

Topological spin-orbit torque in ferrimagnetic Weyl semimetal

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We theoretically find a topological spin-orbit torque (SOT) in the compensated ferrimagnetic Weyl semimetal Ti_2MnAl . The topological SOT arises from all the occupied states, which is free from dissipation. However, it is negligible in metallic magnets and was not reported in Weyl semimetals. We show the enhancement of the topological SOT from the mixed Berry curvature around the Weyl points in Ti_2MnAl . This finding provides a guiding principle for designing highly efficient SOT devices using the topological nature of ferrimagnetic Weyl semimetals.

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Introduction. Efficient electrical control of electron spins is one of the fundamental aims of spintronics [1–4]. Spin-orbit coupling plays a significant role by correlating the electron spin and orbital motion. The spin-orbit coupling enables the electrical generation of spin polarization in heterostructures [3,5–10] or bulk noncentrosymmetric crystals [11–13]. The generated spin polarization acts as an effective magnetic field for magnetic moments, whose effect is called the spin-orbit torque (SOT). The SOT has been intensely studied as a promising method for functional control of magnetization [5,14–17]. However, to obtain a large SOT, one needs to inject a large longitudinal current, which suffers from energy dissipation by Joule heating. Such an SOT is termed the “dissipative SOT.” The problem of dissipation is crucial in metals with a large density of states near the Fermi level E_F , where longitudinal conductivity σ_{xx} is proportional to the density of states. The dissipation comes from the scattering of a large number of electrons around E_F by disorders and phonons. To avoid the dissipation, we need systems that can generate the SOT with a small longitudinal current.

To suppress the dissipation, we focus on the characteristic transport properties of magnetic Weyl semimetals. The magnetic Weyl semimetals feature linear dispersion and point nodes called Weyl points (WPs) [18–20]. Since the density of states near the energy of the WPs is negligibly small, σ_{xx}

tends to be suppressed, leading to the small dissipation in the longitudinal transport. The SOT in bulk magnetic Weyl semimetals has been studied based on the continuum model with isotropic spin-momentum locking and broken inversion symmetry [21,22]. The SOT obtained in these studies needs a finite density of states by shifting E_F away from the WPs, similar to σ_{xx} , corresponding to the dissipative SOT. By contrast, nondissipative transport phenomena, such as the anomalous Hall effect (AHE), arise from the nontrivial band topology in magnetic Weyl semimetals. The band topology is characterized by the Berry curvature from the WPs in momentum space [23,24]. The anomalous Hall conductivity σ_{yx} corresponds to the integral of the Berry curvature over all the occupied electronic states below E_F . Such a mechanism is termed the “topological effect.”

The existence of the SOT driven by all the occupied electronic states below E_F has been pointed out in several magnetic materials [11–13,25–27]. We here refer to such a nondissipative SOT as the “topological SOT.” Theoretically, the topological SOT can be characterized by a mixed Berry curvature [28,29]. The mixed Berry curvature is the extension of the Berry curvature in momentum space to the composite parameter space spanned by momentum and magnetization. In the systems analyzed so far, the magnitude of topological SOT is smaller than the dissipative SOT. To enhance the topological SOT with low Joule heating, materials showing a large mixed Berry curvature are desired.

In this Letter, we find a large topological SOT in a model of magnetic Weyl semimetal Ti_2MnAl . Ti_2MnAl is a candidate for ideal magnetic Weyl semimetal with compensated ferrimagnetic ordering [30], where both time-reversal and inversion symmetries are broken. We show that, in Ti_2MnAl , the topological SOT is significantly larger than dissipative SOT near the energy of WPs with a typical disorder strength. Compared with the SOTs studied in the bulk of noncentrosymmetric antiferromagnets [11–13], we find a giant effective magnetic field in Ti_2MnAl . These properties enable us to

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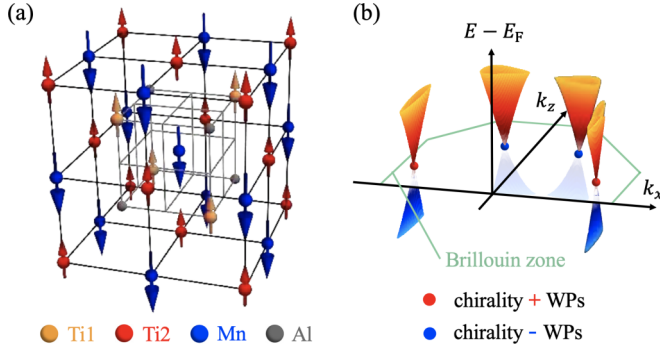


FIG. 1. (a) Crystal structure of Ti₂MnAl. Ti and Mn are responsible for the compensated ferrimagnetic ordering. (b) The energy dispersion around the WPs on $k_y = 0$ plane.

electrically control the compensated ferrimagnetic ordering with low dissipation in the bulk system. By analyzing the distribution of the mixed Berry curvature, we find that the origin of the large topological SOT is the mixed Berry curvature enhanced around the WPs. Our finding of the large topological SOT induced by the mixed Berry curvature from the WPs will provide a guiding principle to choose or design materials for achieving the SOT with ultralow power consumption, based on a wide variety of noncentrosymmetric magnetic Weyl semimetals.

Model. We first explain the electronic properties of Ti₂MnAl [30]. The crystal structure of Ti₂MnAl is an inverse Heusler type with space group $F\bar{4}3m$ [31,32] as shown in Fig. 1(a). Magnetic moments at two Ti sites antiparallelly align to those at the Mn site, showing zero net magnetization. The first-principles calculations of the electronic band structure predict that the WPs are located near E_F , where the density of states is negligibly small. These properties lead to the large anomalous Hall angle $\theta_{\text{AHE}} = \sigma_{yx}/\sigma_{xx}$ [30]. Since not only the time-reversal symmetry but also the inversion symmetry is broken due to its crystalline structure, the SOT is permitted in bulk.

Then, we introduce a tight-binding model of Ti₂MnAl [33]. In reality, all d orbitals mainly contribute to the electronic structure near E_F . For our minimal model to describe the low-energy band structure near E_F , we consider a single isotropic orbital for Ti and Mn as the basis for satisfying the symmetry of the crystalline structure. Here, to distinguish two different Ti sites, we use the notations Ti1 and Ti2 as shown in Fig. 1(a). We neglect the orbital at the Al site for simplicity, while the effect of Al is substantially incorporated via spin-orbit coupling, as discussed later. Our Hamiltonian H consists of a hopping term, on-site energy, spin-orbit coupling, and exchange coupling:

$$\begin{aligned}
 H = & - \sum_{\langle ij \rangle \alpha \beta} \sum_s t_{\alpha\beta} C_{i\alpha s}^\dagger C_{j\beta s} + \sum_{i\alpha} \sum_s \epsilon_\alpha C_{i\alpha s}^\dagger C_{i\alpha s} \\
 & + i \frac{2\lambda_{\text{SOC}}}{a^2} \sum_{\langle ij \rangle \alpha} \sum_{ss'} C_{i\alpha s}^\dagger [(\mathbf{d}_1^{\alpha ij} \times \mathbf{d}_2^{\alpha ij}) \cdot \boldsymbol{\sigma}]_{ss'} C_{j\alpha s'} \\
 & - \sum_{i\alpha} \sum_{ss'} J_\alpha C_{i\alpha s}^\dagger (\boldsymbol{\sigma} \cdot \hat{\mathbf{n}})_{ss'} C_{i\alpha s'}.
 \end{aligned} \quad (1)$$

Here, $C_{i\alpha s}^\dagger$ denotes an electron creation operator of sublattice $\alpha = \text{Ti1, Ti2, and Mn}$ with the spin $s = \uparrow, \downarrow$ at i site. We briefly explain each term in the following (see Appendix A for details of our tight-binding model).

The first term includes the inter- and intrasublattice hoppings. The second term denotes the on-site energy ϵ_α for each sublattice. The third term describes the effect of spin-orbit coupling (SOC) due to the local breaking of inversion symmetry for the intrasublattice hoppings. These hoppings, mediated by the surrounding Al atoms, are described by the two nearest-neighbor hopping vectors $\mathbf{d}_{1,2}^{\alpha ij}$ for the sublattice α between the sites i and j via the Al sites [34]. The effect of Al atoms is thus incorporated in this spin-orbit coupling term. We consider spin-orbit coupling for the Ti2-Ti2 and Mn-Mn hoppings with the same strength λ_{SOC} , and neglect the spin-orbit coupling for the Ti1-Ti1 for simplicity. The fourth term represents the exchange coupling between conduction electron spin $\boldsymbol{\sigma}$ and the ferrimagnetic ordering. The direction of the ferrimagnetic ordering is represented by the unit vector $\hat{\mathbf{n}}$ and is fixed to the z direction. The ferrimagnetism is encoded in the mean-field coupling strength J_α for each sublattice α , where the signs of J_{Ti1} and J_{Ti2} are taken opposite to J_{Mn} .

We explain the electronic structure in this model. The energy spectrum obtained by the Bloch Hamiltonian of H , denoted as $\mathcal{H}(\mathbf{k})$, exhibits the WPs as shown in Fig. 1(b). Red (blue) points correspond to the WPs with positive (negative) chiralities. We obtain eight WPs on the $k_{x,y,z} = 0$ plane, and thus 24 WPs in total. We determine E_F from the 4/6 filling condition. We focus on the behavior of SOT around the energy of the WPs, $-E_0 \leq E_F \leq E_0$, where $E_0 = 50$ meV characterizes the energy scale for the dispersion around the WPs.

Electrically induced spin density. We proceed to the analysis of the SOT. The conduction electron spin driven by the electric field $\mathbf{E}$ acts as an effective magnetic field and exerts the SOT on the magnetic moments at each sublattice. The spin density at sublattice α ($= \text{Ti1, Ti2, Mn}$) is calculated by the Kubo formula [4]:

$$\frac{\langle \sigma_\mu^\alpha \rangle}{E_x} = i\hbar \lim_{q \rightarrow 0} \sum_{n \neq m} \int_{\mathbf{k}} \frac{f_{n\mathbf{k}} - f_{m\mathbf{k}+\mathbf{q}}}{E_{n\mathbf{k}} - E_{m\mathbf{k}+\mathbf{q}}} \frac{S_{nm,\mathbf{k}+\mathbf{q}}^{\alpha,\mu} J_{mn,\mathbf{k}+\mathbf{q}}^x}{E_{n\mathbf{k}} - E_{m\mathbf{k}+\mathbf{q}} + i\eta}. \quad (2)$$

Here, the index μ ($= x, y, z$) denotes the direction of the induced spin density, whereas the direction of $\mathbf{E}$ is fixed to the x direction. We define the spin matrix element $S_{nm,\mathbf{k}+\mathbf{q}}^{\alpha,\mu} = \langle n\mathbf{k} | \hat{\sigma}_\mu^\alpha | m\mathbf{k} + \mathbf{q} \rangle$, where $\hat{\sigma}_\mu^\alpha$ is the electron spin operator at the sublattice α , and $|n\mathbf{k}\rangle$ is the eigenstate with eigenenergy $E_{n\mathbf{k}}$ of $\mathcal{H}(\mathbf{k})$. $J_{mn,\mathbf{k}+\mathbf{q}}^x = \langle m\mathbf{k} + \mathbf{q} | (-e\hat{v}_x) | n\mathbf{k} \rangle$ is the current matrix element where $-e\hat{v} = -\frac{e}{\hbar} \frac{\partial \mathcal{H}}{\partial \mathbf{k}}$ is the current operator. $f_{n\mathbf{k}}$ is the Fermi-Dirac distribution function, with $T = 0$. The integral is taken over the Brillouin zone, with $\int_{\mathbf{k}} = (2\pi)^{-3} \int d^3\mathbf{k}$. We introduce the impurity effect by the first Born approximation, so that the self-energy part η in Eq. (2) is replaced by $\hbar/2\tau$ with the relaxation time τ . To understand the origin of the electrically induced spin density, we decompose Eq. (2) into two parts as (see Appendix B for the derivations)

$$\frac{\langle \sigma_\mu^\alpha \rangle^{\text{dis}}}{E_x} = 2\tau \sum_n \int_{\mathbf{k}} \frac{\partial f_{n\mathbf{k}}}{\partial E} S_{nn,\mathbf{k}}^{\alpha,\mu} J_{nn,\mathbf{k}}^x, \quad (3)$$

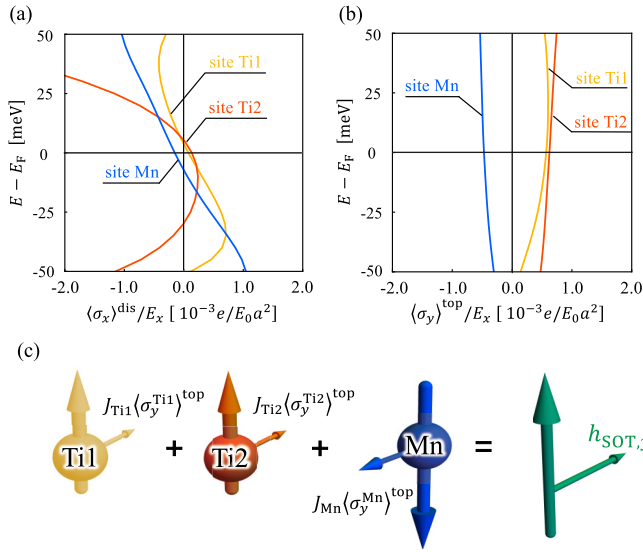


FIG. 2. The energy dependences of the (a) dissipative and (b) topological spin densities on each sublattice. (c) $\langle \sigma_y \rangle^{\text{top}}$ are schematically shown by the arrows on each sublattice. These spin densities are additive on the compensated ferrimagnetic ordering and act as an effective magnetic field h_{SOT} .

$$\frac{\langle \sigma_\mu^\alpha \rangle^{\text{top}}}{E_x} = -2\hbar \sum_{n \neq m} \int_k f_{nk} \frac{\text{Im}[S_{nm,k}^{\alpha,\mu} J_{nm,k}^x]}{(E_{nk} - E_{mk})^2 + (\frac{\hbar}{2\tau})^2}. \quad (4)$$

Equation (3) includes only the electronic states near E_F from $\partial f_{nk}/\partial E$, whereas Eq. (4) describes the contribution from all the electronic states below E_F from f_{nk} . Thus, we classify $\langle \sigma_\mu \rangle^{\text{dis}}$ as the contribution to the “dissipative SOT” and $\langle \sigma_\mu \rangle^{\text{top}}$ as the “topological SOT.” We show later that $\langle \sigma_\mu \rangle^{\text{top}}$ in our model is indeed described by the topological feature, i.e., the mixed Berry curvature.

With the above formulas, we study the energy dependence of $\langle \sigma_\mu^\alpha \rangle^{\text{dis/top}}$, as shown in Figs. 2(a) and 2(b). We here fix the relaxation time $\tau = 3\hbar/E_0 = 39$ fs. This value is appropriate in magnetic Weyl semimetal, which is comparable to the carrier relaxation time ~ 100 fs in $\text{Co}_3\text{Sn}_2\text{S}_2$ estimated from the optical conductivity measurements [35]. In Fig. 2(a), the magnitudes of $\langle \sigma_\mu \rangle^{\text{dis}}$ on all the three sites are suppressed around E_F , namely, around the energy of the WPs. Such an energy dependence is similar to the results obtained with the minimal Weyl Hamiltonian with the isotropic spin-momentum locking in continuum [21,22]. In contrast, as shown in Fig. 2(b), $\langle \sigma_\mu \rangle^{\text{top}}$ for all sublattices are always nonzero around E_F and nearly independent of energy. Such a topological response was not seen in the minimal Weyl Hamiltonian [21,22], which is the artifact of the isotropic spin-momentum locking structure in the Fermi sea around each WP. Since the spin-orbit coupling structure and the WP distribution of Ti_2MnAl considered here are anisotropic in momentum space, it is capable of showing the finite topological response $\langle \sigma_\mu \rangle^{\text{top}}$. We find that, in response to E_x , $\langle \sigma^\alpha \rangle^{\text{dis}}$ points to the x direction while $\langle \sigma^\alpha \rangle^{\text{top}}$ to the y direction.

The topological response leads to a highly efficient conversion from current to spin density, because of the low current

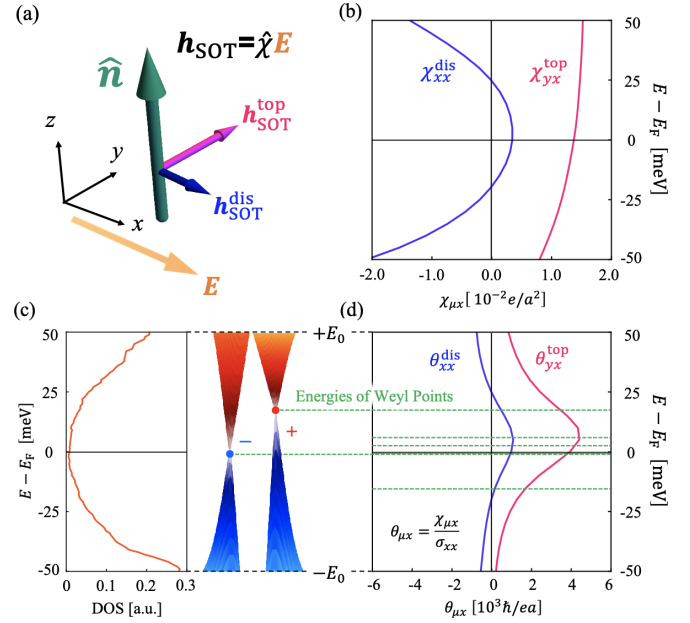


FIG. 3. (a) The effective magnetic fields from dissipative and topological responses. (b) The energy dependences of the response coefficients χ_{xx}^{dis} and χ_{yx}^{top} . (c) The density of states (DOS) and one of the energy dispersion around WPs. (d) The energy dependences of efficiencies of SOT θ_{xx}^{dis} and θ_{yx}^{top} .

density. To compare the spin density generated per unit current on each sublattice with the conventional Rashba system, we evaluate a dimensionless index $\zeta^\alpha = e v_F \langle \sigma_y^\alpha \rangle^{\text{top}} / \langle j_x \rangle^{\text{dis}}$. Here, $\langle j_x \rangle^{\text{dis}}$ is the longitudinal current flowing in the x direction, which is calculated by the formula similar to Eq. (3), by replacing $\hat{\sigma}_\mu$ with $-e\hat{v}_x$. $v_F = 6.3 \times 10^4$ m/s is the Fermi velocity of the Weyl cone. With $E = E_F$, we find these dimensionless indices $\zeta^{\text{Ti1}} = 5.6 \times 10^3$, $\zeta^{\text{Ti2}} = 5.0 \times 10^3$, and $\zeta^{\text{Mn}} = 4.2 \times 10^3$. These values are about 10^4 times larger than the typical value 0.12 in the two-dimensional Rashba system (see Appendix B). In other words, in Ti_2MnAl , we can reduce the current by the factor of $\sim 10^{-4}$ from that in the Rashba systems to induce the same magnitude of spin density.

Effective magnetic field and spin-orbit torque. The above results show that, near E_F , the signs of spin densities at Ti1 and Ti2 and that at Mn are opposite, as schematically shown in Fig. 2(c). These spin densities exert torques on the magnetic moments at the Ti sites and the Mn sites. During the dynamics of these magnetic moments, violation of their compensation is predicted to be energetically prohibited [33]. Thus, we treat the compensated ferrimagnetic ordering $\hat{n}$ as the only kinematical variable in this system. We consider the effective magnetic field acting on $\hat{n}$ in the linear response regime, $h_{\text{SOT}}(\mathbf{r}) = \langle \frac{\partial H}{\partial \hat{n}} \rangle = \sum_\alpha J_\alpha \langle \sigma^\alpha(\mathbf{r}) \rangle$ [36]. Here, $\langle \sigma^\alpha \rangle$ is given by the formula in Eq. (2). This h_{SOT} has a unit of energy density and drives the switching or precessional dynamics of $\hat{n}$. Figure 3(a) depicts the effective magnetic field for dissipative contribution $h_{\text{SOT}}^{\text{dis}}$ (blue arrow) from $\langle \sigma^\alpha \rangle^{\text{dis}}$ and topological contribution $h_{\text{SOT}}^{\text{top}}$ (red arrow) from $\langle \sigma^\alpha \rangle^{\text{top}}$.

First, we investigate the energy dependences of h_{SOT} with a fixed relaxation time $\tau = 3\hbar/E_0$. We characterize h_{SOT}

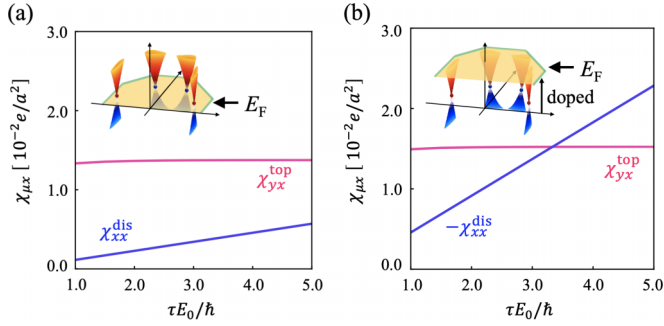


FIG. 4. Relaxation time dependences for response coefficients χ_{xx}^{dis} and χ_{yx}^{top} when (a) E_F is located at the energy of the WPs and (b) E_F is away from the energy of the WPs.

as $\mathbf{h}_{\text{SOT}}^{\text{dis/top}} = \hat{\chi}^{\text{dis/top}} \mathbf{E}$ with the response coefficient tensor $\hat{\chi}^{\text{dis/top}}$, where $\chi_{\mu x}^{\text{dis/top}} = \sum_{\alpha} J_{\alpha} \langle \sigma_{\mu}^{\alpha} \rangle^{\text{dis/top}} / E_x$. In Fig. 3(b), the components of $\hat{\chi}^{\text{dis}}$ and $\hat{\chi}^{\text{top}}$ are shown as functions of energy. Near E_F , $\hat{\chi}^{\text{top}}$ is larger than $\hat{\chi}^{\text{dis}}$ and shows a relatively moderate energy dependence. To characterize the efficiency of the SOT, we introduce a quantity $\theta_{\mu x}^{\text{dis/top}} = h_{\text{SOT}, \mu}^{\text{dis/top}} / \langle j_x \rangle^{\text{dis}} = \chi_{\mu x}^{\text{dis/top}} / \sigma_{xx}$. Figure 3(d) shows the energy dependence of $\theta_{\mu x}^{\text{dis/top}}$. In Fig. 3(d), one can find that θ_{yx}^{top} has a peak near E_F and is dominant over θ_{xx}^{dis} . The peak structure of θ_{yx}^{top} arises from the combination of the large topological SOT and the small σ_{xx} from small density of states around the energy of the WPs [Fig. 3(c)]. By measuring $\mathbf{h}_{\text{SOT}}$ in the unit of Tesla (see Appendix C), we find that θ_{yx}^{top} is of the order of 10^{-13} Tm²/A within the calculated energy range, reaching the value $\theta_{yx}^{\text{top}} = 5.09 \times 10^{-13}$ Tm²/A at E_F . This value is 10–1000 times larger than the SOT efficiency measured in the heterostructures of ferromagnets and heavy metals [7,8], and also those calculated in the bulk antiferromagnets Mn₂Au [11,12] and CuMnAs [13].

Next, we show the τ dependences of $\chi^{\text{dis/top}}$ in the doped case [Fig. 4(a)]. E_F is here located near the energy of the WPs (see the inset). We find that χ_{yx}^{top} is almost independent of τ , which implies the insensitivity to the disorder. Moreover, throughout the entire range of τ , we have calculated, χ_{yx}^{top} remains larger than χ_{xx}^{dis} . In contrast, χ_{xx}^{dis} is proportional to τ as seen from Eq. (3). With regard to magnitude of χ_{xx}^{dis} , the density of states is small enough, and hence χ_{xx}^{dis} becomes also small in comparison with χ_{yx}^{top} .

Then, we study $\chi_{xx}^{\text{dis/top}}$ in the doped case where E_F is 50 meV above the energy of the WPs [Fig. 4(b)]. We find that both the τ dependence and magnitude of χ_{yx}^{top} are almost unchanged from those in Fig. 4(a). In contrast, χ_{xx}^{dis} is still proportional to τ , while its magnitude becomes larger. The magnitude of χ_{xx}^{dis} becomes comparable to that of χ_{yx}^{top} and exceeds it around $\tau = 3.5 \times \hbar/E_0$. This is because the density of states becomes larger when E_F is shifted away from the energy of the WPs.

From the discussions above, we find that χ_{xx}^{dis} is always proportional to τ , and its magnitude depends on E_F , which is consistent with Ref. [21]. Our main finding is that the topological response χ_{yx}^{top} is present irrespective of τ , and is almost independent of the choice of E_F . We note that the

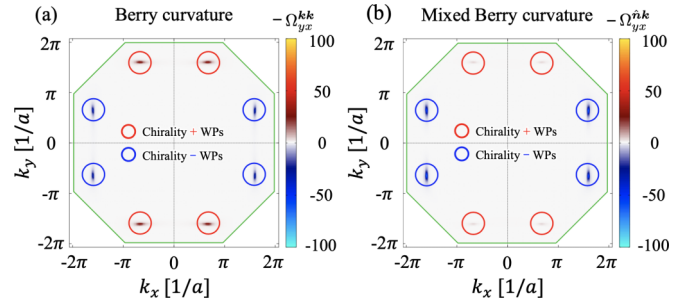


FIG. 5. The mapping of the yx component of (a) the Berry curvature $-\Omega_{yx}^{kk}$ and (b) the mixed Berry curvature $-\Omega_{yx}^{hk}$ on $k_z = 0.1[1/a]$ at E_F . The peaks and valleys of $-\Omega_{yx}^{kk/hk}$ correspond to the positions of the WPs.

present analysis neglects the vertex corrections from the multiple impurity scattering processes for χ_{xx}^{dis} and χ_{yx}^{top} , which is a reliable assumption for qualitative discussions. In particular, χ_{yx}^{top} is a Fermi sea effect unaffected by impurity scatterings, and hence, the absence of vertex corrections does not make a large difference for χ_{yx}^{top} around the energy of WPs. Such behavior of χ_{yx}^{top} is related to the band geometry characterized by the mixed Berry curvature from the WPs, as we shall discuss in the following.

Mixed Berry curvature and discussion. So far, we have found the topological SOT χ_{yx}^{top} is dominant compared to the dissipative SOT χ_{xx}^{dis} in our model of Ti₂MnAl. To understand the origin of χ_{yx}^{top} , we study the mixed Berry curvature [28]. It is a Berry curvature extended in the composite parameter space $(\mathbf{k}, \hat{\mathbf{n}})$ spanned by the momentum $\mathbf{k}$ and the ferrimagnetic ordering $\hat{\mathbf{n}}$. The yx components of the Berry curvature Ω_{yx}^{kk} and the mixed Berry curvature Ω_{yx}^{hk} for the occupied bands are expressed as

$$\begin{aligned} \Omega_{yx}^{kk}(\mathbf{k}) &= 2 \text{Im} \sum_{n \neq m} f_{nk} \frac{\langle n\mathbf{k} | \partial_{k_y} \mathcal{H} | m\mathbf{k} \rangle \langle m\mathbf{k} | \partial_{k_x} \mathcal{H} | n\mathbf{k} \rangle}{(E_{nk} - E_{mk})^2}, \\ \Omega_{yx}^{hk}(\mathbf{k}) &= 2 \text{Im} \sum_{n \neq m} f_{nk} \frac{\langle n\mathbf{k} | \partial_{\hat{n}_y} \mathcal{H} | m\mathbf{k} \rangle \langle m\mathbf{k} | \partial_{k_x} \mathcal{H} | n\mathbf{k} \rangle}{(E_{nk} - E_{mk})^2}. \end{aligned} \quad (5)$$

The topological SOT [Eq. (4)] can be rewritten with the mixed Berry curvature [4,28,29,37], $\mathbf{h}_{\text{SOT}}^{\text{top}} = e\hbar \int_{\mathbf{k}} \hat{\Omega}^{\text{hk}}(\mathbf{k}) \mathbf{E}$, in the clean limit $\tau \rightarrow \infty$.

To understand the contribution of mixed Berry curvature to the topological SOT, we compare the distributions of the Berry curvature $-\Omega_{yx}^{kk}$ and the mixed Berry curvature $-\Omega_{yx}^{hk}$ on the k_x - k_y plane in Fig. 5. We find that the mixed Berry curvature is enhanced around all 24 WPs, which are similar to those of the Berry curvature contributing to the intrinsic AHE. These numerical results suggest that even in the absence of spin-momentum locking, the velocity operator $\partial_{\mathbf{k}} \mathcal{H}$ and the effective magnetic field operator $\partial_{\hat{\mathbf{n}}} \mathcal{H}$ have similar structures near the WPs, yielding the peaks of both the ordinary and mixed Berry curvatures. As a result, the WPs serve as the origin not only of the intrinsic AHE but also of the topological SOT.

Conclusion. Our study clarifies that the ferrimagnetic Weyl semimetal Ti₂MnAl shows a distinct behavior in the SOT. While the previous studies on the SOT in magnetic Weyl

semimetals discussed only the dissipative SOT [21,22], the present study finds that Ti_2MnAl shows the giant topological SOT free from dissipation, which comes from the notably enhanced mixed Berry curvature around the WPs. Moreover, due to the tiny carrier density in Ti_2MnAl , the longitudinal current gets suppressed. As a result of these characteristics, the efficiency of the SOT per the injected current reaches around 10–1000 times those estimated previously in non-centrosymmetric antiferromagnets. These findings imply the importance of the WP and mixed Berry curvature structure around the Fermi level in enhancing the SOT, which serves as a guiding principle in selecting or designing a magnetic material suitable for power-efficient SOT-based devices.

Our findings of the topological SOT in Ti_2MnAl will be advantageous also in the studies on compensated ferrimagnets. While the magnetism in compensated ferrimagnets has been of great interest recently [38], their magnetic orderings are hard to manipulate due to the absence of net magnetization. The topological SOT provides an efficient way to manipulate the magnetic ordering electrically, which may help further understanding and applications of compensated ferrimagnetism in future studies. In particular, since the magnetic moments in Ti_2MnAl are subject to a strong intersublattice coupling [33], we expect that an ultrafast oscillation of $\hat{n}$ may be driven by the topological SOT. This implies the potential applications of Ti_2MnAl to an ultrafast spin torque oscillator [39], where the current-induced oscillation of $\hat{n}$ is converted to an electric signal via the AHE.

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Appendix A: Details of the tight-binding model. We here explain the details of the hopping and spin-orbit coupling terms in our tight-binding model based on our previous study [33]. The primitive unit cell is the fcc type, and the unit vectors are $\mathbf{a}_1 = \frac{a}{2}(1, 1, 0)$, $\mathbf{a}_2 = \frac{a}{2}(0, 1, 1)$, and $\mathbf{a}_3 = \frac{a}{2}(1, 0, 1)$, where a is a lattice constant and we set $a = 1$ for simplicity. In the model Hamiltonian [see Eq. (1)], the first term includes intersublattice hopping with strengths $t_{\text{Ti1-Ti2}}$, $t_{\text{Ti2-Mn}}$, and $t_{\text{Mn-Ti1}}$, and the intrasublattice hopping with strength $t_{\text{Ti1-Ti1}}$, $t_{\text{Ti2-Ti2}}$, and $t_{\text{Mn-Mn}}$. The second term is the on-site energy, and ϵ_α is its strength. The third term is spin-orbit coupling and originates from the intrinsic electric field, which is generated due to the local breaking of inversion symmetry. The dominant asymmetry comes from the imbalance between Ti1 and Al sites. We consider spin-orbit coupling for the Ti2-Ti2 and Mn-Mn hoppings with the same strength λ_{SOC} and neglect the spin-orbit coupling for the Ti1-Ti1 for simplicity. $\mathbf{d}_{1,2}^{\alpha ij}$ are the two nearest-neighbor hopping vectors from site i to j of sublattice α , which was originally introduced in a Fu-Kane-Mele model [34]. The fourth term represents the exchange coupling between conduction electron spin and magnetic moment. $\boldsymbol{\sigma}$ is the Pauli matrix vector representing electron spin. $\hat{n}$ is the unit vector representing the compensated ferrimagnetic ordering. J_α is the coupling strength on sublattice α .

The hopping parameters are set to $t_{\text{Ti1-Ti2}} = 420$ meV, $t_{\text{Ti2-Mn}} = 176.4$ meV, $t_{\text{Mn-Ti1}} = 504$ meV, $t_{\text{Ti1-Ti1}} = 21$ meV,

$t_{\text{Ti2-Ti2}} = 357$ meV, and $t_{\text{Mn-Mn}} = -21$ meV. On-site energies $\epsilon_{\text{Ti1}} = \epsilon_{\text{Ti2}} = \epsilon_{\text{Mn}} = -903$ meV. The strengths of the exchange coupling are $J_{\text{Ti1}} = J_{\text{Ti2}} = 294$ meV and $J_{\text{Mn}} = -714$ meV. The sign difference corresponds to the ferrimagnetic ordering. The strength of spin-orbit coupling is $\lambda_{\text{SOC}} = -84$ meV. The parameters are set so that the energy spectrum, including the WPs, the density of states, and the strength of the anomalous Hall conductivity, become consistent with first-principles calculations [30].

For the momentum integration in the Kubo formula, the momentum space is split into the mesh of $250 \times 250 \times 250$ for all calculations in this paper.

Appendix B: Derivations of Eqs. (3) and (4). We start from Eq. (2):

$$\begin{aligned} \frac{\langle \sigma_\mu^\alpha \rangle}{E_x} &= i\hbar \lim_{q \rightarrow 0} \sum_{n,m} \int_k \frac{f_{nk} - f_{mk+q}}{E_{nk} - E_{mk+q}} \frac{S_{nm,k+q}^{\alpha,\mu} J_{mn,k+q}^x}{E_{nk} - E_{mk+q} + i\eta} \\ &= i\hbar \lim_{q \rightarrow 0} \sum_{n,m} \int_k A_{nm}(\mathbf{k}; \mathbf{q}), \end{aligned} \quad (\text{B1})$$

where the integrand is denoted as $A_{nm}(\mathbf{k}; \mathbf{q})$. The matrix elements for the spin-density operator and current operator are $S_{nm,k+q}^{\alpha,\mu} = \langle n\mathbf{k} | \hat{\sigma}_\mu^\alpha | m\mathbf{k} + \mathbf{q} \rangle$ and $J_{mn,k+q}^x = \langle m\mathbf{k} + \mathbf{q} | (-e\hat{v}_x) | n\mathbf{k} \rangle$.

First, we consider the intraband contribution ($n = m$). The first part of the integrand $A_{nn}(\mathbf{k}; \mathbf{q})$ can be interpreted as the momentum derivative of the Fermi distribution function and the eigenvalue energy,

$$\begin{aligned} A_{nn}(\mathbf{k}; \mathbf{q}) &= \frac{f_{nk} - f_{nk+q}}{k_v - (k_v + q_v)} \frac{k_v - (k_v + q_v)}{E_{nk} - E_{nk+q}} \\ &\quad \times \frac{S_{nn,k+q}^{\alpha,\mu} J_{nn,k+q}^x}{E_{nk} - E_{nk+q} + i\eta} \\ &\xrightarrow{q \rightarrow 0} \frac{\partial f_{nk}}{\partial k_v} \frac{\partial k_v}{\partial E_{nk}} \frac{1}{i\eta} S_{nn,k}^{\alpha,\mu} J_{nn,k}^x \\ &= \frac{\partial f}{\partial E} \bigg|_{nk} \frac{1}{i\eta} S_{nn,k}^{\alpha,\mu} J_{nn,k}^x. \end{aligned} \quad (\text{B2})$$

By introducing a finite relaxation time τ through the first Born approximation, where $\eta = \frac{\hbar}{2\tau}$, Eq. (B1) becomes

$$\begin{aligned} i\hbar \lim_{q \rightarrow 0} \sum_n \int_k A_{nn}(\mathbf{k}; \mathbf{q}) &= i\hbar \sum_n \int_k \frac{\partial f}{\partial E} \bigg|_{nk} \frac{1}{i\frac{\hbar}{2\tau}} S_{nn,k}^{\alpha,\mu} J_{nn,k}^x \\ &= 2\tau \sum_n \int_k \frac{\partial f}{\partial E} \bigg|_{nk} S_{nn,k}^{\alpha,\mu} J_{nn,k}^x, \end{aligned} \quad (\text{B3})$$

which reproduces the Fermi surface term, Eq. (3) in the main text.

Next, let us focus on the interband contribution ($n \neq m$). Taking the $\mathbf{q} \rightarrow 0$ limit, the integrand $A_{nm}(\mathbf{k}; \mathbf{q})$ becomes

$$\begin{aligned} iA_{nm}(\mathbf{k}; \mathbf{q}) &= \frac{f_{nk} - f_{mk+q}}{E_{nk} - E_{mk+q}} \frac{S_{nm,k+q}^{\alpha,\mu} J_{mn,k+q}^x}{E_{nk} - E_{mk+q} + i\eta} \\ &\xrightarrow{q \rightarrow 0} i \frac{f_{nk} - f_{mk}}{E_{nk} - E_{mk}} \cdot \frac{S_{nm,k}^{\alpha,\mu} J_{mn,k}^x}{E_{nk} - E_{mk} + i\eta} \end{aligned}$$

$$\begin{aligned}
&= -\frac{f_{nk} - f_{mk}}{E_{nk} - E_{mk}} \cdot \text{Im} \left[\frac{S_{nm,k}^{\alpha,\mu} J_{mn,k}^x}{E_{nk} - E_{mk} + i\eta} \right] \\
&\approx -\frac{f_{nk} - f_{mk}}{E_{nk} - E_{mk}} \\
&\quad \times \text{Im} [S_{nm,k}^{\alpha,\mu} J_{mn,k}^x] \text{Re} \left[\frac{1}{E_{nk} - E_{mk} + i\eta} \right] \\
&= -2f_{nk} \frac{\text{Im} [S_{nm,k}^{\alpha,\mu} J_{mn,k}^x]}{(E_{nk} - E_{mk})^2 + \eta^2}. \quad (\text{B4})
\end{aligned}$$

Here, the imaginary part of $\frac{1}{E_{nk} - E_{mk} + i\eta}$ is neglected under the assumption $|E_{nk} - E_{mk}| \gg \eta$. The relation $\text{Im}[S_{nm,k}^{\alpha,\mu} J_{mn,k}^x] = -\text{Im}[S_{mn,k}^{\alpha,\mu} J_{nm,k}^x]$ is used for the spin density and current operator matrix elements. Finally, by integrating Eq. (B4) over the Brillouin zone and summing over all bands, Eq. (4) in the main text is derived.

Appendix C: Spin density and its efficiency of the 2d Rashba model. The two-dimensional (2D) Rashba model explains the two-dimensional electron gas (2DEG) with Rashba type spin-orbit coupling. Calculating the SOT by this model, only the $yx(xy)$ component of the dissipative contribution is finite. It is given by the following [40,41]:

$$\frac{\langle \sigma_y \rangle^{\text{dis}}}{E_x} = \frac{2m^*e}{\hbar^2} \lambda \tau. \quad (\text{C1})$$

Here, m^* is the effective mass of 2DEG, and λ is the strength of Rashba-type spin-orbit coupling. With the electron density n , the dc conductivity is also given by $\langle j_x \rangle^{\text{dis}}/E_x = \frac{ne^2}{m^*} \tau$. The dimensionless index ζ , which explains spin density generated per unit current, is given by

$$\zeta = e v_F \frac{\langle \sigma_y \rangle^{\text{dis}}}{\langle j_x \rangle^{\text{dis}}} = \frac{2m^{*2} \lambda v_F}{\hbar^2 n}. \quad (\text{C2})$$

Here, as the effective mass m^* and electron density n of the 2DEG, the typical values of $0.068m_e$ and 10^{15} m^{-2} are chosen, respectively. λ denotes the strength of spin-orbit coupling and is set to $1.51 \times 10^4 \text{ m/s}$ [42]. v_F is the Fermi velocity, given by $v_F = \hbar k_F/m^*$, where Fermi wave vector k_F is assumed to be 0.1 nm^{-1} . Substituting these typical values into Eq. (C2), one can obtain a value $\zeta = 0.12$.

Appendix D: Estimation of the effective magnetic field of Ti_2MnAl . Here, we explain the details of the estimation of the SOT efficiency per current as defined by $\theta_{\mu x}^{\text{dis/top}} = h_{\text{SOT},\mu}^{\text{dis/top}}/\langle j_x \rangle^{\text{dis}} = \chi_{\mu x}^{\text{dis/top}}/\sigma_{xx}$. First, we estimate the magnitude of the electric field in Ti_2MnAl when a current is applied. Given a typical experimental current density of $j = 10^{10} \text{ A/m}^2$ and the longitudinal conductivity of Ti_2MnAl at

E_F , $\sigma_{xx} = 0.24 \frac{e^2}{a\hbar}$, the electric field in Ti_2MnAl is estimated to be 10^5 V/m . This value is used in the following estimation of the magnitude of the effective magnetic field.

Referring to Fig. 2(b), we approximate that the spin densities from all electronic states below E_F on each sublattice have almost the same magnitude: $\langle \sigma^{\text{Ti1}} \rangle^{\text{top}} \approx \langle \sigma^{\text{Ti2}} \rangle^{\text{top}} \approx -\langle \sigma^{\text{Mn}} \rangle^{\text{top}} \equiv s^{\text{top}}$. With this assumption, the effective magnetic field $\mathbf{h}_{\text{SOT}}$

$$\begin{aligned}
\mathbf{h}_{\text{SOT}}^{\text{top}} &= \sum_{\alpha=\text{Ti1,Ti2,Mn}} J_{\alpha} \langle \sigma^{\alpha} \rangle^{\text{top}} \\
&= (J_{\text{Ti1}} \langle \sigma^{\text{Ti1}} \rangle^{\text{top}} + J_{\text{Ti2}} \langle \sigma^{\text{Ti2}} \rangle^{\text{top}} + J_{\text{Mn}} \langle \sigma^{\text{Mn}} \rangle^{\text{top}}) \\
&\approx (J_{\text{Ti1}} + J_{\text{Ti2}} - J_{\text{Mn}}) s^{\text{top}}. \quad (\text{D1})
\end{aligned}$$

We note that $\mathbf{h}_{\text{SOT}}$ defined here has the dimension of energy density, not magnetic field. To convert it to the dimension of the magnetic field, tesla, we need to divide it by the magnetic moment for each site,

$$\mathbf{B}_{\text{SOT}}^{\alpha} = \frac{J_{\alpha} \frac{a^3}{2}}{m_{\alpha}} \langle \sigma^{\alpha} \rangle. \quad (\text{D2})$$

Here, $\frac{a^3}{2}$ represents the volume of the conventional unit cell of Ti_2MnAl , and m_{α} denotes the magnitude of the magnetic moment residing on sublattice α . This $\mathbf{B}_{\text{SOT}}^{\alpha}$ can be expressed with $\mathbf{h}_{\text{SOT}}$ as

$$\begin{aligned}
\mathbf{B}_{\text{SOT}}^{\alpha,\text{top}} &= \frac{J_{\alpha} \frac{a^3}{2}}{m_{\alpha}} \langle \sigma^{\alpha} \rangle^{\text{top}} \\
&\approx \frac{1}{m_{\alpha}} J_{\alpha} \frac{a^3}{2} s^{\text{top}} \\
&= \frac{1}{m_{\alpha}} \frac{J_{\alpha}}{J_{\text{Ti1}} + J_{\text{Ti2}} - J_{\text{Mn}}} \frac{a^3}{2} \mathbf{h}_{\text{SOT}}^{\text{top}}. \quad (\text{D3})
\end{aligned}$$

For instance, the field strength at the Ti1 site reads $|\mathbf{B}_{\text{SOT}}^{\text{Ti1},\text{top}}| \approx 1.15 \times 10^{-3} \text{ T}$, by substituting into Eq. (D3) the quantities $m_{\text{Ti1}} = 1.44 \mu_B$ [31], $J_{\text{Ti1}} = J_{\text{Ti2}} = 294 \text{ meV}$, $J_{\text{Mn}} = -714 \text{ meV}$, lattice constant $a = 6.24 \times 10^{-10} \text{ m}$ [31], the electric field in Ti_2MnAl $E_x = 10^5 \text{ V/m}$, and the response coefficient at E_F , $\chi_{yx}^{\text{top}} = 1.36 \times 10^{-2} \frac{e}{a^2}$, obtained from Fig. 3(b). As discussed in the main text, the spin densities of Ti1 and Ti2 have signs opposite that of Mn. Therefore, the effective magnetic fields acting on the compensated ferrimagnetic order $\mathbf{n}$ are additive across the sublattices [see Fig. 2(c)]. By calculating $\mathbf{B}_{\text{SOT}}^{\alpha,\text{top}}$ for Ti2 and Mn and summing them, the magnetic field acting on the compensated ferrimagnetic ordering $\mathbf{n}$ is found to be $\mathbf{B}_{\text{SOT}}^{\text{n}} \approx 5.09 \times 10^{-3} \text{ T}$, or $\theta_{yx}^{\text{top}} = \mathbf{B}_{\text{SOT},y}^{\text{n}}/j_x = 5.09 \times 10^{-13} \text{ T m}^2/\text{A}$ with respect to the longitudinal current j_x .

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