

Self-organized criticality driven by droplet influx and random fusion

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(Received 17 February 2025; accepted 23 May 2025; published 9 June 2025)

The droplet size distribution typically decays exponentially in solutions formed by liquid-liquid phase separation. Nevertheless, a power-law distribution of nucleolus volumes has been observed in amphibian oocytes, which appears similar to the cluster size distribution in reaction-limited aggregation. In this work, we study the mechanism of power-law distributed droplet sizes and unveil a self-organized criticality driven by droplet influx and random fusion between droplets. Surprisingly, the droplet size dynamics is governed by a Smoluchowski equation similar to the cluster size in aggregation systems. The system reaches a critical state as the area fraction approaches the critical value at which the droplet size has a power-law distribution with an exponent of 1.5. The system is also spatially scale-free, with a divergent correlation length at the critical state, marked by giant droplet-density fluctuations and power-law decay of the pair correlation function.

DOI: [10.1103/PhysRevResearch.7.L022060](https://doi.org/10.1103/PhysRevResearch.7.L022060)

Introduction. Critical phenomena are ubiquitous in various physical systems, marked by a divergent correlation length and scale-free distributions [1–3]. The self-organized criticality (SOC) concept has been successfully applied to various nonequilibrium systems [4–9], in which a system self-organizes to a critical state in which local perturbations trigger responses across all scales. Similarly, fractal structures of clusters and power-law distributions of cluster sizes have been widely observed in the kinetic aggregation of small particles to form large clusters [10–14]. Hyperuniform patterns in which density fluctuations are suppressed at large length scales [15] are also observed in nonequilibrium systems, such as sheared colloidal suspensions [16,17], the jamming transition [18], and spinodal decomposition [19].

Similar to the power-law distributions of cluster sizes in particle-aggregation systems [10,12–14], Brangwynne *et al.* have observed a power-law distribution of nucleolus volumes in *Xenopus laevis* oocytes, in which the formation of nucleoli is governed by liquid-liquid phase separation [20]. Power-law distributions of cluster size and nucleolus volume with an exponent of $\tau = 1.5$ were observed in both cases. However, in the case of aggregation, the power-law distribution happens only in the case of reaction-limited aggregation (RLA) [12,13], in which the number of diffusion-induced collisions before aggregation of two clusters succeeds is sufficiently large to allow clusters to sample all possible mutual bonding configurations. This feature of RLA appears inconsistent with nucleoli since these membraneless organelles are liquidlike and fuse virtually immediately once they touch [20],

a common feature of liquid-liquid phase separation [21–24]. Therefore, the fusion dynamics of nucleoli seems to be better described by diffusion-limited aggregation (DLA); however, a characteristic size of clusters appears in DLA, and the distribution of cluster sizes typically decays exponentially [13,25,26]. Therefore, the mechanism of the power-law-distributed nucleolus volume observed in *X. laevis* oocytes remains elusive. Brangwynne *et al.* empirically found that introducing a slow, constant influx of small droplets in a Monte Carlo simulation of fusing droplets could numerically reproduce the observed power-law distribution of droplet size, which hints that an external injection may be crucial for generating this power-law distribution of droplet sizes.

In this Letter, we introduce a simple model of droplet fusion with a quasistatic addition of small droplets. We add a spherical droplet of a unit area in a random position to a two-dimensional system with a periodic boundary condition and a linear system size L . If any droplets overlap, they fuse into a new spherical droplet with a conserved area and a fixed center of mass. We repeat the fusion process until no droplets overlap. Then, we randomly add a new unit-area droplet and repeat the process. We label the system state by the area fraction ϕ , the total area of all droplets divided by the system area, which gradually increases as more and more droplets are added to the system. Interestingly, the system approaches a critical state as $\phi \rightarrow \phi_c$, where the droplet size distribution becomes scale-free and decays as a power law (Fig. 1), which has not been reported yet, to our knowledge. We propose an argument to demonstrate that the fusion kernel, the probability for two droplets to fuse per unit ϕ , can be approximated as $K(S_1, S_2) \sim S_1 + S_2$ and confirm this prediction numerically. Interestingly, this type of kernel leads exactly to an exponent of 1.5 for the droplet size distribution in the context of kinetic aggregation [10,27,28], in agreement with the distributions of nucleolus volumes [20]. We also calculate the spatial density variance of droplets inside a finite subsystem and find that

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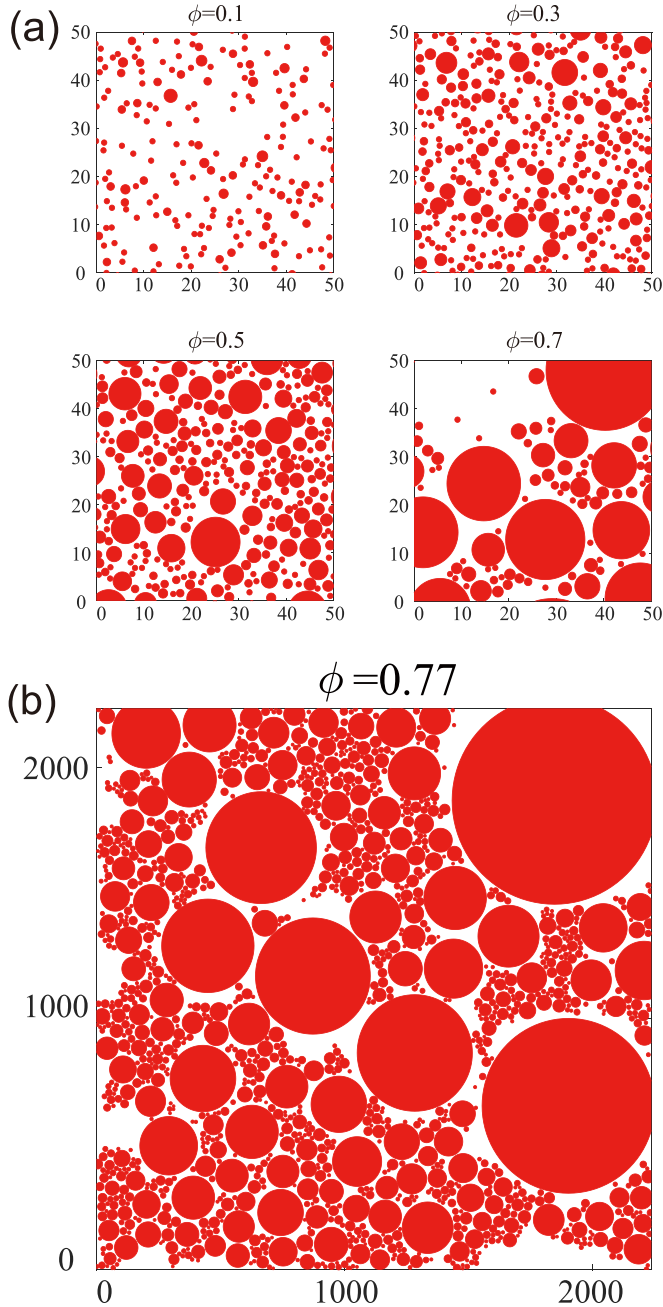


FIG. 1. We randomly add small droplets quasistatically to the system. As the area fraction of droplets ϕ increases, the system approaches a critical state at which scale-free patterns of droplets emerge. (a) Local configurations of the system at area fractions $\phi = 0.1, 0.3, 0.5$, and 0.7 . (b) The whole-system configuration at $\phi = 0.77$ near the critical state.

it decreases more slowly than the inverse of the subsystem area, which means the system exhibits a diverging correlation length. Furthermore, the average droplet fusion time approaches a finite value at criticality ϕ_c , contrasting with conventional self-organized critical systems in which such times diverge.

Results. We introduce a characteristic droplet size S^* defined as follows: The total area of droplets with S smaller than S^* is half the total area of all droplets. As we continuously

add droplets to the system, S^* increases as ϕ increases and diverges as $\phi \rightarrow \phi_c$, with the following finite-size scaling:

$$S^* \sim L^2 f[(\phi_c - \phi)^{-\gamma}/L^2]. \quad (1)$$

The scaling function $f(x)$ exhibits two distinct regimes: (1) For $x \ll 1$, we expect a scaling behavior for an infinitely large system, $S^* \sim (\phi_c - \phi)^{-\gamma}$, which means that $f(x) \sim x$; (2) for $x \gg 1$, finite-size effects dominate, with $f(x)$ approaching a constant, yielding $S^* \sim L^2$. The excellent data collapse demonstrates that $\phi_c = 0.80 \pm 0.01$ and $\gamma = 3.4 \pm 0.1$, unveiling the scale invariance and self-organized critical behaviors in our system [Fig. 2(a)].

We use the Smoluchowski equation to describe the ϕ dependence of the droplet size distribution,

$$\begin{aligned} \frac{dn(S, \phi)}{d\phi} &= \frac{1}{2} \sum_{S_1} K(S_1, S - S_1) n(S_1, \phi) n(S - S_1, \phi) \\ &\quad - n(S, \phi) \sum_{S_1} K(S, S_1) n(S_1, \phi) + \delta_{S,1}, \end{aligned} \quad (2)$$

where $K(S_1, S_2)$ is the fusion kernel, the probability for two droplets with size S_1 and S_2 to fuse per unit ϕ . Here, $n(S, \phi)$ is the number density of the droplet size, defined as the number of droplets with size S normalized by the system area L^2 at a given ϕ . In this paper, we interchangeably use the terms "number density of the droplet size" and "droplet size distribution." Note that $\phi = \sum_S S n(S, \phi)$. The last term on the right side of Eq. (2) represents the addition of new droplets with unit area.

In the thermodynamic limit $L \rightarrow \infty$, $n(S, \phi)$ must be system size independent. Because the dimension of the kernel is length squared, we expect it to be a linear function of the droplet size S in the simplest scenario. Another way to think about kernel scaling is to enlarge the system by a factor λ to create an imaginary system. For a small interval $\Delta\phi$, the number density of newly created droplets in the original system is given by $\Delta n(S_1 + S_2) = \Delta\phi K(S_1, S_2) n(S_1) n(S_2)$. In the enlarged system, this expression becomes $\Delta n'(\lambda^2 S_1 + \lambda^2 S_2) = \Delta\phi K(\lambda^2 S_1, \lambda^2 S_2) n'(\lambda^2 S_1) n'(\lambda^2 S_2)$. Since $n'(\lambda^2 S) = n(S)/\lambda^2$ by definition, we have $K(\lambda^2 S_1, \lambda^2 S_2) = \lambda^2 K(S_1, S_2)$. Because the two droplets are equivalent, we propose the following form of the fusion kernel:

$$K(S_1, S_2) \sim S_1 + S_2, \quad (3)$$

which we verify numerically (Fig. 3).

Kernels with such a sum form have an analytical solution in the context of kinetic aggregation of colloidal particles [10]. We propose that the same result should apply to the droplet size distribution, leading to the following asymptotic form of $n(S, \phi)$:

$$n(S, \phi) = \frac{\phi}{\sqrt{\pi S_c}} S^{-\frac{3}{2}} \exp\left(-\frac{S}{S_c}\right), \quad (4)$$

which satisfies dynamical scaling [13,27]:

$$n(S, \phi) = \phi S_c^{-2} \psi\left(\frac{S}{S_c}\right), \quad (5)$$

where $\psi(x) = x^{-3/2} e^{-x}/\sqrt{\pi}$.

Equation (5) suggests that the characteristic droplet size S^* is proportional to S_c , which can be seen from its definition:

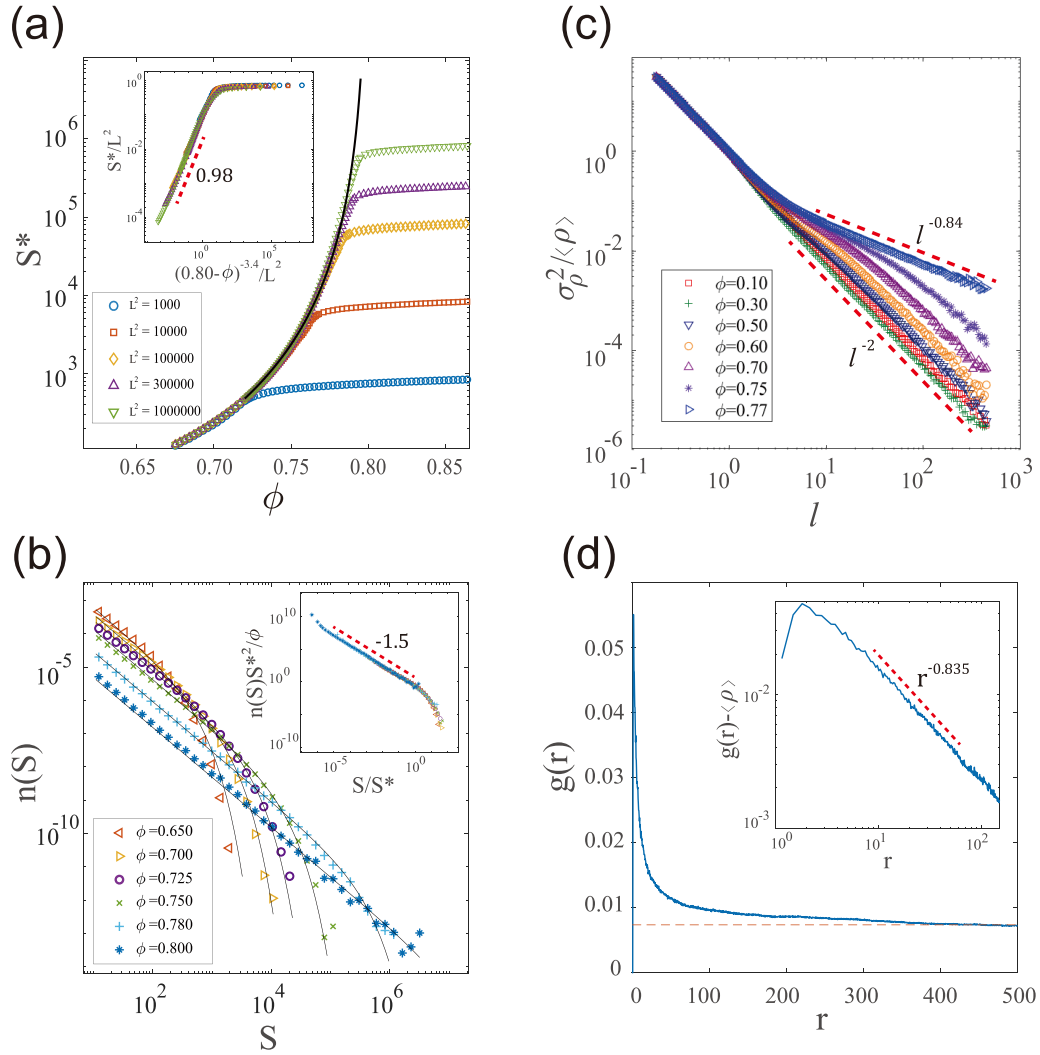


FIG. 2. (a) The characteristic droplet size S^* versus area fraction ϕ for systems of different sizes ($L^2 = 10^3$ to 10^6). The main panel shows S^* , which satisfies the scaling $(\phi_c - \phi)^{-\gamma}$ (dashed line) with $\phi_c = 0.80$ and $\gamma = 3.4$. The inset demonstrates data collapse when plotting S^*/L^2 versus $(\phi_c - \phi)^{-\gamma}/L^2$, confirming the universal scaling function $f(x)$ with two distinct regimes. (b) Droplet size distributions at various area fractions. The solid lines represent the theoretical prediction (4). The inset shows the normalized distribution, and the dashed line has a slope of 1.5 in the log-log plot. (c) The variance in the spatial density of droplets inside a subsystem as a function of its linear size l , normalized by the average spatial density $\langle \rho \rangle$. As ϕ approaches the critical value, $\sigma_\rho^2(l) \sim l^{-\eta}$, with $\eta = 0.84$. (d) Pair correlation function $g(r)$ near the critical point. The inset shows $g(r) - \langle \rho \rangle$ versus r in a log-log plot, from which we find that $g(r) - \langle \rho \rangle \sim r^{-\eta}$, with η being consistent with the value in (c).

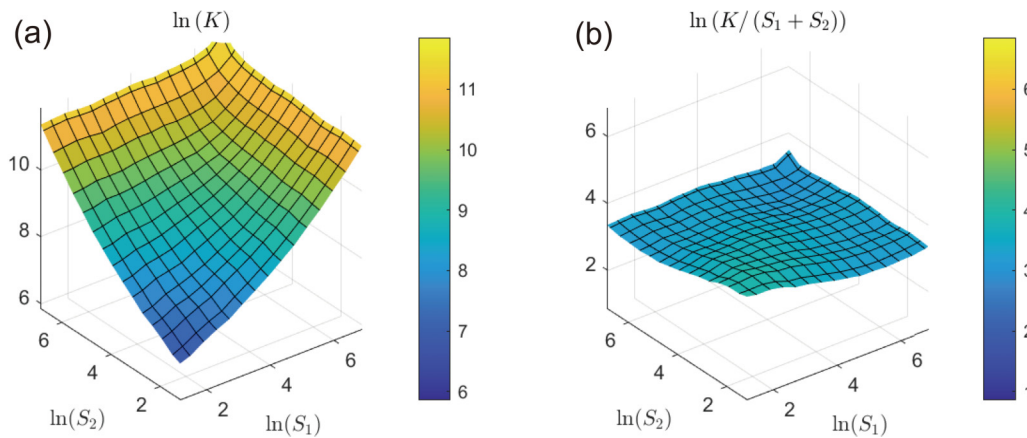


FIG. 3. (a) The fusion kernel $K(S_1, S_2)$ in the range $0.7 < \phi < 0.75$. (b) The fusion kernel normalized by $S_1 + S_2$ is approximately independent of S_1 and S_2 . The pronounced flatness of the kernel after normalization confirms the sum-form scaling $K(S_1, S_2) \sim S_1 + S_2$.

$\frac{\phi}{2} = \int_{S^*}^{\infty} n(S, \phi) S dS = \int_{S^*}^{\infty} \frac{\phi}{\sqrt{\pi}} \left(\frac{e^{-S/S_c}}{\sqrt{S S_c}} \right) dS$. By substituting $x = S/S_c$, we obtain $\int_{S^*/S_c}^{\infty} x^{-1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$. Numerical calculation gives $S_c = 4.40 S^*$. To verify the validity of Eq. (5), we compute S_c from the measured S^* . Intriguingly, the theoretical curves (solid line) calculated using Eq. (5) demonstrate excellent agreement with our simulation results [Fig. 2(b)]. We further confirm that the scaling form should also work if we replace S_c by S^* [inset of Fig. 2(b)]. The power-law exponent below the characteristic cutoff is 1.5, as predicted. The power-distributed droplet size is also valid when system-spanning droplets exist. In this case, the size distribution for non-system-spanning droplets follows the power law with an exponent of 1.5.

To verify whether there is a diverging correlation length, we cut the system into a lattice of square blocks with a linear size l and compute the spatial density ρ of droplets inside each block (the number of droplets in the block divided by

l^2). We calculate the spatial density variance across all blocks. For ϕ far below ϕ_c , the variance $\sigma_\rho^2 \sim l^{-2}$, according to the central limit theorem [15]. Interestingly, near the critical state, the variance decays as a power-law function of l , but with a different exponent,

$$\sigma_\rho^2(l) \sim l^{-\eta}, \quad (6)$$

where $\eta = 0.84$ [Fig. 2(c)]. We also compute the isotropic pair correlation function $g(r)$, the particle density with a distance r from the particle at the center, using the droplet center near ϕ_c . One expects that $g(r)$ should decay as a power-law function of the distance with the same exponent η ,

$$g(r) - \langle \rho \rangle \sim r^{-\eta}. \quad (7)$$

Here, $\langle \rho \rangle$ is the average droplet density (the total droplet number divided by the system area). This can be derived as follows [15]:

$$\begin{aligned} \sigma_\rho^2(l) &= \frac{\langle \delta N(l)^2 \rangle}{l^{2d}} \\ &= l^{-2d} \iint w_l(\mathbf{r}_1, \mathbf{r}_2) \langle \delta \rho(\mathbf{r}_1) \delta \rho(\mathbf{r}_2) \rangle d\mathbf{r}_1 d\mathbf{r}_2 \\ &= l^{-2d} \iint w_l(\mathbf{r}_1, \mathbf{r}_2) \langle \rho(\mathbf{r}_1) \delta \rho(\mathbf{r}_2) \rangle d\mathbf{r}_1 d\mathbf{r}_2 \\ &= l^{-2d} \iint w_l(\mathbf{r}_1, \mathbf{r}_2) \langle \rho \rangle [g(|\mathbf{r}_2 - \mathbf{r}_1|) - \langle \rho \rangle] d\mathbf{r}_1 d\mathbf{r}_2 \\ &= l^{-2d} \langle \rho \rangle \int \left(\int w_l(\mathbf{r}_1, \mathbf{r}_1 + \Delta \mathbf{r}) d\mathbf{r}_1 \right) [g(|\Delta \mathbf{r}|) - \langle \rho \rangle] d\Delta \mathbf{r} \\ &= l^{-2d} \langle \rho \rangle \int [l^d v(\Delta \mathbf{r}/l)] \{l^{-\eta} [g(|\Delta \mathbf{r}|) - \langle \rho \rangle]\} l^d d(\Delta \mathbf{r}/l) \\ &= l^{-\eta} \langle \rho \rangle \int v(\mathbf{x}) [g(|\mathbf{x}|) - \langle \rho \rangle] d\mathbf{x} \\ &\sim l^{-\eta}. \end{aligned} \quad (8)$$

Here, d is the system's spatial dimension, $\mathbf{r}$ denotes the displacement vector from the center of the square block, and $\delta \rho(\mathbf{r}) = \rho(\mathbf{r}) - \langle \rho \rangle$ represents density fluctuations from the average value. The window function $w_l(\mathbf{r}_1, \mathbf{r}_2)$ indicates whether both points lie within a block of size l , formally defined as

$$w_l(\mathbf{r}_1, \mathbf{r}_2) = \begin{cases} 1 & \text{if } \|\mathbf{r}_1\|_\infty \leq l/2, \|\mathbf{r}_2\|_\infty \leq l/2, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

where $\|\cdot\|_\infty$ denotes the maximum norm (i.e., the largest absolute value among all coordinate components). The integral $\int w_l(\mathbf{r}_1, \mathbf{r}_1 + \Delta \mathbf{r}) d\mathbf{r}_1$ corresponds to the overlap volume between two blocks of size l displaced by $\Delta \mathbf{r}$. This equals the total block volume l^d multiplied by a scaling factor $v(\Delta \mathbf{r}/l) \in [0, 1]$, where v represents the fractional overlap. We confirm Eq. (7) numerically [Fig. 2(d)].

We also investigate the fusion dynamics of droplets and define the average fusion time n_f as the mean number of merge events triggered when a new droplet of unit area is

introduced. When the area fraction changes by $\Delta \phi$, $L^2 \Delta \phi$ unit-area droplets are added to the system, inducing $L^2 \Delta \phi n_f$ coalescence events. The resulting net change in droplet number $L^2 \Delta \langle \rho \rangle = L^2 \Delta \phi - L^2 \Delta \phi n_f$ leads to

$$n_f = 1 - \frac{d\langle \rho \rangle}{d\phi}. \quad (10)$$

The average droplet density $\langle \rho \rangle$ has a nonmonotonic dependence on ϕ . Notably, in the thermodynamic limit $L \rightarrow \infty$, $\langle \rho \rangle$ exhibits a linear decrease near the critical point ϕ_c (Fig. 4). Interestingly, we find that $n_f \approx 1.32$ as $\phi \rightarrow \phi_c$, different from traditional self-organized criticality models, in which the mean avalanche duration diverges at criticality [4].

Discussion. In this work, we introduce a simple model of droplet fusion in which we add small droplets quasistatically. As the area fraction ϕ approaches the critical point, the system becomes scale-free and exhibits power-law distributed droplet sizes. Using an argument based on dimensional analysis, we propose the fusion kernel as a sum form, $K(S_1, S_2) \sim S_1 + S_2$.

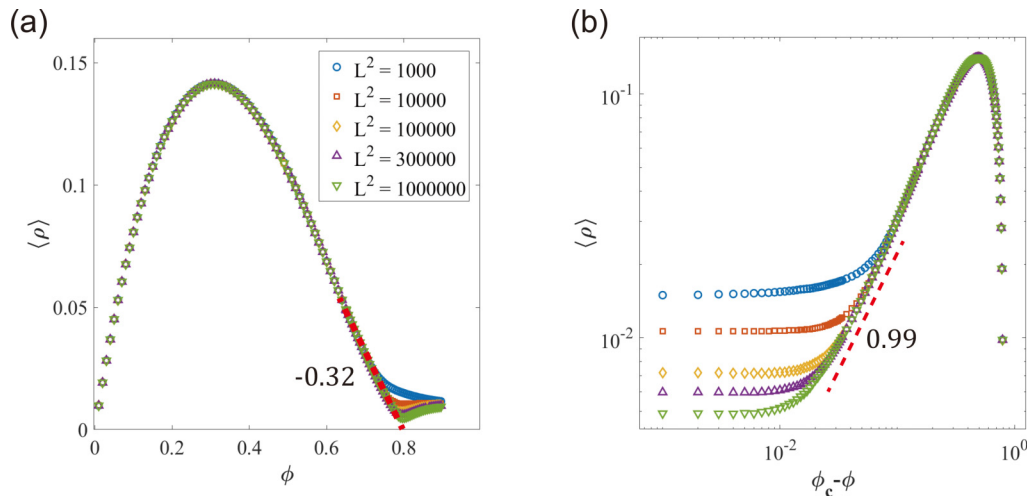


FIG. 4. (a) Nonmonotonic dependence of the average droplet density $\langle \rho \rangle$ on the area fraction ϕ for different system sizes ($L^2 = 10^3$ to 10^6), showing a linear decrease (slope = -0.32) near criticality. (b) The log-log plot shows the linear scaling between $\langle \rho \rangle$ and $\phi_c - \phi$.

The same argument should hold in three dimensions, where S represents the volume of a droplet. The sum-form kernel has been shown to lead to a power-law-distributed cluster size with an exponent of 1.5 [10,13]. Our findings rationalize the exponent of 1.5 observed in the power-law distribution of nucleolus volume [20], emerging from self-organized criticality driven by droplet influx and fusion processes. Similar power-law distributions have also been observed in collective human activities [29]. Nucleoli function primarily as factories for ribosome subunit biogenesis, and they are RNA/protein bodies within the nucleus [30,31]. The nucleus of amphibian oocytes is much larger than somatic nuclei; therefore, it is a model system to study the shape and dynamics of nucleoli. We hypothesize that the addition of new nucleoli in amphibian oocytes may be generated by significant rDNA transcription to provide the egg with large numbers of ribosomes.

Interestingly, the number fraction of N -mers in hyperbranched polymers follows a power-law function similar to the number density of droplets in our case [32]. We argue that in the case of hyperbranched polymers, the probability of two polymers (N_1 -mer and N_2 -mer) reacting per unit

time is also proportional to the sum of their total number: $N_1 + N_2$. This process is described by a sum kernel, leading to the same asymptotic distribution. Power-law-distributed bubble sizes have also been observed in motility-induced phase separation [33], in which new bubbles are nucleated at the expense of the larger ones, in line with the reverse Ostwald scenario [34].

The self-organized criticality we uncover for this simple system resembles driven disordered systems such as sheared amorphous solids, in which the systems approach a critical state as the driving force approaches the critical value [8,9,35–38]. In our model, the driving force is the quasistatic addition of new droplets, which drives the system into a novel critical state, which, to our knowledge, has yet to be found.

Acknowledgments. This research was funded by the National Natural Science Foundation of China (Grant No. 12474190), the National Key Research and Development Program of China (Grant No. 2024YFA0919600), and Peking-Tsinghua Center for Life Sciences grants.

Data availability. The data supporting this study's findings are available within the article.

- [1] H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, New York, 1971).
- [2] P. C. Hohenberg and B. I. Halperin, Theory of dynamic critical phenomena, *Rev. Mod. Phys.* **49**, 435 (1977).
- [3] S.-K. Ma, *Modern Theory of Critical Phenomena* (Routledge, New York, 2018).
- [4] P. Bak, C. Tang, and K. Wiesenfeld, Self-organized criticality: An explanation of the $1/f$ noise, *Phys. Rev. Lett.* **59**, 381 (1987).
- [5] P. Bak, C. Tang, and K. Wiesenfeld, Self-organized criticality, *Phys. Rev. A* **38**, 364 (1988).
- [6] D. L. Turcotte, Self-organized criticality, *Rep. Prog. Phys.* **62**, 1377 (1999).
- [7] P. Bak, *How Nature Works: The Science of Self-Organized Criticality* (Springer, New York, 2013).
- [8] J. Lin, E. Lerner, A. Rosso, and M. Wyart, Scaling description of the yielding transition in soft amorphous solids at zero temperature, *Proc. Natl. Acad. Sci. USA* **111**, 14382 (2014).
- [9] J. Lin, T. Gueudré, A. Rosso, and M. Wyart, Criticality in the approach to failure in amorphous solids, *Phys. Rev. Lett.* **115**, 168001 (2015).
- [10] G. K. von Schulthess, G. B. Benedek, and R. W. D. Blois, Measurement of the cluster size distributions for high functionality antigens cross-linked by antibody, *Macromolecules* **13**, 939 (1980).
- [11] D. A. Weitz, J. S. Huang, M. Y. Lin, and J. Sung, Dynamics of diffusion-limited kinetic aggregation, *Phys. Rev. Lett.* **53**, 1657 (1984).

- [12] D. A. Weitz, J. S. Huang, M. Y. Lin, and J. Sung, Limits of the fractal dimension for irreversible kinetic aggregation of gold colloids, *Phys. Rev. Lett.* **54**, 1416 (1985).
- [13] D. A. Weitz and M. Y. Lin, Dynamic scaling of cluster-mass distributions in kinetic colloid aggregation, *Phys. Rev. Lett.* **57**, 2037 (1986).
- [14] R. C. Ball, D. A. Weitz, T. A. Witten, and F. Leyvraz, Universal kinetics in reaction-limited aggregation, *Phys. Rev. Lett.* **58**, 274 (1987).
- [15] S. Torquato, Hyperuniform states of matter, *Phys. Rep.* **745**, 1 (2018).
- [16] L. Corte, P. M. Chaikin, J. P. Gollub, and D. J. Pine, Random organization in periodically driven systems, *Nat. Phys.* **4**, 420 (2008).
- [17] S. Wilken, R. E. Guerra, D. J. Pine, and P. M. Chaikin, Hyperuniform structures formed by shearing colloidal suspensions, *Phys. Rev. Lett.* **125**, 148001 (2020).
- [18] S. Wilken, R. E. Guerra, D. Levine, and P. M. Chaikin, Random close packing as a dynamical phase transition, *Phys. Rev. Lett.* **127**, 038002 (2021).
- [19] S. Wilken, A. Chaderjian, and O. A. Saleh, Spatial organization of phase-separated DNA droplets, *Phys. Rev. X* **13**, 031014 (2023).
- [20] C. P. Brangwynne, T. J. Mitchison, and A. A. Hyman, Active liquid-like behavior of nucleoli determines their size and shape in *Xenopus laevis* oocytes, *Proc. Natl. Acad. Sci. USA* **108**, 4334 (2011).
- [21] C. P. Brangwynne, C. R. Eckmann, D. S. Courson, A. Rybarska, C. Hoege, J. Gharakhani, F. Jülicher, and A. A. Hyman, Germline p granules are liquid droplets that localize by controlled dissolution/condensation, *Science* **324**, 1729 (2009).
- [22] A. A. Hyman, C. A. Weber, and F. Jülicher, Liquid-liquid phase separation in biology, *Annu. Rev. Cell Dev. Biol.* **30**, 39 (2014).
- [23] S. F. Banani, H. O. Lee, A. A. Hyman, and M. K. Rosen, Biomolecular condensates: Organizers of cellular biochemistry, *Nat. Rev. Mol. Cell Biol.* **18**, 285 (2017).
- [24] F. Jülicher and C. A. Weber, Droplet physics and intracellular phase separation, *Annu. Rev. Condens. Matter Phys.* **15**, 237 (2024).
- [25] P. Meakin, Diffusion-limited droplet coalescence, *Phys. A (Amsterdam, Neth.)* **165**, 1 (1990).
- [26] J. Berry, C. P. Brangwynne, and M. Haataja, Physical principles of intracellular organization via active and passive phase transitions, *Rep. Prog. Phys.* **81**, 046601 (2018).
- [27] P. G. J. van Dongen and M. H. Ernst, Dynamic scaling in the kinetics of clustering, *Phys. Rev. Lett.* **54**, 1396 (1985).
- [28] P. G. J. Dongen and M. H. Ernst, Cluster size distribution in irreversible aggregation at large times, *J. Phys. A* **18**, 2779 (1985).
- [29] J. C. Bohorquez, S. Gourley, A. R. Dixon, M. Spagat, and N. F. Johnson, Common ecology quantifies human insurgency, *Nature (London)* **462**, 911 (2009).
- [30] F.-M. Boisvert, S. Van Koningsbruggen, J. Navascués, and A. I. Lamond, The multifunctional nucleolus, *Nat. Rev. Mol. Cell Biol.* **8**, 574 (2007).
- [31] M. Feric, N. Vaidya, T. S. Harmon, D. M. Mitrea, L. Zhu, T. M. Richardson, R. W. Kriwacki, R. V. Pappu, and C. P. Brangwynne, Coexisting liquid phases underlie nucleolar subcompartments, *Cell* **165**, 1686 (2016).
- [32] M. Rubinstein and R. H. Colby, *Polymer Physics* (Oxford University Press, Oxford, 2003).
- [33] X.-q. Shi, G. Fausti, H. Chaté, C. Nardini, and A. Solon, Self-organized critical coexistence phase in repulsive active particles, *Phys. Rev. Lett.* **125**, 168001 (2020).
- [34] E. Tjhung, C. Nardini, and M. E. Cates, Cluster phases and bubble phase separation in active fluids: Reversal of the Ostwald process, *Phys. Rev. X* **8**, 031080 (2018).
- [35] P. Chauve, T. Giamarchi, and P. Le Doussal, Creep and depinning in disordered media, *Phys. Rev. B* **62**, 6241 (2000).
- [36] A. Rosso, A. K. Hartmann, and W. Krauth, Depinning of elastic manifolds, *Phys. Rev. E* **67**, 021602 (2003).
- [37] A. Rosso, P. Le Doussal, and K. J. Wiese, Avalanche-size distribution at the depinning transition: A numerical test of the theory, *Phys. Rev. B* **80**, 144204 (2009).
- [38] M. Ozawa, L. Berthier, G. Biroli, A. Rosso, and G. Tarjus, Random critical point separates brittle and ductile yielding transitions in amorphous materials, *Proc. Natl. Acad. Sci. USA* **115**, 6656 (2018).