

Direct quantum process characterization via erasure-based closed timelike curves

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Characterizing complex quantum evolutions in the microscopic world is crucial for understanding fundamental physics and advancing quantum information applications. In conventional quantum process tomography, individual process matrix elements are obtained indirectly through reconstruction. Here, we use sequential system-ancilla couplings and quantum erasure to directly access these elements for arbitrary quantum processes. Within the pseudo-density-matrix framework, these operations realize an effective erasure-based closed timelike curve (E-CTC) interaction. During E-CTC-assisted measurements, the protocol combines selective computational-basis addressing with fixed superposition preparation and projections. Although the configuration count is not reduced for general processes, these fixed settings can simplify control when computational-basis states are readily addressable. We experimentally demonstrate our scheme in a photonic platform by characterizing several two- and four-dimensional quantum processes with average fidelities above 99%. Our work offers a basis-rotation-free approach to high-dimensional quantum process characterization within the erasure-based CTC framework.

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I. INTRODUCTION

With the development of large-scale quantum information processing, the efficient and faithful characterization of quantum dynamics in large Hilbert spaces is becoming an urgent and essential issue. The conventional method is quantum process tomography [1], which can reconstruct the dynamics process from a series of projection measurements with several input states by employing a complex indirect reconstruction algorithm. However, when utilized in large Hilbert spaces, it faces significant challenges in the number and diversity of required operations, as well as in algorithmic complexity. Explicitly, for a d -dimensional system, it requires $O(d^4)$ measurement results using a complete set of $d + 1$ mutually unbiased bases, provided that such a set exists in dimension d . The process matrix elements are then inferred from the measured probabilities through a reconstruction procedure. In contrast, direct characterization methods are more efficient in accessing certain information in a quantum evolution, such as coherence [2], fidelity [3], and propagator [4] estimation. Methods for the direct characterization of a quantum process have been developed through quantum error-detection techniques [5–9] and sequential weak values [10,11]. However, their practical applications in large-scale, high-dimensional systems remain limited by the challenges of preparing

sophisticated entangled states or implementing complex coupling operations. In addition, the weak values in approximation would lead to intrinsic high-order errors and less precision [12].

In general, the arrow of time imposes a constraint that necessitates the separability of pre- and postprocess operations, thus restricting our ability to manipulate and analyze quantum dynamics. This limitation can be circumvented in the presence of closed timelike curves (CTCs) [13,14]. CTCs are solutions of Einstein's equations, which allow observers to travel backward in time and interact with the past. Intriguingly, when quantum processes are embedded within a CTC, exotic dynamics emerge, offering resolutions to the well-known grandfather paradox in Deutsch's CTC (D-CTC) [15] and postselected CTC (P-CTC) [16,17] models. The power of interaction between past and future in CTC provides us with new tools to handle quantum information before and after the quantum process collectively. It has been proven that CTCs have substantial benefits for many information processing tasks [18–25]. However, unlike instantaneous interaction between distinct systems, it is still unknown whether the interaction between temporal past and future exists in the real physical world. Instead, the CTCs were experimentally simulated based on D-CTC [26] and P-CTC [17,25] frameworks.

In this work, we demonstrate that the ability of past-future interaction in CTC can be utilized for direct quantum process characterization (DQPC). First, we propose an erasure-based CTC (E-CTC) scheme, where an ancilla is used to interact with the system twice and a subsequent quantum erasure [27,28] leads to the postselection on the past-future interaction. In contrast to D-CTC and P-CTC, the self-consistency of E-CTC is naturally satisfied. Subsequently, with the E-CTC, we present a method to directly characterize a quantum

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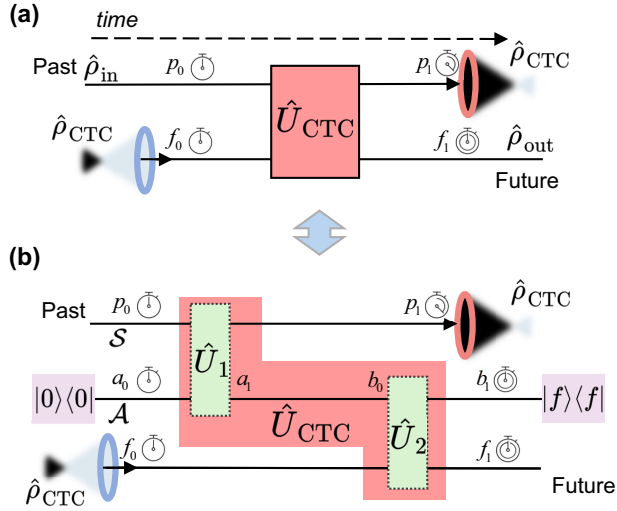


FIG. 1. Closed timelike curve and its erasure-based implementation. (a) Model of a quantum state interacting with the past of itself. The dashed arrow indicates the direction of time. The black and blue cones show that a quantum state travels back to its past. (b) Scheme of the erasure-based CTC interaction. An ancilla $\mathcal{A}$ sequentially interacts with system through $\hat{U}_1$ and $\hat{U}_2$, which is finally erased with projection $|f\rangle\langle f|$.

process through selective computational-basis addressing with fixed superposition preparation and projections. Then, we experimentally demonstrated our method by characterizing several two-dimensional (2D) and four-dimensional (4D) processes in photonic system.

II. THEORETICAL FRAMEWORK

We first introduce our erasure-based scheme to implement the CTC interaction according to the pseudo-density-matrix (PDM) formalism [29–32]. In this framework, the quantum systems in different times are described as d -dimensional quantum states in independent past and future Hilbert spaces denoted as $\mathbb{H}_{p_j}$ and $\mathbb{H}_{f_j}$, respectively, as shown in Fig. 1(a). The subscript $j = \{0, 1\}$ represents the time order with direction $0 \rightarrow 1$ in chronology-respecting past and future temporal spaces. The temporal PDM describing the correlation between past and future spaces is defined as $\hat{\rho}_{p_j f_j}$. In the presence of CTCs, a quantum state $\hat{\rho}_{\text{CTC}}$ in $\mathbb{H}_{p_1}$ travels back to $\mathbb{H}_{f_0}$ and interacts with the past version of itself $\hat{\rho}_{\text{in}}$ in $\mathbb{H}_{p_0}$ through a unitary operator $\hat{U}_{\text{CTC}}$. Thus, we have the relation

$$\hat{\rho}_{p_1 f_1} = \hat{U}_{\text{CTC}} \hat{\rho}_{p_0 f_0} \hat{U}_{\text{CTC}}^{-1}. \quad (1)$$

The state after the CTC evolution is expressed as $\hat{\rho}_{\text{out}} = \hat{\mathcal{E}}_{\text{CTC}}(\hat{\rho}_{\text{in}})$. However, such a causally correlated interaction is currently believed to be physically infeasible due to the causality constraint: The future f_0 , originating from the past p_1 , cannot in turn influence p_1 .

The diagram for our erasure-based CTC scheme is illustrated in Fig. 1(b). For subsequent application in DQPC, we

aim to implement the evolution of $\hat{U}_{\text{CTC}}$ defined by

$$\begin{aligned} \hat{U}_{\text{CTC}} &= e^{-i\hat{O}_p \otimes \hat{O}_f} \\ &= \sum_{j,k=0}^{d-1} e^{-iO_p^j O_f^k} |j\rangle_{p_1} \langle j|_{p_0} \otimes |k\rangle_{f_1} \langle k|_{f_0}, \end{aligned} \quad (2)$$

where $\hat{O}_p$ and $\hat{O}_f$ are arbitrary Hermitian operators with the decomposition in eigenbasis $\hat{O}_p = \sum_{j=0}^{d-1} O_p^j |j\rangle_{p_1} \langle j|_{p_0}$ and $\hat{O}_f = \sum_{k=0}^{d-1} O_f^k |k\rangle_{f_1} \langle k|_{f_0}$, respectively. It should be noted that, through the universal quantum circuit synthesis method, we can further construct arbitrary CTC evolutions.

An ancilla $\mathcal{A}$ is introduced to interact sequentially with the past and future states of the system $\mathcal{S}$. We first consider that the dimension of the ancilla is also d , such that the ancilla can be expressed in a basis of orthogonal states $\{|m\rangle_s\}$, where m is an integer ranging from 0 to $d-1$. The subscript $s = \{a_0, a_1, b_0, b_1\}$ denotes the temporal space of the ancilla. The steps for implementing the erasure-based CTC interaction are as follows.

(1) The ancilla state is prepared in $|0\rangle_{a_0}$ and coupled with the past of system p_0 through

$$\hat{U}_1 = \sum_{j,m=0}^{d-1} |j\rangle_{p_1} \langle j|_{p_0} \otimes |\overline{m+j}\rangle_{a_1} \langle m|_{a_0}, \quad (3)$$

where we define $\overline{m+j} := (m+j) \bmod d$.

(2) The ancilla interacts with the future of system f_0 via

$$\hat{U}_2 = \sum_{n,k=0}^{d-1} e^{-i\varphi_{nk}} |k\rangle_{f_1} \langle k|_{f_0} \otimes |n\rangle_{b_1} \langle n|_{b_0}, \quad (4)$$

where the phases $\varphi_{nk} = O_p^n O_f^k$ are used to realize the d^2 phases in Eq. (2). The two-step evolutions are unitary, and thus reversible.

(3) The correlation between $\mathcal{S}$ and $\mathcal{A}$ is erased by projecting the ancilla on the state

$$|f\rangle_{b_1} = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} |m\rangle_{b_1}. \quad (5)$$

Then, the resulting map from $\mathbb{H}_{p_0} \otimes \mathbb{H}_{f_0}$ to $\mathbb{H}_{p_1} \otimes \mathbb{H}_{f_1}$ is $\hat{\rho}_{p_1 f_1} = \hat{M} \hat{\rho}_{p_0 f_0} \hat{M}^\dagger$, with $\hat{M} = \hat{U}_{\text{CTC}} / \sqrt{d}$ (see Sec. IB of the Supplemental Material [33]). The factor $1/\sqrt{d}$ in $\hat{M}$ arises from the ancilla erasure projection. The resulting $\hat{\rho}_{p_1 f_1}$ is left unnormalized.

Therefore, by employing a d -dimensional ancilla, an arbitrary past-future coupling $\hat{U}_{\text{CTC}}$ defined in Eq. (2) can be constructed with a constant success probability of $P_{\text{suc}} = 1/d$. Intuitively, information from the past is coherently transferred to the ancilla through preselection and the $\hat{U}_1$ operation, establishing quantum correlations between them. Combined with $\hat{U}_2$, future interacts with past mediated by ancillary correlation. In particular, if the numbers of nonzero elements of O_p^j and O_f^k are reduced to d_1 and d_2 , respectively, the ancilla dimension can be correspondingly reduced to $d' = \max\{d_1, d_2\} + 1$, resulting in a success probability of

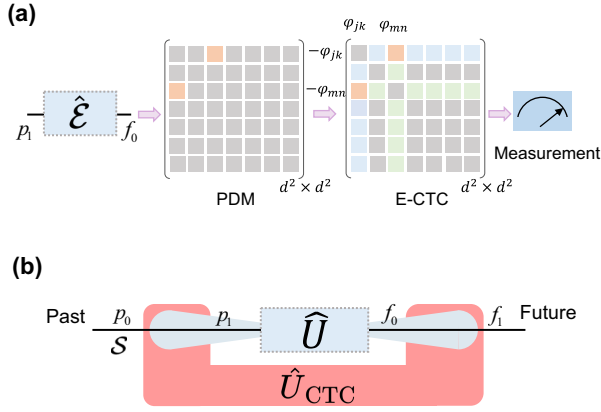


FIG. 2. Scheme for direct quantum process characterization. (a) Procedure for characterizing an arbitrary process. A d -dimensional quantum process $\hat{\mathcal{E}}$ is represented as a $d^2 \times d^2$ PDM, with the colored elements indicating the phases applied with E-CTC. (b) Quantum circuit diagram for E-CTC interaction for a coherent quantum process $\hat{U}$.

$P_{\text{suc}} = 1/d'$. While the erasure-induced evolution can also be described in terms of two-time correlations, we use the E-CTC picture as an operational interpretation of the effective past-future connection. The underlying circuit respects chronological order, and its implementation and reconstruction formulas follow from sequential system-ancilla couplings and quantum erasure. At a conceptual level, this past-future interaction picture is related to D-CTC and P-CTC models, although the underlying constructions are different [15,17]. Beyond their conceptual role, CTC ideas have also motivated metrology protocols that can be implemented in chronology-respecting experiments [24,25].

Then, due to the E-CTC interaction, the input-output relation can be written as (see Sec. IC of the Supplemental Material [33])

$$\hat{\rho}_{\text{out}} = \text{Tr}_{p_0}[(\hat{\rho}_{\text{in}}^T \otimes \hat{f}_{f_1}) \hat{M}' \hat{\rho}_{p_1 f_0}^{T_{p_1}} \hat{M}'^\dagger], \quad (6)$$

where

$$\hat{M}' = \frac{1}{\sqrt{d}} \sum_{j,k=0}^{d-1} e^{-i\varphi_{jk}} |j\rangle_{p_0} \langle j|_{p_1} \otimes |k\rangle_{f_1} \langle k|_{f_0}. \quad (7)$$

The temporal PDM $\hat{\rho}_{p_1 f_0}$ in $\mathbb{H}_{p_1} \otimes \mathbb{H}_{f_0}$ describes the relation between p_1 and f_0 , thus implying self-consistency. The superscript T_{p_1} means partial transposition in $\mathbb{H}_{p_1}$ with respect to orthogonal basis $\{|j\rangle_{p_1}\}$.

Next, we perform direct characterization of the d -dimensional quantum process $\hat{\mathcal{E}}$ from p_1 to f_0 by embedding it within the E-CTC interaction. The schematic of this method is shown in Fig. 2(a). The PDM $\hat{\rho}_{p_1 f_0}$ fully describing $\hat{\mathcal{E}}$ can be defined as

$$\hat{\rho}_{p_1 f_0} = \sum_{x,y,s,t=0}^{d-1} \rho_{sy,xt} |x\rangle_{p_1} \langle y|_{f_0} \langle t|_{f_0} \langle s|_{p_1}, \quad (8)$$

where $\{\rho_{sy,xt}\}$ are the elements of PDM.

We prepare the system $\mathcal{S}$ in the state

$$|f\rangle_{p_0} = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} |m\rangle_{p_0}, \quad (9)$$

and subsequently project it in the state

$$|f\rangle_{f_1} = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} |m\rangle_{f_1}. \quad (10)$$

Then, the resulting output probability after E-CTC interaction is given by (see Sec. ID of the Supplemental Material [33])

$$P = \frac{P_{\text{suc}}}{d^2} \sum_{j,k,m,n=0}^{d-1} e^{-i(\varphi_{jk} - \varphi_{mn})} \rho_{jk,mn}. \quad (11)$$

To get the elements $\{\rho_{jk,mn}\}$, we set the two corresponding phases to $\varphi_{jk} \in \{0, \pm\frac{\pi}{2}\}$ and $\varphi_{mn} \in \{0, \pi\}$ [34], with the other phases set to zero. Consequently, we require just a qutrit ancilla and have $P_{\text{suc}} = 1/3$. The corresponding output probability given by Eq. (11) is denoted as $P_{\varphi_{jk}, \varphi_{mn}}^{jk,mn}$. The real and imaginary parts of nondiagonal elements of $\{\rho_{jk,mn}\}$ can be calculated as

$$\text{Im}(\rho_{jk,mn}) = \frac{3d^2}{8} \left(P_{\frac{\pi}{2}, 0}^{jk,mn} - P_{-\frac{\pi}{2}, 0}^{jk,mn} - P_{\frac{\pi}{2}, \pi}^{jk,mn} + P_{-\frac{\pi}{2}, \pi}^{jk,mn} \right), \quad (12)$$

$$\text{Re}(\rho_{jk,mn}) = \frac{3d^2}{8} \left(2P_{0,0} - 2P_{0,\pi}^{jk,mn} - P_{\frac{\pi}{2}, 0}^{jk,mn} - P_{-\frac{\pi}{2}, 0}^{jk,mn} + P_{\frac{\pi}{2}, \pi}^{jk,mn} + P_{-\frac{\pi}{2}, \pi}^{jk,mn} \right). \quad (13)$$

The diagonal elements can be obtained by

$$\rho_{jk,jk} = \langle k|_{f_1} \langle j|_{p_0} \hat{\rho}_{p_1 f_0}^{T_{p_1}} |j\rangle_{p_0} |k\rangle_{f_1}, \quad (14)$$

which is the output probability obtained by inputting $|j\rangle_{p_0}$ and measuring on $|k\rangle_{f_1}$ without applying E-CTC interaction. The total number of experimental configurations required to fully characterize the quantum process is $(5d^4 - 3d^2 + 2)/2$. For general processes, our protocol does not reduce the configuration count.

In particular, we consider coherent quantum processes, a significant and widely used class of operations that preserve the purity of quantum states. The PDM of coherent evolution $\hat{U}$ can be expressed as $\hat{\rho}_{p_1 f_0} = |\hat{\mathcal{C}}(\hat{U})\rangle \langle \hat{\mathcal{C}}(\hat{U})|^{T_{p_1}}$, with

$$|\hat{\mathcal{C}}(\hat{U})\rangle = \sum_{m,n=0}^{d-1} u_{mn} |m\rangle_{p_1} |n\rangle_{f_0}, \quad (15)$$

representing the Choi-Jamiołkowski isomorphism [35] of $\hat{U}$. In this case, our scheme can be further simplified to directly obtain u_{jk} by setting only one phase to $\varphi_{jk} \in \{\frac{\pi}{2}, -\frac{\pi}{2}\}$ [36], with the other phases set to zero. Thus, a qubit ancilla is enough, with $P_{\text{suc}} = 1/2$. The schematic diagram is shown in Fig. 2(b). The system is initially prepared in state $|f\rangle_{p_0}$ and finally measured in state $|f\rangle_{f_1}$. The output probability can be obtained by (see Sec. ID of the Supplemental Material [33])

$$P_{\varphi_{jk}}^{jk} = \frac{1}{2d^2} \left| (e^{-i\varphi_{jk}} - 1)u_{jk} + \sum_{m,n=0}^{d-1} u_{mn} \right|^2. \quad (16)$$

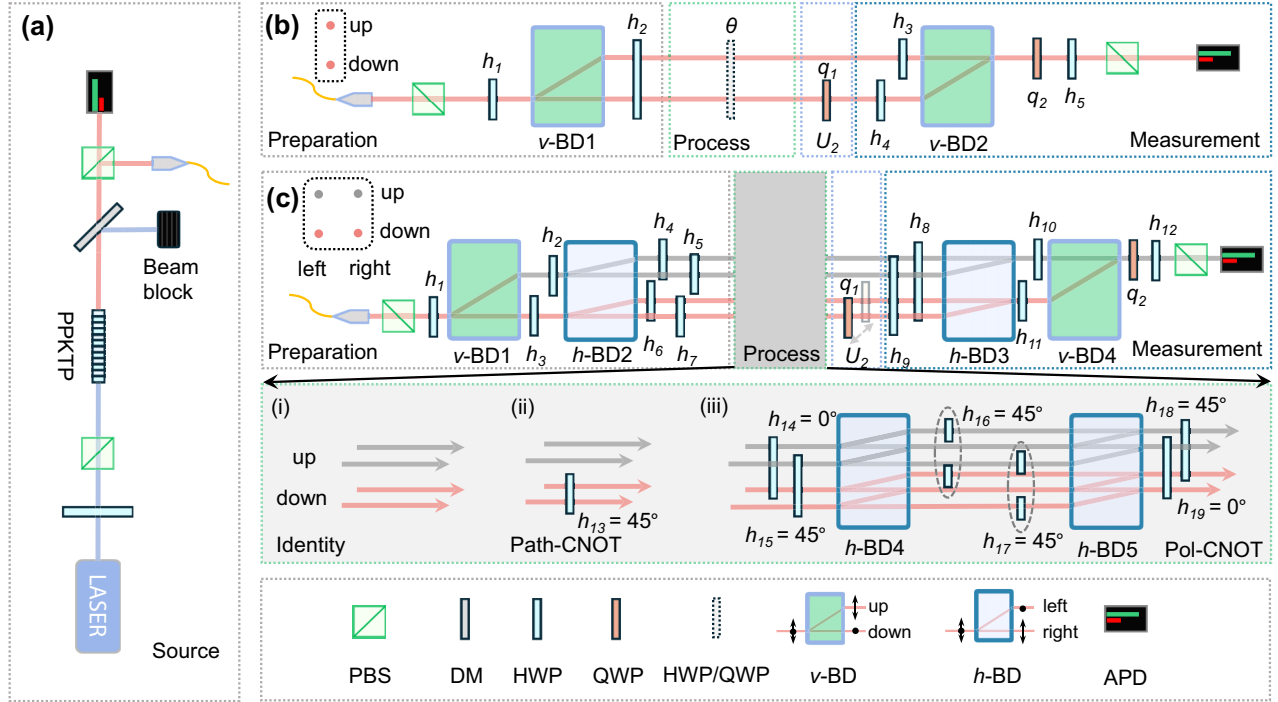


FIG. 3. Experimental setup for direct quantum process characterization. (a) A type-II spontaneous parametric down-conversion in PPKTP crystal is used for heralded single-photon generation. Setups for direct characterization of two-dimensional [panel (b)] and four-dimensional [panel (c)] quantum processes, including (i) identity $\hat{I}_4$, (ii) path-CNOT, and (iii) pol-CNOT operations. Both setups consist of four segments: state preparation, quantum process, $\hat{U}_2$ operation, and measurement. Each segment is implemented using a combination of BD, HWP, QWP, and PBS. For clarity, the HWP h_{16} and h_{17} are shown as two separate HWPs within the dotted areas. DM, dichroic mirror; v-BD, beam displacer that vertically displaces light; h-BD, beam displacer that horizontally displaces light; APD, avalanche photodiode.

Then, the matrix elements can be solved as

$$\text{Re}(u_{jk}) = \frac{d\sqrt{P_0} \left[2 \pm \sqrt{-r_1^2 - (r_2 - 2)^2 + 2r_1(r_2 + 2)} \right]}{2\sqrt{2}}, \quad (17)$$

$$\text{Im}(u_{jk}) = \frac{d\sqrt{P_0}(r_1 - r_2)}{2\sqrt{2}}, \quad (18)$$

where $r_1 = P_{\frac{\pi}{2}}^{jk}/P_0$ and $r_2 = P_{-\frac{\pi}{2}}^{jk}/P_0$. To determine the sign of “ $\pm$ ” in $\text{Re}(u_{jk})$, we can further measure $|u_{jk}|^2 = |\langle k|_{f_1} \langle j|_{p_0} | \mathcal{E}(\hat{U}) |^2$, which corresponds to a probability that can be experimentally measured by preparing the input state in $|j\rangle_{p_0}$ and projecting on $|k\rangle_{f_1}$ without applying the E-CTC interactions. We assign a sign “+” or “−” if the measured value of $|u_{jk}|^2$ is larger or smaller than that of $d^2 P_0(r_1 + r_2)/2$, respectively. Consequently, the total number of experimental configurations required to determine all elements $\{u_{jk}\}$ is $3d^2 + 1$.

Conventional quantum process tomography varies input states and measurement bases, whereas our E-CTC-assisted measurements keep superposition preparation and projections fixed. Different matrix elements are accessed through selective addressing and ancilla operations drawn from a small, fixed set. The ancilla has three levels for general processes or two for coherent processes, independent of d . This can simplify control when computational-basis states are readily addressable. Each matrix element

is obtained from a small, fixed number of probabilities using Eqs. (12)–(14) and (16)–(18). No matrix inversion or optimization over the full Choi matrix is required.

III. EXPERIMENTAL IMPLEMENTATION

We experimentally demonstrate the direct characterization of 2D and 4D unitary quantum processes based on a linear photonic platform. The experimental setup of the heralded single-photon source is shown in Fig. 3(a). A continuous-wave laser with a wavelength of 405 nm is used to pump a periodically poled potassium titanyl phosphate (PPKTP) crystal to generate photon pairs centered at 810 nm via type-II spontaneous parametric down-conversion. The photon pairs are then separated by a polarization beam splitter (PBS), with one photon detected as a trigger signal and the other as a heralded single photon.

The experimental setups for the 2D and 4D quantum processes are depicted in Figs. 3(b) and 3(c), respectively, each comprising four modules: state preparation, quantum process, $\hat{U}_2$ operations with $\varphi_{jk} = \pm \frac{\pi}{2}$, and measurement. The ancilla qubit is encoded in the states of a photon in the up and down paths, with the basis states defined as $|0\rangle_A \equiv |\text{up}\rangle$ and $|1\rangle_A \equiv |\text{down}\rangle$. The basis states of the 2D system $\mathcal{S}$ are encoded in horizontal and vertical polarization states as $|0\rangle_S \equiv |H\rangle$ and $|1\rangle_S \equiv |V\rangle$. In the 4D system, we combine the

polarization and the left/right paths to encode the basis states as $|0\rangle_S \equiv |\text{left}\rangle|H\rangle$, $|1\rangle_S \equiv |\text{left}\rangle|V\rangle$, $|2\rangle_S \equiv |\text{right}\rangle|H\rangle$, and $|3\rangle_S \equiv |\text{right}\rangle|V\rangle$.

For convenience, the state preparation and the $\hat{U}_1$ operation shown in Fig. 1(b) are combined into a single state preparation stage by using half-wave plates (HWP) and beam displacers (BDs) (see Secs. II A and III A of the Supplemental Material [33]). Considering the preparation of d basis states $|j\rangle_{p_0}$ for measuring $|u_{jk}|^2$, along with d different $\hat{U}_1$ operations, the total numbers of prepared states for 2D and 4D experiments are 4 and 8, respectively.

The 2D quantum processes are implemented using an HWP or a quarter wave plate (QWP) at different optical axis angles θ (see Sec. II B of the Supplemental Material [33]). As depicted in the insets of Fig. 3(c), the 4D quantum processes are realized using interferometers composed of BDs and HWPs (see Sec. III B of the Supplemental Material [33]), including the identity operation $\hat{I}_4$, the path-CNOT (controlled-NOT with path as the control and polarization as the target), and the pol-CNOT (controlled-NOT with polarization as the control and path as the target).

The $\hat{U}_2$ operations with $\varphi_{jk} = \pm\pi/2$ after quantum processes are realized using a quarter wave plate q_1 . This QWP is fixed at the bottom in the 2D experiments and is located at either the bottom-left or bottom-right position in the 4D experiments (see Sec. IIC of the Supplemental Material [33]). Finally, the combination of BDs and wave plates forms the measurement modules of the 2D and 4D experiments (see Secs. IID and IIIC of the Supplemental Material [33]).

IV. RESULTS

First, we present the experimental results for the direct characterization of 2D quantum processes. Based on Eqs. (17) and (18), the real and imaginary parts of the matrix elements $\{u_{jk}\}$ for the HWP and QWP operations with optical axis angle θ are plotted in Figs. 4(a) and 4(b), respectively. The experimental results agree well with the theoretical curves derived from the corresponding ideal matrices. The performance is quantified by the fidelity between the experimental $\hat{U}$ and the ideal $\hat{U}_{\text{theo}}$, given by $F(\hat{U}, \hat{U}_{\text{theo}}) = |\text{Tr}[\hat{U}^\dagger \hat{U}_{\text{theo}}]|^2 / (\text{Tr}[\hat{U}^\dagger \hat{U}] \text{Tr}[\hat{U}_{\text{theo}}^\dagger \hat{U}_{\text{theo}}])$ [37]. The average fidelities over the optical axis angles for the HWP and QWP operations are calculated as $\bar{F}_H = 99.8\% \pm 0.2\%$ and $\bar{F}_Q = 99.9\% \pm 0.1\%$, respectively. For each 2D quantum process, our method requires 13 experimental configurations.

We then show the experimental results for the direct characterization of three 4D quantum processes, including $\hat{I}_4$, path-CNOT, and pol-CNOT, in Figs. 5(a), 5(b), and 5(c), respectively, with blue and green bars representing the real and imaginary parts of the matrix elements of $\{u_{jk}\}$, and red bars indicating ideal values for comparison. The corresponding fidelities with respect to the ideal matrices are $99.97\% \pm 0.02\%$, $99.54\% \pm 0.07\%$, and $99.93\% \pm 0.11\%$, respectively. Exploiting prior knowledge of unitarity, our protocol requires $3d^2 + 1$ experimental configurations and therefore 49 configurations for $d = 4$. This reduction in configuration count relative to conventional full-process tomography is enabled by prior knowledge of unitarity, as in other unitary-specific

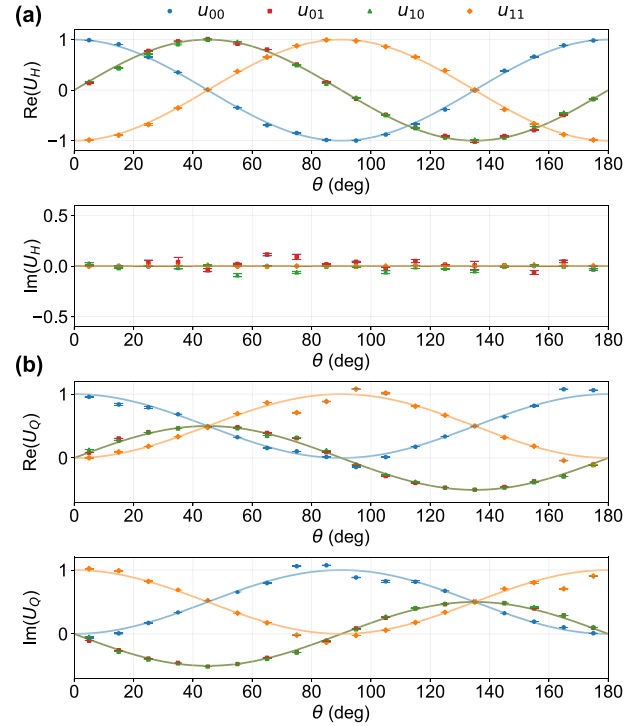


FIG. 4. Direct characterization of two-dimensional quantum process implemented using HWP and QWP. (a) The imaginary and real parts of matrix elements for $\hat{U}_H$ in different optical axis angles. (b) The imaginary and real values of matrix elements for $\hat{U}_Q$ in different optical axis angles. The solid lines are theoretical values of matrix elements from $\hat{U}_H$ and $\hat{U}_Q$, and the markers are experimental results. The error bars are calculated through 50 Poisson samplings.

characterization schemes that also achieve $O(d^2)$ measurement complexity [38,39].

V. DISCUSSION

In conclusion, we developed an erasure-based closed time-like curve framework within the temporal pseudo-density-matrix formalism. With the E-CTC interaction, we proposed a scheme for the direct characterization of arbitrary quantum processes. We experimentally demonstrated the protocol on two- and four-dimensionally photonic processes with average fidelities above 99%.

A key advantage of our scheme lies in its potential to simplify experimental control. While the number of experimental configurations for general processes is not reduced compared to standard tomography, our scheme benefits from selective computational-basis addressing with fixed superposition preparation and projections. Specifically, as shown in Eq. (9), the initial required state for E-CTC-assisted measurements is an equal superposition across the computational-basis states. The projective measurements on the ancilla and system are likewise performed in this equal superposition basis, as given by Eqs. (5) and (10). Furthermore, the coupling operations are conditioned on system states in the computational basis, as defined in Eqs. (3) and (4). These superposition settings remain fixed between configurations, although their implementation still requires

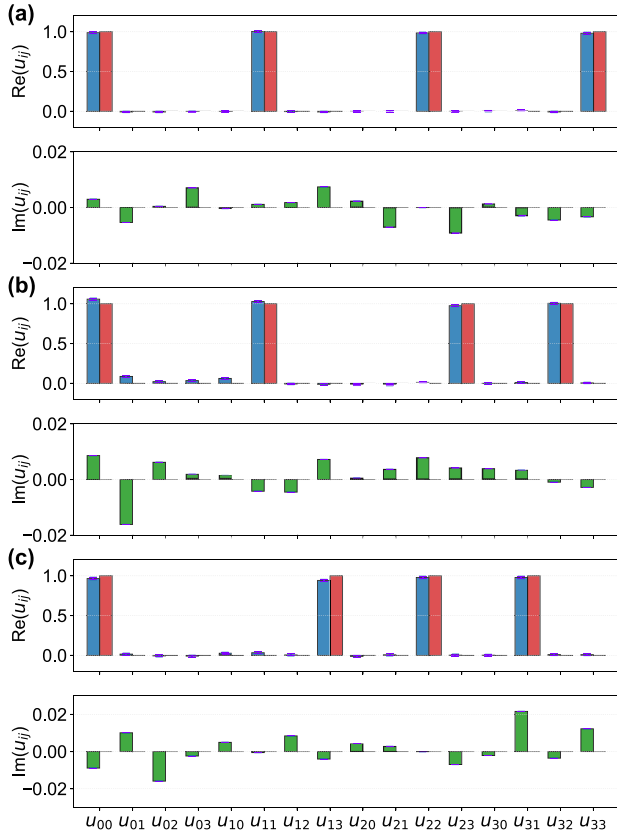


FIG. 5. Direct characterization of four-dimensional quantum processes. The corresponding real and imaginary parts of matrix $\hat{U}_4$ [panel (a)], path-CNOT [panel (b)], and pol-CNOT [panel (c)] gates. The blue and green outer bars represent experimental results. The red bars indicate the theoretical values of ideal processes, which are zero for imaginary parts of the processes investigated in experiments. The error bars are calculated through 50 Poisson samplings and displayed in purple.

fixed basis transformations. Additional computational-basis preparations and projections provide the diagonal elements and the magnitudes needed to resolve the sign ambiguity for coherent processes. This feature can simplify experimental implementation and reduce operational overhead, especially when computational-basis states are readily addressable.

Taking the photonic platform as an example, performing rotation operations in high-dimensional path (spatial) [40,41] and temporal [42,43] Hilbert spaces necessitates the use of complex, precisely controlled interferometers, whereas frequency-encoded rotations [44,45] can use electro-optic modulation and pulse shaping or nonlinear frequency conversion. Therefore, our method can facilitate the characterization

of quantum processes in such systems by varying selective addressing and ancilla operations while keeping superposition preparation and projections fixed during E-CTC-assisted measurements. For path-encoded systems, selective addressing is implemented through local operations on selected spatial modes. Beyond these control features, our protocol directly extracts individual matrix elements from measured probabilities, without matrix inversion or optimization over the full Choi matrix.

Closed timelike curves represent a fundamental concept arising from the efforts to understand our Universe. The paradoxes inherent in time travel have compelled existing quantum CTC models to adopt various strategies to enforce self-consistency. Specifically, Deutsch's CTC achieves consistency by requiring a fixed-point quantum state ρ_{CTC} determined by the CTC interaction, while the postselection-based CTC attains it through quantum teleportation to the past. In contrast to these imposed consistency conditions, our E-CTC model possesses an intrinsic self-consistency, which naturally arises from the temporal correlations encoded in the pseudo-density-matrix formalism, albeit at the cost of non-determinism. The E-CTC picture therefore describes an effective temporal interaction in a chronology-respecting experiment, without implying a physical closed timelike curve or a violation of causality. We believe that this model can contribute to understanding the interplay among determinism, quantum erasure, and quantum causality [46] in theory and experiment.

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R.Y. and Y.-X.G. conceived the original idea. R.Y., G.Y., and Y.-X.G. designed the experiment. R.Y., G.Y., W.Z., and D.-J.G. implemented the experiment. R.Y., G.Y., X.L., J.-N.Z., and Y.C. performed the data analysis. R.Y., G.Y., W.Z., Y.-X.G., and S.-N.Z. wrote the paper with input from all the authors. Y.-X.G. and S.-N.Z. supervised the project. All the authors discussed the results and contributed to the paper.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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