

Experimentally enhancing high-dimensional subchannel discrimination via optimal local filtering

Yen-An Shih,¹ Wan-Guan Chang^{2,3}, Gelo Noel M. Tabia^{4,3,*}, Hao-Cheng Weng^{5,1}, Tsung-Ying Tsai,¹ Chih-Sung Chuu^{1,†}, Huan-Yu Ku^{6,‡}, and Costantino Budroni⁷¹Department of Physics and Center for Quantum Science and Technology, National Tsing Hua University, Hsinchu 30003, Taiwan²Institute of Information Science, Academia Sinica, Taipei 115, Taiwan³Department of Physics and Center for Quantum Frontiers of Research & Technology (QFort), National Cheng Kung University, Tainan 701, Taiwan⁴Foxconn Quantum Computing Research Center, Taipei 114, Taiwan⁵Quantum Engineering Technology Labs and H. H. Wills Physics Laboratory, University of Bristol, Bristol BS8 1UB, United Kingdom⁶Department of Physics, National Taiwan Normal University, Taipei 116059, Taiwan⁷Department of Physics “E. Fermi” University of Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

(Received 16 September 2024; accepted 25 April 2025; published 13 August 2025)

The subchannel discrimination task involves distinguishing between different branches of a quantum evolution. The guessing probability of this task can be enhanced using quantum resources such as entanglement and steerability. In this study, we explore how to optimize the task’s guessing probability within the one-sided device-independent quantum information processing framework, which is naturally associated with steering, and restricted to one-way local operations and classical communication. From a theoretical standpoint, we demonstrate how to separately optimize local filters and measurements, and provide an optimality condition that directly links the enhancement of guessing probability to the maximum steerability achievable through local filters. We then experimentally demonstrate the enhancement of the discrimination task on a two-ququart photonic system by implementing the optimal local filters.

DOI: [10.1103/PhysRevResearch.7.L032035](https://doi.org/10.1103/PhysRevResearch.7.L032035)

Introduction. Manipulating quantum resources through distillation, filtering, or amplification [1–3] is essential for achieving quantum advantage in quantum information tasks [4], e.g., state discrimination [5–12], which underpins quantum communication [13,14]. For instance, entanglement enhances the guessing probability in channel discrimination [15–18]. This quantum advantage is characterized by entanglement robustness, a measure of entanglement [19–22]. However, the maximum enhancement may involve global operations on a two-party system. The optimal solution can be formulated as a semidefinite program (SDP) in terms of a quantum tester [23], implementable as a global state preparation and measurement. Such operations are difficult to implement when the parties are spatially distributed in line with communication assumptions.

From this practical perspective, then, it makes sense to restrict ourselves to local operations and classical communication (LOCC). Further restricting to one-way LOCC (1W-LOCC) and its associated one-sided device-independent

(1S-DI) framework [24] provides a complete characterization of the advantage for the channel discrimination, and its generalization the subchannel discrimination, in terms of quantum steerability [25–30] (see also Refs. [31–36] for qubit-qubit experiments). More precisely, the steering robustness [27] quantifies the maximal quantum advantage over all possible subchannel discrimination tasks, which is directly mapped to a steering game or, equivalently, a steering inequality. Thus, beyond the general interest for 1S-DI or 1W-LOCC protocols, this restriction is motivated by practical considerations.

Recently, the distillation of steering under 1W-LOCCs has been studied both theoretically [37–41] and experimentally [41–44]. In the one-copy stochastic scenario, the problem has been fully resolved [38–41,43]. Although the link between steerability and task advantage is evident, steering distillation does not always guarantee benefits for the task associated with the given resource. In extreme cases, the distilled resource may fail to surpass the optimal classical guessing probability. Here, we present a method for optimizing local operations in the 1S-DI or 1W-LOCC scenario using SDP and provide an optimality condition that also guarantees that the maximum enhancement of the discrimination task coincides with the maximum quantum steering distillation. Finally, we demonstrate a proof-of-principle experiment using local filters on a two-ququart photonic system, achieving the optimal enhancement of the state discrimination task and the maximum steering distillation in a single experiment. Our amplification strategy achieves 98% discrimination success, aligned with the theoretical 100%, with minimal setup changes.

*Contact author: gelonoel-tabia@gs.ncku.edu.tw†Contact author: cschuu@phys.nthu.edu.tw‡Contact author: huan.yu@ntnu.edu.tw

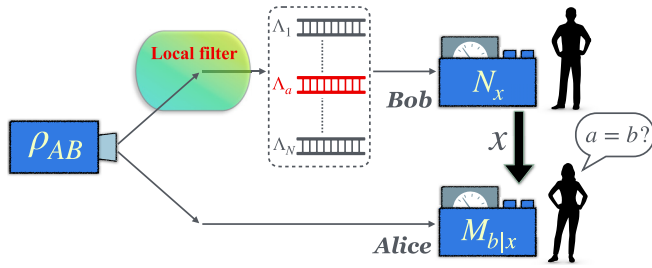


FIG. 1. Subchannel discrimination in 1W-LOCC scenarios. The entangled state ρ_{AB} is shared by Alice and Bob. Bob's system undergoes a quantum evolution with branches (subchannels) $\{\Lambda_a\}_a$. After this, Bob performs the measurement $\{N_x\}_x$ and sends the output x to Alice, which performs the measurement $\{M_{b|x}\}_b$ accordingly. Alice then uses the outcome b to formulate the guess about which subchannel Λ_a was applied, i.e., $P_{\text{guess}} = P(a = b)$.

Background knowledge. A positive operator-valued measure (POVM), $\{M_a\}_a$, is a collection of positive operators $M_a \geq 0$ that sum up to identity $\sum_a M_a = \mathbb{1}$. A POVM represents a quantum measurement: the probability for the outcome a is $p(a) = \text{Tr}(\rho M_a)$. State transformation after measurement is described by *quantum instrument* $\Lambda = \{\Lambda_a\}_a$, i.e., a set of completely positive (CP) trace nonincreasing (TNI) maps such that $\sum_a \Lambda_a$ is a quantum channel [CP trace preserving (TP)]. The probability and output state are given by $p(a) = \text{Tr}(\Lambda_a(\rho))$ and $\rho_a = \Lambda_a(\rho)/p(a)$, respectively.

An instrument Λ defines a subchannel discrimination task as the following multi-round game. In every round, the referee applies Λ to half of a shared state ρ_{AB} , the B part, which results in the generation of the output a and the application of the transformation Λ_a . Alice and Bob try to guess a by performing measurements on the system. Note that the usual channel discrimination task $\{p_a, \Phi_a\}_a$, where each channel Φ_a is applied with a probability p_a , is a special case [45].

The most general protocol involves entangling measurements on Alice and Bob's systems and can be explicitly calculated via SDP methods [23] based on the quantum comb formalism [46]. Here, we consider a different situation where the physical operations and device characterization are restricted, namely, in the 1S-DI or 1W-LOCC scenario. Beyond the interest for minimal experimental assumptions and characterization of the devices, which are the main traits of device-independent approaches to quantum information, the restriction to LOCC operations is also dictated by practical reasons as noted by Piani and Watrous [27]. Entangling operations can be difficult, or even practically impossible to implement. A further restriction to 1W-LOCC allows for a complete characterization of the resources available and the optimal guessing probability via the steerability properties of the state [27]. The 1W-LOCC protocol starts with the application of Λ to Bob's side. Then, Bob performs the local measurement $\{N_x\}_x$ and communicates x to Alice. Then, she performs the measurement $\{M_{b|x}\}_b$ and uses the outcome b to formulate her guess. The overall protocol is schematically illustrated in Fig. 1. The guessing probability is then $P_{\text{guess}} := p(a = b)$, namely,

$$P_{\text{guess}} = \sum_{x,a} \text{Tr}((M_{a|x} \otimes N_x)(\text{id}_A \otimes \Lambda_a)(\rho_{AB})), \quad (1)$$

with id_A being the identity channel. Note that only local operations $\{N_x\}_x$ and $\{M_{b|x}\}_b$ and a classical communication from Bob to Alice are involved, i.e., 1W-LOCC operations. In the 1S-DI framework, only Bob's system is characterized, i.e., we have no assumption on Alice's setup. Thus, one can mathematically formulate the 1S-DI or 1W-LOCC task by tracing out Alice

$$\begin{aligned} & \text{Tr}((M_{a|x} \otimes N_x)(\text{id}_A \otimes \Lambda_a)(\rho_{AB})) \\ &= \text{Tr}_B(N_x \Lambda_a(\text{Tr}_A(M_{a|x} \otimes \mathbb{1}_{\rho_{AB}}))) \\ &= \text{Tr}_B(N_x \Lambda_a(\sigma_{a|x}^B)) = \text{Tr}_B(\Lambda_a^\dagger(N_x) \sigma_{a|x}^B) \\ &= \text{Tr}_B(G_{a,x} \sigma_{a|x}^B), \end{aligned} \quad (2)$$

where $G_{a,x} := \Lambda_a^\dagger(N_x)$ and $\sigma_{a|x}^B := \text{Tr}_A(M_{a|x} \otimes \mathbb{1}_{\rho_{AB}})$ is the (unnormalized) reduced state obtained from the uncharacterized measurement of $M_{a|x}$ on Alice's side. These reduced states form a *state assemblage* (SA)—the standard notion of a steering resource [24]. Equation (2) provides already a clear picture of how the subchannel discrimination game can be optimized in the 1S-DI or 1W-LOCC scenario, namely, by optimizing Bob's measurements $\{N_x\}_x$ and Bob's SA $\{\sigma_{a|x}\}_{a,x}$. In order to discuss the latter, it is helpful to recall some basic definitions related to quantum steering.

A *measurement assemblage* (MA) is a collection $\mathbf{M} = \{M_{a|x}\}_{a,x}$ with $\{M_{a|x}\}_a$ a measurement. The SA $\sigma = \{\sigma_{a|x}\}_{a,x}$ satisfies $\sigma_{a|x} \geq 0$ and $\sum_a \sigma_{a|x} = \rho_B$ for all x . Each $\sigma_{a|x}$ is the (unnormalized) reduced state as in Eq. (2). A SA is *unsteerable* if we can write $\sigma_{a|x} = \sum_\lambda D(a|x, \lambda) \sigma_\lambda$, for $\{\sigma_\lambda\}_\lambda$ with $\sigma_\lambda \geq 0$, $\sum_\lambda \sigma_\lambda = \rho_B$ and $D(a|x, \lambda)$ a fixed deterministic postprocessing. Otherwise, the SA σ is *steerable*. Similarly, a MA is *jointly measurable* if $M_{a|x} = \sum_\lambda D(a|x, \lambda) G_\lambda$ with a valid POVM $\{G_\lambda\}_\lambda$. Steerability is witnessed whenever

$$\sum_{a,x} \text{Tr}(F_{a|x} \sigma_{a|x}) > 1, \quad (3)$$

for some positive operators $\{F_{a|x}\}_{a,x}$ that satisfy some normalization constraints (see Ref. [27] and Appendix A). For each pair (a, x) Bob receives a state $\sigma_{a|x}/\text{Tr}(\sigma_{a|x})$, on which he evaluates the operator $F_{a|x}$. We refer to this protocol as a *steering game* and the value of the right-hand side of Eq. (3) as its local bound [47]. The analogy between $\{F_{a|x}\}_{a,x}$ and $\{G_{a,x}\}_{a,x}$ in Eq. (2) is apparent. Indeed, Piani and Watrous [27] provided an operational interpretation of the maximum score over all possible games $\{F_{a|x}\}_{a,x}$ as the maximum advantage that steerable resources provide over unsteerable ones in subchannel discrimination tasks. More precisely, $G_{a,x} = \alpha F_{a|x}$ for $\alpha := \|\sum_{a,x} F_{a|x}\|^{-1}$, where α is the maximum guessing probability in the discrimination task when using only unsteerable resources. For the optimal game $\{F_{a|x}^*\}_{a,x}$

$$P_{\text{guess}} = \alpha^* \sum_{a,x} \text{Tr}(F_{a|x}^* \sigma_{a|x}^B) = \alpha^*(1 + \text{SR}(\sigma)), \quad (4)$$

where SR is the *steering robustness*, see Ref. [27] and Appendix A. The steering resource SR, then, quantifies the maximum advantage over all subchannel discrimination games.

The resource SR cannot increase, on average, with 1W-LOCC operations [24], but it can when one postselects on

a fixed outcome of a stochastic operation, i.e., performs a *local filter*. We say that two assemblages are connected by a local filter, i.e., $\sigma' = \mathcal{E}_{\omega_0}(\sigma)/p_{\text{guess}}$, where $p_{\text{guess}} = \text{Tr}(\sum_a \mathcal{E}_{\omega_0}(\sigma_{a|x})) = \text{Tr}(\mathcal{E}_{\omega_0}(\rho_B))$ and $\mathcal{E}_{\omega_0}$ is an element of a quantum instrument $\{\mathcal{E}_\omega\}_\omega$, i.e., a local filter, and we write $\sigma \xrightarrow{\text{LF}} \sigma'$. Notice that this operation is consistent with the 1S-DI or 1W-LOCC framework. Local filters on Bob's side can stochastically increase SR up to its maximum [38], namely, to the *incompatibility robustness* (IR) of the *steering equivalent observables* (SEO) [48]. Here, the SEO $\mathbf{B} = \{B_{a|x}\}_{a,x}$ is a valid MA defined as $B_{a|x} = \rho_B^{-1/2} \sigma_{a|x} \rho_B^{-1/2} \forall a, x$ with $\rho_B := \sum_a \sigma_{a|x}$. Similarly to SR, IR is also defined in terms of an optimal witness of incompatibility. The optimal filter can be efficiently computed via a SDP [38].

Theoretical results. The optimization of subchannel discrimination in the 1S-DI or 1W-LOCC scenario, as shown by Eq. (2), consists in optimizing Bob's operations ($\{N_x\}_x$) and Bob's resources ($\{\sigma_{a|x}\}_{a,x}$). It is straightforward to see that the former is a SDP (see Appendix A). Here, we show that the latter, namely, the optimization of the SA for a fixed steering game via local filtering, can be efficiently solved via SDP (Result 2). Alternating between these two optimizations (seesaw method) provides a heuristic way of optimizing the subchannel discrimination in the 1S-DI or 1W-LOCC scenario. The seesaw method does not guarantee the global optimality of the solution, i.e., with respect to all local measurements and all SA. However, such optimality is guaranteed whenever the filter provides a score for the steering game coinciding with the maximal steering robustness that can be stochastically distilled via local filters. A sufficient condition for this is given in Result 3.

The first observation concerns the maximal amplification of the guessing probability by local filters:

Observation 1. Given two SAs, σ and σ' connected by a local filter, i.e., $\sigma \xrightarrow{\text{LF}} \sigma'$, and a subchannel discrimination task in the 1S-DI or 1W-LOCC scenario, i.e., Eqs. (1) and (2), we have

$$P_{\text{guess}}(\sigma') - P_{\text{guess}}(\sigma) \leq \alpha(1 + \text{IR}(\mathbf{B})) - \alpha = \alpha \text{IR}(\mathbf{B}), \quad (5)$$

where $P_{\text{guess}}(\sigma)$ is the optimal guessing probability for the subchannel discrimination task obtained with the SA σ , and similarly for σ' , α is the guessing probability with no steering resources, and $\text{IR}(\mathbf{B})$ is the incompatibility robustness of the SEO of σ ; see Supplemental Material I [49].

A proof can be obtained by Eq. (4) and the observation in Refs. [38,40] that the maximal steerability obtained by local filters is IR of SEO, whereas the infimum in the same equivalence class (connection by local filter) is given by 0, i.e., the unsteerable SA.

Beyond this general upper bound, we obtain the filter to maximize the score of any steering game.

Result 2. Given a steering game $\{F_{a|x}\}$ and a SA σ , the maximal score obtainable by only applying local filtering on Bob's side, namely,

$$\max_{\Phi \in \text{CPTNI}} \frac{1}{p_{\text{guess}}} \sum_{a,x} \text{Tr}(F_{a|x} \Phi(\sigma_{a|x})), \quad (6)$$

with $p_{\text{guess}} = \sum_a \text{Tr}(\Phi(\sigma_{a|x}))$ the guessing probability of passing the filter, can be determined by the SDP:

$$\begin{aligned} \max_{\eta, C_\Phi} \quad & \sum_{a,x} \text{Tr}(F_{a|x} \eta_{a|x}) \\ \text{sub. to } & \eta_{a|x} = \text{Tr}_{B'}((\sigma_{a|x}^{B'})^t \otimes \mathbb{1}^B C_\Phi^{(B'B)}), \quad \forall a, x, \\ & \text{Tr}\left(\sum_a \eta_{a|x}\right) = 1, \quad C_\Phi^{(B'B)} \geq 0. \end{aligned} \quad (7)$$

To prove this, notice that C_Φ is the Choi matrix [50] of the filter Φ , up to a constant factor necessary to make it TNI. By Eq. (6), this factor cancels out. Indeed, the condition $\text{Tr}(\sum_a \eta_{a|x}) = 1$ guarantees that $\eta_{a|x} := \Phi(\sigma_{a|x})/p_{\text{guess}}$ is properly normalized. To conclude, one can explicitly construct the Choi matrix of the physical filter C_Φ^{norm} from the solution C_Φ^{SDP} as $C_\Phi^{\text{norm}} = C_\Phi^{\text{SDP}}/\lambda_{\text{max}}(\text{Tr}_{\text{out}}(C_\Phi))$, where λ_{max} denotes the maximal eigenvalue. Other normalizations are possible, but this provides the highest probability of passing the filter. Since the identity channel is a feasible solution, the SDP is strictly feasible and it provides a game score that is equal to or better than the original one.

The separate, and alternating, optimization of the SA σ (Result 2) and the operations $\{N_x\}_x$ (Appendix B) does not guarantee the global optimality of the solution. It is guaranteed, however, when the game score obtained saturates the bound in Observation 1. A sufficient condition for this is provided in the following.

Result 3. Consider the SA σ obtained from the bipartite maximally entangled state $|\Phi\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^N |ii\rangle$ and two projective measurements, each with two outcomes. Consider the steering game giving a score $1 + \text{SR}(\sigma)$ [see Eq. (4)]. Then, the optimal game score obtained after the filter in Result 2 and the one obtained after the filter maximizing $\text{SR}(\Phi(\sigma))$ coincide. This filter corresponds to a projection onto the qubit subspace.

The proof is obtained by explicit analytical calculations of the solution of Eq. (7) and the maximal robustness obtainable by local filters [38], which are shown to coincide; see SM II for details. There are several physical implications of Result 3. First, there exist a subchannel discrimination task that is optimal for both given SA and distilled SA. Thus, the optimal distillability of steerability can be efficiently observed experimentally without the need for task or game exchange. Besides, the Bell state is the optimal state to demonstrate quantum steerability for two binary projective measurements. However, it has been demonstrated that the high-dimensional maximally entangled state is not always optimal; see Supplementary note 3 in Ref. [38].

To conclude, in the case of Result 3, not only we can prove the optimality of the solution of Result 2, but we are also able to design an experiment that demonstrates both the optimal advantage in 1S-DI or 1W-LOCC subchannel discrimination and a witness of steerability. This is the scenario we explore from the experimental perspective.

Experimental demonstration. Figure 1 provides a schematic overview of the protocol that underlies our experimental implementation. To experimentally demonstrate the enhancement of the guessing probability, we consider

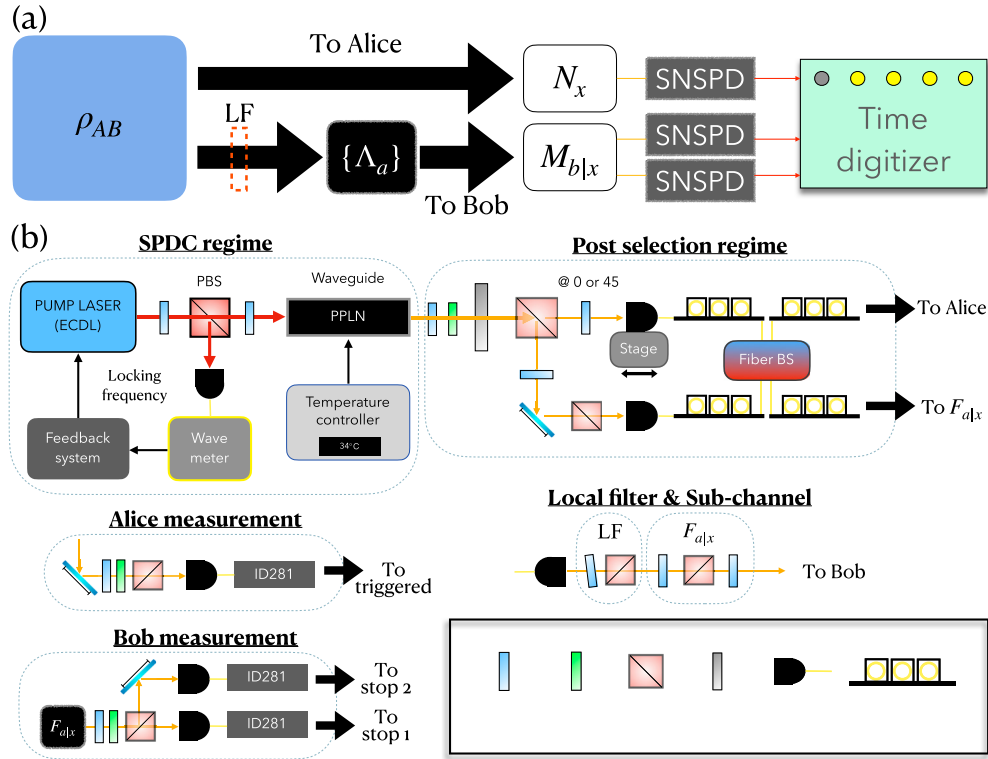


FIG. 2. (a) Experimental amplification of high-dimensional subchannel discrimination via optimal local filtering (LF). ρ_{AB} consists of a SPDC-based entangled-photon source and postselection. Λ_a is a branch of quantum evolution and is composed of the steering game $\{F_{a|x}\}_{a,x}$. $\{N_x\}_x$ and $\{M_{b|x}\}_{b,y}$ are Alice and Bob measurement bases, respectively. (b) Experimental setup. HWP: half wave plate; QWP: quarter wave plate; PBS: polarization beam splitter; BP: bandpass filter; Coupler: fiber coupling lens; PC: polarization controller.

a family of high-dimensional subchannel discrimination tasks. According to the construction of Piani and Watrous [27], a steering game $\{F_{a|x}\}_{a,x}$ always defines subchannel discrimination tasks Λ by

$$\Lambda_a(\rho) = \alpha \sum_{x=1}^{|X|} \text{Tr}(F_{a|x}\rho) |x\rangle\langle x|. \quad (8)$$

By construction $\sum_a \Lambda_a$ is CP-TNI. If it is not TP, we can always add extra outcomes to make it TP minimally perturbing the guessing probability; see Ref. [27] for details.

For this task, the choice of Bob's measurement $\{N_x\}_x = \{|x\rangle\langle x|\}_x$ is optimal [27], so no optimization of Bob's measurement is needed. The MA $\{M_{a|x}\}_{a,x}$ on Alice's side gives rise to the SA $\sigma = \{\sigma_{a|x}\}_{a,x}$. In the 1S-DI or 1W-LOCC framework, Alice's side is not characterized, and we can only optimize the SA σ via local filters on Bob's side, as previously discussed.

In our setup, we realize the two-ququart maximally entangled state with two pairs of entangled photons at different times [51]. Each pair is prepared in the polarization Bell state, $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|H_A H_B\rangle + |V_A V_B\rangle)$, and their joint state is equivalent to the four-dimensional entangled state $|\Psi_{AB}^4\rangle = \frac{1}{2} \sum_{i=0}^3 |i_A i_B\rangle$ if we relabel $|H_j H_j\rangle$, $|H_j V_j\rangle$, $|V_j H_j\rangle$, and $|V_j V_j\rangle$ as $|0_j\rangle$, $|1_j\rangle$, $|2_j\rangle$, and $|3_j\rangle$ with $j = A, B$. For consistency, measurements, filters, and subchannels are realized as tensor products; see SM III.

Alice's measurements $M_{a|x}$ are given by $M_{0|0} = |0_A\rangle\langle 0_A| + |2_A\rangle\langle 2_A|$ and $M_{0|1} = \sum_{i=0}^1 |\xi_i\rangle\langle \xi_i|$, where $\xi_0 = \cos \theta_1 |0_A\rangle +$

$\sin \theta_1 |2_A\rangle$ and $\xi_1 = \cos \theta_2 |1_A\rangle + \sin \theta_2 |3_A\rangle$ for $\theta_{1,2} \in [0, \frac{\pi}{4}]$, and $M_{1|x} = \mathbb{I}_4 - M_{0|x}$ for $x = 0, 1$. Bob measures in the combined computational basis $N_x = |x\rangle\langle x|$ for $x = 0, 1, 2, 3$.

We define the subchannel discrimination task Λ via Eq. (8) and the steering game $F_{a|x}^{\theta_1, \theta_2} = f_{a|x}^{\theta_1} \oplus f_{a|x}^{\theta_2}$ (direct sum over the spans of $\{|0\rangle, |1\rangle\}$ and $\{|2\rangle, |3\rangle\}$) with

$$f_{a|x}^{\theta} = \begin{pmatrix} \frac{1 + \cos[\frac{\pi}{4}((-1)^x - 4a) - \theta]}{2(\sqrt{2} + 2)} & -\frac{\sin[\frac{\pi}{4}((-1)^x - 4a) - \theta]}{2(\sqrt{2} + 2)} \\ -\frac{\sin[\frac{\pi}{4}((-1)^x - 4a) - \theta]}{2(\sqrt{2} + 2)} & \frac{1 - \cos[\frac{\pi}{4}((-1)^x - 4a) - \theta]}{2(\sqrt{2} + 2)} \end{pmatrix}. \quad (9)$$

Since we have two projective measurements with binary outcomes, Result 3 implies that the optimal filter is the one saturating the steering robustness. It can be determined via Result 2. For the subchannels determined by $F_{a|x}^{\frac{\pi}{4}, \theta_2}$, the optimal filter is $\rho \mapsto K_B \rho K_B^\dagger / \text{Tr}(\rho K_B^\dagger K_B)$, with $K_B = |0_B\rangle\langle 0_B| + |1_B\rangle\langle 1_B|$, which boosts the guessing probability up to 100%.

Figure 2 illustrates the experimental setup (see SM IV for more details). The subchannels for the discrimination task [Eqs. (8) and (9)] are implemented through their Kraus operators (see SM V for explicit calculations). In turn, the Kraus operators are implemented by two half wave plates (HWPs) and one PBS, with the angles of the wave plates determined by the Jones calculation. The filtering operation K_B is realized using a PBS to achieve the nearly perfect guessing probability. All measurements, including the quantum state tomography and projection, are accomplished by a set of HWPs, quarter wave plates (QWPs), and PBS. The integration time for

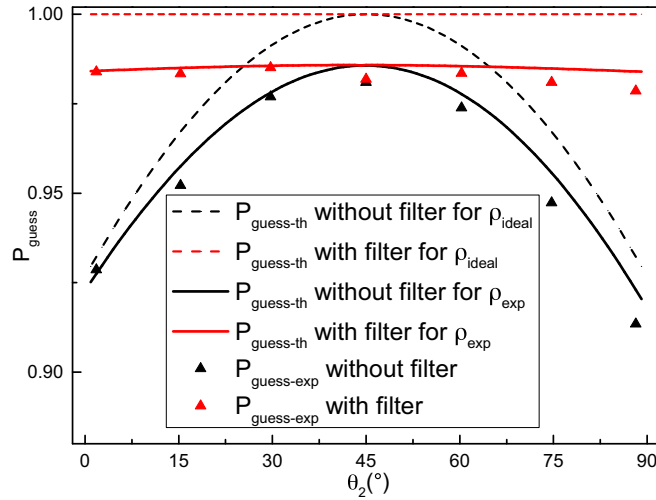


FIG. 3. Guessing probability of subchannel discrimination tasks for different θ_2 . The black and red triangles correspond to the tasks before and after applying the optimal filters, respectively. The dash lines are the theoretical values without (black) and with (red) local filters assuming an ideal maximally entangled state. The solid lines are the numerical results using the prepared states in our experiment. The triangles are the measured guessing probabilities in the experiment.

collecting each coincidence measurement is set to be 10 s. This setup allows us to measure the guessing probability for the subchannel discrimination task by simulating it, i.e., in each round a single Kraus operator of a fixed Λ_a is implemented. The filter is implemented exactly, since the optimal one consists of a single Kraus operator. Thus, we are able to measure the guessing probability and its amplification. Particularly, our experimental setup does not require reconfiguring the subchannel structure or altering any measurements; the desired filtering and guessing probability amplification can be achieved simply by a PBS. This aligns with the physical implications outlined in Result 3.

Figure 3 shows the guessing probability of the successful subchannel discrimination P_{guess} measured for a family of high-dimensional subchannel discrimination tasks depending on θ_2 . Experimentally, P_{guess} is given by the ratio of the number of successful events to the total number of events (see SM V). Without the local filters, the discrimination is imperfect, even with the maximally entangled state (black triangles). However, after applying the local filters, we observe a significant increase of the guessing probability approaching unity (red triangles). For instance, an increase of P_{guess} from 91.3% to 97.9% is observed in the task near $\theta_2 = \frac{\pi}{2}$. The slight discrepancy between the measured P_{guess} (triangle dots) and the P_{guess} predicted by the theory results from the imperfect preparations of the states and the subchannels. The fidelity between ideal and experimental states is 0.9735. Besides, the trace distances between the ideal and experimental χ matrices (see SM VI) for the first and second subchannels are 0.0711 and 0.0744, respectively. In this context, we provide a unified framework that optimally improves the guessing probability with minimal setup changes. Thus, a complete reconfiguration of the experimental apparatus, e.g., uncertainty estimation, is unnecessary.

Conclusions. We investigate, from a theoretical and experimental perspective, the optimization of subchannel discrimination tasks in the 1S-DI or 1W-LOCC scenario, namely, when only one party possesses trusted devices and is able to perform local quantum operations. As noted already by Piani and Watrous [27], this scenario is motivated by practical considerations related to the ease of implementing the necessary operations and the characterization of the resources provided by the theory of steering. Under those restrictions, the optimization of the task consists in optimizing Bob's measurements and its SA. Bob simply chooses the optimal measurements, whereas the probabilistic SA optimization involves local filters. We show that both can be optimized via a SDP (Result 2), providing a heuristic optimization based on a seesaw method. Moreover, we provide sufficient conditions for the heuristic optimization to provide the true optimal strategy (Result 3). Moreover, such a result also indicates scenarios where one can simultaneously demonstrate the optimal enhancement of the discrimination task and the steerability of the SA. For the experimental demonstration, we investigate a one-parameter family of subchannel discrimination tasks defined on a two-ququart system, compute the optimal filter, and implement it experimentally. In contrast to previous works on the optimization of quantum resources [52–56], we optimize the guessing probability for a specific task. The practicality of our method goes beyond the 1S-DI or 1W-LOCC scenario: since it requires a minimal modification of the experimental setup—only one extra element (local filter) must be added—it may find applications in general scenarios. We hope that our work will stimulate further theoretical and experimental investigations of the enhancement of discrimination tasks.

Acknowledgments. C.-S.C. is supported by the National Science and Technology Council, Taiwan (Grant No. NSTC 113-2119-M-007-012). H.-Y.K. is supported by the Ministry of Science and Technology, Taiwan (Grant No. MOST 112-2112-M-003-020-MY3), and Higher Education Sprout Project of National Taiwan Normal University (NTNU).

Y.-A.S. and W.-G.C. have contributed equally to this work.

Appendix A: SDP formulation of steering and incompatibility robustness. Here, we recall some basic definitions. The steering robustness σ , $\text{SR}(\sigma)$, is defined as the minimal t such that $\{\frac{\sigma_{a|x} + t\xi_{a|x}}{1+t}\}_{a,x}$ has a LHS model for some SA ξ . It is computed via the SDP (in dual form) [27]:

$$\begin{aligned} & \text{maximize} && \sum_{a,x} \text{Tr}(F_{a|x} \sigma_{a|x}) = \text{SR}(\sigma) + 1 \\ & \text{subject to} && \sum_{a,x} D(a|x, \lambda) F_{a|x} \leq \mathbb{1}, \text{ and } F_{a|x} \geq 0, \quad (\text{A1}) \end{aligned}$$

where $D(a|x, \lambda) := \delta_{a, \lambda_x}$ is the deterministic conditional probability. The objective becomes optimal, steering inequality with the quantum-classical threshold 1. Thus, $\{F_{a|x}\}_{a,x}$ is interpreted as the optimal steering game to detect the robustness. Obviously, this steering game provides a maximal score precisely for steering robustness (plus 1). On the other hand, this steering game can be reformulated as the subchannel discrimination task by Eq. (7) in the main body. Then, once Alice and Bob implement the subchannel discrimination task associated with the steering game, the optimal guessing probability can be obtained by Eq. (3).

For a measurement assemblage $\mathbf{M} = \{M_{a|x}\}_{a,x}$ the dual SDP problem can be expressed as

$$\begin{aligned} \text{maximize} \quad & \sum_{a,x} \text{Tr}(\omega_{a|x} M_{a|x}) = \text{IR}(\mathbf{M}) + 1 \\ \text{subject to} \quad & \eta \geq \sum_{a,x} D(a|x, \lambda) \omega_{a|x} \\ & \omega_{a|x} \geq 0, \quad \text{tr}(\eta) = 1. \end{aligned} \quad (\text{A2})$$

Similar to the steering robustness, $\{\omega_{a|x}\}_{a,x}$ can be seen as the optimal incompatibility game and the objective function is the optimal violation value of the incompatibility inequality.

Given a SA σ with marginal state $\sum_a \sigma_{a|x} = \rho_B \forall x$, we can always convert it to a MA $\{B_{a|x}\}$ called steering-equivalent observable (SEO) [48] by

$$B_{a|x} = \rho_B^{-1/2} \sigma_{a|x} \rho_B^{-1/2}. \quad (\text{A3})$$

It has been shown that if an SA is unsteerable, the corresponding SEO must be jointly measurable (and vice versa). The incompatibility robustness of the SEO from the σ determines the optimal steering distillability by the local filter. The Kraus operator of the optimal local filter can be constructed by $\sqrt{\eta} \sqrt{\rho_B}^{-1/2}$, where η is the feasible solution from the dual SDP of the incompatibility robustness [38]. The steering robustness of the distilled SA is equivalent to the incompatibility robustness of SEO.

Appendix B: Seesaw Optimization of local operations. Here, we briefly review the seesaw method to optimize Bob's local operations in the 1S-DI or 1W-LOCC subchannel

discrimination protocol. Let us rewrite the optimization procedure explicitly, by combining Eqs. (1) and (2):

$$\begin{aligned} P_{\text{guess}}^{\text{max, filter}} &= \max_{\{N_x\}_x, \Phi} \frac{P_{\text{guess}}}{p_\Phi} \\ &= \frac{1}{p_\Phi} \sum_{a,x} \text{Tr}_B(\Lambda_a^\dagger(N_x) \Phi(\sigma_{a|x}^B)), \end{aligned} \quad (\text{B1})$$

where $P_{\text{guess}}^{\text{max, filter}}$ is the maximum guessing probability conditioned on passing the filter and $p_\Phi = \text{Tr}(\sum_a \Phi(\sigma_{a|x}))$ the probability of passing the filter Φ . The seesaw algorithm consists in alternating the optimization of $\{N_x\}_x$, for fixed Φ , and of Φ , for fixed $\{N_x\}_x$. The latter is defined explicitly in Result 2, whereas the former can be written as

$$\begin{aligned} \max_{\{N_x\}_x} \frac{1}{p_\Phi} \sum_{a,x} \text{Tr}_B(\Lambda_a^\dagger(N_x) \Phi(\sigma_{a|x}^B)), \\ N_x \geq 0, \quad \text{for all } x, \\ \sum_x N_x = \mathbb{1}, \end{aligned} \quad (\text{B2})$$

which is clearly an SDP. Alternating the optimization in Eq. (B2) with the one in Eq. (7) provides a heuristic method to optimize over local operations in the 1S-DI or 1W-LOCC scenario. Note that for the special case of subchannel discrimination tasks defined from steering games as in Eq. (8), as the ones considered in our experiment, the optimization in Eq. (B2) is not necessary, as the optimal measurement is known [27].

-
- [1] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Mixed-state entanglement and quantum error correction, *Phys. Rev. A* **54**, 3824 (1996).
 - [2] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Concentrating partial entanglement by local operations, *Phys. Rev. A* **53**, 2046 (1996).
 - [3] M. Horodecki, P. Horodecki, and R. Horodecki, Mixed-state entanglement and distillation: Is there a “bound” entanglement in nature? *Phys. Rev. Lett.* **80**, 5239 (1998).
 - [4] E. Chitambar and G. Gour, Quantum resource theories, *Rev. Mod. Phys.* **91**, 025001 (2019).
 - [5] J. Bae and L.-C. Kwek, Quantum state discrimination and its applications, *J. Phys. A: Math. Theor.* **48**, 083001 (2015).
 - [6] C. Carmeli, T. Heinosaari, and A. Toigo, Quantum incompatibility witnesses, *Phys. Rev. Lett.* **122**, 130402 (2019).
 - [7] P. Skrzypczyk, I. Šupić, and D. Cavalcanti, All sets of incompatible measurements give an advantage in quantum state discrimination, *Phys. Rev. Lett.* **122**, 130403 (2019).
 - [8] R. Uola, T. Kraft, J. Shang, X.-D. Yu, and O. Gühne, Quantifying quantum resources with conic programming, *Phys. Rev. Lett.* **122**, 130404 (2019).
 - [9] P. Skrzypczyk and N. Linden, Robustness of measurement, discrimination games, and accessible information, *Phys. Rev. Lett.* **122**, 140403 (2019).
 - [10] R. Takagi and B. Regula, General resource theories in quantum mechanics and beyond: Operational characterization via discrimination tasks, *Phys. Rev. X* **9**, 031053 (2019).
 - [11] X. Yuan, Y. Liu, Q. Zhao, B. Regula, J. Thompson, and M. Gu, Universal and operational benchmarking of quantum memories, *npj Quantum Inf.* **7**, 108 (2021).
 - [12] K. Flatt, H. Lee, C. R. I. Carceller, J. B. Brask, and J. Bae, Contextual advantages and certification for maximum-confidence discrimination, *PRX Quantum* **3**, 030337 (2022).
 - [13] C. H. Bennett, Quantum cryptography using any two nonorthogonal states, *Phys. Rev. Lett.* **68**, 3121 (1992).
 - [14] R. Renner, Security of quantum key distribution, *Int. J. Quantum Inf.* **06**, 1 (2008).
 - [15] M. Piani and J. Watrous, All entangled states are useful for channel discrimination, *Phys. Rev. Lett.* **102**, 250501 (2009).
 - [16] J. Bae and D. Chruściński, Operational characterization of divisibility of dynamical maps, *Phys. Rev. Lett.* **117**, 050403 (2016).
 - [17] S. Pirandola, R. Laurenza, C. Lupo, and J. L. Pereira, Fundamental limits to quantum channel discrimination, *npj Quantum Inf.* **5**, 50 (2019).
 - [18] T. Cooney, M. Mosonyi, and M. M. Wilde, Strong converse exponents for a quantum channel discrimination problem and quantum-feedback-assisted communication, *Commun. Math. Phys.* **344**, 797 (2016).
 - [19] G. Vidal and R. Tarrach, Robustness of entanglement, *Phys. Rev. A* **59**, 141 (1999).
 - [20] M. B. Plenio and S. Virmani, An introduction to entanglement measures, *Quantum Info. Comput.* **7**, 1 (2007).
 - [21] O. Gühne and G. Tóth, Entanglement detection, *Phys. Rep.* **474**, 1 (2009).

- [22] J. Bae, D. Chruściński, and M. Piani, More entanglement implies higher performance in channel discrimination tasks, *Phys. Rev. Lett.* **122**, 140404 (2019).
- [23] J. Bavaresco, M. Murao, and M. T. Quintino, Strict hierarchy between parallel, sequential, and indefinite-causal-order strategies for channel discrimination, *Phys. Rev. Lett.* **127**, 200504 (2021).
- [24] R. Gallego and L. Aolita, Resource theory of steering, *Phys. Rev. X* **5**, 041008 (2015).
- [25] A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935).
- [26] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Steering, entanglement, nonlocality, and the Einstein-Podolsky-Rosen paradox, *Phys. Rev. Lett.* **98**, 140402 (2007).
- [27] M. Piani and J. Watrous, Necessary and sufficient quantum information characterization of Einstein-Podolsky-Rosen steering, *Phys. Rev. Lett.* **114**, 060404 (2015).
- [28] D. Cavalcanti and P. Skrzypczyk, Quantum steering: a review with focus on semidefinite programming, *Rep. Prog. Phys.* **80**, 024001 (2017).
- [29] R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne, Quantum steering, *Rev. Mod. Phys.* **92**, 015001 (2020).
- [30] Y. Xiang, S. Cheng, Q. Gong, Z. Ficek, and Q. He, Quantum steering: Practical challenges and future directions, *PRX Quantum* **3**, 030102 (2022).
- [31] K. Sun, X.-J. Ye, Y. Xiao, X.-Y. Xu, Y.-C. Wu, J.-S. Xu, J.-L. Chen, C.-F. Li, and G.-C. Guo, Demonstration of Einstein-Podolsky-Rosen steering with enhanced subchannel discrimination, *npj Quantum Inf.* **4**, 12 (2018).
- [32] D. J. Saunders, S. J. Jones, H. M. Wiseman, and G. J. Pryde, Experimental EPR-steering using Bell-local states, *Nat. Phys.* **6**, 845 (2010).
- [33] Y.-Y. Zhao, H.-Y. Ku, S.-L. Chen, H.-B. Chen, F. Nori, G.-Y. Xiang, C.-F. Li, G.-C. Guo, and Y.-N. Chen, Experimental demonstration of measurement-device-independent measure of quantum steering, *npj Quantum Inf.* **6**, 77 (2020).
- [34] S.-L. Chen, H.-Y. Ku, W. Zhou, J. Tura, and Y.-N. Chen, Robust self-testing of steerable quantum assemblages and its applications on device-independent quantum certification, *Quantum* **5**, 552 (2021).
- [35] S. Slussarenko, D. J. Joch, N. Tischler, F. Ghafari, L. K. Shalm, V. B. Verma, S. W. Nam, and G. J. Pryde, Quantum steering with vector vortex photon states with the detection loophole closed, *npj Quantum Inf.* **8**, 20 (2022).
- [36] H.-Y. Ku, J. Kadlec, A. Černoch, M. T. Quintino, W. Zhou, K. Lemr, N. Lambert, A. Miranowicz, S.-L. Chen, F. Nori, and Y.-N. Chen, Quantifying quantumness of channels without entanglement, *PRX Quantum* **3**, 020338 (2022).
- [37] M. T. Quintino, N. Brunner, and M. Huber, Superactivation of quantum steering, *Phys. Rev. A* **94**, 062123 (2016).
- [38] H.-Y. Ku, C.-Y. Hsieh, S.-L. Chen, Y.-N. Chen, and C. Budroni, Complete classification of steerability under local filters and its relation with measurement incompatibility, *Nat. Commun.* **13**, 1 (2022).
- [39] C.-Y. Hsieh, H.-Y. Ku, and C. Budroni, Characterisation and fundamental limitations of irreversible stochastic steering distillation, *arXiv:2309.06191*.
- [40] H.-Y. Ku, C.-Y. Hsieh, and C. Budroni, Measurement incompatibility cannot be stochastically distilled, *arXiv:2308.02252*.
- [41] Q.-X. Zhang, X.-X. Fang, and H. Lu, Experimental distillation of tripartite quantum steering with an optimal local filtering operation, *Photonics Res.* **12**, 552 (2024).
- [42] T. Pramanik, Y.-W. Cho, S.-W. Han, S.-Y. Lee, Y.-S. Kim, and S. Moon, Revealing hidden quantum steerability using local filtering operations, *Phys. Rev. A* **99**, 030101(R) (2019).
- [43] R. V. Nery, M. M. Taddei, P. Sahium, S. P. Walborn, L. Aolita, and G. H. Aguilar, Distillation of quantum steering, *Phys. Rev. Lett.* **124**, 120402 (2020).
- [44] Z.-Y. Hao, Y. Wang, J.-K. Li, Y. Xiang, Q.-Y. He, Z.-H. Liu, M. Yang, K. Sun, J.-S. Xu, C.-F. Li, and G.-C. Guo, Filtering one-way Einstein-Podolsky-Rosen steering, *Phys. Rev. A* **109**, 022411 (2024).
- [45] Indeed, this protocol corresponds to the subchannel discrimination defined by $\{p_a \Phi_a\}_a$. This is a quantum instrument: the trivial POVM $\{p_a 11\}_a$ followed by the application of the channel Φ_a .
- [46] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Theoretical framework for quantum networks, *Phys. Rev. A* **80**, 022339 (2009).
- [47] In analogy with nonlocal games, where Alice and Bob must generate an output given an input, here Bob, for each classical input (a, x) and quantum state $\sigma_{a|x}/\text{Tr}(\sigma_{a|x})$, must make a measurement that will be used to estimate the expectation value of $F_{a|x}$.
- [48] R. Uola, C. Budroni, O. Gühne, and J.-P. Pellonpää, One-to-one mapping between steering and joint measurability problems, *Phys. Rev. Lett.* **115**, 230402 (2015).
- [49] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevResearch.7.L032035> for detailed theoretical derivations and proofs, experimental parameters of the optical setup, and methods of data analysis.
- [50] M.-D. Choi, Completely positive linear maps on complex matrices, *Linear Algebra and Its Applications* **10**, 285 (1975).
- [51] H.-P. Lo, C.-M. Li, A. Yabushita, Y.-N. Chen, C.-W. Luo, and T. Kobayashi, Experimental violation of Bell inequalities for multi-dimensional systems, *Sci. Rep.* **6**, 22088 (2016).
- [52] P. G. Kwiat, S. Barraza-Lopez, A. Stefanov, and N. Gisin, Experimental entanglement distillation and 'hidden' non-locality, *Nature (London)* **409**, 1014 (2001).
- [53] J.-W. Pan, C. Simon, C. Brukner, and A. Zeilinger, Entanglement purification for quantum communication, *Nature (London)* **410**, 1067 (2001).
- [54] M. Horodecki, P. Horodecki, and R. Horodecki, General teleportation channel, singlet fraction, and quasidistillation, *Phys. Rev. A* **60**, 1888 (1999).
- [55] F. Rzepčdek, T. Schiet, L. P. Thinh, D. Elkouss, A. C. Doherty, and S. Wehner, Optimizing practical entanglement distillation, *Phys. Rev. A* **97**, 062333 (2018).
- [56] H.-Y. Ku, H.-C. Weng, Y.-A. Shih, P.-C. Kuo, N. Lambert, F. Nori, C.-S. Chu, and Y.-N. Chen, Hidden nonmacrorealism: Reviving the Leggett-Garg inequality with stochastic operations, *Phys. Rev. Res.* **3**, 043083 (2021).