

Superradiant scattering from evanescent waves

M. Pardal ¹, R. A. Fonseca ^{1,2} and J. Vieira ¹¹GoLP/IPFN, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisbon, Portugal²ISCTE - Lisbon University Institute, Lisbon, Portugal

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Evanescent waves, such as the ones present at the interface between a laser and an overdense plasma, provide extremely localized fields that are able to induce rapid accelerations in charged particles. We show that such accelerations can generate high-energy, low-divergence, directional photon beams even for moderately relativistic electrons ($\gamma \simeq 10$). This process can also lead to superradiant emission of bright, ultrashort pulses of light, as long as the electron bunch is suitably modulated. Although never directly observed, this mechanism should leave clear signatures in laser-plasma acceleration experiments with minimal setup adaptations.

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Introduction. Advanced x-ray [1] sources are central to probe and image ultrafast processes occurring at the atomic and molecular scale [2–4]. Large-scale free-electron lasers [5] and synchrotrons [6–8] are among the most successful, but efforts have been made to develop smaller and more cost-effective radiation sources in recent years, relying on a plethora of radiation generation mechanisms. For example, high harmonic generation by a plasma mirror [9–13] and Thomson scattering of plasma electrons [14–16], can lead to emission of harmonics up to the x-ray region of an ultra-intense laser pulse. Furthermore, ultrarelativistic electrons can be used to obtain transition radiation in the x-ray region [17–20] and betatron radiation from high-energy electrons ($\simeq 1$ GeV) accelerated by ultra-intense lasers ($a_0 > 1$) in laser plasma accelerators [21–25] routinely achieves KeV x-ray beams [26].

Nevertheless, mechanisms capable of producing x rays from moderately relativistic electron beams ($\gamma \sim 10$) or moderately intense laser pulses ($a_0 < 1$) can open new exciting research opportunities by democratizing access to high-energy photon beams. A notable example, Thomson scattering of a relativistic electron beam by a laser pulse [27,28], can generate double Doppler-shifted versions of the incident laser, producing nm wavelength pulses from a 800 nm laser. In this case, for a fixed γ , the emitted wavelength is set by the laser's wavelength, but could it be possible to develop a mechanism to obtain emission at even smaller wavelengths? Here, we address this question by exploring a new light-emitting process based on the scattering of an electron from an evanescent wave. Evanescent waves are at the core of plasmonics, where they play a key role in applications ranging from photon induced near-field microscopy [29] to superresolution microscopy thanks to superlensing effects [30]. These waves were also exploited towards radiation emission in graphene,

thanks to a Thomson-scattering effect experienced by electrons that propagate parallel to the evanescent wave [31]. These surface modes are ubiquitous in laser-plasma interaction experiments, namely in those with plasma mirrors, which are commonly used to improve the contrast of high-power laser pulses [32].

The degree of spatial localization of surface waves in the direction normal to the surface is set by the material skin depth. The skin depth λ_{ev} can be more than an order of magnitude smaller than the wavelength, $\lambda_0 = 2\pi c/\omega_0$, of the laser that excites the evanescent mode. For example, according to the linear dispersion relation for electromagnetic waves in plasma, $\lambda_{\text{ev}} = c/\sqrt{\omega_p^2 - \omega_0^2}$, where ω_0 is the central frequency of the incident laser pulse, $\omega_p = \sqrt{e^2 n_0/m_e \epsilon_0}$ is the plasma frequency, e the elementary charge, m_e the electron mass, ϵ_0 the vacuum electric permittivity, and n_0 the plasma density. Thus, it can be as short as $\simeq 10$ nm for a plasma density $n_0 \sim 2 \times 10^{23} \text{ cm}^{-3}$. This is more than two orders of magnitude below the central wavelength of a typical laboratory laser pulse, where $\lambda_0 \simeq 800$ nm.

In this paper, we take direct advantage of the extreme spatial localization of evanescent waves to generate directed x rays and demonstrate, with theory and through particle-in-cell (PIC) simulations for OSIRIS [33,34] complemented by the Radiation Diagnostic for OSIRIS (RaDiO) [35], that electrons with $\gamma \simeq 10$ –100 that scatter across a surface wave can produce KeV-MeV radiation. Furthermore, we also find that suitably modulated electron bunches can produce a superradiant optical shock. The creation of optical shocks by tilted electron bunches was investigated by Bolotowsky and Ginzburg in transition radiation [36], but here we show that, because the radiation from each bunch electron interferes constructively, its intensity scales favorably with the square of bunch particles as in superradiance. More generally, we found that the intersection position between the electron bunch and the evanescent wave can act as a virtual particle, whose trajectory defines key radiation properties, just as if it were a single real particle [37]. This can bring previously unexplored spectral features and coherence in evanescent scattering configurations.

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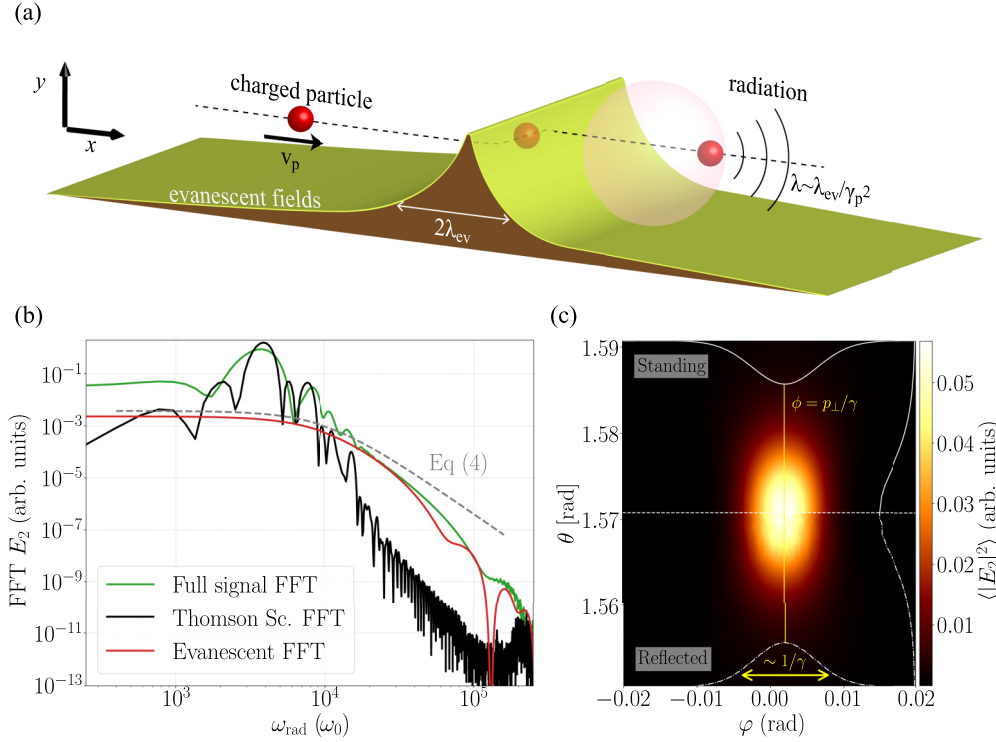


FIG. 1. Simulation results of the radiation generated during evanescent mode scattering of a single electron. (a) Simulation setup and evanescent field. A single particle collides with a localized evanescent field and suffers sharp acceleration, producing radiation. (b) Frequency spectrum of the emitted radiation in the plasma mirror setup. Contributions from the evanescent and pre-evanescent regimes are plotted separately from the full spectrum. (c) Spatial intensity distribution of the emitted evanescent radiation at a far away spherical detector for the standing evanescent wave (top) and for the plasma mirror setup (bottom).

Single particle evanescent radiation. The electromagnetic vector potential of a standing evanescent wave, $\mathbf{A}_{\text{ev}}$, can adequately characterize the evanescent field structure. In its simplest form, $\mathbf{A}_{\text{ev}}$ may then be given by:

$$\mathbf{A}_{\text{ev}} = A_{0,\text{ev}}[1 - \text{sign}(x)(e^{-|x|/\lambda_{\text{ev}}} - 1)] \times \cos(\omega_{\text{ev}}t)\mathbf{e}_y \equiv A_{\text{ev}}(x, t)\mathbf{e}_y, \quad (1)$$

where $A_{0,\text{ev}}$ is the vector potential peak amplitude, ω_{ev} is the evanescent wave frequency, λ_{ev} is its typical scale length (also denoted by skin depth), x is the distance along the surface normal, and $\mathbf{e}_y$ is a unit vector tangent to the surface. The green surface in Fig. 1(a) is a schematic illustration of the evanescent wave structure.

Owing to its highly localized spatial structure, evanescent waves can impart a strong transverse acceleration to an electron that propagates across the surface fields. Such an intense, albeit localized acceleration, underlies radiation emission in evanescent wave-scattering processes. The acceleration can be deduced from the electron trajectory. Figure 1(a) depicts the trajectory of an electron (red sphere) moving across an evanescent wave (green surface). This motion follows from the conservation of the y component of the canonical momentum $\mathcal{P}_y$. For an electron initially at rest, $\mathcal{P}_y = p_y - eA_{0,\text{ev}} = 0 \Rightarrow p_y = eA_{0,\text{ev}}$. Here, $p_y = m_e\gamma_p v_y$ is the electron momentum along y , and v_y its transverse velocity. Solving the corresponding equation of motion, under the assumption that the electron velocity (v_x) along x and its relativistic factor ($\gamma_p = 1/\sqrt{1 - v_p^2/c^2}$) remain constant during the interaction, to-

gether with the assumption that the evanescent wave oscillates slowly ($\omega_{\text{ev}} \ll v_x/\lambda_{\text{ev}}$) leads to:

$$\frac{dv_y}{dt} \simeq \frac{v_x v_{y0,\text{ev}} \exp(-v_x|t|/\lambda_{\text{ev}})}{\lambda_{\text{ev}}} \equiv a_y, \quad (2)$$

where we used $x = v_p t$ to relate the longitudinal electron position with the interaction time t . Equation (2) shows that the transverse electron acceleration grows as λ_{ev} decreases. Hence, even though an evanescent wave may not change the electron trajectory appreciably (due to its high spatial localization), it can still impart a strong acceleration to produce high-energy radiation.

To make quantitative predictions, we consider the radiated energy in the far field (I_{rad}), per unit frequency ω , per solid angle Ω , which, for an arbitrary electron trajectory $\mathbf{r}(t)$, is generally given by the Liénard-Wiechert potentials [38–40]. The single-particle spectrum emitted along the forward x direction ($\theta = \pi/2$, $\varphi = 0$), is then given by (see Supplemental Material [41] for detailed calculations):

$$\left| \frac{d^2 I_{\text{rad}}}{d\omega d\Omega} \right|_{\mathbf{n}=\mathbf{e}_x} = \frac{e^2}{4\pi^2 c} \left| \frac{4\beta_{y0}\gamma_p^2}{1 + \omega^2 \lambda_{\text{ev}}^2 / (2\gamma_p^2 v_x^2)} \right|^2, \quad (3)$$

with $\mathbf{n} = [\sin(\theta)\cos(\varphi), \sin(\theta)\sin(\varphi), \cos(\theta)]$ the unit vector that points to the observation direction, c is the speed of light and t the time of emission. In addition, $\boldsymbol{\beta} = \mathbf{v}/c$ is the electron velocity $\beta_{y0} = eA_{\text{ev},0}/(m_e\gamma_p c)$.

Equation (3) shows that an electron moving across an evanescent wave produces a broadband Lorentzian spectrum.

In the time domain, the spectrum given by Eq. (3) corresponds to an evanescent field $E_{\text{rad}} \propto \exp(-c|t|2\gamma_p^2/\lambda_{\text{ev}})$. This unusual field profile can be regarded as a flying electromagnetic copy of the surface wave, compressed by a double Doppler shift.

While the laser intensity controls the peak radiated intensity, the corresponding spectral width, $\Delta\omega$, is proportional to $\Delta\omega \propto 2\gamma_p^2(c/\lambda_{\text{ev}})$. Accordingly, solid plasma targets with various densities, or different nonionized materials, each with a characteristic λ_{ev} , can produce measurably different spectra. As a figure of merit, the typical radiation wavelength, λ_{rad} , is then proportional to the evanescent wave skin depth compressed by a double Doppler shift factor. Hence, with $\lambda_{\text{ev}} \simeq 10$ nm, even a moderately relativistic electron, with a relativistic factor $\gamma_p = 10$, could radiate at $\lambda_{\text{rad}} \propto \lambda_{\text{ev}}/(2\gamma_p^2) \simeq 0.1$ nm.

Equation (3) also shows that evanescent wave scattering produces radiation with noticeably distinct properties from other known radiation mechanisms, sharing some similarities, however, with transition radiation whose energy spectrum is given by the $d^2I_{\text{rad}}/(d\omega d\Omega) \propto \omega_p^4 \gamma_p^6 \theta^2 / [\omega^4 (1 + \gamma_p^2 \theta^2)^4]$ [40]. The radiated spectrum decays with ω^{-4} in both cases. However, whereas in transition radiation the spectral intensity always decays with ω^{-4} , in evanescent radiation it only decays with ω^{-4} for $\omega \gg 2\gamma_p^2 \sqrt{\omega_p^2 - \omega_0^2}$. Thus, in the case of evanescent radiation, the decay in intensity happens only at higher frequencies. The number of radiated photons is also higher in the case of evanescent radiation when compared with transition radiation, for particles with the same energy colliding with similar plasma targets. Additionally, transition radiation does not lead to on-axis emission, whereas evanescent wave scattering produces highly directional radiation, along the main propagation axis (x in the present case).

Figures 1(b)–1(c) illustrate the typical radiation spectrum of evanescent wave scattering [Fig. 1(b)] and the time-integrated radiated intensity [Fig. 1(c)] from two-dimensional (2D) numerical simulations with the particle-in-cell (PIC) code OSIRIS and with the Radiation Diagnostic for OSIRIS (RaDiO). A virtual spherical detector records the radiation in time. The detector is placed in the far field and is aligned with the longitudinal x direction. Figure 1(b) shows simulation results where a single electron with $\gamma_p = 80$ scatters through an evanescent mode with $A_{\text{ev},0} = 0.2m_e c/e$ and $\lambda_{\text{ev}} \simeq 0.17 c/\omega_0$. We considered two types of evanescent modes, one created by a plasma mirror reflecting a linearly polarized laser pulse and an externally imposed standing mode like the one described by Eq. (1). The laser pulse's normalized vector potential is $a_0 = eA_0/(m_e c) = 0.2$ and spot size $W_0 = 6\lambda_0$. The plasma mirror density is $n_0 = 32n_{\text{crit}}$, where $n_{\text{crit}} = \omega_0^2 m_e \epsilon_0 / e^2$ is the critical plasma density. The initial electron transverse momentum, $p_y(t=0) = 0.16m_e c$, matches the normalized laser vector potential at its injection position.

In the plasma mirror configuration, the electron first scatters in the standing wave created by the superposition of the incident and reflected laser (Thomson scattering), before reaching the evanescent wave at the plasma surface (evanescent wave scattering). The full radiation spectrum (green) in Fig. 1(b) then contains Thomson scattering (black)

and evanescent wave-scattering (red) components. Thomson scattering leads to an intensity peak at the double Doppler-shifted laser frequency ($\propto \omega_0 \gamma_p^2 = 6.4 \times 10^3 \omega_0$). Evanescent wave scattering dominates at higher frequencies. For instance, at $\omega_{\text{rad}} \simeq 10^6 \omega_0$, the evanescent field radiation spectral power is two orders of magnitude higher than that of the Thomson scattering. Figure 1(b) also shows that the predictions of Eq. (3) (dashed gray) are in excellent agreement with the evanescent wave-scattering spectrum calculated numerically. The time-integrated spatial intensity distribution is fully contained in the region of $1/\gamma_p$, and is offset in the direction of the particle's perpendicular momentum, as depicted in Fig. 1(c).

Evanescent radiation superradiance. Evanescent wave-scattering radiation from an electron bunch (with N_e bunch electrons) generally leads to incoherent emission. Here, the radiated intensity is directly proportional to the number of bunch electrons, $I_{\text{rad}} \propto N_e$. However, in certain conditions, the spatial shape of the bunch can bring temporal coherence phenomena and superradiance to evanescent wave scattering, where the radiated intensity scales with the number of particles squared $I_{\text{rad}} \propto N_e^2$.

Consider a thin electron bunch where all electrons follow along the same trajectory, except for temporal and spatial offsets. We then assume that the trajectory of an electron that arrives at the evanescent wave at $t = t_i$, is given by $\mathbf{r}_i(t) = \mathbf{r}(t - t_i) + \mathbf{r}_{\text{vp}}(t_i) \equiv \mathbf{r}(t') + \mathbf{r}_{\text{vp}}(t_i)$. Here, $\mathbf{r}(t')$ is the electron displacement relative to a spatial offset $\mathbf{r}_{\text{vp}}(t_i)$, and $t' = t - t_i$ is the time-elapsed relative to t_i , and $\mathbf{r}_{\text{vp}}(t_i)$ is the instantaneous position where the electron bunch intersects the evanescent wave. To determine the corresponding radiated spectrum, we can integrate the Liénard-Wiechert expression for all bunch electrons by assuming a continuous charge distribution. For a one-dimensional bunch density profile, this integration yields:

$$\frac{d^2 I_{\text{rad}}}{d\omega d\Omega} = \frac{N_e^2}{T^2} \mathcal{F}_{\text{sing}}(\omega) \left| \int \exp \{i\omega[t_i - \mathbf{n} \cdot \mathbf{r}_{\text{vp}}(t_i)/c]\} dt_i \right|^2, \quad (4)$$

where $\mathcal{F}_{\text{sing}}$ is the spectral intensity emitted by a single electron. Electrons radiate the most at the position $\mathbf{r}_{\text{vp}}$ where they intersect the evanescent mode. Equation (4) shows that the crossing point then acts as a virtual particle and light source whose trajectory defines the temporal coherence properties of the emitted radiation as if it were a single charged particle [37]. The recent generalized superradiance concept [42] is a complementary case to Eq. (4) where both the particle position and momenta are modulated.

A clear example of this effect appears when we consider a tilted electron bunch with slope m traveling along x with velocity v_b . In this case, $\mathbf{r}_{\text{vp}} = v_{\text{vp}} t_i \mathbf{e}_y$, where $v_{\text{vp}} = m v_b$ is the vertical velocity of the virtual particle. According to Eq. (4), the radiation spectrum emitted by a flat-top, tilted electron bunch with duration T , is given by:

$$\frac{d^2 I_{\text{rad}}}{d\omega d\Omega} = \frac{N_e^2}{4} \mathcal{F}_{\text{sing}}(\omega) \text{sinc}^2 \left\{ \frac{\omega T [1 - (v_{\text{vp}}/c) \sin(\theta) \sin(\varphi)]}{2} \right\}, \quad (5)$$

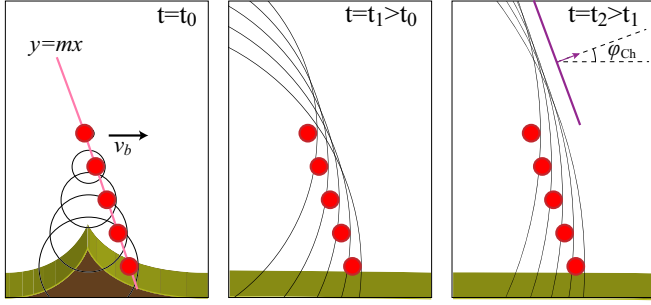


FIG. 2. A tilted particle beam collides with an evanescent wave, radiation from each particle is represented as an expanding circumference.

where we integrated from $t_i = [-T/2, T/2]$. We also assumed that the number of bunch particles is proportional to the bunch duration, $N_e \propto T$. For $\theta = \pi/2$, Eq. (5) recovers the typical superradiant scaling of the radiated power with the number of bunch electrons squared when $\varphi = \arcsin(c/v_{vp}) \equiv \varphi_{Ch}$ (see Supplemental Material [41] for more details). The critical angle φ_{Ch} corresponds to the Cherenkov cone angle [43] in vacuum. Figure 2 provides a clear pictorial demonstration that this superradiance effect is due to the onset of an optical shock, as in the classic, single-particle Cherenkov radiation effect.

The photon flux of evanescent radiation. It is useful to determine the total number of photons with emitted by the particles, $N_{ph}(\omega_r)$, at a given frequency ω_r within a bandwidth $\Delta\omega$. For a single particle, N_{ph} can be obtained from the Liénard-Wiechert potentials by integrating the radiation energy spectrum $dI/d\omega d\Omega$ along a frequency interval:

$$N_{ph}(\omega_r) = \frac{1}{\hbar\omega_r} \int_{\omega_r}^{\omega_r+\Delta\omega} \frac{dI}{d\omega d\Omega} d\omega \simeq \frac{dI}{d\omega d\Omega}(\omega_r) \frac{\Delta\omega}{\hbar\omega_r}. \quad (6)$$

Using the Liénard-Wiechert expression, and introducing $\mathcal{I}_{sing}(\omega_r)$ to replace the integral:

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} &= \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} \frac{\mathbf{n} \times [(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^2} e^{i\omega[t - \mathbf{n} \cdot \mathbf{r}(t)/c]} dt \right|^2 \\ &= \frac{e^2}{4\pi^2 c} \mathcal{I}_{sing}(\omega_r), \end{aligned} \quad (7)$$

N_{ph} becomes:

$$N_{ph}(\omega_r) = \frac{e^2}{4\pi^2 c} \frac{\Delta\omega}{\hbar\omega_r} \mathcal{I}_{sing}(\omega_r) = \frac{\alpha}{4\pi^2} \frac{\Delta\omega}{\omega_r} \mathcal{I}_{sing}(\omega_r). \quad (8)$$

Assuming a frequency-dependent bandwidth such that $\Delta\omega/\omega_r$ remains constant at $\Delta\omega/\omega_r = 10^{-3} = 0.1\%$, the number of photons per 0.1% bandwidth becomes:

$$\begin{aligned} N_{ph}(\omega_r) [\text{photons/mrad}^2 \text{ per } 0.1\% \text{BW}] \\ \simeq 1.84 \times 10^{-13} \mathcal{I}_{sing}(\omega_r). \end{aligned} \quad (9)$$

For a bunch of particles, the total number of photons depends on the nature of the radiation emission process: assuming incoherent emission, the number of photons scales with the number of particles in the beam N_p , while assuming coherent emission, the number of photons scales with the

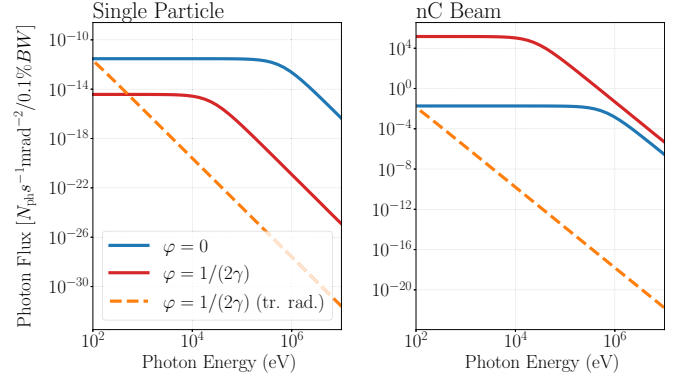


FIG. 3. Expected photon flux on axis and at the Cherenkov angle for a single particle (left) and for a nano-Coulomb beam of particles (right), assuming quadratic intensity scaling with the number of particles at the Cherenkov angle. The orange dashed line shows the expected number of photons for the same particles emitting transition radiation.

number of particles in the beam squared N_p^2

$$\begin{aligned} N_{ph}(\omega_r) [\text{photons/mrad}^2 \text{ per } 0.1\% \text{BW}] \\ = \begin{cases} N_p 1.84 \times 10^{-13} \mathcal{I}_{sing}(\omega_r) & (\text{incoherent}) \\ N_p^2 1.84 \times 10^{-13} \mathcal{I}_{sing}(\omega_r) & (\text{coherent}) \end{cases}. \end{aligned} \quad (10)$$

Figure 3 shows the expected photon flux from a single particle both on axis and at the Cherenkov angle obtained from numerical integration of Eq. (10) for a particle with $\gamma = 100$ colliding with an evanescent field with $A_{0,ev} = 0.2m_e c$ and $\lambda_{ev} = 40$ nm.

Because each particle radiates mostly on axis, the expected number of photons decreases when we look at larger angles, being close to four orders of magnitude smaller than the on axis. However, if we get a reasonably large number of particles involved in the process, for example using a nano-Coulomb particle bunch, the expected number of photons quickly increases faster at the Cherenkov angle than on axis, due to the quadratic scaling expected in superradiant processes. This of course, assumes that all particles in the beam radiate in phase for all frequencies at the Cherenkov angle, which may be difficult to achieve in real-life scenarios.

Superradiant optical shock formation. Figure 4(a) shows the onset of the optical shock in the virtual detector produced by a tilted electron bunch ($\gamma_p = 80$) with slope $m = 36$, which scatters from an evanescent wave (under the same parameters as in Fig. 1) in a 2D PIC simulation. The velocity of the superluminal virtual particle is $v_{vp}/c \simeq 36$, and the corresponding Cherenkov angle is $\theta_{Ch} \simeq 27$ mrad. This prediction is in excellent agreement with simulation results shown in Fig. 4(a). The optical shock leads to a sharp intensity spike. Interestingly, despite appearing in a region where single electrons nearly do not radiate [see Fig. 1(c)], the peak intensity at the optical shock is higher than the peak intensity on axis, which is where single bunch electrons produce most of their radiation. This is a direct consequence of the quadratic superradiant scaling, which happens only at the Cherenkov angle. Figure 4(b) shows that at $\varphi = \varphi_{Ch}$, the radiation intensity grows

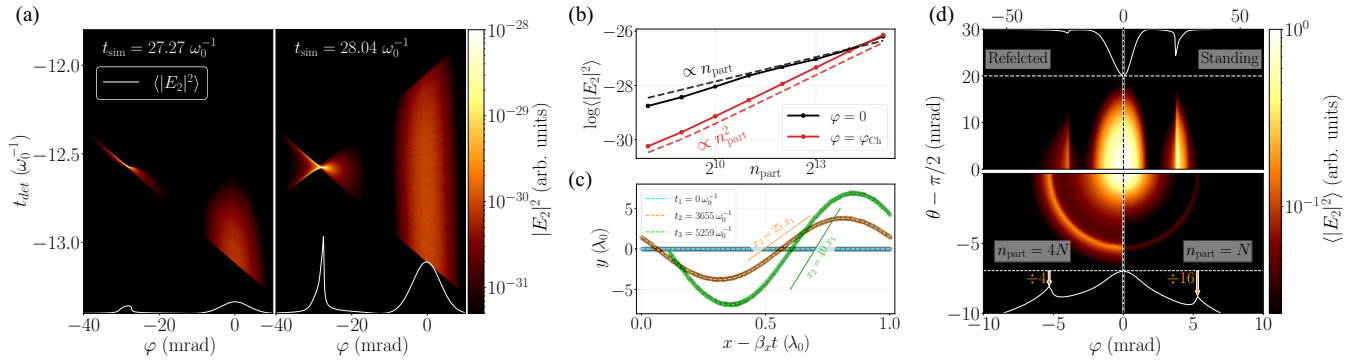


FIG. 4. Evanescent radiation from particle beams. (a) Snapshots of the spatiotemporal profile of radiation from a tilted particle beam colliding with a standing evanescent wave, and time-integrated profile (white line). (b) Integrated radiation intensity scaling with the number of particles on axis and at the Cherenkov angle in the setup of Fig. 4(a). (c) Particle beam shape after increasing propagation times. (d) Radiation spatial intensity profile for each evanescent mode (top left: plasma mirror evanescent wave due to reflection; top right: standing evanescent wave) and time-integrated radiation spectrum of a spiral-shaped electron bunch colliding with a standing evanescent wave, modulated by a circularly polarized laser pulse. White lines show line outs for $\theta = \pi/2$ in log scale (bottom).

quadratically with the number of particles whereas on axis it grows linearly with the number of particles.

Figures 4(c)–4(d) demonstrate a possible all-optical beam-shaping mechanism, which uses a laser pulse (with $a_0 \ll 1$) to modulate a relativistic electron bunch and control the onset of superradiance. In the most simple configuration, the laser pulse, which copropagates with the electron bunch, exerts a spatially periodic force that pushes bunch electrons along the polarization direction, thereby modulating the electron bunch at λ_0 . The modulation persists when the laser vanishes. Hence, a linearly or a circularly polarized laser pulse can introduce a sinusoidal or helical modulation on a pencil electron beam. Interestingly, the sinusoidal modulation on the bunch can be approximated as a sequence of tilted bunches with opposite slopes.

Figure 4(c) illustrates how a linearly polarized laser pulse imparts a sinusoidal modulation on a thin, pencil-like electron bunch with length $L \sim \lambda_0$, with $\gamma = 80$ and longitudinal thermal spread $u_x = 10^{-3} m_e c$. We used a Gaussian laser with wavelength λ_0 , waist $W_0 \sim 30\lambda_0$, and peak normalized vector potential is $a_0 = 0.1$ [dashed line in Fig. 4(a)].

The evanescent mode appears either self-consistently as the plasma target reflects the laser pulse or as an externally imposed standing evanescent wave like the ones generated by surface plasmons. Figure 4(d) (top) shows that the corresponding time-integrated radiation profile consists of two bright horizontal lobes appearing at $\varphi = \pm 22$ mrad. The angular location of the intensity peaks coincides exactly with the predictions of Eq. (5). Each tilted section of the bunch crosses the evanescent mode with a velocity $v_{vp} \sim c$. Thus, for $\theta = \pi/2$, $\varphi_{Ch} = 22$ mrad, exactly as shown in the simulations. In addition, the time-integrated radiation profile strongly contrasts with Fig. 1(c) for a single electron, emphasizing the important role of the spatial bunch shape on radiation emission.

Figure 4(d) (bottom) shows the time integrated radiation spectrum obtained from a 3D PIC simulation with similar setup as in sinusoidal bunch case but this time using a circularly polarized laser pulse to create a helical modulation. The angular position of the resulting ring structure coincides exactly with the predicted Cherenkov cone angle. The inten-

sity in the ring region grows with the number of electrons in the beam, N , squared, as opposed to the on-axis radiation, which grows linearly with the number of bunch electrons, in perfect agreement with the predictions of Eq. (5).

Conclusion. In conclusion, our scheme appears as a promising mechanism for high-frequency radiation emission, with a clear spectral signature that should be experimentally measurable. Furthermore, the radiation from scattering electrons on evanescent modes only occurs where the target exists. This radiation thus carries information about the target shape, this property could be exploited as a diagnostic of evanescent wave scattering or, conversely, to image targets with high (sub-incident-laser-wavelength) resolution. The mechanism described in this paper, capable of producing x rays by solely relying on moderately relativistic electrons can open new exciting research opportunities.

In this work, we used the reflection properties of plasma mirrors to create the evanescent field, but other alternatives are possible. Surface plasmons [44] are an interesting example, which provide potentially more localized evanescent fields with interesting features and radiation regimes. More intense evanescent modes, for example appearing when electrons are entirely removed from a solid target (e.g., by using an ultra-intense laser pulse), can produce stronger signals. Since evanescent waves in plasma can be excited at low laser intensities $I \ll 10^{18} \text{ W/cm}^2$, a KeV x-ray source based on evanescent wave scattering could be experimentally realized in standard laser wakefield accelerator (LWFA) experiments. This mechanism could be particularly appealing in the context of high repetition rate (KHz) LWFAs [23,45], where the energy of betatron photons, at the $\simeq 1$ eV level, is too low for applications.

Additionally, the laser-based electron modulator can shape a moderately relativistic electron beam in multiple ways. Hence, a suitable superposition of modes with different frequencies and polarization can control the virtual particle trajectory and therefore control superradiance, coherence and the spectral properties of radiation. For example, orthogonally polarized laser modes at ω_0 and $2\omega_0$ could create a virtual particle whose trajectory resembles the well-known figure-eight

motion of an electron in a laser field, which is at the core of high harmonic generation in lasers.

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