

Revisiting the physics of hole-conjugate fractional quantum Hall channels

D. C. Glattli^{✉,*}, C. Boudet, A. De, and P. Roulleau

Nanoelectronics Group, *Université Paris-Saclay*, CEA, CNRS, *SPEC*, 91191 Gif-sur-Yvette Cedex, France



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The fractional quantum Hall effect provides the first example of multiple quantum phases hosting quasi-particles not being fermions or bosons, but anyons. It is therefore important to investigate whether all these phases can be practical to detect these anyons. Here, we propose revisiting the intriguing physics of hole-conjugate Fractional Quantum Hall (FQH) phases with counter-propagating edge channels in the filling factor range $1/2 \leq \nu \leq 1$ whose properties may prevent anyon interference detection. In the seminal KFP model by Kane, Fisher, and Polchinski, upstream neutral and downstream charge modes, along with random incoherent interchannel tunneling, are considered essential for explaining the observed conductance at $\nu = 2/3$. In this work, we introduce a minimal model that qualitatively reproduces the edge physics of the $\nu = 2/3$ FQH phase without invoking neutral and charge modes. By inserting N fictive reservoirs—either Landauer reservoirs (LRs) or Energy-Preserving Reservoirs (EPRs)—along the counter-propagating channels to simulate edge channel mixing, we show that the conductance rapidly approaches the KFP fixed point of $2/3 \times (e^2/h)$ for both EPRs and LRs. While LRs produce equal charge and thermal relaxation lengths, EPRs exhibit an infinite thermal relaxation length, consistent with recent observations. Our model also predicts current noise in small Hall bars that varies linearly with voltage at large bias, with an apparent Fano factor $\simeq 0.19/\sqrt{N+1}$ for N LRs or an exponentially vanishing Fano factor for EPRs. Moreover, with a Quantum Point Contact at the center of the Hall bar, the model predicts the recently observed $0.5 e^2/h$ conductance plateau. Finally, early experiments reporting upstream excitations are found to be conveniently explained by upstream diffusive heat flow and δT noise, rather than by specific collective neutral modes. The results obtained in this study can be generalized to more complex hole-conjugate fractional quantum Hall phases.

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I. INTRODUCTION

The Fractional Quantum Hall Effect (FQHE) arises in two-dimensional electron system (2DES) under high perpendicular magnetic field B , at low temperature, when the filling factor $\nu = en_s h/B$ value is fractional— n_s being the electron density. At simple rational values of ν , the 2DES forms topologically insulating states. Conduction occurs on the edge via topological chiral edge channels. For $\nu = p/(2p+1)$, i.e., $1/3, 2/5, 3/7, \dots$, there are p co-propagating edge channels. For $\nu = p/(2p-1)$, with $p \geq 2$, counter-propagating edge channels occur. Early theoretical work [1,2] modeled the $\nu = 2/3$ edge by an outer chiral integer edge channel with conductance e^2/h and an inner counter-propagating fractional edge channel with conductance $-1/3 \times (e^2/h)$. However, this picture gives a total conductance of $4/3 \times (e^2/h)$, while all observations find $2/3 \times (e^2/h)$. Solving this puzzle, the Kane-Fisher-Polchinski (KFP) model [3,4] showed, in a Chiral Luttinger Liquid (CLL) picture, that the combination of Coulomb coupled counter-propagating modes with spatially

random disorder on the edges leads to a downstream chiral charge mode with conductance $2/3$ and a bosonic upstream neutral mode. Later, experiments led by the Weizmann group using current noise measurements [5] found evidence of upstream excitation, therefore assigned to the predicted neutral mode. The puzzling physics of the $2/3$ edge has then led to a flurry of theoretical models [6–14] inspired by the KFP model to improve it or to adapt it to situations where more complex reconstruction of the edge occurs [15,16]. Concurrently, experimental works based on conductance, noise, or thermal conductance have explored the nature of the edge [17–25]. Recently, artificial counter-propagating $2/3$ edge channels have been realized by coupling left and right regions of different (1 and $1/3$) filling factors [25–27]. However, the KFP model, although widely accepted, lacks physical intuition for nonexperts. It is difficult to separate what comes from the bosonic modes entering in the CLL picture from what comes from the disorder that mixes the counter-propagating modes by tunneling. Also, it becomes clear from a study of the propagation of neutral and charge modes that neutral modes cannot propagate beyond a micron size charge equilibration length (see Ref. [12], Appendix D, and Fig. 6). In the present work, we will present a new intuitive and powerful approach to model hole-conjugate edge channels, which solves this issue and yields predictions compatible with all observations. We show that charge equilibration between counter-propagating channels is the only relevant ingredient.

*Contact author: christian.glattli@cea.fr

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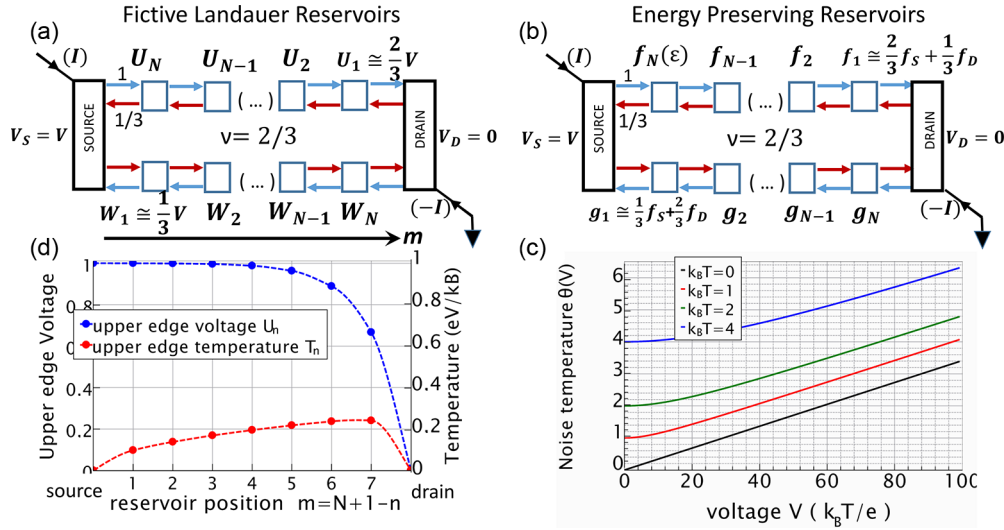


FIG. 1. (a) Hall bar with N fictive Landauer reservoirs inserted on upper and lower edges. (b) Same as panel (a) but with energy-preserving reservoirs. (c) Noise temperature $\Theta(V) = S_S/4k_B G_N$ of the Hall bar versus voltage for several drain-source contact temperatures. The noise is computed for $N = 7$ LR. (d) Landauer reservoirs: spatial voltage (blue) and temperature (red) distribution along the upper edge for $N = 7$. $m = N + 1 - n$ denotes the spatial position from left to right.

II. THE MODEL

Here, we provide a novel approach using basic tools of mesoscopic physics. The model discussed in the following should be better viewed as a toy model. It is not intended to provide a microscopic description of channel mixing; it is intended to explore, in an easy way, what the minimal ingredients are that lead to results compatible with observations.

We consider a Hall bar, with physical source and drain Ohmic contacts on the left and right ends [see Figs. 1(a) and 1(b)]. To enable charge equilibration between counter-propagating channels, N fictitious charge reservoirs are inserted along the upper and lower edges. The use of Ohmic contacts has been recently considered in Refs. [28,29] to study the heat flow along quantum Hall edge; however, we do not consider here charging effects. We concentrate, as a first step, only on the charge equilibration properties of the reservoirs. We consider two generic cases schematically shown in Fig. 1. In Fig. 1(a), the reservoirs are fictive Ohmic contacts described as Landauer reservoirs (LRs). In Fig. 1(b), the reservoirs are Energy-Preserving Reservoirs (EPRs), as introduced in Ref. [30]. In both cases, the 1 and $-1/3$ outer and inner channels ballistically propagate between two consecutive reservoirs. We recall that for an LR, quasiparticles are absorbed whatever their energy and are emitted following a Fermi distribution having the temperature and the chemical potential of the reservoir (here expressed in voltage units). LRs thus provide full equilibration of both charge and energy between the counter-propagating channels. An opposite situation occurs for EPRs: An excitation hitting the reservoir at energy ϵ is absorbed and re-emitted at the same energy, and charge equilibration alone occurs. Note that, using reservoirs, we will not address a third possible regime: coherent channel mixing [27,31].

In the following, we find that both LR and EPR lead to the same conductance values for N reservoirs, rapidly converging with N toward the KFP $2/3 \times (e^2/h)$ conductance fixed point.

For LR, we also recover the zero and negative thermal conductances, respectively, predicted for $\nu = 2/3$ or $3/5$ by the KFP model and observed in Refs. [18,20,23]. We emphasize that the case of energy-preserving channel equilibration was overlooked in the literature, and our result shows that dissipative processes are not necessary to reach the KFP conductance fixed point. However, beyond conductance, comparing LRs and EPRs leads to quantitatively different temperature distribution and current noise.

III. CONDUCTANCE AND CURRENT NOISE

We first concentrate on predictions for conductance and current noise for a $2/3$ Hall bar for both LRs and EPRs.

For LRs, the fictive contacts are evenly distributed and floating at potential U_n (W_n) along the upper or lower edge, with no external current entering them. The charging energy is considered negligible. For convenience, we set the conductance quantum $G_0 = e^2/h$ equal to unity. The Landauer-Büttiker relation for the n th fictive reservoir on the upper edge channel is

$$0 = \frac{4}{3}U_n - U_{n+1} - \frac{1}{3}U_{n-1}. \quad (1)$$

A similar relation holds for the potentials W_n of the lower edge. For energy-preserving reservoirs, which describe elastic processes, an electron from the integer channel absorbed in the reservoir at energy ϵ is re-emitted as an electron in the same channel or as a quasiparticle in the fractional channel at the same energy. Current conservation at energy ϵ yields

$$0 = \frac{4}{3}f_n(\epsilon) - f_{n+1}(\epsilon) - \frac{1}{3}f_{n-1}(\epsilon), \quad (2)$$

with $f_n(\epsilon)$ the nonequilibrium energy distribution of the n th EPR of the upper edge channel. A similar set of equations holds for the energy distributions $g_n(\epsilon)$ of the lower edge channel. Here, we must clarify the use of energy distribution

$f_n(\epsilon)$ for the $-1/3$ fractional channel. As one considers only perfect ballistic transmission between reservoirs, the amount of current carried in the energy interval $d\epsilon$ is $-e^* f_n(\epsilon) d\epsilon$, $e^* = e/3$, consistent with the $-e^2/3h$ inner channel conductance. We think that treating the distribution as Fermi-like and disregarding chiral Luttinger Liquid aspects is, at this stage, a reasonable simplification not qualitatively changing the results below.

Using appropriate Landauer-Büttiker boundary conditions on source and drain contacts at potentials V_S and V_D , respectively, we find the following voltage distribution for LR (see Appendix A):

$$U_n = \frac{V_S(1 - e^{-n\lambda}) + V_D(e^{-n\lambda} - e^{-(N+1)\lambda})}{1 - e^{-\lambda(N+1)}}, \quad (3)$$

with $\lambda = \ln 3$. Similarly, the energy distribution for the EPRs is

$$f_n(\epsilon) = \frac{f_S(\epsilon)(1 - e^{-n\lambda}) + f_D(\epsilon)(e^{-n\lambda} - e^{-(N+1)\lambda})}{1 - e^{-\lambda(N+1)}}, \quad (4)$$

with $f_{S(D)}(\epsilon)$ the Fermi distributions of the source (drain) contacts at temperature T and voltage $V_S(V_D)$, respectively. For the lower edge channel, exchanging source and drain voltages (energy distributions) in Eq. (3) [Eq. (4)] leads to similar expressions for W_n [or $g_n(\epsilon)$]. The conductance for N reservoirs is found to be

$$G_N = \frac{2}{3} \frac{1 + \frac{1}{3}e^{-\lambda N}}{1 - \frac{1}{3}e^{-\lambda N}} \quad (5)$$

for both LR and EPRs. In general, as long as conductance properties are concerned, both dissipative and nondissipative reservoirs remarkably give the same conductance. The first five values of G_N when N goes from 0 to $N = 4$ are, respectively, $2(2/3)$, $\frac{5}{4}(2/3)$, $\frac{14}{13}(2/3)$, $\frac{41}{40}(2/3)$, and $\frac{122}{121}(2/3)$. With just two fictive reservoirs, the $2/3$ KFP conductance fixed point is reached within 7.7% accuracy.

It is interesting to compare the current flowing through the upper and lower edge channels. For the upper edge, we find a *downstream* current converging toward $(2/3)e^2V/h$:

$$I_{\text{downstr}} = \frac{2}{3}V \frac{1}{1 - e^{-(N+1)\lambda}}, \quad (6)$$

while for the lower edge we find an *upstream* current exponentially vanishing with N :

$$I_{\text{upstr}} = \frac{2}{3}V \frac{e^{-(N+1)\lambda}}{1 - e^{-(N+1)\lambda}}. \quad (7)$$

We can repeat the same approach for the $3/5$ FQHE edge that is made of an outer (1) downstream integer channel and two inner ($-1/5$) upstream fractional channels. For LR, Eq. (1) now becomes

$$0 = \frac{7}{5}U_n - U_{n+1} - \frac{2}{5}U_{n-1}, \quad (8)$$

and similarly in the case of EPR, Eq. (2) becomes

$$0 = \frac{7}{5}f_n(\epsilon) - f_{n+1}(\epsilon) - \frac{2}{5}f_{n-1}(\epsilon). \quad (9)$$

For the computation of LR voltages and EPR energy distribution, Eqs. (3) and (4) hold with now $\lambda = 5/2$. The

conductance for N reservoirs and $3/5$ Hall bar becomes

$$G_N = \frac{3}{5} \frac{1 + \frac{2}{5}e^{-\lambda N}}{1 - \frac{2}{5}e^{-\lambda N}}. \quad (10)$$

We now concentrate on the case of dissipative Landauer reservoirs to investigate dissipation and noise at $\nu = 2/3$. For simplicity, we set the voltages $V_S = V$ and $V_D = 0$. Along the upper channel, from left to right, we observe from Eq. (3) that the voltages V_n remain almost equal to $V_S = V$ all along the channel and drop rapidly to $V_D = 0$ near the drain contact [see Fig. 1(d)]. Conversely, from right to left, the lower channel voltage remains almost zero and rapidly rises to V near the source. This feature has several implications: We expect strong dissipation near the drain (source) for the upper (lower) edge and almost no dissipation along the Hall bar. Indeed, we find that the power dissipated in the n th upper edge fictive contact is (see Appendix A) $P_n = \frac{1}{2}(\frac{4}{3}V)^2 e^{-2\lambda n} / (1 - e^{-(N+1)\lambda})^2$. For the lower edge, the power $\bar{Q}_n = P_n$. This confirms the existence of “hot spots” near the drain and the source for, respectively, the upper and lower edge channels, as found in previous theoretical models [6,9,10]. To check that the calculated power expressions make sense, we have computed the total power P in the Hall bar, summing the drain, the source, and the upper and lower edge contributions. We find a total power $P = I \cdot V$, as it should be. Knowing the set of P_n , we can calculate the temperature distribution T_n (T'_n) for the upper (lower) edge channel using the universal quantum thermal conductance κ_0 . Equal thermal conductance for elementary 1 and $-1/3$ chiral channels was predicted in Ref. [32]. Fixing the temperature of the physical drain and source reservoirs equal to T , the temperature T_n of the fictive reservoirs is obtained by solving the standard heat flow equations (see Appendix A). The solution is

$$(k_B T_n)^2 = (k_B T)^2 + \frac{3}{(2\pi)^2} (eV)^2 [1 - n/(N+1) - e^{-2\lambda n}]. \quad (11)$$

The same temperature distribution $T'_n = T_n$ is found for the lower edge. For the upper edge, from right to left, we observe a square-root-like increase of the spatial temperature distribution followed by a sudden drop to T near the drain [see Fig. 1(d)]. A similar symmetric spatial variation occurs for the lower edge. Importantly, the temperature increase along the edge channel signals a diffusive *upstream* heat flow contrasting with the downstream charge flow. As our model does not require introducing a specific neutral collective mode, the counter-propagating heat flow plays its role. Our toy model thus provides a new perspective on the nature of the upstream excitations. We will show later that the upstream heat flow explains the upstream signatures observed in the experiment [5]. We emphasize that for $\nu = 2/3$ the upstream heat flow occurs because of the finite voltage bias inducing dissipation in each fictive reservoir. In contrast, in absence of the current, Sec. IV will show that the thermal conduction is diffusive and nonchiral as found in Ref. [32].

Knowing the temperature distribution along the edge of the $2/3$ Hall bar, we now can compute the nonequilibrium hot electron thermal noise for $V \neq 0$. To proceed, we consider all independent thermal current noise sources gen-

erated between each pair of fictive reservoirs. For the pair $(n, n-1)$ on the upper edge, the current noise spectral density is $S_I^{n,n-1} = 2(k_B T_n + \frac{1}{3}k_B T_{n-1})$, with the T_n given by Eq. (11). A fluctuation $\delta I_{n,n+1}$ between an $(n, n+1)$ fictive contact pair is found giving rise to the fluctuation $\delta I_S^{n,n+1} = \frac{2}{9}\delta I_{n,n+1}e^{-(N-1-n)\lambda}/(1-e^{-(N+1)\lambda})$ at the source contact and, from current conservation, an equal fluctuation in the drain contact. Adding the uncorrelated mean square fluctuations of the noise of all pairs $(n, n+1)$ provides the total drain current noise. Setting source and drain temperatures to $T = 0$ for clarity, we find that the total current noise power can be expressed as

$$S_{I_S} = F_{LR} 2eI, \quad (12)$$

with the (fake) Fano factor F_{LR} given by

$$F_{LR} = \frac{\sqrt{3}}{\pi\sqrt{N+1}} \left[\frac{1}{3} + \frac{2}{27} \sum_{m=2}^N e^{-2(m-1)\lambda} \times \left(\sqrt{m} + \frac{1}{3}\sqrt{m-1} \right) \right]. \quad (13)$$

Summation in Eq. (13) is valid for large N and gives $F_{LR} \simeq 0.192\,982\,03\dots/\sqrt{N+1}$. For large lengths, the nonequilibrium hot electron thermal noise vanishes like the square root of $\sqrt{N+1}$, the Hall bar length. Similar length dependence was predicted using different theoretical approaches [10,33] and experimentally observed [25]. This is in agreement with the diffusive heat flow leading to zero thermal conductance for infinite length predicted in [32] for $\nu = 2/3$. We observe from the summation in Eq. (13), where m stands for $m = N - n$, that the main noise contributions do not come from the “hot spots” (near the drain for the upper edge) but from noise sources situated close to the source having the smallest noise temperature T_n (n close to N). For finite temperature T , the asymptotic linear variation of the noise with applied voltage V becomes accurate for $eV \gtrsim 35k_B T$ [see Fig. 1(c)].

We now turn to the dissipation and the current noise for the case of energy-preserving reservoirs instead of LR. Because of the nature of the fictive EPRs, we do not expect any dissipation along the upper and lower channels except at source and drain that are physical LR. Indeed, the sum of the power dissipated in the source and the drain is found equal to the Joule power IV (see Appendix B). Thus, contrasting with the Landauer reservoir case where the thermal and charge equilibration lengths were found to be equal, the thermal length is here infinite. These two opposite thermal length behaviors have been experimentally observed [21,34,35]. For EPR, we expect a current noise different from the LR case. The nonequilibrium energy distributions f_n and g_n given by Eq. (4) are occupation probabilities taking values between 0 and 1. This leads to fluctuations of the state occupation number at energy ϵ equal to $f_n(\epsilon)[1 - f_n(\epsilon)]$. This results in the current noise spectral density associated with an $(n+1, n)$ EPR pair equal to $S_I^{n+1,n} = 2\frac{e^2}{h}(k_B\Theta_{n+1} + \frac{1}{3}k_B\Theta_n)$, where we have defined the fictive temperatures $k_B\Theta_n = \int_{-\infty}^{\infty} d\epsilon f_n(\epsilon)[1 - f_n(\epsilon)]$.

The total noise observed in the drain or source can be calculated as we did above for LR, replacing T_n by Θ_n . In contrast with the LR case, the noise is found exponentially

TABLE I. Fake Fano factors for different lengths N . For small $N = 5-7$, F_{LR} is not obtained from the large N expression [Eq. ((13))] but from exact calculation at finite N .

N	F_{LR}	$F_{LR}\sqrt{N+1}$	F_{EPR}	$3^N \cdot F_{EPR}$
5	0.079 11	0.1937	0.002 698	0.6556
6	0.073 05	0.1933	0.000 908	0.6621
7	0.068 27	0.1931	0.000 304	0.6649
∞	0	0.192 98...	0	2/3

vanishing with the size N (see Appendix B). For the fictive temperatures growing linearly with V , we can define the fake Fano factor F_{EPR} . For large N , we found $F_{EPR} \simeq 0.666(1/3)^N$ contrasting with $F_{LR} \propto N^{-1/2}$. Such exponential dependence was reported in Ref. [35]. Table I gives Fano factor values for some finite N values.

IV. THERMAL CONDUCTANCE

We now address the thermal conductance at $\nu = 2/3$ and $\nu = 3/5$. As the prediction of thermal conductances in these two regimes, confirmed by observations, was an important check of the KFP model, it is interesting to see whether our model reproduces these predictions. Only the case of LR will be considered here as, EPR exchanging energy but no heat, thermal conductance is not defined. To proceed, we consider a situation where no voltage bias is applied between the left and right Ohmic contacts. Instead, different temperatures are applied between left and right contacts with temperatures T_L and T_R , respectively. The quantity of interest here is no longer the current but the heat flow.

We consider the thermal flow through the upper edge. Similar calculation holds for the lower edge. Contrary to the above calculation, as there is no current, no dissipation occurs in any fictive reservoir. The temperature T_n of the n th reservoir is expected between T_L and T_R . Expressing that the sum of the heat entering and exiting the n th reservoir cancels, we get for $\nu = 2/3$:

$$0 = 2T_n^2 - T_{n-1}^2 - T_{n+1}^2. \quad (14)$$

The solution is of the form $T_n^2 = A + Bn$, with A and B determined by the boundary conditions $T_{n=0} = T_R$ and $T_{n=N+1} = T_L$. This gives

$$T_n = \sqrt{T_R^2 + (T_L^2 - T_R^2)n/(N+1)}. \quad (15)$$

The heat flow is $\dot{Q} = \kappa_0(T_L^2 - T_R^2)/2(N+1)$. For $\nu = 2/3$, the thermal conduction is thus found to be diffusive. Chiral features are absent and the thermal conductance vanishes in the large N limit as $1/\sqrt{N+1}$. This agrees with the predictions of Ref. [32] based on the KFP approach.

In contrast, for $\nu = 3/5$, Ref. [32] predicts a surprising ballistic-like thermal conductance given by $-\kappa_0 T$, i.e., corresponding to upstream heat flow. In the present model, which is based on a series of Landauer reservoirs randomizing charge and energy between them, it seems unlikely that a ballistic-like thermal conductance can appear. However, this is what we are going to show below. Writing the heat balance equation for

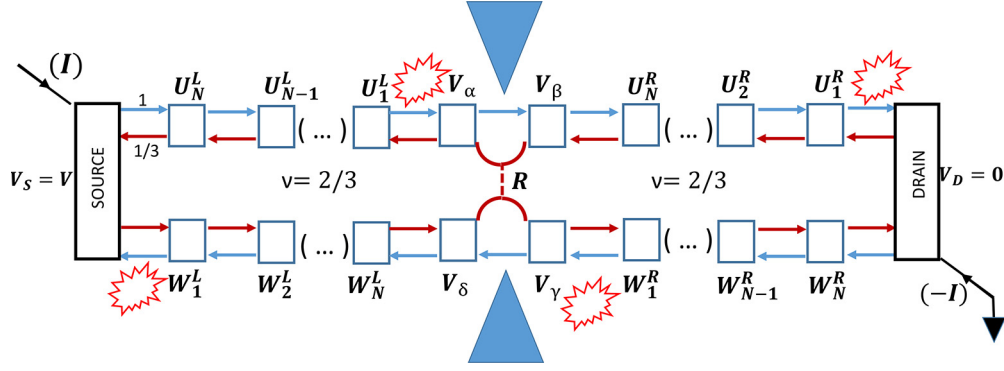


FIG. 2. $2/3$ Hall bar with a QPC in its middle. N LRs are inserted along the edges on each side of the QPC. In red are indicated the “hot spots” where most dissipation occurs.

$\nu = 3/5$ yields

$$0 = 3T_n^2 - T_{n+1}^2 - 2T_{n-1}^2. \quad (16)$$

The complete solution is $T_n^2 = A + B2^n$, with A and B being determined by the boundary conditions for $n = 0$ and $n = N + 1$. This gives the distribution of the square temperature along the edge:

$$T_n^2 = \frac{T_R^2 - 2^{-N-1}T_L^2 + 2^{n-N-1}(T_L^2 - T_R^2)}{1 - 2^{-N-1}}. \quad (17)$$

The heat flow going from reservoir n to reservoir $n - 1$ is $\dot{Q}_n = (\kappa_0/2)(T_n^2 - T_{n-1}^2)$. Its expression is

$$\dot{Q}_n = -(\kappa_0/2) \frac{T_R^2 - T_L^2 2^{-(N+1)}}{1 - 2^{-(N+1)}}. \quad (18)$$

The heat flow does not depend on n as expected for energy conservation and is found to flow upstream. For large N , we get $-\kappa_0 T_R^2/2$. This is equivalent to the ballistic heat flow of single ballistic upstream channel in perfect agreement with Ref. [32].

V. QUANTUM POINT CONTACT CONDUCTANCE AT $\nu = 2/3$

We now address practical situations exploiting the present model. While we have considered above a homogeneous Hall bar, we now investigate the conductance when a quantum point contact is located in the middle of the Hall bar and induces tunneling between the upstream upper and lower fractional ($-1/3$) inner edge channels while the downstream integer channel is transmitted. To proceed, we introduce N fictive reservoirs on the left side of the QPC (for both the lower and upper edges) and, symmetrically, N reservoirs on the right side (see Fig. 2). We seek for a solution of the upper and lower potentials $U_n^{L(R)}$ and $W_n^{L(R)}$, respectively, on the right (R) and left (L) sides of the QPC. The boundary conditions, given by Landauer-Büttiker relations at the source and the drain and at the QPC (see Appendix C), provide a determination of the unknown potentials. For large N , the QPC conductance is found to be

$$G = \frac{2}{3} \frac{e^2}{h} \frac{1 + \mathcal{R}/2}{1 + \mathcal{R}}, \quad (19)$$

where $\mathcal{R}$ is the reflection probability of the fractional inner channel. Note that, as we consider partitioning a fractional channel, Luttinger liquid corrections may apply and we should better consider $\mathcal{R}$ as the ratio of the backscattered current to the current incoming on the QPC. The graph of QPC conductance versus $\mathcal{R}$ is given in Appendix C. Interestingly, for full reflection, we get the half conductance quantum $\frac{e^2}{2h}$. A similar result was found in Ref. [24] in a model attempting to explain the observed 0.5 conductance plateau and in the theoretical works of Refs. [14,36]. We find the same conductance $G(\mathcal{R})$ for energy-preserving reservoirs instead of LRs, confirming that conductance alone cannot distinguish between dissipative and nondissipative regimes.

It is interesting to look at the potential distribution along the edges for the LR case. Starting from the source contact at V , the upper edge voltage remains almost equal to V but quickly drops near QPC to reach $V_\alpha = V \frac{1+\mathcal{R}/2}{1+\mathcal{R}}$. Then, on the upper right of the QPC, starting at $V_\beta = V_\alpha$, the voltages remain almost constant and rapidly drop near the drain at $V_D = 0$. Similarly, starting from the drain, the lower edge voltages remain almost zero and rapidly increase near the QPC to reach $V_\gamma = V \frac{\mathcal{R}/2}{1+\mathcal{R}}$. On the left of the QPC, the voltages remain almost equal to $V_\delta = V_\gamma$ to finally rapidly increase near the source to V . The rapid drops of potentials being associated with dissipation, we can identify two new “hot spots” on both sides of the QPC as shown in Fig. 2.

VI. DELTA-T NOISE VERSUS NEUTRAL MODE PARTITION NOISE

As another application of the model, we now consider the setup of Fig. 3, which is similar to Bid’s experiment reporting counter-propagating neutral modes [5]. This seminal experiment had a strong impact on the community as it was the first to report the manifestation of upstream effects. However, exploiting the results of Ref. [12], it is clear that neutral modes are unable to propagate more than very few charge relaxation lengths *at all frequencies*. A calculation of the neutral and charge mode attenuation lengths is shown in Fig. 7. Then, it is unlikely that the neutral mode can be the *direct* source of partition noise in the experiment [5] where the distance between the source of neutral modes is about 40 times the charge equilibration length leading to an attenuation factor of

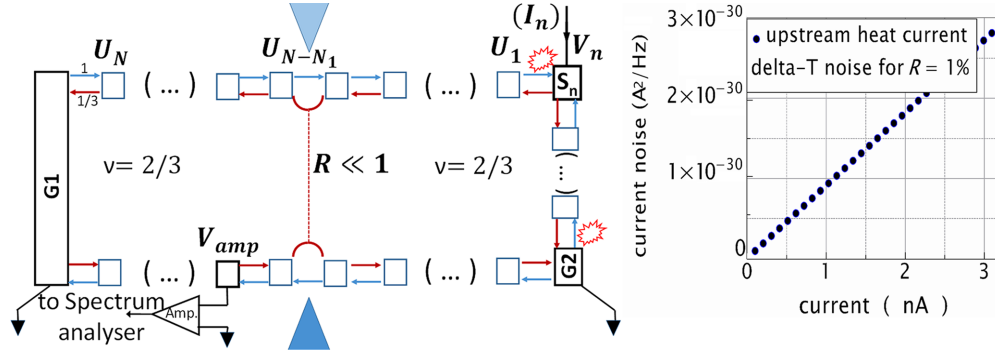


FIG. 3. Left: Setup similar to Bid’s experiment [5]. Weak reflection $\mathcal{R} \ll 1$ is considered. The upper upstream heat flow heats the upper side of the QPC. Right: Calculated delta- T noise.

4×10^{-18} . Here, we show that the thermal gradient due to the counter-propagating heat flow found above (Sec. III) is responsible for the observed noise in Ref. [5]. In the setup of Fig. 3, the source contact (S_n), located at the upper right of a QPC, is at potential V_n and injects the current I_n . The left and right contacts (G_1 and G_2) of the Hall bar are grounded. A fourth physical contact (V_{amp}) on the lower left side of the QPC records the voltage fluctuations $S_{V_{amp}}$, from which the current noise $S_I = S_{V_{amp}}(2e^2/3h)^2$ can be inferred.

For simplicity, $k_B T = 0$ and the QPC weakly backscatters the $-1/3$ inner edge ($\mathcal{R} \ll 1$) leaving the Hall bar almost unperturbed. The Hall bar edges are modeled by $N \gg 1$ fictive LRs. According to the above results, the current I_n flows from S_n to contact G_2 . A hot spot forms at G_2 and on the left side of S_n . This generates an *upstream heat flow* flowing along the upper edge and heating the upper side of the QPC. For N_1 LRs between the QPC and G_1 , the temperature T_{up} of the upper channel at the QPC is, according to Eq. (11),

$$k_B T_{up} = \frac{\sqrt{3}}{2\pi} e V_n \sqrt{\frac{N_1}{N}}. \quad (20)$$

In contrast, the lower channel, connected to zero-temperature zero-voltage contacts remains at $T = 0$. This situation generates a “delta- T ” shot noise [37–41] at the QPC whose one side is at T_{up} and the other side at $T = 0$. A similar conclusion, but in a full CLL approach, was found in Ref. [6]. T_{up} is given by (see Ref. [38])

$$S_I = 2(1/3) \left(\mathcal{R} \frac{2e^2}{3h} \right) \ln(2) k_B T_{up}, \quad (21)$$

where the $1/3$ factor comes from the tunneling of quasiparticle of charge $e^* = e/3$. For $N_1 = N/2$, zero temperature, and transmission 0.99 ($\mathcal{R} = 1\%$), $I_n = 3$ nA gives $S_I \simeq 2.8 \times 10^{-30} \text{ A}^2/\text{Hz}$ —a value close to the reported experimental value. If LRs are now replaced by EPRs, the distribution function f_{N-N_1} at the location of the QPC is close to the $T = 0$ Fermi distribution of contact G_1 . Therefore, *no delta- T noise* is expected.

VII. SUMMARY

To conclude, the present model provides a novel, intuitive, and powerful way to obtain the conductance, the noise, the thermal conductance, and more general counter-intuitive

features of hole-conjugate FQH edge channels. We have addressed particularly the $2/3$ and $3/5$ fractional edges. The model also predicts the anomalous $0.5e^2/h$ QPC conductance plateau at $\nu = 2/3$. The model suggests a paradigm shift in our understanding of the edge physics: Taking into account interchannel charge equilibration is the only requirement to get all the striking and counter-intuitive features of hole-conjugate FQHE. Collective charge and neutral modes are ignored as a first step for simplicity. When the conductor is biased by a finite current, the model gives a new interpretation of upstream excitations as a counter-propagating heat flow detectable by its delta- T noise. A Bid-type experiment [5] observing noise would signal a dissipative equilibrium regime (i.e., LRs) generating delta- T noise. In contrast, observing no noise would signal no thermal equilibration (i.e., EPRs). Finally, our model has taught us that CLL physics, which emphasizes the role of collective modes, may not be the initial step necessary for describing $2/3$ physics. Including capacitance to ground and mutual reservoir capacitances in our model can be done later to incorporate neglected CLL physics and describe the AC regime [12,25,42] and chiral Luttinger effects.

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APPENDIX A: HALL BAR CONDUCTANCE FOR N FICTIVE LRS

To compute the Hall bar conductance with LRs inserted between source and drain, we use Landauer-Büttiker relations. The boundary conditions are as follows: A current I sent by the external circuit enters the physical source contact, and accordingly, a current $-I$ enters the drain contact. The voltages of the source (drain) contacts are V_S (V_D), respectively. The fictive LR voltages are U_n (W_n) for upper (lower) edges. We

have

$$I = \frac{4}{3}V_S - \frac{1}{3}U_N - W_1, \quad (\text{A1a})$$

$$0 = \frac{4}{3}U_n - U_{n+1} - \frac{1}{3}U_{n-1}, \quad (\text{A1b})$$

$$0 = \frac{4}{3}W_n - W_{n+1} - \frac{1}{3}W_{n-1}, \quad (\text{A1c})$$

$$-I = \frac{4}{3}V_D - U_1 - \frac{1}{3}W_N, \quad (\text{A1d})$$

with n running from 1 to N following a counterclockwise (upstream) order. Seeking for a solution $\propto \exp(-n\lambda)$, we find $\lambda = 0$ or $\ln(3)$. The general solution is a linear combination of these two solutions whose coefficients are determined by the source and drain boundary conditions (A1a) and (A1d), which leads to

$$U_n = \frac{V_S(1 - e^{-n\lambda}) + V_D(e^{-n\lambda} - e^{-(N+1)\lambda})}{1 - e^{-\lambda(N+1)}}, \quad (\text{A2})$$

with $\ln(\lambda) = 3$. Using Eq. (A1d) [or Eq. (A1a)], we find the conductance $G_N = (2/3) \frac{1+(1/3)^{(N+1)}}{1-(1/3)^{(N+1)}}$ for N reservoirs, including the case $N = 0$. A similar computation can be done for $\nu = 3/5$, with $\lambda = 5/2$, and will not be repeated here.

To compute the temperature of each floating fictive reservoir, we set the voltages $V_S = V$ and $V_D = 0$ for simplicity and we first calculate the Joule power P_n dissipated in each reservoir:

$$\begin{aligned} P_n &= \frac{1}{2} \left[(U_{n+1} - U_n)^2 + \frac{1}{3} (U_n - U_{n-1})^2 \right] \\ &= \frac{1}{2} \left(\frac{4}{3} V \right)^2 e^{-2n\lambda} / (1 - e^{-(N+1)\lambda})^2. \end{aligned} \quad (\text{A3})$$

A similar calculation holds for the n th reservoir of the lower edge with power $Q_n = P_n$. To check that the power calculated makes sense, we compute the total power P dissipated in the Hall bar in the limit $N \rightarrow \infty$. The Joule power dissipated at the drain and the source by the upper and the lower edge channels are, respectively, $P_D = P_S = \frac{1}{2} [V(1 - e^{-\lambda})^2]$:

$$P = P_D + P_S + 2 \sum_{n=1}^N P_n = V^2 \left[\left(\frac{2}{3} \right)^2 + \frac{1}{8} \left(\frac{4}{3} \right)^2 \right] = IV, \quad (\text{A4})$$

as it should be.

To obtain the temperature distribution along the edge, we have to solve the following heat flow equation:

$$P_n = \frac{\kappa_0}{2} (T_n^2 - T_{n-1}^2 + T_n^2 - T_{n+1}^2). \quad (\text{A5})$$

As $P_n \propto e^{-2n\lambda}$, the general solution can be written as $T_n^2 = Ae^{-2n\lambda} + Bn + C$, where A , B , and C are determined from the drain and source thermal boundary conditions. The solution, valid for large N , is given by Eq. (11) in the main text. For small N , values of T_n are given in Table II for $k_B T = 0$ and $V = 1$.

Once we have the set of T_n values, we can compute the (hot) thermal (chiral) current noise source $S_I^{n+1,n} = 2k_B [T_{n+1} + \frac{1}{3}T_n]$ between pair of reservoirs $(n+1, n)$ on the

TABLE II. Distribution of temperature T_n for the n th Landauer reservoir n , for $N = 4-7$. The T_n are calculated for unit voltage V and $T = 0$.

N	T_1	T_2	T_3	T_4	T_5	T_6	T_7
4	0.2297	0.2121	0.1747	0.1237			
5	0.2345	0.2232	0.1949	0.1593	0.1126		
6	0.2382	0.2310	0.2082	0.1805	0.1474	0.1042	
7	0.2409	0.2367	0.2177	0.1949	0.1688	0.1378	0.0974

upper edge and calculate, for each independent pair, the induced source and drain fluctuations. A similar calculation is done for the lower edge. We found that the current noise $S_{I_S}^{(n+1,n)}$ viewed at the source contact due to the $(n+1, n)$ pair is $S_{I_S}^{(n+1,n)} = S_I^{n+1,n} \left(\frac{2}{9} \frac{e^{-(N-n-1)\lambda}}{(1-e^{-(N+1)\lambda})} \right)^2$. Adding all independent current noise of each pair for both upper and lower edges and adding the drain and source contact contributions, we get the total noise of the Hall bar. It is interesting to note the following. The hottest temperature for the upper (lower) edge occurs close to the drain (source), forming “hot spots” (see Fig. 1), but the current noise contribution is exponentially suppressed by a factor $\simeq e^{-2N\lambda}$. On the contrary, the current noise sources close to the source (drain) contact contribute with the largest weight while they have the lowest noise temperature. This corresponds to “noise spots.” However, here no shot noise occurs in the present toy model, only thermal noise with temperature T_n rising linearly with V . As the temperatures T_n near the source scale as $1/\sqrt{N+1}$, the nonequilibrium noise vanishes for large length.

We, thus, get the effective noise temperature $\tilde{\Theta}_N(V)$ of the whole $2/3$ Hall bar defined as $S_I^N = 4G_N k_B \tilde{\Theta}_N(V)$, for N fictive probes when a drain-source voltage V is applied. For zero drain and source temperatures, $\tilde{\Theta}_N(V) \propto V$. To characterize the noise, we define the effective (or fake) Fano factor by $2eI = 2eG_N V$, i.e., $F_{LR} = 2k_B \tilde{\Theta}(V)/eV$.

Table I, in the main text, gives values of the fake Fano factor. The values for $N=5-7$ result from an exact numerical calculation for N reservoirs. We see that for $N \leq 5$, the large $N \rightarrow \infty$ limit given by Eq. (13) is almost reached.

APPENDIX B: HALL BAR CONDUCTANCE FOR N FICTIVE EPRs

The calculation of the unknown distributions f_n and g_n for the n th reservoir of the upper and lower edges is very similar to the calculation of potentials for LRs. The boundary conditions are the Fermi distributions $f_S(\epsilon)$ and $f_D(\epsilon)$ at, respectively, the source and drain reservoirs. We get the set of following linear equations for the upper edge:

$$0 = \frac{4}{3}f_N(\epsilon) - f_S(\epsilon) - \frac{1}{3}f_{N-1}(\epsilon), \quad (\text{B1a})$$

$$0 = \frac{4}{3}f_n(\epsilon) - f_{n+1}(\epsilon) - \frac{1}{3}f_{n-1}(\epsilon), \quad 1 \leq n \leq N-2, \quad (\text{B1b})$$

$$0 = \frac{4}{3}f_1(\epsilon) - f_2(\epsilon) - \frac{1}{3}f_D(\epsilon). \quad (\text{B1c})$$

A similar set of equation holds for the lower edge. The general solution is

$$f_n(\epsilon) = \left(1 - \frac{e^{\lambda(N+1-n)} - 1}{e^{\lambda(N+1)} - 1}\right) f_S(\epsilon) + \frac{e^{\lambda(N+1-n)} - 1}{e^{\lambda(N+1)} - 1} f_D(\epsilon) \quad (\text{B2})$$

for $1 \leq n \leq N$, where, as above, $\lambda = \ln(3)$.

The nonequilibrium distribution f_n is thus a linear combination of the source and drain Fermi distributions. In the limit of large N , $f_1 \rightarrow \frac{2}{3}f_S + \frac{1}{3}f_D$, $f_N \rightarrow f_S$. Similarly, $g_1 \rightarrow \frac{1}{3}f_S + \frac{2}{3}f_D$ and $g_N \rightarrow f_D$. The current at the source is

$$I_D = \int_{-\infty}^{\infty} \left[\frac{4}{3} f_S(\epsilon) - \frac{1}{3} f_N(\epsilon) - g_1(\epsilon) \right] d\epsilon = \frac{2}{3} \int_{-\infty}^{\infty} [f_S(\epsilon) - f_D(\epsilon)] d\epsilon = \frac{2}{3} V. \quad (\text{B3})$$

The conductance is thus insensitive to the nature of the reservoirs, LR or EPRs.

We now investigate where dissipation occurs in the $2/3$ Hall bar with EPRs. Because of the nature of the fictive BRs, we do not expect any dissipation along the upper and lower channels except at source and drain that are physical LR. Indeed, let us calculate the power P_S and P_D dissipated in the source and in the drain. For simplicity, we consider the reservoirs at zero temperature and choose the zero of energy at the chemical potential of the grounded drain contact. For large N , we have

$$P_S = \int_{-\infty}^{\infty} d\epsilon (\epsilon - V) \left[\frac{1}{3} \{f_N(\epsilon) - f_S(\epsilon)\} + \{g_1(\epsilon) - f_D(\epsilon)\} \right] = \frac{1}{3} \int_{-\infty}^{\infty} d\epsilon (\epsilon - V) \{f_S(\epsilon) - f_D(\epsilon)\} = \frac{1}{2} \frac{2}{3} V^2. \quad (\text{B4})$$

A similar expression gives $P_D = P_S$. The total Joule heating is $P = P_D + P_S = IV$ and thus exclusively occurs in the physical Ohmic drain and source contacts. The study of dissipation indicates that no generation of thermal current fluctuations can occur in the fictive EPRs. However, the distributions f_n and g_n can generate nonthermal population fluctuations. We can define a fictive temperature Θ_n associated with the distribution f_n as

$$\Theta_n = \int_0^{eV} d\epsilon f_n(\epsilon) [1 - f_n(\epsilon)] = eV \frac{(1 - e^{-n\lambda})(e^{-n\lambda} - e^{-(N+1)\lambda})}{(1 - e^{-(N+1)\lambda})^2}. \quad (\text{B5})$$

There is an important difference between the Θ_n for EPRs and the T_n for LR. While the latter have to obey the heat flow equation (A5), the former are entirely defined by the f_n . Figure 4 compares EPR and LR temperatures.

The noise for the case of EPR reservoirs is computed in a way similar to the LR case, except the T_n are now replaced by the Θ_n . Because the exponential factor relating the noise of a contact pair $(n+1, n)$ to the current noise it produces at the source $\delta I_S^{n,n+1} = \frac{2}{9} \delta I_{n,n+1} e^{-(N-1-n)\lambda} / (1 - e^{-(N+1)\lambda})$ reduces further the small EPR noise, the final noise of the Hall bar is found exponentially vanishing with the sample size. Some values are indicated in Table I in the main text.

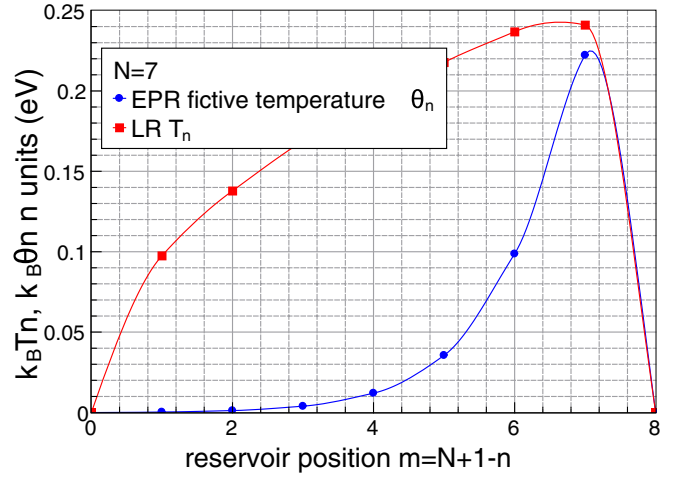


FIG. 4. Effective EPR temperatures (blue dots) and nonequilibrium LR temperatures (red dots) calculated for $N = 7$. The lines interpolating the points are guides for the eyes.

APPENDIX C: QPC CONDUCTANCE

While the above studies were considering a homogeneous Hall bar, we consider a QPC inducing tunneling between the upper and lower $(-1/3)$ inner edge channels. We first consider the case of fictive Landauer reservoirs. The computation of conductance for case of EPRs is very similar and leads to same conductance values. Provided the numbers of reservoirs on the left side and on the right side are infinite, the QPC location has not to be located right in the middle of the Hall bar. Departure from the conductance calculated below will be observed only if the QPC is very close to the drain or to the source. To proceed, we introduce N fictive reservoirs on the left side of the QPC (for both the lower and upper edges) and symmetrically N reservoirs on the right side. We seek for a solution of the upper and lower potentials $U_{L(R)}$ and $W_{L(R)}$, respectively, on the right (R) and left (L) sides of the QPC. We present the solution in the limit of very large number of reservoirs. We make use of the following property. Consider a part of a $2/3$ edge channel with potential V_A on the upstream side and V_B on the downstream side and N LR whose index n rises following the upstream direction. The voltage V_n of the n th reservoir is given by

$$V_n = \frac{V_A(1 - e^{-n\lambda}) + V_B(e^{-n\lambda} - e^{-(N+1)\lambda})}{1 - e^{-(N+1)\lambda}}. \quad (\text{C1})$$

We apply this relation to solve the boundary condition at the drain, the source, and the four contacts $(\alpha, \beta, \gamma, \delta)$ surrounding the QPC (see Fig. 5). Namely, the Landauer-Büttiker relation at the drain is

$$I = \frac{4}{3} V_S - \frac{1}{3} U_N^L - W_1^L \quad (\text{C2})$$

and becomes, using the large N limit of Eq. (C1),

$$I \simeq \frac{4}{3} V_S - \frac{1}{3} V_S - \left(\frac{2}{3} V_\alpha + \frac{1}{3} V_S \right); \quad (\text{C3})$$

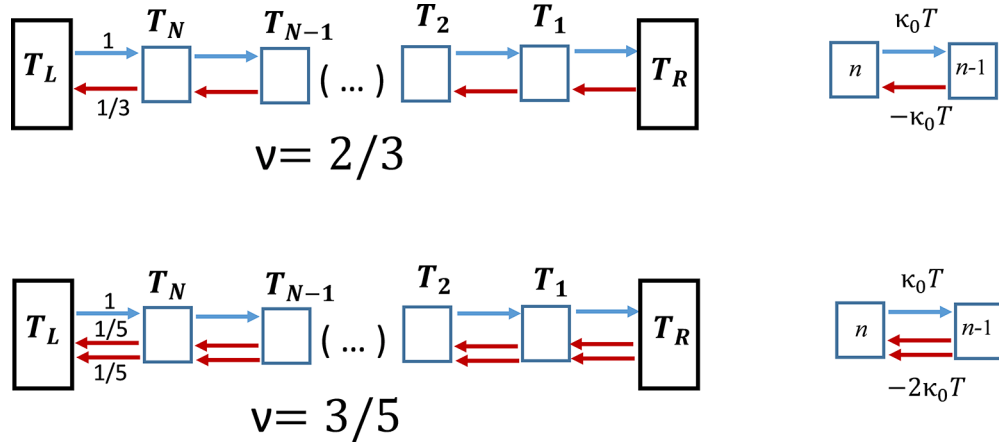


FIG. 5. Thermal conductance. On the top is schematically represented the $2/3$ upper edge with the chiral (1 and $-1/3$) conductance channels on the left and the $\kappa_0 T$ and $-\kappa_0 T$ thermal conductance channels on the right. The total thermal conductance is $1 - 1 = 0$. In the bottom figure, a similar schematic drawing of the conductance and thermal edge channels is represented for $\nu = 3/5$. The total thermal conductance is $1 - 1 = -1$ corresponding to upstream heat flow.

similarly, the Landauer-Büttiker relations applied to contact (α) on the upper left of the QPC give

$$0 = \frac{4}{3}V_\alpha - U_1^L - \frac{1}{3}[(1 - \mathcal{R})V_\beta + \mathcal{R}V_\delta] \\ = \frac{4}{3}V_\alpha - \left(\frac{2}{3}V_S + \frac{1}{3}V_\alpha\right) - \frac{1}{3}[(1 - \mathcal{R})V_\beta + \mathcal{R}V_\delta], \quad (\text{C4})$$

where we have used the reflection coefficient $\mathcal{R}$ of the fractional inner channel and the large N limit of Eq. (C1). Repeating the same procedure at contacts β , γ , and δ and at the drain provides an efficient way to compute the conductance $G = \frac{2}{3}(1 + \mathcal{R}/2)/(1 + \mathcal{R})$ in the presence of the QPC. Figure 6 shows the conductance variation. For comparison, we have included the case of a smooth edge and smooth QPC for which the $2/3$ edge separates into two reconstructed $1/3$ edge channels and an $e^2/3h$ plateau occurs. For full reflection, $\mathcal{R} = 1$, the upper edge voltages at the QPC are $V_\alpha = V_\beta = 3V_S/4$, while the lower edge voltages are $V_\gamma = V_\delta = V_S/4$. The voltage drop at the QPC is $V_{\text{QPC}} = V_S/(1 + \mathcal{R})$. This drop could be observed using photoassisted shot noise that presents a zero-temperature noise singularity at the

Josephson relation $e^*V_{\text{QPC}} = hf$, with f the microwave irradiation frequency [43,44]. We can reproduce the same findings for energy-preserving reservoirs instead of LRs, introducing the nonequilibrium energy distributions f_α , f_β , f_γ , and f_δ for EPRs around the QPC. As done for voltages in the case of LRs, we use the generic relation

$$f_n = \frac{f_A(1 - e^{-n\lambda}) + f_B(e^{-n\lambda} - e^{-(N+1)\lambda})}{1 - e^{-\lambda(N+1)}}, \quad (\text{C5})$$

where f_A denotes some known distribution on the upstream end of a chain of N EPRs and f_B the distribution at the downstream end. Applying this relation to simplify the equations expressing the current conservation at each energy for drain, source, and QPC EPRs allows us to compute the conductance, which is found identical to the case of LRs.

APPENDIX D: NEUTRAL MODE DECAY

Here, we use the dissipative plasmonic transmission line model of counter-propagating $2/3$ edge channels as introduced in Ref. [12]. Figure 7 shows the distributed capacitive

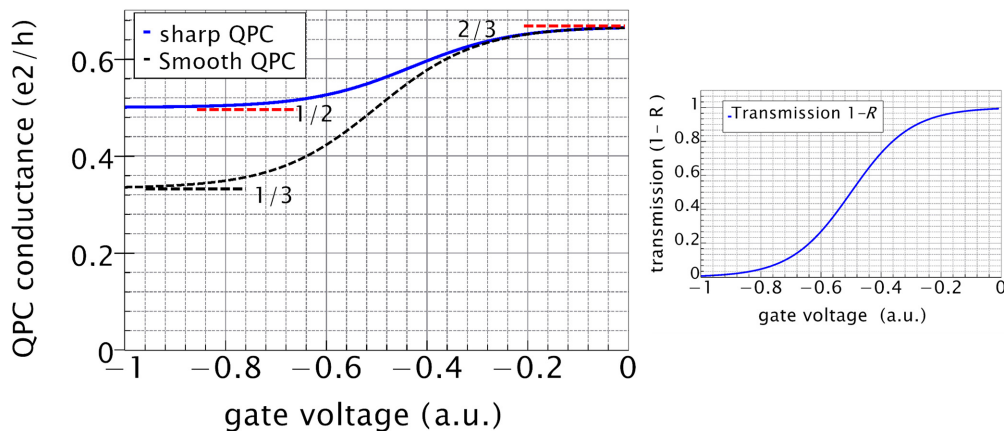


FIG. 6. QPC conductance versus gate voltage. For sharp edge $G = (2/3)(1 + \mathcal{R}/2)/(1 + \mathcal{R})$ (blue curve) and for smooth reconstructed edge $G = (1/3)(1 + (1 - \mathcal{R}))$ (black dotted curve). We chose a standard variation of $\mathcal{R}$ versus gate voltage as shown in the right figure.

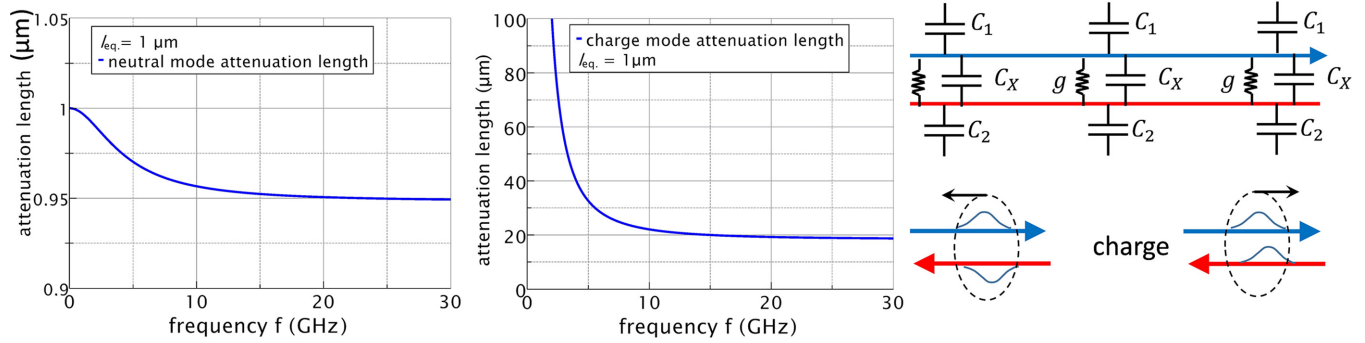


FIG. 7. Neutral and charge mode attenuation. Right: Capacitive transmission line model, as in Ref. [12], where distributed capacitances stand for self- and interchannel interaction. A tunneling conductance allows for charge equilibration between channels over a length l_{eq} . Left: Neutral mode attenuation length calculated for $l_{eq} = 1 \mu\text{m}$. Center: Same for the charge mode.

model, modeling the interaction with C_X , C_1 , and C_2 , the capacitance per unit length for the interchannel coupling and the self-interaction within the outer and inner edges, respectively. A uniform interchannel tunneling conductance $g = e^2/2hl_{eq}$ is used to mimic the KFP disorder channel mixing; l_{eq} is the charge mixing length, typically of μm size. The model is equivalent to the bosonic KFP model and shows upstream neutral and downstream charge modes. The attenuation lengths for neutral and charge modes are shown in Fig. 7.

We observe that the attenuation length of the neutral mode is smaller than l_{eq} at all frequencies, while the length is order of magnitude larger for the charge mode and is infinite in the DC limit. The charge mode attenuation has been recently measured in Ref. [45]. It is, thus, unlikely that neutral modes reach the QPC and generate noise in the experiment [5], while our suggestion of counter-propagating heat flow yielding a finite temperature difference at the QPC may likely generate the noise observed in this experiment.

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