \prod_{i=1}^n (1 - P_i)$. In Fig. 3(d), we estimate P_n from simulations for the bulk of the survival distribution (see Sec. SIV of the Supplemental Material [14] for additional details). We find that it is approximately constant, i.e., does not depend on n , and very small. We thus approximate $S(n) \propto e^{-Pn}$. The decay rates measured in Fig. 3(b) are also displayed as dashed lines, showing good agreement with direct estimates of P (symbols).

The trapping approach not only captures the survival kinetics, but can also be used to predict the evolution of $\text{MSD}(n)$. To derive an equation for this quantity, we write $\text{MSD}(n)$ as a sum of two contributions. The first comes from tracers that have not yet been confined by step n . We observe that for such tracers $\text{MSD}(n|\text{survived}) = 2Dn$, with D being ρ dependent (for additional details, see Sec. SV of the Supplemental Material [14]). Thus, their contribution to the

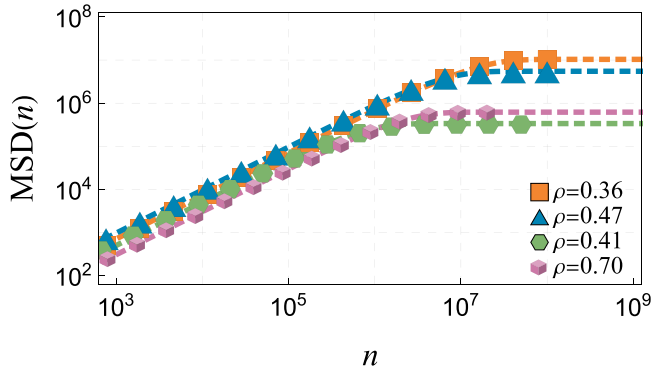


FIG. 4. Successful prediction of $\text{MSD}(n)$ across lattices and dimensions. The MSD of the Sokoban random walk (markers for various lattices) compared to the prediction provided by Eq. (3), shown in dashed lines. Additional densities are presented in Sec. SV of the Supplemental Material [14].

total $\text{MSD}(n)$ is $2Dne^{-nP}$. The second contribution comes from tracers that were confined prior to step n . We again use the law of total probability to integrate over the contributions of tracers that were confined at each $n' < n$. This integral yields $2D(\frac{1}{P} - (\frac{1}{P} + n)e^{-nP})$. Summing the two contributions, we obtain (see Sec. SV of the Supplemental Material [14])

$$\text{MSD}(n) \approx 2D(1 - e^{-nP})/P. \quad (3)$$

Equation (3) enables prediction of the macroscopic ensemble property $\text{MSD}(n)$ using two microscopic parameters: the trapping probability P and diffusion constant D . Crucially, these parameters can be easily estimated from the short-time dynamics to predict $\text{MSD}(n)$ at all times, which was previously impossible. Doing so for all four lattices and using Eq. (3) provides high-accuracy prediction of $\text{MSD}(n)$ (see Fig. 4 and Sec. SV of the Supplemental Material [14]). Thus, trapping succeeds in capturing behavior in both 2D and 3D, where the “snowplow effect” failed.

Trapping and the “snowplow effect” are two competing mechanisms of confinement. While the former describes a long and continual process, the latter is an instantaneous one. Thus, it is possible that a tracer will trap itself while “snowplowing.” In 2D, trapping typically does not occur until the macroscopic snowplowed cage is essentially established; as a result, the two descriptions agree on MSD_∞ , explicitly linking the macroscopic parameters (C, γ) that control the snowplow prediction in Eq. (1) to the microscopic kinetic parameters (D, P) that control the trapping dynamics. Specifically, taking $n \rightarrow \infty$ in Eq. (3) and comparing with Eq. (1), we have $\frac{2D}{P} = (\frac{1-\rho}{C\rho})^{\frac{2}{\gamma}}$. In 3D, trapping arrests the walk before a macroscopic wall can form, so the snowplow estimate becomes an upper bound rather than the realized terminal MSD.

Classical versus Sokoban trapping: exponential kinetics revisited. Sokoban trapping is distinct from the classical trapping problem [16,17]. There, a tracer wanders around in an arena that is randomly populated with traps that annihilate the tracer upon contact. Since annihilation can only occur when a newly discovered site turns out to be a trap, the number of distinct sites visited is key to understanding kinetics in this problem. Conversely, in the Sokoban, trapping

is an emergent property, and obstacle configurations become traps only when obstacle pushing is introduced. Moreover, the tracer can get trapped even when stepping into a site that has already been discovered in the past [see discussion on the example in Fig. 3(c)]. In addition, unlike classical trapping, a Sokoban tracer can destroy traps, e.g., by pushing the free obstacle in Fig. 3(c) one site downward, thus neutralizing the trap indefinitely. We therefore argue that pushing promotes a new, self-generated, trapping mechanism that goes beyond the existing theory of classical trapping.

For classical trapping, at short-intermediate timescales, one finds exponential survival kinetics, $S(n) \propto e^{-an}$, in 3D and exponential kinetics with logarithmic corrections, $S(n) \propto e^{-an/\ln(n)}$, in 2D [16]. At asymptotically long times, survival is governed by rare initial configurations of the traps, giving rise to stretched-exponential survival $S(n) \propto e^{-an^{d/d+2}}$ [16–18], where d is the dimension. Very recently, Singh *et al.* proved that the long-time asymptotics of the survival of the Sokoban on the one-dimensional lattice is $S(n) \propto \mathcal{K}(n)e^{-an^{1/3}}$, with $\mathcal{K}(n) \propto n^{7/6}$ [15,19]. This modified stretched-exponential asymptotics, reminiscent of classical trapping, led Singh *et al.* to postulate $S(n) \propto \mathcal{K}(n)e^{-an^{1/2}}$ asymptotics for the Sokoban on the 2D square lattice.

Singh *et al.* focused on obstacle densities higher than those considered here when characterizing the survival kinetics. For the 2D square lattice, we focused on $\rho \leq 0.4$, while Ref. [15] presents results for $\rho = 0.5, 0.6$. Performing detailed simulations of the Sokoban for these higher densities (10^6 walk realizations versus 10^4 above), we find full agreement with the results in Ref. [15], affirming strong deviations from exponential kinetics (see Sec. VI of the Supplemental Material [14]). However, as the obstacle density is lowered, deviations from exponential decay become progressively less pronounced, such that for the densities considered here, exponential decay provides an almost perfect description of the survival kinetics: emerging after a short transient and persisting out to times nearly nine standard deviations beyond the mean ($S(n) \approx 10^{-5}$; see Sec. VII of the Supplemental Material [14]). These findings call for the development of a theory that will be able to explain Sokoban survival kinetics in dimensions 2 and up, possibly revealing a density-dependent crossover between exponential kinetics and the modified stretched-exponential asymptotics postulated by Singh *et al.*

Conclusions and outlook. We studied the Sokoban random walk, a minimal model for transport in the presence of tracer-media interactions, across lattices and dimensions. In two-dimensional Euclidean tilings, we find that the exploration length scale is well described by Eq. (1) and the macroscopic “snowplow” picture: Obstacles cleared from the explored region accumulate at its perimeter and ultimately form an impenetrable wall, yielding a finite MSD_∞ for any $\rho > 0$. In three dimensions, this macroscopic reasoning breaks down: Despite boundary condensation, confinement occurs at length scales that can be orders of magnitude smaller than the snowplow estimate, indicating an additional arrest mechanism.

At high obstacle densities, the walker either is completely confined from the get-go or has very limited ability to explore beyond its immediate surroundings. We thus focused

on lower obstacle densities for which $\sqrt{\text{MSD}_\infty} \gg 1$. There, we identified the missing mechanism as *emergent trapping*. Confinement is triggered by rare, irreversible local rearrangements that close the “door” behind the tracer. At low obstacle densities, the resulting trapping probability is approximately constant in step number, leading to exponential survival. This perspective yields a closed-form prediction for the time-dependent mean-squared displacement. The prediction is encapsulated in Eq. (3), in terms of two microscopic quantities accessible from short-time dynamics: the diffusion coefficient $D(\rho)$ and the instantaneous trapping probability $P(\rho)$. Trapping and the “snowplow” effect are complementary mechanisms that together provide a coherent picture of confinement. In this unified view, snowplowing provides an upper bound on the ultimate size of the explored region, whereas trapping provides the kinetic route to arrest. This distinction becomes decisive in 3D, where transience prevents the sustained boundary processing required for snowplow-controlled confinement.

A key implication is that an interacting, history-dependent, process can nevertheless admit a coarse-grained description. Indeed, recent experiments and models of environmental memory captured the effect of pushing via an effective diffusion coefficient $D(\rho)$ [9]. Our results show that this is generally insufficient: Pushing also generates a distinct kinetic

pathway to arrest through emergent trapping. Once the pair $\{D(\rho), P(\rho)\}$ is known from short-time dynamics, Eq. (3) determines the subsequent mean-squared displacement, providing a minimal predictive characterization of transport despite the underlying complexity.

More broadly, the Sokoban walk serves as a counterexample in which pushing hinders, rather than enhances, transport by creating self-generated traps. This underscores that “more pushing” need not imply faster spreading, and that the outcome depends on the microscopic laws of motion. An important next step is therefore to predict $D(\rho)$ and $P(\rho)$, for pushing and other tracer-media interactions—from local geometry and rearrangement statistics, enabling systematic exploration of alternative laws of motion and delineating regimes where interactions eliminate percolation, restore it, or generate qualitatively new dynamics.

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Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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