

Joule expansion of a quantum gas

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We revisit the classic Joule-expansion experiments, now with a quantum-degenerate atomic Bose gas. In contrast to the classical-gas experiments, where no temperature change was measured, here we observe and quantitatively explain both cooling and heating effects, which arise, respectively, due to quantum statistics and interparticle interactions.

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The Joule expansion of a gas, first performed by Gay-Lussac in 1807 [1] and then independently by Joule in 1845 [2], is a classic experiment in thermodynamics that was crucial for proving the equivalence of work and heat and establishing the first law of thermodynamics [3]. In these experiments, a gas is initially confined to a part of a thermally insulated vessel and then allowed to freely expand and fill the entire volume without doing work on its surroundings. While the original experiments were celebrated for measuring no change in the gas temperature, it was later understood that this expansion is isothermal only for ideal (noninteracting) gases, and should lead to cooling or heating for gases with, respectively, attractive or repulsive interactions [4,5]. Subsequent experiments sought to measure this interaction effect, but were unable to do so; in classical gases, the expected fractional temperature change, set by the ratio of interaction to kinetic energy, is small [6], and easily masked by any leakage of heat into or from the walls of the vessel [5,7].

In this Letter, we revisit Joule-expansion experiments with a quantum-degenerate atomic Bose gas, in which the interaction energy can exceed the kinetic one, and for which a purely quantum-statistical isoenergetic cooling is expected in the absence of interactions. Our setup consists of two concentric optical box traps [8,9], as outlined in Fig. 1(a), and the strength of contact interactions in our ³⁹K gas is characterized by the *s*-wave scattering length *a*. Previously, the related quantum Joule-Thomson (JT) effect [10] was observed in a single optical box [11,12], by mapping the JT isoenthalpic rarefaction onto the removal of particles from the trap, but this

mapping is valid only in the special cases $a \rightarrow 0$ and $a \rightarrow \infty$ (see also [13]). Our setup allows studies of Joule expansion for any interaction strength, and by tuning *a* via a magnetic Feshbach resonance, we observe both isoenergetic cooling for

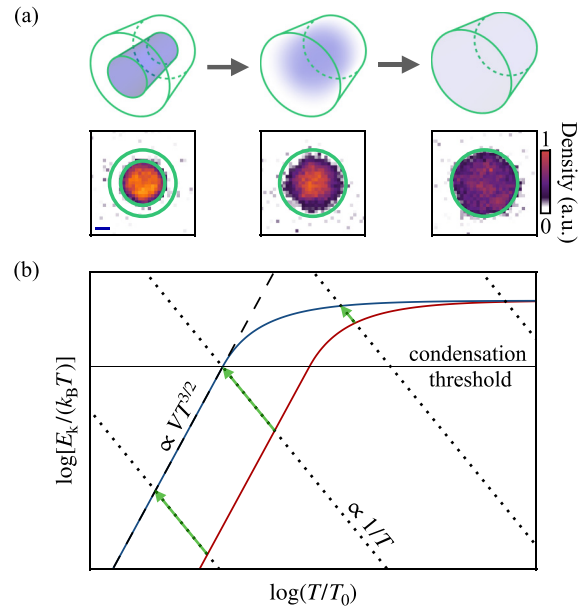


FIG. 1. Joule expansion of a Bose gas. (a) Our setup consists of two cylindrical optical box traps, shown in green. We prepare a quantum-degenerate gas in the inner box, and, by switching off this trap, allow the gas to freely expand and fill the outer one. The bottom row shows absorption images of the clouds before, during, and after the expansion. The scale bar (blue) corresponds to 10 μm. (b) Thermodynamic diagram for the ideal Bose gas. Here, E_k is the kinetic energy per particle, T is the temperature, and T_0 is an arbitrary constant introduced just to define the dimensionless temperature T/T_0 . The red and blue solid curves are isochores for the same atom number N but different volumes V , so different condensation temperatures. Below the condensation temperature $E_k \propto VT^{5/2}$ (dashed line), while in the classical-gas limit $E_k = (3/2)k_B T$. The dotted lines are isoenergetic contours, and the green arrows show the Joule expansion for various initial temperatures in the smaller volume (red).

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vanishing interactions ($a \rightarrow 0$) and isoenergetic heating due to repulsive interactions ($a > 0$).

Our experiments begin with a quasihomogeneous gas of ^{39}K atoms held in the inner box trap. We prepare the gas in the lowest hyperfine state and control the scattering length a by tuning the magnetic field B in the vicinity of the resonance at 402.7 G [20]. We then rapidly switch off the inner trap, allowing the gas to fill the outer one, and measure the temperature after the gas has thermalized [21].

We start with the ideal-gas case ($a \rightarrow 0$) and in Fig. 1(b), graphically explain the origin of the purely quantum-statistical cooling of a homogeneous Bose gas. For the same atom number N and two different volumes V , we plot the dimensionless $E_k/(k_B T)$ versus the dimensionless T/T_0 (solid lines), where E_k is the kinetic energy per particle (here, equal to the total energy per particle), T is the temperature, and T_0 an arbitrary reference. The dotted lines are isoenergetic contours, and the green arrows show the cooling effect of Joule expansion from the smaller (red) to the larger (blue) volume, which vanishes in the high- T limit; the ratios of final to initial temperatures are given by the horizontal components of the green vectors.

Quantitatively,

$$\frac{E_k}{k_B T} = \begin{cases} \frac{3}{2} \frac{\zeta(5/2)}{\zeta(3/2)} \left(\frac{T}{T_c}\right)^{3/2} & \text{for } T \leq T_c \\ \frac{3}{2} \frac{g_{5/2}(z)}{g_{3/2}(z)} & \text{for } T > T_c \end{cases}, \quad (1)$$

where $T_c \propto (N/V)^{2/3}$ is the critical temperature for Bose-Einstein condensation, ζ is the Riemann function, with $\zeta(5/2)/\zeta(3/2) \approx 0.51$, g_α is a polylogarithm of order α , and $z = \exp[\mu/(k_B T)]$ is the fugacity, where μ is the chemical potential. For $T > T_c$, the fugacity satisfies

$$g_{3/2}(z) = \frac{N\lambda^3}{V}, \quad (2)$$

where $\lambda = h/\sqrt{2\pi m k_B T}$ is the thermal wavelength and m the atom mass. Below T_c , Eq. (1) gives $E_k/(k_B T) \propto VT^{3/2}$ [dashed line in Fig. 1(b)], while above T_c the dependence on V and T is implicit in z . For the classical-gas $z \ll 1$; in this limit $g_\alpha(z) \rightarrow z$ and $E_k = (3/2)k_B T$ is independent of V , so the cooling effect vanishes.

From hereon, we denote the initial volume and temperature (in the inner box) V and T , and the final volume and temperature (in the outer box) V_f and T_f . Denoting the critical temperature in the inner box T_c , the critical temperature in the outer one is $T_{c,f} = T_c(V/V_f)^{2/3}$.

If the gas is condensed both before and after the expansion, Eq. (1) gives

$$\frac{T_f}{T} = \left(\frac{V}{V_f}\right)^{2/5}. \quad (3)$$

This relation holds if $T_f \leq T_{c,f}$, which, using Eq. (3), gives the condition for the initial temperature

$$\frac{T}{T_c} \leq \left(\frac{V}{V_f}\right)^{4/15}. \quad (4)$$

For higher T/T_c , the fractional cooling, $1 - T_f/T$, gradually reduces and vanishes in the classical-gas limit.

Experimentally, we prepare an equilibrium interacting gas of density $n \approx 1.7 \mu\text{m}^{-3}$ in the inner box, with $V \approx$

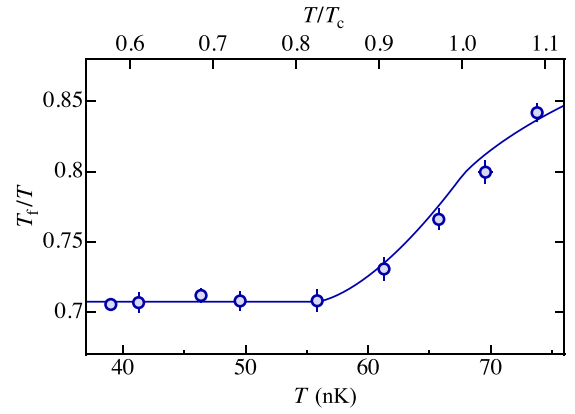


FIG. 2. Quantum-statistical cooling of a noninteracting Bose gas due to Joule expansion. Here, T is the initial temperature, T_c the condensation temperature for the initial gas density, and T_f the final temperature. The fractional temperature change, $1 - T_f/T$, is generally greater for lower T/T_c , and constant if the gas is partially condensed both before and after the expansion. Our measurements agree with numerical calculations (solid line) that take into account the fact that the gas in our optical box trap is not perfectly homogeneous [21]; in a perfect box, the low- T plateau of T_f/T would, as per Eqs. (3) and (4), have a value of 0.68 and extend to $T/T_c \approx 0.78$.

$3.6 \times 10^4 \mu\text{m}^3$, and tune $a \rightarrow 0$ before the expansion. After the gas fills the outer box, with $V_f \approx 2.6V$, we increase the scattering length to $a = 300 a_0$ (where a_0 is the Bohr radius) to facilitate thermalization (see also [21]).

In Fig. 2, we plot the measured T_f/T versus both T and T/T_c . Our observations agree with numerical calculations (solid line) that take into account the fact that the gas in our optical box is not perfectly homogeneous [21]. For comparison, the low- T/T_c plateau is at $T_f/T \approx 0.71$, whereas according to Eq. (3), it would be at $T_f/T \approx 0.68$, and the plateau extends to $T/T_c \approx 0.83$, whereas according to Eq. (4), it would extend to $T/T_c \approx 0.78$.

We now turn to experiments with an interacting quantum gas, where the total energy per particle, conserved during the Joule expansion, is

$$E = E_k + E_{\text{int}}, \quad (5)$$

where E_{int} is the interaction energy per particle. In the Hartree-Fock approximation [22],

$$E_{\text{int}} = (2 - \eta^2) \frac{2\pi \hbar^2 n a}{m}, \quad (6)$$

where η is the condensed fraction; the factor $2 - \eta^2$ quantifies two-body correlations in the quantum gas. Relative to the ideal-gas case, the presence of repulsive interactions ($a > 0$) during the isoenergetic expansion raises T_f , because, as the density drops, E_{int} is partially converted into E_k .

In experiments, we now increase the initial density to $n \approx 7 \mu\text{m}^{-3}$, and focus on the low- T/T_c regime, where the purely quantum-statistical cooling effect is constant. In Fig. 3(a), we show measurements of T_f/T for two different values of a , which agree with our numerical calculations [21]. In Fig. 3(b), we plot the same data versus the initial E_k/E_{int} , which shows that, for our parameters, the interaction effect overcomes the

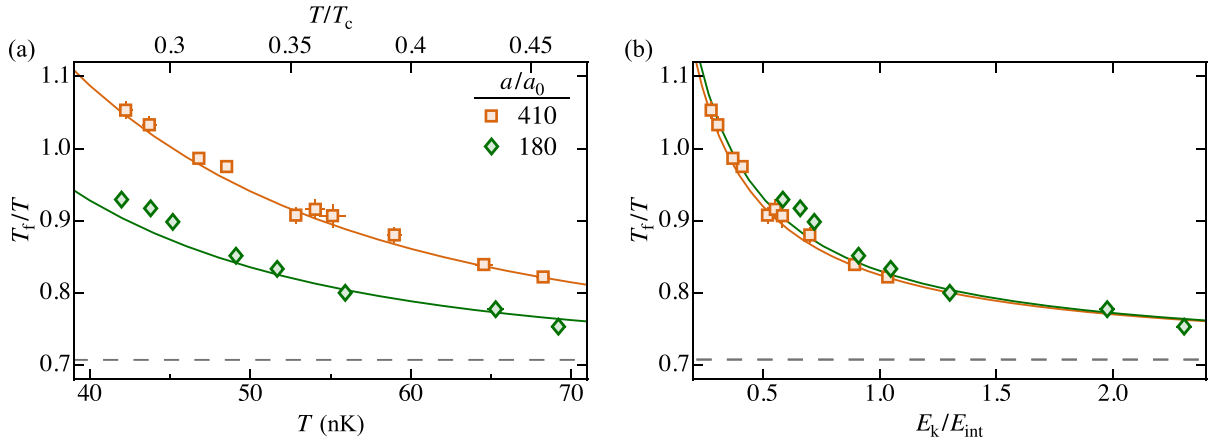


FIG. 3. Joule expansion of an interacting Bose gas. (a) The quantum-statistical cooling effect, indicated by the dashed line, is countered by a heating effect due to repulsive interactions. Our measurements for two values of the scattering length a are reproduced in numerical calculations [21], shown by the solid lines. (b) Plotting the same data versus the ratio of the initial kinetic and interaction energies shows that the interaction effect overcomes the quantum-statistical one (so $T_f > T$) for $E_{\text{int}}/E_k \gtrsim 3$.

quantum-statistical one (so $T_f > T$) for $E_{\text{int}}/E_k \gtrsim 3$. Note that the two (theoretical) curves do not perfectly coincide because for the same initial η the change in two-body correlations due to the expansion depends on a .

In conclusion, we have revisited the classic Joule-expansion experiments with a quantum Bose gas and quantitatively revealed the richer physics that arises through an interplay of quantum statistics, interparticle interactions, and quantum correlations. With current technology [9,23], our methods could also be extended to slow (adiabatic)

expansions, which would allow realizations of closed thermodynamic cycles and quantum engines in box-like geometries [24–26].

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Data availability. The data that support the findings of this article are openly available [27].

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