

Control protocol for dynamic synthesis of qubit and qudit gates using photonic pulses and magnetic fields

A. F. Urquijo Rodríguez ^{1,*} Edgar A. Gómez ² and H. Vinck-Posada ¹

¹*Grupo de Superconductividad y Nanotecnología, Departamento de Física, Universidad Nacional de Colombia, Carrera 45 No. 26-85, Bogotá 111321, Colombia*

²*Grupo de Investigación en Física Teórica y Computacional, Programa de Física, Universidad del Quindío, Armenia 630004, Colombia*



(Received 28 May 2025; accepted 16 September 2025; published 10 December 2025)

We propose a theoretical control protocol designed for the dynamic synthesis of single-qubit and four-level qudit quantum gates using external parameters, such as photonic Gaussian pulses and magnetic fields, in a microcavity-quantum well system. Our approach takes advantage of tunable coherent light-matter interactions that can be modulated by the magnetic field between the exciton and negative trion coupled to the lowest photonic mode. We demonstrate that it is possible to achieve precise manipulation of populations of encoded quantum states through the unitary evolution of the system. In particular, we illustrate our optimization method for generating a single-qubit gate with a mean fidelity of 99.99 as well as the realization of an iswap gate in the four-level qudit case with a fidelity of 99.6.

DOI: [10.1103/b8q1-hn3f](https://doi.org/10.1103/b8q1-hn3f)

I. INTRODUCTION

Exciton polaritons are quasiparticles formed from a statistical blend of cavity photons and bound electron-hole states within a mesoscopic semiconductor system. These quasiparticles exhibit intriguing properties owing to their bosonic nature [1] and the small effective masses derived from their photonic components. The most notable properties are the Bose-Einstein condensation near room temperature [2,3], polariton lasing [4], and vorticity in the quantum fluid regime [5]. Furthermore, exciton polaritons are expected to function in the quantum regime [6], theoretically displaying entanglement and quantum correlation signatures [7,8], which makes them promising candidates for applications in quantum computation [9–12] or quantum simulators. Recent studies have focused on manipulating the quantum properties of polaritonic systems by applying an external magnetic field. This manipulation influences the excitonic component of the polariton, allowing for the tuning of the exciton energy, altering the Zeeman splitting [13,14], and enhancing the exciton oscillator strength [15]. The quantum characteristics of this platform can be further enhanced by considering the many-body effects induced by the Coulomb interactions within the semiconductor. This can lead to the emergence of more exotic quasiparticles, such as trions, in doped semiconductors [16]. Trions, which are the bound states of three fermions, have also been shown to coherently interact with the light modes

confined within a cavity [17,18]. This opens up the possibility of magnetic tuning (electrons, excitons, and trions) inside the microcavities, enabling the use of exciton-trion systems as multilevel qudits. In recent years, there has been considerable interest in generating quantum gates on semiconductor platforms and achieving quantum control using external parameters, for example, spin-photon hybrid systems, which are considered electron-spin qubits in quantum dots within optical microcavities [19–21], photonic systems with more than one degree of freedom in charged quantum dots inside optical microcavities [22], and qubits based on conduction-band electron spins in quantum dots embedded within a microcavity [23]. Recently, researchers demonstrated the feasibility of implementing quantum gates through controlled parametric interactions between the control and target polariton modes [24–26]. However, few theoretical studies have focused on the influence of external magnetic fields and ultrashort pulse engineering on microcavity-quantum well systems and how they can be utilized to control quantum state transitions [27–29]. The aim of this study is to introduce a theoretical approach for synthesizing single-qubit and four-level qubit quantum gates through the quantum evolution of a trion-exciton-polariton system utilizing external magnetic fields and a set of coherent pumping pulses as the physical control parameters. The remainder of this paper is organized as follows. In Sec. II, we present a theoretical model for describing a semiconductor quantum well (QW) immersed in an optical cavity within the framework of a finite system. Specifically, we examined the properties of exciton and trion states using a finite-system Hamiltonian model. In addition, we discuss the optimization algorithm for the quantum gates in detail. In Sec. III, we provide a detailed analysis of the numerical results, focusing on the effects of the magnetic field on the Coulomb interaction energy, exciton and trion energies, and the synthesis of single-qubit and four-qudit gates. Finally, concluding remarks are

*Contact author: afurquijor@unal.edu.co

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

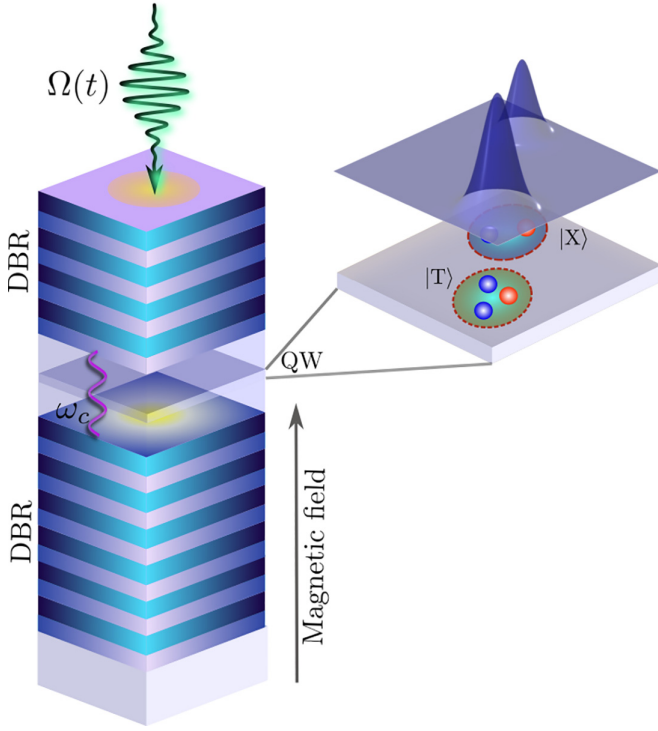


FIG. 1. Illustration of the physical system: A single QW is immersed in a semiconductor microcavity formed by two distributed Bragg reflectors (DBR) and exposed to a perpendicular magnetic field. Within this material, trion and exciton resonances can emerge owing to the interaction between the charged carriers and the confined lowest photonic mode. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

presented in Sec. V. In Appendix A, we present the mathematical details of Coulomb matrix element computation. The single-particle-state ordering used in this study is briefly discussed in Appendix B.

II. THEORETICAL FRAMEWORK

A. Finite-system Hamiltonian

The system considered in this study consisted of a single QW embedded in a planar semiconductor microcavity subjected to an external perpendicular magnetic field. Exciton and negative trion states can arise in semiconductor systems owing to the Coulomb correlation between the electrons and holes. Both quasiparticles can interact with the lowest photonic mode confined inside the cavity, as shown in Fig. 1. We limit our consideration to the tight confinement limit in QW and ignore band mixing effects. Furthermore, we explicitly included the spin degree of freedom of the particle in our model to consider the optical selection rules that govern the light-matter interaction in the system. Using basis states composed of the direct product of the Landau levels and QW lowest z subband wave functions, we can write the system Hamiltonian in the second quantization framework as follows

[30]:

$$H = \sum_i \{ \mathcal{E}_i^{(e)} e_i^\dagger e_i + \mathcal{E}_i^{(h)} h_i^\dagger h_i \} + \hbar \omega a^\dagger a + \frac{\beta}{2} \left[\sum_{ijkl} \langle ij|V|kl \rangle e_i^\dagger e_j^\dagger e_k^\dagger e_l - 2 \sum_{i\bar{j}k\bar{l}} \langle i\bar{j}|V|k\bar{l} \rangle e_i^\dagger h_{\bar{j}}^\dagger h_{\bar{k}} e_l \right] + \hbar g \sum_i \{ e_{i\uparrow}^\dagger h_{i\downarrow}^\dagger a + a^\dagger h_{i\downarrow} e_{i\uparrow} \}, \quad (1)$$

where e_j and $h_{\bar{j}}$ are the fermionic annihilation operators for the electrons and holes, respectively. The subscript j labels the single-particle states $\{|n_j, l_j, s_j\rangle\}$, where n_j is the radial quantum number, l_j is the z projection of the angular momentum, and s_j is the z component of the j th spin particle. Note that the overline notation denotes the states for holes; that is, $l_j^{(h)} = -l_j^{(e)}$ and $s_j^{(h)} = -s_j^{(e)}$. In addition, a corresponds to the annihilation operator of the lowest photonic mode with left circular polarization. The Hamiltonian described in Eq. (1) encompasses several processes. The first part considers the single-particle energies, which incorporate the effects of the cyclotron, Zeeman, and QW subband energies for both electrons and holes, expressed as follows:

$$\mathcal{E}_i^{e(h)} = \frac{\hbar \omega_c^{e(h)}}{2} (2n_i + |l_i| \pm l_i + 1) \pm g_{e(h)} \mu_B B s_i + \frac{\hbar^2 \pi^2}{2m_{e(h)} L} + E_g. \quad (2)$$

In this context, E_g represents the energy gap, whereas $\omega_c^{e(h)} = eB/m_{e(h)}$ represents the cyclotron frequency of electrons (holes). The effective gyromagnetic factor for electrons (holes) is denoted by $g_{e(h)}$ and μ_B denotes the Bohr magneton. The width of the quantum well is denoted by L . The masses of the electrons and holes are represented by m_e and m_h , respectively. The second part of the Hamiltonian accounts for the Coulomb interaction between electrons and holes. Importantly, this interaction term is influenced by the magnetic field through the Coulomb amplitude strength β and the matrix elements $\langle ij|V|kl \rangle$ (refer to Appendix A), which are defined as follows:

$$\langle ij|V|st \rangle = \delta_{l_i+l_j, l_s+l_t} \int_0^\infty dq F_\alpha(q) \vartheta_{i,s}(q) \vartheta_{j,t}(q). \quad (3)$$

Here, α denotes the dimensionless scale ratio L/l_b , where l_b is the magnetic length, $\vartheta_{a,b}(q)$ are integral terms involving radial contributions, and $F_\alpha(q)$ is the form factor given by

$$F_\alpha(q) = \frac{\pi^2 [20\alpha^3 \pi^2 q^3 + 3\alpha^5 q^5 + 32\pi^4 (\alpha q + e^{-\alpha q} - 1)]}{\alpha^2 (4\pi^2 q + \alpha^2 q^3)^2}. \quad (4)$$

Finally, the last line of the Hamiltonian involves the light-matter interaction between the electron and hole pairs and the photonic mode confined inside the cavity with an amplitude of g . Using an exact diagonalization technique, we can investigate the structure of the excitons of the system and the negative trion resonances. Such properties can be achieved using a suitable bare-state basis to diagonalize the system Hamiltonian. Furthermore, owing to the conservation of total angular momentum, a block-diagonal approach can

be employed to perform the diagonalization process in a fixed angular momentum subspace. For excitons, we use an electron-hole pair basis, represented by $|\Psi_X\rangle = |i\rangle_e \otimes |\bar{j}\rangle_h$. Conversely, negative trion states can be obtained by employing a set of states, $|\Psi_T\rangle = |i, j\rangle_e \otimes |\bar{k}\rangle_h$. This proposal accounts for the initial residual electron density in the semiconductor conduction band. For both cases, we constructed a set of states with a fixed total angular momentum. In addition, we established a cutoff criterion for single-particle states (see Appendix B) based on the convergence of the first few excited states of the system.

B. Trion-exciton polariton effective model

Understanding the properties of a finite system is challenging because of the numerous energy levels involved in Hamiltonian formulation. However, under certain experimental conditions, it is possible to excite the system to produce exciton ground state (X) and negative trion (T) singlet bright states in the QW. To develop a simpler model that retains the Coulomb correlation information of matter quasiparticles, we can employ an effective Hamiltonian model to account for the interaction of X and T with the photons inside the cavity. Assuming a sufficiently large mean spatial separation between the X and T states, the Coulomb interaction between these states can be disregarded. In our theoretical approach, this can be achieved by computing the eigenstates corresponding to the exciton and trion ground states, using the Hamiltonian given by Eq. (1). Consequently, we can express the matter-dressed states as

$$|X\rangle = \sum_{i,\bar{j}} \varphi_X(i, \bar{j}) e_i^\dagger h_{\bar{j}}^\dagger |0\rangle_e \otimes |0\rangle_h, \quad (5)$$

$$|T\rangle = \sum_{i,j,\bar{k}} \varphi_T(i, j, \bar{k}) e_i^\dagger e_j^\dagger h_{\bar{k}}^\dagger |0\rangle_e \otimes |0\rangle_h. \quad (6)$$

Here, we denote $|0\rangle_{e(h)}$ as the corresponding vacuum state for electrons (holes), and $\varphi_{X(T)}$ as the linear combination coefficients of the Coulomb correlated states for the exciton and negative trion, respectively. Considering that the bright trion singlet has well-defined spin symmetry, this state decays to $|1\downarrow\rangle$ after the electron-hole combination through the recombination channel with left-polarized light. Therefore, we can describe the light-matter state basis spanned by $|\alpha\rangle \otimes |\beta\rangle \otimes |n\rangle$, where $|n\rangle \in \{|0\rangle, \dots, |N\rangle\}$, $|\alpha\rangle \in \{|G\rangle, |X\rangle\}$, and $|\beta\rangle \in \{|1\downarrow\rangle, |T\rangle\}$, where $|1\downarrow\rangle$ is the electron state in the first orbital with a down z -spin component, and n corresponds to the photon number. The effective Hamiltonian system can be represented as follows ($\hbar = 1$):

$$\begin{aligned} H_{\text{eff.}} = & \omega a^\dagger a + (\delta_X + \omega) \sigma^\dagger \sigma + (\delta_T + \omega) \tau^\dagger \tau \\ & + g_X \mathcal{P}_X(B) (\sigma^\dagger a + a^\dagger \sigma) + g_T \mathcal{P}_T(B) (\tau^\dagger a + a^\dagger \tau) \\ & + \Omega(t) (e^{-i\omega_L t} a^\dagger + e^{i\omega_L t} a), \end{aligned} \quad (7)$$

where ω is the photonic mode frequency. Here, $\delta_X = \omega_X - \omega$ is the energy detuning between the exciton and the photonic mode. $\delta_T = \omega_T - \omega_e - \omega$ is the energy detuning between the trion reference energy and the photonic mode. Furthermore, ω_X , ω_T , and ω_e are the eigenfrequencies of the exciton, trion, and electron ground states, respectively. The raising and lowering operators for matter are defined as

$\sigma = |G\rangle\langle X| \otimes \mathbb{I}_T \otimes \mathbb{I}_a$ and $\tau = \mathbb{I}_X \otimes |1\downarrow\rangle\langle T| \otimes \mathbb{I}_a$, which connect the excited state with the corresponding ground state of each quasiparticle. The effective Hamiltonian given by Eq. (7) includes terms such as the free energy of photons, exciton energy, and trion energy measured for the free electron state. Furthermore, the effects of the magnetic field on the light-matter interaction amplitudes were considered using dipole transition matrix elements [31,32]. This is because $\mathcal{P}_X(B) = \langle 0|_e \otimes \langle 0|_h h_{\bar{i}\downarrow} e_{i\uparrow} |X\rangle$ and $\mathcal{P}_T(B) = \langle 1\downarrow| \otimes \langle 0|_h h_{\bar{i}\downarrow} e_{i\uparrow} |T\rangle$ together with the corresponding effective light-matter interaction constants g_X and g_T . Note that, in our model, these effective interaction constants are considered different, as reported from experimental and theoretical perspectives [17,33]. Finally, the last term in the effective Hamiltonian refers to external coherent photonic pumping with frequency ω_L and time-dependent amplitude profile $\Omega(t)$ acting on the system. It is noteworthy that, in our model, we consider the same physical system capable of implementing either a qubit or a qudit, contingent on the experimental conditions and the subspace chosen for dynamic control. In the qubit regime, we concentrate on a reduced two-level subspace formed by either exciton-photon or trion-photon coupling, depending on which excitation is resonantly targeted. In fact, it has been experimentally demonstrated [34] that exciton and trion states in quantum wells can be selectively excited using resonant optical pumping owing to their distinct energies and selection rules. In the qudit regime, the system can be simultaneously populated under optical doping or nonresonant excitation conditions [35].

C. Optimization algorithm for quantum gates

In this subsection, we consider the effective Hamiltonian to explore how the magnetic field and external coherent photonic pumping can be used to perform controlled quantum operations on the system. To achieve this, we restrict the Hilbert space of our effective model to a four-qudit-state basis using the following state mapping: $|G, 1\downarrow, 0\rangle \rightarrow |0\rangle$, $|G, T, 0\rangle \rightarrow |1\rangle$, $|X, 1\downarrow, 0\rangle \rightarrow |2\rangle$, and $|X, T, 0\rangle \rightarrow |3\rangle$. Additionally, a quantum control method was applied to determine the best set of parameters for external pulses. More precisely, for external pulses given by a train of Gaussian pulses, it is defined as

$$\Omega(t) = \sum_{i=1}^M A_i \exp\left(-\frac{(t-t_i)^2}{2\xi_i^2}\right), \quad (8)$$

where parameters such as A_i , t_i , and ξ_i denote the amplitude, center, and width of the i th Gaussian pulse, respectively. Figure 2 shows a flowchart of the optimization algorithm used to obtain the quantum gate. Basically, each qudit bare basis state $|I\rangle \in \{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$ is numerically evolved using the time-dependent Schrödinger's equation, with an initial arbitrary set of parameters for the external pulse and magnetic field in the effective Hamiltonian model. The time-evolution operator is then obtained by projecting the final states onto their corresponding initial states, expressed as $U_{IJ} = \langle I(t=0)|J(t=t_f)\rangle$, where $|I(t)\rangle$ represents the state of a two-qubit basis at time t . We evaluated the quantum infidelity function $\mathcal{J} = 1 - |\text{Tr}[U^\dagger V]|^2/N^2$, where V is the target quantum gate and N is the number of quantum states involved in the op-

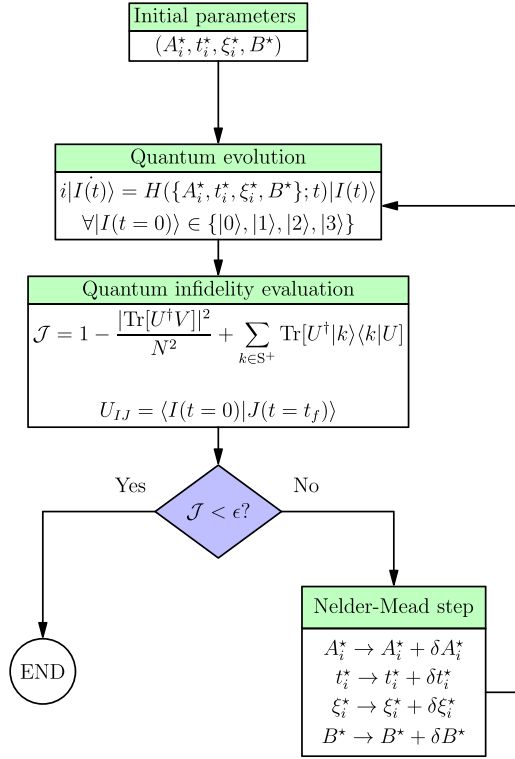


FIG. 2. The flowchart illustrates the stages involved in the optimization algorithm for a quantum gate. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

eration (two qubits and four qudits). It is important to note that we incorporate a penalty term to reduce the presence of states $k \in S^+$, where S^+ represents states that are not part of the qudit. The Nelder-Mead [36] multidimensional optimization algorithm is employed to iteratively update the pumping parameters and magnetic field to minimize the quantum infidelity $\mathcal{J}$.

III. RESULTS AND DISCUSSION

A. Exciton and negative trion resonances

In this subsection, we discuss the impact of the external magnetic field on the Coulomb interaction in detail. In particular, we focus on its direct consequence on both the energy spectrum and configuration of the exciton and trion states. For this task, we consider that the electron states are confined to the initial three Landau levels, specifically $n = 0, 1, 2$, within the conduction band. Similarly, hole states are limited to the $n = 0, 1, 2$ Landau levels in the heavy-hole valence band. The Fermi level is precisely defined as the midgap reference point. Additionally, we consider for our numerical simulations the typical parameter values reported for GaAs QW systems [37]. More precisely, $E_g = 1500$ meV, $\beta = 3.18\sqrt{B}$ meV, $L = 10$ nm, $m_e = 0.0665m_0$, $m_h = 0.235m_0$, $g_e(B) = -0.01667 + 0.0052B$, $g_h(B) = -0.05B$, $g_X = 3$ meV, and $g_T = 0.5$ meV. We begin evaluating the impact on the Coulomb interaction when the magnetic field is changed through the form factor given by Eq. (4). Figure 3(a) shows the form factor distribution as a function of q coordinates with different values of the

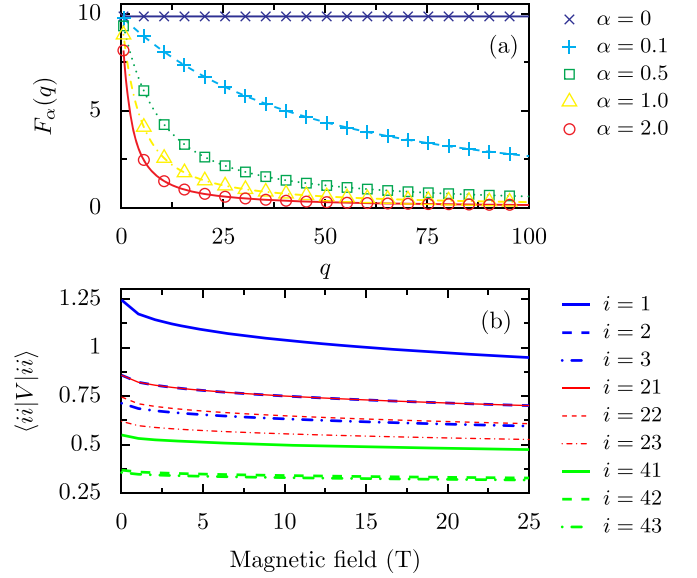


FIG. 3. Panel (a) shows the form factor $F_\alpha(q)$ as a function of q coordinates for different values of the α length ratio. The case of $\alpha = 0$ corresponds to the two-dimensional limit. Panel (b) shows the dependence of Coulomb matrix elements $\langle ii|V|ii\rangle$ on the external magnetic field. We considered the first three single-particle states at each level. Label i refers to the corresponding Landau-level basis ordering. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

characteristic length scale ratio α . In general, it was found that the form factor decreased significantly, whereas α increased because of the confinement effect exerted by the magnetic field on the particles. Consequently, Coulomb matrix elements are attenuated. Therefore, the form factor behaves as an amplitude that modulates the contributions of the radial function integrals. It is worth noting that $\alpha \rightarrow 0$ recovers the two-dimensional limit, in agreement with other theoretical results [38–40]. Further analysis of the Coulomb matrix elements is presented in Fig. 3(b). In particular, we show Coulomb matrix elements of the form $\langle ii|V|ii\rangle$ as a function of the magnetic field for the first three single-particle states at each Landau level. Interestingly, we observed that the matrix elements decreased, in agreement with the behavior of the form factor, as shown in Fig. 3(a). However, it is worth noting that the potential energy of the system increased as the magnetic field increased because the Hamiltonian [see Eq. (1)] incorporates Coulomb strength through β term. Figure 4 shows the ground-state energy for the negative trion in subspaces with angular momentum $l_z = 0$ (singlet state with energy E_S) and $l_z = -1$ (triplet state with energy E_T), as well as the ground-state energy for the exciton plus a free electron (E_{X+e^-}) in the QW system. We observed that at low magnetic fields, the Coulomb interaction energy is comparable to the cyclotron energy, as is well known in semiconductor physics [41]. In contrast, at high values of the magnetic field, it was found that the cyclotron energy dominates the Coulomb interaction energy; therefore, the system becomes uncoupled with an energy spectrum according to the Landau energy levels. The inset shows the trion binding energies $E_b^{S(T)} = E_{S(T)} - E_{X+e^-}$ for a singlet or triplet state as a function of the applied magnetic field. We ob-

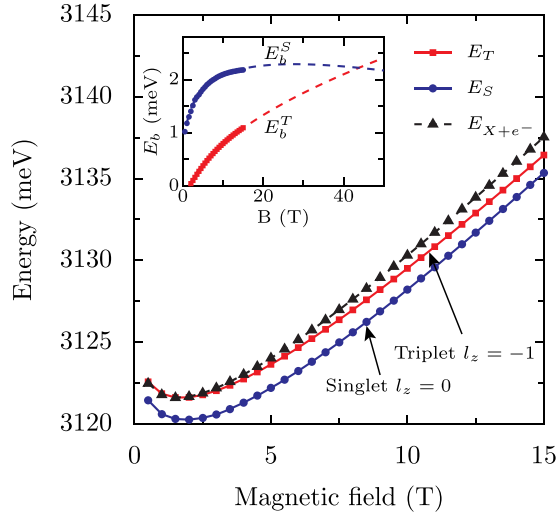


FIG. 4. Lowest-energy states of the negative trion in a GaAs QW as a function of the magnetic field. The main panel shows the triplet E_T , singlet E_S , and exciton plus free electron energies E_{X+e^-} . The inset shows the numerical calculations for the binding energies E_b^S (blue circle line) and E_b^T (red circle line) of the negative trion states relative to the exciton ground state and the free electrons as a function of the magnetic field. Extrapolation of the numerical results for the binding energies E_b^S and E_b^T are shown as blue and red dashed lines, respectively. A crossing between these two energies is predicted at $B \approx 42$ T. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

serve that both energies increase monotonically, in complete agreement with experimental observations [35,42,43]. Additionally, we extrapolated the numerical results for the binding energies E_b^S (blue dashed line) and E_b^T (red dashed line) in the high-magnetic-field regime. Interestingly, we found that our proposed model captures important features, such as the crossing between the binding energies at $B \approx 42$ T, which predicts a change in the negative trion ground state from the singlet to triplet state. This phenomenon has been reported both experimentally [35] and theoretically [44]. This analysis shows that by modifying the Coulomb interaction, the magnetic field influences both the energy spectrum and the configuration of the exciton and trion states. These tunable features enable precise control of the quantum behavior of the system, which is essential for designing and implementing specific quantum gates. In particular, the ability to adjust energy levels and light-matter coupling strength via magnetic fields provides a robust mechanism for tailoring gate operations.

B. Single-qubit quantum gates

The following demonstrates how photonic pulses and an external magnetic field can be used for the gate synthesis. More precisely, we applied the optimization method described in Sec. II C to determine a set of optimal parameters A_i , t_i , and ξ_i , as well as the external magnetic field B that tunes the system evolution in a specific quantum gate. To achieve this, we label the qubit states as $|G, 1\rangle \rightarrow |0\rangle$, $|X, 0\rangle \rightarrow |1\rangle$ for the exciton, and $|1 \downarrow, 1\rangle \rightarrow |0\rangle$, $|T, 0\rangle \rightarrow |1\rangle$ for the trion. The quantum average infidelity was also considered using the

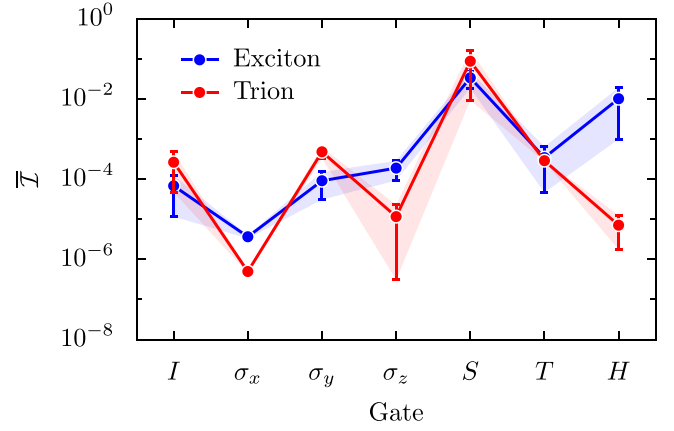


FIG. 5. Numerical results for the quantum average infidelity using the optimization technique for creating single-qubit quantum gates for excitons (depicted by the blue circle line) and trions (represented by the red circle line). Error bars correspond to the standard deviation over the $N = 1000$ random initial states. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

following expression:

$$\bar{\mathcal{I}} = 1 - \frac{1}{N} \sum_{n=1}^N |\langle \psi_n | U^\dagger V | \psi_n \rangle|^2. \quad (9)$$

Figure 5 shows the quantum average infidelity of the single-qubit gate synthesis process. Here, we computed the quantum average infidelity over $N = 1000$ random initial states $|\psi_n\rangle = \cos(\theta_n/2)|0\rangle + \exp(i\phi_n)\sin(\theta_n/2)|1\rangle$, where $\theta_n \in [0, \pi]$ and $\phi_n \in [0, 2\pi]$. It is noteworthy that $\langle \psi_n | U^\dagger$ corresponds to the quantum evolution of the initial state under the Hamiltonian given by Eq. (7) and the optimal parameters obtained using the optimization algorithm. The term $V|\psi_n\rangle$ corresponds to the evolution of the initial state through the target quantum gate. In particular, we consider the maximum number of pumping pulses $M = 5$ to study the following gates: identity (I), Pauli- X (σ_x), Pauli- Y (σ_y), Pauli- Z (σ_z), square root of Z (S), $\pi/8$ gate (T), and Hadamard gate (H). It should be noted that in both cases, quantum average infidelity was close to zero. This implies that the action of the photonic pumping pulse and the external magnetic field is optimal for controlling the desired states. For example, an interesting situation occurs when a photonic pumping pulse is neglected. The Pauli- X gate can be obtained by transforming the effective Hamiltonian [Eq. (7)] to Pauli matrix. In this representation, the temporal evolution of the state is determined by rotation around a unitary vector on a modified Bloch sphere. This vector can be written as

$$\mathbf{n} = \frac{1}{\sqrt{G_X^2 + (\delta_X/2)^2}} \left(G_X, 0, \frac{\delta_X}{2} \right), \quad (10)$$

where $G_X = g_X \mathcal{P}_X(B)$ is the light-matter amplitude and δ_X is the detuning between the light and exciton states (a similar for the vector $\mathbf{n}$ can be obtained for the trion-photon case). The unitary vector $\mathbf{n}$ living in the XZ plane can be reoriented by applying an optimized magnetic field. Thus, the system reaches the resonance condition, inducing a dynamic

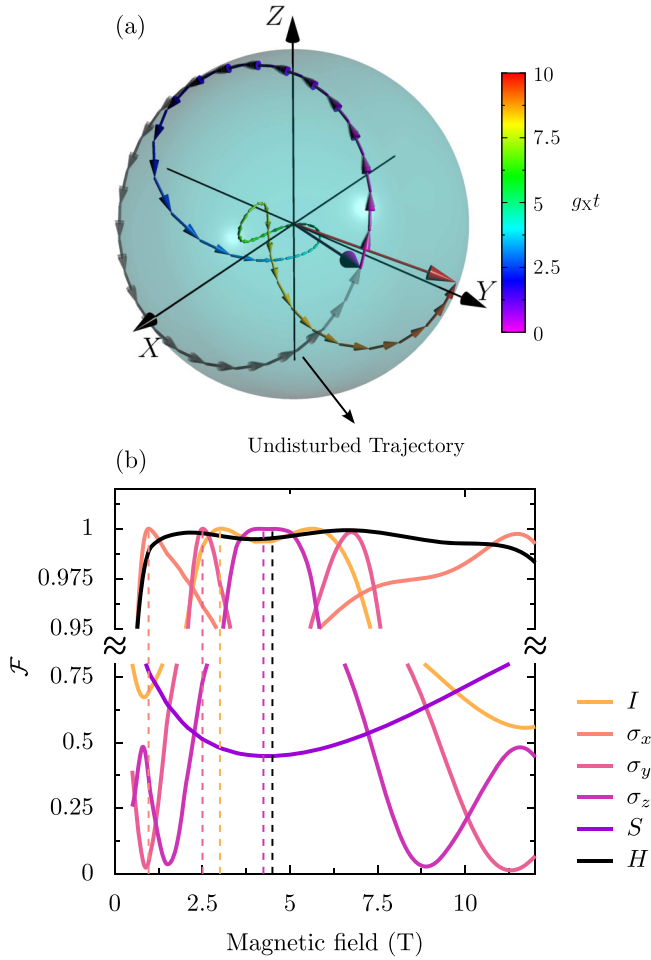


FIG. 6. Panel (a) shows the time evolution of the initial state $|\psi_0\rangle = \cos(\pi/5)|G, 1\rangle + \sin(\pi/5)e^{i\pi/3}|X, 0\rangle$ on the modified Bloch sphere for obtaining the optimal Pauli-Y gate. The undisturbed trajectory corresponds to the time evolution of the initial condition with an optimal magnetic field $B = 2.51$ T without photonic pumping. The trajectory goes inside the Bloch sphere because intermediary states are activated during the time evolution via the pumping action. Panel (b) shows fidelity as a function of the external magnetic field. The vertical dashed lines indicate the optimal magnetic field for each gate. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

rotation around the X axis. However, in order to produce rotations around an arbitrary orientation on the modified Bloch sphere, the system must be perturbed by an external photonic pumping pulse. The general situation is illustrated in Fig. 6(a), where we consider as the arbitrary initial state $|\psi_0\rangle = \cos(\pi/5)|0\rangle + e^{i\pi/3}\sin(\pi/5)|1\rangle$ and the time evolution on the modified Bloch sphere is tracked [45]. When the photonic pumping pulse is neglected and the optimized magnetic field takes the value $B = 2.51$ T, a circular trajectory (undisturbed trajectory) on the modified Bloch sphere is registered around a unitary vector on the XZ plane. Conversely, when the optimal photonic pumping pulse was considered, the registered trajectory on the modified Bloch sphere became significantly more complex. Initially, the trajectory followed the corresponding rotation as if it had no photonic pumping

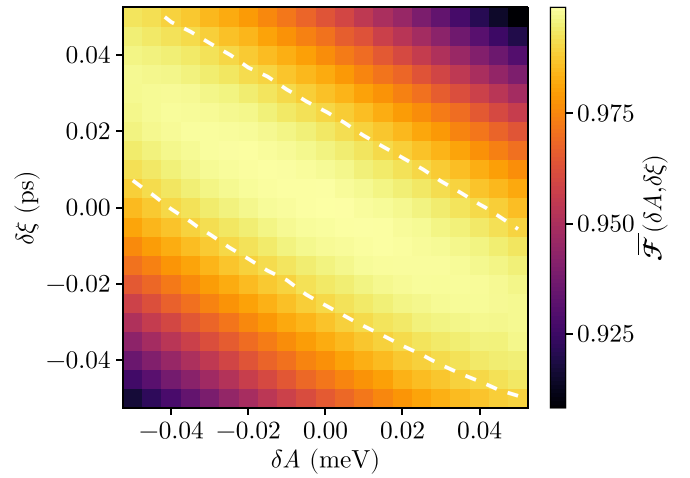


FIG. 7. Numerical results of the quantum average fidelity map for the σ_y gate. The optimal magnetic field B is fixed, whereas the amplitudes A_i and widths ξ_i of the photonic pulses are perturbed around their optimal values: $A_i \rightarrow A_i + \delta A$ and $\xi_i \rightarrow \xi_i + \delta \xi$. Furthermore, the center of each Gaussian pulse was randomly shifted and the mean fidelity was computed by averaging over 200 samples. The inner region, delimited by the white dashed lines, shows the 0.99 tolerance region.

pulse; then, the trajectory was obtained in the modified Bloch sphere. This behavior is a consequence of population transfer to other states on a computational basis. Finally, the trajectory reached the surface of the modified Bloch sphere, successfully producing a rotation on Y axis. Figure 6(b) shows the quantum fidelity as a function of the external magnetic field $\mathcal{F} = |\langle\psi_0|U^\dagger(B)V|\psi_0\rangle|^2$. It was found that, in general, the fidelity approaches one for a limited region of the applied magnetic field, implying that is very sensitive to changes in the magnetic field, except for Hadamard or Pauli-Z gates, which are more robust to changes in the magnetic field because the quantum fidelity is approximately one for a wide range of magnetic field values. Similar results, not shown here, were obtained in gate synthesis when the trion-photon contributions were considered [46]. Figure 7 shows the average fidelity map $\bar{\mathcal{F}}(\delta A, \delta \xi)$ as a function of the small deviations in the amplitudes A and widths ξ of the optimal photonic pulse, corresponding to the synthesized σ_y gate, with the optimal magnetic field value at $B = 2.51$ T. Each data point in Fig. 7 is computed as a numerical time average over a random deviation of the pulse centers around the optimal values t_i , which are allowed to vary within 5%. The results show that, while the gate performance is sensitive to changes in the pulse parameters, the degradation of fidelity is smooth and continuous, and a broad plateau of high fidelity ($\bar{\mathcal{F}} \geq 0.99$) remains clearly identifiable. This plateau, which is bounded by dashed white lines in Fig. 7, defines a robust operating region in the parameter space where the gate operation is preserved with minimal infidelity.

C. Qudit quantum gate

We numerically investigated the production of the iswap four-level-qudit gate by controlling the detunings δ_X and δ_T , and the modulation terms of the light-matter amplitudes

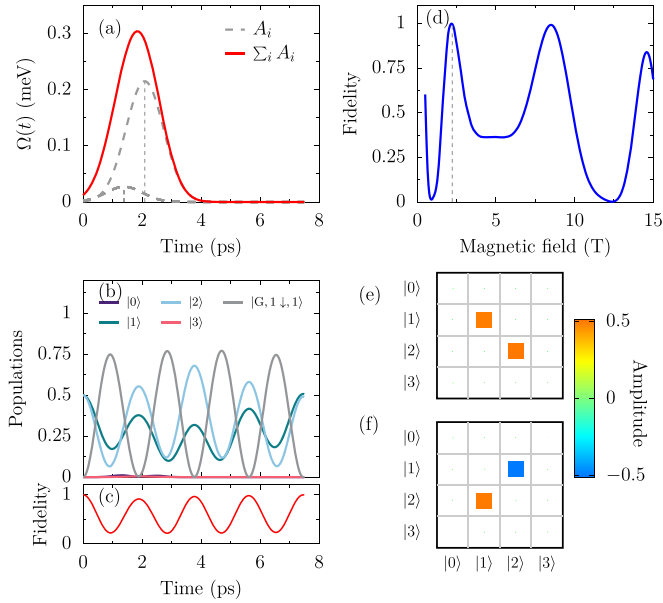


FIG. 8. Panel (a) shows the optimal amplitude profile of a photonic pumping pulse. Panel (b) shows the time evolution of the qudit populations with the initial condition $|\phi_0\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$ together with all the optimal parameters (A_i, t_i, ξ_i, B) for the corresponding iswap gate. The quantum state $|G, 1 \downarrow, 1\rangle$ acts as an intermediary state, facilitating the coherent transfer of the population generated by the pumping pulse toward the qudit states. Panel (c) shows the fidelity as a function of time. Panel (d) shows the fidelity as a function of the external magnetic field together with the optimal photonic pumping pulse. The vertical dashed line indicates the optimal magnetic field for which quantum fidelity is maximized. Panels (e) and (f) show the real and imaginary parts of quantum state tomography for the time-evolved state $U|\phi_0\rangle$. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

$g_X \mathcal{P}_X(B)$ and $g_T \mathcal{P}_T(B)$ through the magnetic field and photonic pumping pulse. Figure 8(a) shows the amplitude profile $\Omega(t)$ as a function of time. The amplitude profile shown by the solid red line corresponds to an asymmetric Gaussian shape acting on the system. This shape profile is the result of the optimization algorithm for each term in the train of Gaussian pulses, shown as gray dashed lines. Figure 8(b) shows the time evolution of the populations when the initial state is considered as $|\phi_0\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$ together with the optimized amplitude profile mentioned above and the optimal magnetic field $B = 2.22$ T. We observe that the occupation probability of states $|1\rangle$ and $|2\rangle$ shows a coherent exchange behavior influenced by the occupation of the auxiliary state $|G, 1 \downarrow, 1\rangle$, whereas the states $|0\rangle$ and $|3\rangle$ remain with negligible occupation probability. This fact evidences how control of qudit states can be achieved through the light-matter interaction with intermediary states. Furthermore, it was found that the time evolution of fidelity as a function of time between the target quantum gate and the dynamically synthesized gate exhibits quasiperiodic behavior, reaching a maximum of 0.996 at $t = 7.5$ ps, as shown in Fig. 8(c). However, Fig. 8(d) shows the fidelity as a function of the external magnetic field and it is found that it depends strongly on the applied magnetic field. There are values for which the fidelity takes values close to

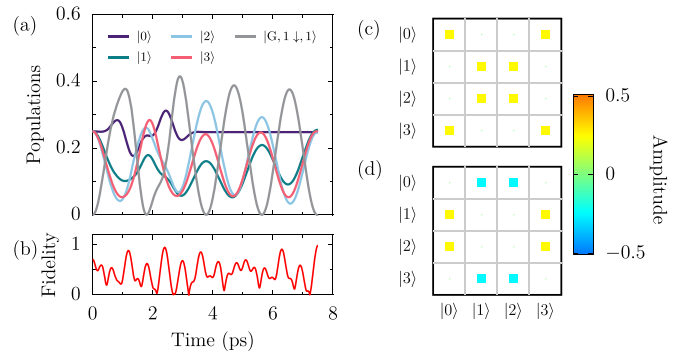


FIG. 9. Panel (a) shows the time evolution of the system populations for the initial condition $|\phi_0\rangle = \frac{1}{\sqrt{4}}(|0\rangle + |1\rangle + |2\rangle + |3\rangle)$. Panel (b) shows fidelity as a function of time. Panels (c) and (d) display the real and imaginary parts of the quantum state tomography of the system state when the fidelity reaches its maximum. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

unity. For example, $\mathcal{F} = 0.996$ at $B = 2.22$ T and $\mathcal{F} = 0.991$ at $B = 8.5$ T, which confirms that the optimization procedure was successfully used to achieve the proposed quantum gate. Figures 8(e) and 8(f) show the real and imaginary parts of quantum state tomography for the time-evolved state $U|\phi_0\rangle$ at $t = 7.5$ ps when the quantum gate is successfully achieved. At this time, the time-evolved state exhibits the expected quantum structure of the iswap operation, because both the populations and phases of the state are recovered. It should be noted that the time-evolution operator U should be obtained through an optimization procedure. Additionally, we test the optimization method on the qudit gate by considering a four-level pure state as an initial condition. Figure 9(a) illustrates the temporal evolution of the populations associated with the initial state $|\phi_0\rangle = \frac{1}{\sqrt{4}}(|0\rangle + |1\rangle + |2\rangle + |3\rangle)$ under optimized control parameters for the iswap gate. Figure 9(b) shows the fidelity as a function of time. Here, the fidelity remains practically close to one. Figures 9(c) and 9(d) show the quantum state tomography of the final state that closely aligns with the anticipated outcome of the ideal iswap operation. This effectively confirms that the synthesized gate functions as a genuine qudit operation, rather than merely facilitating a two-level population exchange. We validated the performance of our method for the synthesized gate by successive applications of the time-evolution operator U on the states $|1\rangle$ and $|2\rangle$ [47] as shown in Figs. 10(a) and 10(b). It is confirmed that the population interchanges between $|1\rangle$ and $|2\rangle$ states, as is well known in the iswap gate. We observed a systematic reduction in the occupations during successive applications of the U operator as a consequence of numerical errors in the optimization algorithm. While the current manuscript focuses on idealized unitary evolution, it is worth mentioning that in realistic systems, several effects should be incorporated into the theoretical model, such as the finite lifetimes of excitons and trions, photon leakage due to finite cavity reflectivity, and decoherence effects that inevitably degrade quantum coherence. Therefore, all of these effects reduce the achievable fidelity of the gate operations and it could be investigated by incorporating some of these environmental effects into our

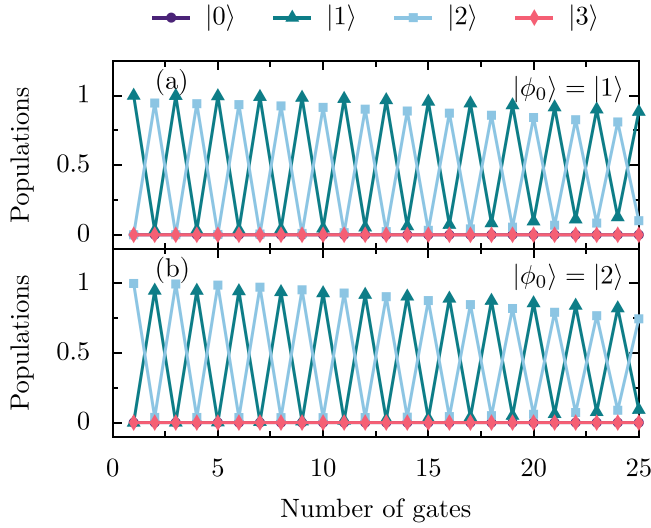


FIG. 10. Panels (a) and (b) show the performance of the iswap synthesized gate through numerical calculations of the populations for the quantum states $|1\rangle$ and $|2\rangle$ along each application of the U operator. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

control framework. More precisely, the time evolution of the system can be modeled as an open quantum system within the Lindblad master equation approach, with different collapse operators describing the corresponding irreversible processes. In particular, the proposed optimization method should include the time evolution of the density matrix up to the final gate time, which defines a nonunitary quantum channel $\mathcal{U}_g$ that can be determined by the applied control parameters, for example, the pulse train A_i, t_i, ξ_i and the magnetic field B . The figure of merit is the average gate fidelity between the target unitary gate V and the actual channel $\mathcal{U}_g$, restricted to the computational subspace of dimension d . This theoretical extension, utilizing a master equation approach, delineates a clear methodology for integrating realistic loss mechanisms and optimizing control under conditions pertinent to experimental settings. An initial numerical investigation, which considers dissipative channels, demonstrates that the coherent dynamics of our polariton-based gates are influenced by the population loss of quantum states. However, further exploration of this aspect is planned as part of our future research.

TABLE I. Comparison of the number of gate operations that can be executed within the coherence time across various experimental platforms. Superconducting transmons, for instance, offer coherence windows that accommodate approximately 1000 operations. In contrast, exciton-based photonic platforms, despite their ultrafast nature, are currently restricted to just a few tens of operations due to short polariton lifetimes and radiative decay. Values for the operation and coherence times were taken from literature as follows: for the exciton-trion (GaAs QW), operation time was obtained from our numerical computations and coherence time from Ref. [56]; for the exciton-biexciton platform, both the operation and coherence times were obtained from Ref. [58]; and in the superconducting transmon case, the operation and coherence times were obtained from Refs. [47,57,59].

Platform	Operation time (T_{op})	Coherence time T_{coh}	No. of operations
Exciton-trion (GaAs QW)	$\sim 1\text{--}10$ ps	2–135 ps (cav. photon lifetime)	2–13
Exciton-biexciton (QDs)	$\sim 5\text{--}10$ ps	5–130 ps (exciton lifetime)	1–13
Superconducting transmon	$\sim 10\text{--}150$ ns	50–300 μ s	$\sim 10^3$

IV. EXPERIMENTAL FEASIBILITY

The proposed protocol for generating quantum gates using polariton-based systems in AlGaAs/GaAs cavities with embedded GaAs quantum wells is experimentally feasible based on current technological capabilities. The system relies on coherent ultrafast optical pumping and external magnetic fields to manipulate negative trion states [48,49]. Experimental evidence supports the achievability of key parameters, including Rabi splittings of 4–15 meV [50], optical pulse durations in the picosecond range [51,52], and drive amplitudes around 0.3 meV [53,54]. The required magnetic field strengths of 2–5 T are also within practical limits, with recent experiments demonstrating submillitesla precision in field measurements [55]. While dissipative effects due to finite lifetimes of excitons, trions, and cavity photons can degrade gate fidelities, state-of-the-art cavities with quality factors around 10^5 have achieved polariton lifetimes of approximately 270 ps [56]. This is significantly longer than the proposed gate times of around 10 ps, suggesting that the protocol can be implemented within the coherence time of the system. However, further optimization may be necessary to address fidelity degradation caused by dissipative effects. Potential solutions could involve developing a dissipative version of the optimization approach to achieve a dynamical equilibrium regime, thereby compensating for losses and maintaining high gate fidelities. While superconducting transmon qubits offer high fidelity and longer coherence times [57], they operate at extremely low temperatures and have slower gate operation times compared to optical platforms [47]. In contrast, quantum-dot exciton-biexciton systems and the proposed exciton-trion polariton platform achieve much faster operation times in the picosecond range, but currently face challenges with lower fidelity, shorter coherence times, and limited experimental validation [58]. Table I provides an approximate comparison of operation times, coherence times, and the number of operations achievable by each platform.

V. CONCLUSIONS

In this study, we introduced a theoretical control protocol to synthesize single-qubit and four-level quantum gates using the Hamiltonian dynamics of a trion-exciton-polariton system in semiconductor QWs. Our approach leverages the effects of external electromagnetic fields on charge carriers within the active media of an optical cavity to manipulate the phase

and amplitude of the encoded state populations. For qubits, we encoded the states into the lowest polariton, whereas for qudits, we selected the lowest matter states without photon contributions. We found that our methodology enables precise modeling of the Coulomb interactions between charge carriers and allows incorporation of the effects of the magnetic field on the system properties with high fidelity. We examined the behavior of the Coulomb matrix elements in the QW system as a function of the magnetic field, and found that the amplitude of each matrix element depends significantly on the behavior of the form factor $F_\alpha(q)$, resulting in blue shifts due to the applied magnetic field in the exciton and trion energies, as well as in their corresponding ground-state structures. It was also demonstrated that the fidelity is sensitive to magnetic field changes for each quantum-controlled gate, underscoring the importance of magnetic control in the synthesis process. In particular, we constructed an iswap gate in a four-level qudit with 99.6% fidelity, confirming that the light-matter interaction can be controlled by the magnetic field and allows population transfer between qudit states to achieve the desired interchange in gate operation.

ACKNOWLEDGMENTS

The authors acknowledge technical and computational support from Grupo de Óptica e Información Cuántica (GOIC-UNAL). A.F.U.R., H.V.-P., and E.A.G. thank financial support from project “Ampliación del uso de la mecánica cuántica desde el punto de vista experimental y su relación con la teoría, generando desarrollos en tecnologías cuánticas útiles para metrología y computación cuántica a nivel nacional,” BPIN 2022000100133, from Ministerio de Ciencia, Tecnología e Innovación (Minciencias), Gobierno de Colombia–Sistema General de Regalías (SGR) under Grant No. BPIN 2022000100133. E.A.G. acknowledges the financial support from the Universidad del Quindío under Project No. 1141.

A.F.U.R. analyzed and interpreted the data, contributed analysis tools, and wrote the paper. E.A.G. conceived and designed the analysis, analyzed and interpreted the data, and wrote, reviewed, and edited the manuscript. H.V.-P. conceived and designed the analysis, and analyzed and interpreted the data.

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this study.

DATA AVAILABILITY

The data supporting this study’s findings are available within the article.

APPENDIX A: COMPUTATION OF COULOMB MATRIX ELEMENTS

Let us consider the Hamiltonian that describes the electrostatic interaction of charge carriers inside a semiconductor QW, together with the presence of an external magnetic field. In particular, we introduce dimensionless particle coordinates

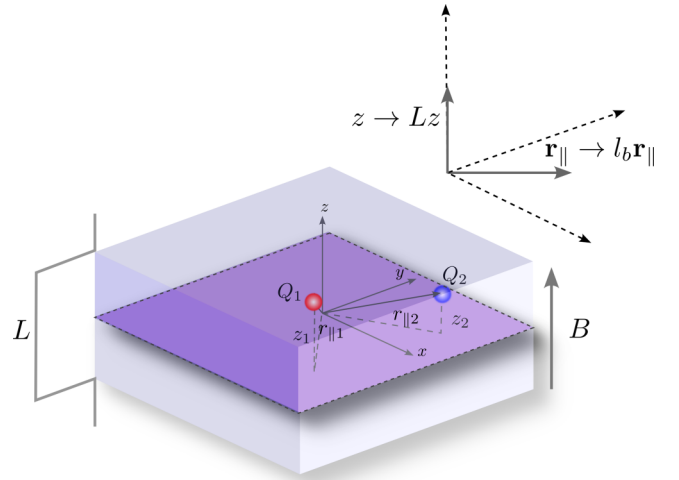


FIG. 11. The schematic illustrates the geometry of the system, wherein the charged particles are confined within the QW under the influence of an external perpendicular magnetic field, denoted as B . The radial coordinate r and transverse coordinate z define the particle positions. The figure on the right shows the scaling of the dimensionless coordinates. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

($\mathbf{r}_\parallel, z$), as shown in Fig. 11:

$$V(\mathbf{r}_1 - \mathbf{r}_2) = \frac{Q^2}{4\pi\epsilon_0 l_b} \frac{1}{\sqrt{|\mathbf{r}_\parallel 1 - \mathbf{r}_\parallel 2|^2 + \alpha^2(z_1 - z_2)^2}}, \quad (\text{A1})$$

where $l_b = \sqrt{2\hbar/QB}$ denotes the in-plane magnetic length, α is defined as the ratio of the width of the QW to the magnetic length L/l_b , Q is the fundamental electron charge, B is the magnetic field intensity, ϵ is the relative permittivity of the material, and ϵ_0 is the electrical permittivity of vacuum. Considering the second quantization framework, we choose as basis functions the product of the eigenfunctions of a single electron subject to an external perpendicular magnetic field to describe the in-plane problem and the ground state of an infinite symmetric QW in the transverse direction, as follows:

$$\begin{aligned} \phi_i(\mathbf{r}) &= c_{n_i, l_i} r^{|l_i|} e^{-\frac{r^2}{2}} L_{n_i}^{|l_i|}(r^2) e^{il_i \theta} \cos(\pi z) \\ &= c_{n_i, l_i} R_{n_i, l_i}(r) e^{il_i \theta} \zeta(z). \end{aligned} \quad (\text{A2})$$

Here, n_i and l_i denote the radial and angular quantum numbers of the states $|i\rangle$ and $L_{n_i}^{|l_i|}(r)$ are the associated Laguerre polynomials. The normalization constant is given by $c_{n_i, l_i} = \sqrt{2n_i! / [\pi(n_i + |l_i|)!]}$. We assume the first QW subband approximation owing to the large energy gap between the first excited state and the QW ground state compared to the typical Coulomb strength in thin QWs. In addition, the spin degrees of freedom (s_i) were neglected because the influence of Coulomb interaction preserves their inherent properties. We define $\langle ij|V|st\rangle$ as the two-body Coulomb matrix element as

$$\langle ij|V|st\rangle = \int d^3r_1 d^3r_2 \phi_i^*(\mathbf{r}_1) \phi_j^*(\mathbf{r}_2) V(\mathbf{r}_1 - \mathbf{r}_2) \phi_t(\mathbf{r}_2) \phi_s(\mathbf{r}_1). \quad (\text{A3})$$

Note that $\mathbf{r}$ is a 3D vector, and labels (1, 2) denote the position vectors of each particle. By using the Coulomb potential, Eq. (A1), and single-particle wave functions using Fourier integrals [40],

$$\phi_k(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3q \psi_k(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}, \quad (\text{A4})$$

$$V(\mathbf{r}_1 - \mathbf{r}_2) = \frac{1}{(2\pi)^3} \int d^3q \tilde{V}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{r}_1 - \mathbf{r}_2)}, \quad (\text{A5})$$

where $\psi_k(\mathbf{q})$, $\tilde{V}(\mathbf{q})$ correspond to the single-particle wave function and the Coulomb potential Fourier transform, respectively. The Coulomb matrix element takes the following form [60]:

$$\begin{aligned} \langle ij|V|st \rangle &= \frac{1}{(2\pi)^9} \int d^3q \int d^3q_1 \int d^3q_2 \psi_i^*(\mathbf{q}_1) \psi_j^*(\mathbf{q}_2) \\ &\times \psi_t(\mathbf{q}_2 + \mathbf{q}) \psi_s(\mathbf{q}_1 - \mathbf{q}) \tilde{V}(\mathbf{q}). \end{aligned} \quad (\text{A6})$$

It is straightforward to see that the last expression can be rewritten in a more compact form by grouping terms as follows:

$$\langle ij|V|st \rangle = \frac{1}{(2\pi)^3} \int d^3q A_{i,s}(\mathbf{q}) B_{j,t}(\mathbf{q}) \tilde{V}(\mathbf{q}), \quad (\text{A7})$$

where $A_{i,s}(\mathbf{q})$ and $B_{j,t}(\mathbf{q})$ are the convolution integrals given by

$$A_{i,s}(\mathbf{q}) = \frac{1}{(2\pi)^3} \int d^3q_1 \psi_i^*(\mathbf{q}_1) \psi_s(\mathbf{q}_1 - \mathbf{q}), \quad (\text{A8})$$

$$B_{j,t}(\mathbf{q}) = \frac{1}{(2\pi)^3} \int d^3q_2 \psi_j^*(\mathbf{q}_2) \psi_t(\mathbf{q}_2 + \mathbf{q}). \quad (\text{A9})$$

Now, the previous integrals can be transformed into the space coordinate representation and together with the Fourier transform of the 3D dimensionless Coulomb potential:

$$\tilde{V}(\mathbf{q}) = \frac{4\pi\alpha}{q_z^2 + \alpha^2 q^2}. \quad (\text{A10})$$

We obtain the final form of the Coulomb matrix element, which can be expressed as

$$\langle ij|V|st \rangle = \delta_{l_i+l_j, l_s+l_t} \int_0^\infty dq F_\alpha(q) \vartheta_{i,s}(q) \vartheta_{j,t}(q), \quad (\text{A11})$$

with

$$F_\alpha(q) = \frac{\pi^2 [20\alpha^3 \pi^2 q^3 + 3\alpha^5 q^5 + 32\pi^4 (\alpha q + e^{-\alpha q} - 1)]}{\alpha^2 (4\pi^2 q + \alpha^2 q^3)^2} \quad (\text{A12})$$

and

$$\begin{aligned} \vartheta_{a,b}(q) &= c_{n_a, l_a} c_{n_b, l_b} \int_0^\infty dr r^{|l_a|+|l_b|+1} e^{-r^2} \\ &\times L_{n_a}^{|l_a|}(r^2) L_{n_b}^{|l_b|}(r^2) J_{|l_a-l_b|}(qr). \end{aligned} \quad (\text{A13})$$

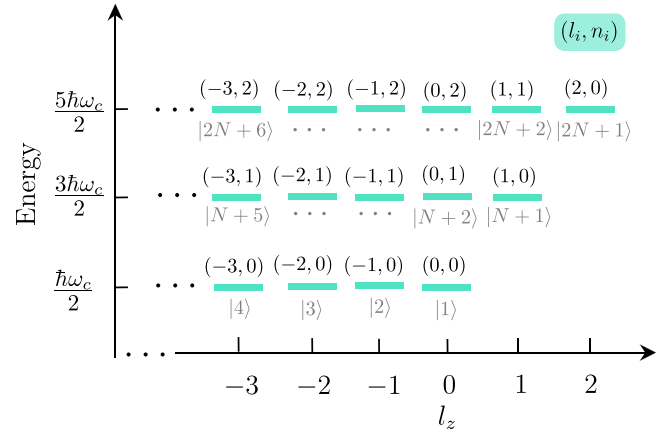


FIG. 12. The diagram illustrates the ordering of the Landau basis, in which each layer comprises N degenerate electronic states. The Dirac notation $|i\rangle$ denotes the ordering number, whereas the pair (n_i, l_i) specifies the associated radial and angular quantum numbers. For the interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

Here, $J_{|l_a-l_b|}(qr)$ denotes the Bessel function of the first type. It is important to highlight the explicit conservation of angular momentum, as shown in Eq. (A11) because the scattering processes involved in the Coulomb interaction preserve the angular momentum of particles. Owing to the highly oscillatory nature of the functions $\eta_{a,b}(q)$ at elevated angular momentum values, Eq. (A11) was computed numerically using the quadrature method [61] to ensure high precision. The finite structure of the Coulomb matrix element is significantly influenced by the form factor $F_\alpha(q)$, which is an envelope function that exhibits a magnetic-field-dependent decay rate.

APPENDIX B: LANDAU BASIS ORDERING

In this Appendix, we elucidate the method by which the ordering of quantum states is utilized in all the calculations in this study. Specifically, Fig. 12 illustrates the basis for ordering Landau levels. We assume that the states are evenly distributed across the three levels, with N states per level ($N = 20$ for the numerical simulations). Furthermore, each layer comprises degenerate electronic states, with the energy determined by (neglecting the spin) $\hbar\omega_c^{(e)}(B)/2 \cdot (2n_i + |l_i| - l_i + 1)$, where $\omega_c(B)$ represents the electron cyclotron frequency. Each state is identified by the ordering number $|i\rangle$ and its corresponding quantum number (n_i, l_i) , which denote the radial and angular momentum numbers, respectively.

- [1] H. Deng, H. Haug, and Y. Yamamoto, Exciton-polariton Bose-Einstein condensation, *Rev. Mod. Phys.* **82**, 1489 (2010).
 [2] A. Imamoglu, R. J. Ram, S. Pau, and Y. Yamamoto, Nonequilibrium condensates and lasers without inversion: Exciton-polariton lasers, *Phys. Rev. A* **53**, 4250 (1996).

- [3] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, Bose-Einstein condensation of exciton polaritons, *Nature (London)* **443**, 409 (2006).

- [4] T. Byrnes, N. Y. Kim, and Y. Yamamoto, Exciton–polariton condensates, *Nat. Phys.* **10**, 803 (2014).
- [5] A. V. Yulin, I. A. Shelykh, E. S. Sedov, and A. V. Kavokin, Vorticity of polariton condensates in rotating traps, *Phys. Rev. B* **108**, 155301 (2023).
- [6] S. Savasta, O. D. Stefano, V. Savona, and W. Langbein, Quantum complementarity of microcavity polaritons, *Phys. Rev. Lett.* **94**, 246401 (2005).
- [7] K. G. Lagoudakis, M. Wouters, M. Richard, A. Baas, I. Carusotto, R. André, L. S. Dang, and B. Deveaud-Plédran, Quantized vortices in an exciton–polariton condensate, *Nat. Phys.* **4**, 706 (2008).
- [8] P. Schwendimann, C. Ciuti, and A. Quattropani, Statistics of polaritons in the nonlinear regime, *Phys. Rev. B* **68**, 165324 (2003).
- [9] S. Ghosh and T. C. H. Liew, Quantum computing with exciton-polariton condensates, *npj Quantum Inf.* **6**, 16 (2020).
- [10] S. B. Anantharaman, K. Jo, and D. Jariwala, Exciton–photronics: From fundamental science to applications, *ACS Nano* **15**, 12628 (2021).
- [11] S. S. Demirchyan, I. Y. Chestnov, A. P. Alodjants, M. M. Glazov, and A. V. Kavokin, Qubits based on polariton Rabi oscillators, *Phys. Rev. Lett.* **112**, 196403 (2014).
- [12] D. Solnyshkov, O. Bleu, and G. Malpuech, All optical controlled-not gate based on an exciton–polariton circuit, *Superlattices Microstruct.* **83**, 466 (2015).
- [13] A. V. Larionov, V. D. Kulakovskii, S. Höfling, C. Schneider, L. Worschech, and A. Forchel, Polarized nonequilibrium Bose-Einstein condensates of spinor exciton polaritons in a magnetic field, *Phys. Rev. Lett.* **105**, 256401 (2010).
- [14] P. Walker, T. C. H. Liew, D. Sarkar, M. Durska, A. P. D. Love, M. S. Skolnick, J. S. Roberts, I. A. Shelykh, A. V. Kavokin, and D. N. Krizhanovskii, Suppression of Zeeman splitting of the energy levels of exciton-polariton condensates in semiconductor microcavities in an external magnetic field, *Phys. Rev. Lett.* **106**, 257401 (2011).
- [15] B. Pietka, D. Zygunt, M. Król, M. R. Molas, A. A. L. Nicolet, F. Morier-Genoud, J. Szczytko, J. Łusakowski, P. Zięba, I. Tralle, P. Stepnicki, M. Matuszewski, M. Potemski, and B. Deveaud, Magnetic field tuning of exciton-polaritons in a semiconductor microcavity, *Phys. Rev. B* **91**, 075309 (2015).
- [16] M. T. Portella-Oberli, J. Berney, L. Kappei, F. Morier-Genoud, J. Szczytko, and B. Deveaud-Plédran, Dynamics of trion formation in $\text{In}_x\text{Ga}_{1-x}\text{As}$ quantum wells, *Phys. Rev. Lett.* **102**, 096402 (2009).
- [17] R. Rapaport, E. Cohen, A. Ron, E. Linder, and L. N. Pfeiffer, Negatively charged polaritons in a semiconductor microcavity, *Phys. Rev. B* **63**, 235310 (2001).
- [18] F. Rana, O. Koksai, M. Jung, G. Shvets, A. N. Vamivakas, and C. Manolatu, Exciton-trion polaritons in doped two-dimensional semiconductors, *Phys. Rev. Lett.* **126**, 127402 (2021).
- [19] H.-R. Wei and F.-G. Deng, Universal quantum gates on electron-spin qubits with quantum dots inside single-side optical microcavities, *Opt. Express* **22**, 593 (2014).
- [20] J.-Z. Liu, H.-R. Wei, and N.-Y. Chen, A heralded and error-rejecting three-photon hyper-parallel quantum gate through cavity-assisted interactions, *Sci. Rep.* **8**, 1885 (2018).
- [21] Y.-H. Han, C. Cao, L. Zhang, X. Yi, P.-P. Yin, L. Fan, and R. Zhang, High-fidelity hybrid universal quantum controlled gates on photons and quantum-dot spins, *Int. J. Theor. Phys.* **60**, 1136 (2021).
- [22] B.-Y. Xia, C. Cao, Y.-H. Han, and R. Zhang, Universal photonic three-qubit quantum gates with two degrees of freedom assisted by charged quantum dots inside single-sided optical microcavities, *Laser Phys.* **28**, 095201 (2018).
- [23] P. Dong and Z.-L. Cao, Quantum computation with quantum-dot spin qubits inside a cavity, *Phys. Lett. A* **373**, 1527 (2009).
- [24] O. Kyriienko and T. C. H. Liew, Exciton-polariton quantum gates based on continuous variables, *Phys. Rev. B* **93**, 035301 (2016).
- [25] L. S. Ricco, I. A. Shelykh, and A. Kavokin, Qubit gate operations in elliptically trapped polariton condensates, *Sci. Rep.* **14**, 4211 (2024).
- [26] J. Barrat, A. Tzortzakakis, M. Niu, X. Zhou, G. Paschos, D. Petrosyan, and P. Savvidis, Qubit analog with polariton superfluid in an annular trap, *Sci. Adv.* **10**, eado4042 (2024).
- [27] C. A. Jiménez-Orjuela, H. Vinck-Posada, and J. M. Villas-Bôas, Dark excitons in a quantum-dot-cavity system under a tilted magnetic field, *Phys. Rev. B* **96**, 125303 (2017).
- [28] A. de Freitas, L. Sanz, and J. M. Villas-Bôas, Coherent control of the dynamics of a single quantum-dot exciton qubit in a cavity, *Phys. Rev. B* **95**, 115110 (2017).
- [29] E. Paspalakis and J. Boviatsis, Ultrashort electromagnetic pulse control of intersubband quantum well transitions, *Nanoscale Res. Lett.* **7**, 478 (2012).
- [30] A. F. Urquijo-Rodríguez, E. A. Gómez, B. A. Rodríguez, and H. Vinck-Posada, Quantum control of polariton emission in a microcavity-quantum well system under magnetic field, *J. Phys.: Condens. Matter* **36**, 505804 (2024).
- [31] E. Laird, F. M. Marchetti, D. K. Efimkin, M. M. Parish, and J. Levinsen, Rydberg exciton-polaritons in a magnetic field, *Phys. Rev. B* **106**, 125407 (2022).
- [32] A. Kudlis, I. Aleksandrov, Z. Alisultanov, K. Varga, I. Shelykh, and V. Shahnazaryan, Theory of magnetotriion-polaritons in transition metal dichalcogenide monolayers, *npj 2D Mater. Appl.* **8**, 77 (2024).
- [33] M. Combescot and J. Tribollet, Trion oscillator strength, *Solid State Commun.* **128**, 273 (2003).
- [34] E. A. Zhukov, D. R. Yakovlev, M. Bayer, M. M. Glazov, E. L. Ivchenko, G. Karczewski, T. Wojtowicz, and J. Kossut, Spin coherence of a two-dimensional electron gas induced by resonant excitation of trions and excitons in $\text{CdTe}(\text{Cd}, \text{Mg})\text{Te}$ quantum wells, *Phys. Rev. B* **76**, 205310 (2007).
- [35] I. Bar-Joseph, Trions in GaAs quantum wells, *Semicond. Sci. Technol.* **20**, R29 (2005).
- [36] J. A. Nelder and R. Mead, A simplex method for function minimization, *Comput. J.* **7**, 308 (1965).
- [37] B. A. Rodriguez and A. Gonzalez, Spin polarization and magnetoluminescence of confined electron-hole systems, *Phys. Rev. B* **63**, 205324 (2001).
- [38] P. Hawrylak, Far infrared absorption by screened d-states in quantum wells in a strong magnetic field, *Solid State Commun.* **88**, 475 (1993).
- [39] P. Pietiläinen and T. Chakraborty, Interacting-electron states and the persistent current in a quantum ring, *Solid State Commun.* **87**, 809 (1993).

- [40] T. Chakraborty and P. Pietiläinen, Electron-electron interaction and the persistent current in a quantum ring, *Phys. Rev. B* **50**, 8460 (1994).
- [41] A. H. MacDonald and D. S. Ritchie, Hydrogenic energy levels in two dimensions at arbitrary magnetic fields, *Phys. Rev. B* **33**, 8336 (1986).
- [42] D. M. Whittaker and A. J. Shields, Theory of X^- at high magnetic fields, *Phys. Rev. B* **56**, 15185 (1997).
- [43] C. Riva, F. M. Peeters, and K. Varga, Positively charged magnetoexcitons in a semiconductor quantum well, *Phys. Rev. B* **64**, 235301 (2001).
- [44] P. Redliński and J. Kossut, Negative trions in CdTe quantum wells in the presence of a magnetic field—a numerical study, *Semicond. Sci. Technol.* **17**, 237 (2002).
- [45] Annotation 1: The numerical results were obtained by considering the effective Hamiltonian as described in Eq. (7) while disregarding the contributions from trions. Specifically, we omitted all terms that involve trion operators.
- [46] Annotation 2: The numerical results for trion-photon contributions are obtained by considering the effective Hamiltonian as described in Eq. (7) and neglecting all terms that involve exciton operators.
- [47] X. Wu, S. L. Tomarken, N. A. Petersson, L. A. Martinez, Y. J. Rosen, and J. L. DuBois, High-fidelity software-defined quantum logic on a superconducting qubit, *Phys. Rev. Lett.* **125**, 170502 (2020).
- [48] A. Manassen, E. Cohen, A. Ron, E. Linder, and L. N. Pfeiffer, Exciton and trion spectral line shape in the presence of an electron gas in GaAs/AlAs quantum wells, *Phys. Rev. B* **54**, 10609 (1996).
- [49] P. Dawson, I. Galbraith, A. I. Kucharska, and C. T. Foxon, Low-power, all-optical nonlinear absorption in asymmetric double quantum wells, *Appl. Phys. Lett.* **58**, 2889 (1991).
- [50] D. Choi, M. Park, C.-Y. Sung, H. Choi, and Y.-H. Cho, Realization of exciton-polariton condensation in GaAs-based microcavity grown by metalorganic chemical vapor deposition, *Phys. Rev. Res.* **4**, 033188 (2022).
- [51] A. V. Sekretenko, S. S. Gavrilov, and V. D. Kulakovskii, Polariton-polariton interactions in microcavities under a resonant 10 to 100 picosecond pulse excitation, *Phys. Rev. B* **88**, 195302 (2013).
- [52] V. D. Kulakovskii, A. A. Demenev, A. S. Brichkin, and N. A. Gippius, Self-shaping of cavity-polariton systems created by resonant broadband excitation, *Phys. Rev. B* **98**, 205424 (2018).
- [53] A. M. Weiner, Ultrafast optical pulse shaping: A tutorial review, *Opt. Commun.* **284**, 3669 (2011), .
- [54] A. Hayat, C. Lange, L. A. Rozema, A. Darabi, H. M. van Driel, A. M. Steinberg, B. Nelsen, D. W. Snoke, L. N. Pfeiffer, and K. W. West, Dynamic Stark effect in strongly coupled microcavity exciton polaritons, *Phys. Rev. Lett.* **109**, 033605 (2012).
- [55] I. I. Ryzhov, G. G. Kozlov, D. S. Smirnov, M. M. Glazov, Y. P. Efimov, S. A. Eliseev, V. A. Lovtcius, V. V. Petrov, K. V. Kavokin, A. V. Kavokin, and V. S. Zapasskii, Spin noise explores local magnetic fields in a semiconductor, *Sci. Rep.* **6**, 21062 (2016).
- [56] Y. Sun, P. Wen, Y. Yoon, G. Liu, M. Steger, L. N. Pfeiffer, K. West, D. W. Snoke, and K. A. Nelson, Bose-Einstein condensation of long-lifetime polaritons in thermal equilibrium, *Phys. Rev. Lett.* **118**, 016602 (2017).
- [57] A. P. M. Place *et al.*, New material platform for superconducting transmon qubits with coherence times exceeding 0.3 milliseconds, *Nat. Commun.* **12**, 1779 (2021).
- [58] S. J. Boyle, A. J. Ramsay, F. Bello, H. Y. Liu, M. Hopkinson, A. M. Fox, and M. S. Skolnick, Two-qubit conditional quantum-logic operation in a single self-assembled quantum dot, *Phys. Rev. B* **78**, 075301 (2008).
- [59] M. A. Rol, C. C. Bultink, T. E. O'Brien, S. R. de Jong, L. S. Theis, X. Fu, F. Luthi, R. F. L. Vermeulen, J. C. de Sterke, A. Bruno, D. Deurloo, R. N. Schouten, F. K. Wilhelm, and L. DiCarlo, Restless tuneup of high-fidelity qubit gates, *Phys. Rev. Appl.* **7**, 041001 (2017).
- [60] J. Z. García, Analytic Coulomb matrix elements in a three-dimensional geometry, *J. Math. Phys.* **46**, 122104 (2005).
- [61] G. V. Milovanovic and M. P. Stanić, Numerical integration of highly oscillating functions, *Analytic Number Theory, Approximation Theory, and Special Functions: In Honor of Hari M. Srivastava*, edited by G. V. Milovanović and M. T. Rassias (Springer, New York, 2014), pp. 613–649.