

Quantum metrology with higher-order exceptional points in atom-cavity magnonics

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Exceptional points (EPs), which arose early from non-Hermitian physics, significantly amplify the system's response to minor perturbations, and they act as a useful concept to enhance measurement in metrology. In particular, such a metrological enhancement grows dramatically with the EP's order. However, the Langevin noises intrinsically existing in the non-Hermitian systems diminish this enhancement. In this study, we propose a protocol for quantum metrology with the construction of higher-order EPs (HOEPs) in an atom-cavity system through Hermitian magnon-photon interaction. The construction of HOEPs utilizes the atom-cavity non-Hermitian-like dynamical behavior but avoids the external Langevin noises via the Hermitian interaction. A general analysis is exhibited for the construction of arbitrary n th-order EP (EP $_n$). As a demonstration of the superiority of these HOEPs in quantum metrology, we work out an EP $_3/4$ -based atomic sensor with sensitivity being orders of magnitude higher than that achievable in an EP $_2$ -based atomic sensor. We further unveil the mechanism behind the sensitivity enhancement from HOEPs. The experimental establishment for this proposal is suggested with potential candidates. This EP-based atomic sensor, taking advantage of the atom-light interface, offers new insight into quantum metrology with HOEPs.

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Introduction. In the past few decades, new approaches for precision measurement have been proposed based on exceptional points (EPs), where the n th-order EP (EP $_n$) indicates the coalescence of the system's n ($n \geq 2$) eigenvalues and their corresponding eigenstates [1–5]. The system exhibits a nonlinear response to external perturbations ϵ near EP $_n$, with the response scaling as $\epsilon^{1/n}$ for $\epsilon \ll 1$ [6–15]. This power-law behavior, increasing dramatically with the EPs' order n , highlights the system's advantage as sensors based on higher-order EPs (HOEPs) in metrology [16–19].

Despite the promise of EPs, the presence of intrinsic Langevin noise in non-Hermitian systems poses an obstacle to the sensitivity enhancements [20–27]. To solve this issue, several strategies have been proposed, including the use of postselection techniques to discard quantum noise [28], and Hamiltonian dilation by integrating the non-Hermitian system and the reservoir as a large Hermitian system [29,30]. Recent demonstrations have shown that EPs can also be constructed within Hermitian systems that include nonlinear coupling [31–33]. In such systems, the EPs delineate the dynamical

phase transition between parametric oscillation and parametric amplification. Consider a quadratic bosonic system with the Hamiltonian

$$\hat{\mathcal{H}} = \sum_i \delta_i \hat{a}_i^\dagger \hat{a}_i + \sum_{i,j} (\kappa_{ij} \hat{a}_i^\dagger \hat{a}_j + g_{ij} \hat{a}_i^\dagger \hat{a}_j^\dagger + \text{H.c.}). \quad (1)$$

The first two terms in (1) are particle-conserving, manifesting as periodic oscillations in the system's dynamics, while the third term is particle-nonconserving, leading to exponential growth in the dynamical behavior. EPs appear at the parametric amplification threshold of the system where the exponential growth terms reach a certain equilibrium with the periodic oscillation terms. These EPs emerge solely from the system's non-Hermitian-like behaviors, rather than from losses and gains, thus they are not subject to Langevin noise limitations.

In the frame of Hamiltonian (1), we propose a cavity-mediated atomic sensor that exploits HOEPs with nonlinear responses to external perturbations while immunizing Langevin noise. The cavity mode interacts with the collective spin-wave excitations (*magnons*) through either SU(1,1)- or SU(2)-type Raman interaction [34–39], and it establishes couplings among multiple atomic ensembles to form HOEPs. The HOEPs are anticipated to enhance the sensing performance of atoms in probing various fields in demand, such as electric fields [40], magnetic fields [41], optical fields [42], and even exotic fields [43]. The ideology considered to construct HOEPs in the work includes two important aspects: (i) Although the Hamiltonian with Raman interaction

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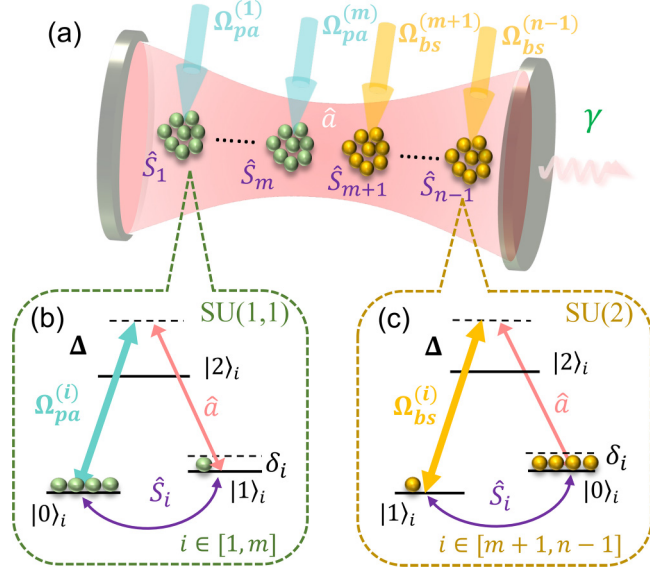


FIG. 1. (a) Atom-cavity system to realize EPn. $n-1$ atomic ensembles are placed inside a cavity, where m ($1 \leq m \leq n-1$) ensembles interact with the cavity mode through SU(1,1)-type Raman interactions (b), and the remaining $n-m-1$ ensembles interact with the cavity mode through SU(2)-type Raman interactions (c). Parts (b) and (c) depict the atomic energy levels and mechanisms of the SU(1,1)- and SU(2)-type atom-light interactions, respectively.

in our system is Hermitian, the system's dynamical matrix exhibits non-Hermitian behaviors satisfying pseudo-Hermiticity [44–46] and particle-hole symmetry [15,47,48]. The spontaneous breaking of these symmetries induces the emergence of HOEPs through tuning the interaction strengths and detunings. (ii) The essential condition for the system to achieve the highest-order EP requires the irreducibility of a dynamical matrix characterizing the entire atom-cavity system. This irreducibility can be determined in terms of the Casimir invariants of the SU(1,1) and SU(2) transformations [49,50]. As an illustration, an EP3-based atomic sensor is devised, showing sensitivity that is orders of magnitude superior to an EP2-based atomic sensor. This sensitivity enhancement arises completely from the HOEP-induced signal amplification at the given working points.

The model. The proposed atom-cavity system is illustrated in Fig. 1. Each atomic ensemble consists of N_a atoms with a pair of lower metastable states $|0\rangle_i$ and $|1\rangle_i$, as well as an excited state $|2\rangle_i$. Here $i = 1, \dots, n-1$ labels a particular atomic ensemble. Coherence between the states $|0\rangle_i$ and $|1\rangle_i$ can be established and represented by the atomic spin wave $\hat{S}_i = \sum_k N_a |0\rangle_{kk} \langle 1|_i$. Using the Holstein-Primakoff transformation [51] and assuming that the mean number of atoms transferred to the state $|1\rangle_i$ is much smaller than N_a , we can view the collective spin-wave excitations in the atomic ensembles as quasiparticles known as *magnons* and express them in terms of bosonic annihilation operator $\hat{b}_i$, i.e., $\hat{S}_i = (N_a - \hat{b}_i^\dagger \hat{b}_i)^{1/2} \hat{b}_i \approx \sqrt{N_a} \hat{b}_i$. As an experimental choice with alkali atoms, e.g., ^{87}Rb , the states $|0\rangle$ and $|1\rangle$ correspond to $|5S_{1/2}, F=1, m_F=\pm 1\rangle$, and $|2\rangle$ corresponds to $|5P_{1/2}, F'=1, m_{F'}=0\rangle$. An external field can be applied to lift the degeneracy of the $m_F = \pm 1$ states and enable distinct transition

channels between these states. The transitions $m_F = \mp 1$ to $m_{F'} = 0$ can be coupled using circularly polarized σ_+ pump and σ_- cavity fields, respectively.

Two types of Raman interactions—SU(1,1)- and SU(2)—[34–39] are exploited between the cavity field and the magnon fields. For convenience, we assume that the first m atomic ensembles interact with the cavity mode via SU(1,1)-type Raman interactions. They are initially prepared through optical pumping in state $|0\rangle_i$ with $i \in [1, m]$ [see Fig. 1(b)]. Pump light $\Omega_{pa}^{(i)}$ couples the state $|0\rangle_i$ and the excited state $|2\rangle_i$ with a large single-photon detuning Δ to simultaneously generate correlated magnon field $\hat{b}_i$ and cavity field $\hat{a}$. By adiabatically eliminating the excited state $|2\rangle_i$, we have the Hamiltonian $\hat{H}_{\text{SU}(1,1)} \propto \hat{b}_i^\dagger \hat{a}^\dagger + \text{H.c.}$ that obeys the Lie algebra of SU(1,1). The remaining $n-m-1$ atomic ensembles experience SU(2)-type Raman interactions. They are initially prepared through optical pumping in another metastable state, here for convenience relabeled as $|0\rangle_i$ with $i \in [m+1, n-1]$ in our theoretical treatment [see Fig. 1(c)]. The magnon field $\hat{b}_i$ interchanges with the cavity field $\hat{a}$ via pump light $\Omega_{bs}^{(i)}$ detuned by Δ . By performing adiabatically elimination of $|2\rangle_i$, we end at Hamiltonian $\hat{H}_{\text{SU}(2)} \propto \hat{b}_i^\dagger \hat{a} + \text{H.c.}$ that obeys the Lie algebra of SU(2).

The Hamiltonian of the whole system is then given by

$$\hat{H} = \hat{H}_0 + \sum_{i=1}^m g_i (\hat{b}_i^\dagger \hat{a}^\dagger + \hat{a} \hat{b}_i) + \sum_{i=m}^{n-1} \kappa_{i-m} (\hat{b}_i^\dagger \hat{a} + \hat{a}^\dagger \hat{b}_i), \quad (2)$$

where $\hat{H}_0 = \sum_{i=1}^{n-1} \hat{H}_i = \sum_{i=1}^{n-1} (\delta_i + \epsilon_i) \hat{b}_i^\dagger \hat{b}_i$ is the free-field Hamiltonian for all magnon modes, δ_i is the two-photon detuning, and ϵ_i represents the external perturbation acting on the particular magnon mode i ; $g_i = \sqrt{N_a} |\Omega_{pa}^{(i)} g_{pa}| / \Delta$ and $\kappa_{i-m} = \sqrt{N_a} |\Omega_{bs}^{(i)} g_{bs}| / \Delta$ (here κ_0 is set to 0) are the effective coupling coefficients of SU(1,1)- and SU(2)-type Raman interaction, respectively. g_{pa} and g_{bs} represent the atomic transition coefficients. We utilize such an atom-cavity system as a quantum sensor to detect the external perturbations ϵ_i . In the following context, we set all atomic ensembles to experience the same perturbation on the detuning $\epsilon_i \equiv \epsilon$. The result of $\epsilon_i \neq \epsilon_j$ is shown in [52].

Emergence of higher-order exceptional point. Despite the Hermiticity of the system's Hamiltonian, nonconservative SU(1,1) terms $\hat{a}^\dagger \hat{b}_i^\dagger + \hat{b}_i \hat{a}$ lead to a non-Hermitian-like dynamics of the system. We can formulate the system's dynamics based on the Heisenberg equation:

$$i \frac{\partial}{\partial t} \vec{\Phi}(t) = H_D^{(n,m)} \vec{\Phi}(t). \quad (3)$$

Here $\vec{\Phi} = (\hat{b}_1, \hat{b}_1^\dagger, \dots, \hat{b}_{n-1}, \hat{b}_{n-1}^\dagger, \hat{a}, \hat{a}^\dagger)^T$ is the field vector and $H_D^{(n,m)}$ is the non-Hermitian dynamical matrix of our system. $H_D^{(n,m)}$ inherently follows the particle-hole symmetry, i.e., $\mathcal{C} H_D^{(n,m)} \mathcal{C}^{-1} = -H_D^{(n,m)}$, with $\mathcal{C} = \mathbf{I}_n \otimes \sigma_x$ being the conjugate parity operation $\{\hat{c} \leftrightarrow \hat{c}^\dagger\}$, as well as the pseudo-Hermiticity, i.e., $\eta H_D^{(n,m)} \eta^{-1} = H_D^{(n,m)\dagger}$ with $\eta = \mathbf{I}_n \otimes \sigma_z$ for the parity operation $\{\hat{c}^\dagger \leftrightarrow -\hat{c}^\dagger\}$. Here, $\hat{c}$ represents the magnon/photon annihilation operator $\{\hat{b}_i, \hat{a}\}$, $\sigma_{x,y,z}$ are the Pauli matrices, and $\mathbf{I}_n$ is the $n \times n$ identity matrix. The presence of these symmetries enables the non-Hermitian matrix $H_D^{(n,m)}$ to manifest real eigenvalues, which correspond to dynamically stable

solutions in the system. The spontaneous breaking of these symmetries induces a dynamical phase transition, leading to the emergence of EPs of at least order 2.

Achieving the highest-order EPn in the atom-cavity system necessitates that $H_D^{(n,m)}$ be irreducible, indicating that the system cannot be decomposed into independent subsystems under linear transformations. This can be examined in terms of the free magnon Hamiltonians ($\hat{H}_i$ and $\hat{H}_j$, $i \neq j$) and the corresponding Casimir invariants [49,50], i.e., $\hat{b}_i^\dagger \hat{b}_i - \hat{b}_j^\dagger \hat{b}_j$ for SU(1,1)-type interaction and $\hat{b}_i^\dagger \hat{b}_i + \hat{b}_j^\dagger \hat{b}_j$ for SU(2)-type interaction. For two magnon modes (i, j) engaging in different types of interactions with the cavity mode, i.e., $i \in [1, m]$, $j \in [m+1, n-1]$, $\hat{H}_i + \hat{H}_j$ must not be proportional to the $\hat{b}_i^\dagger \hat{b}_i - \hat{b}_j^\dagger \hat{b}_j$. For two magnon modes (i, j) both engaging in the same type of interactions with the cavity mode, i.e., either $i, j \in [1, m]$ or $i, j \in [m+1, n-1]$, $\hat{H}_i + \hat{H}_j$ must not be proportional to $\hat{b}_i^\dagger \hat{b}_i + \hat{b}_j^\dagger \hat{b}_j$. In general, to achieve the highest-order EPn, the two-photon detunings adhere to the condition $\delta'_1 \neq \dots \neq \delta'_m \neq -\delta'_{m+1} \neq \dots \neq -\delta'_{n-1}$, where $\delta'_i = \delta_i + \epsilon_i$.

For illustration, we consider a system with two atomic ensembles. The two magnon modes interact with the cavity field through SU(1,1) and SU(2) interactions, respectively. The dynamical matrix is as follows:

$$H_D^{(3,1)} = \begin{pmatrix} \delta'_1 \sigma_z & \mathbf{0} & ig_1 \sigma_y \\ \mathbf{0} & \delta'_2 \sigma_z & \kappa_1 \sigma_z \\ ig_1 \sigma_y & \kappa_1 \sigma_z & \mathbf{0} \end{pmatrix}. \quad (4)$$

This system can achieve EP3 when the dynamical matrix (4) is irreducible under the essential condition $\delta_1 = \delta_2 = 0$ and $\epsilon_1 \neq -\epsilon_2$ [see Figs. 2(a) and 2(b)]. At this EP3, the eigenvalues exhibit cubic-root responses to perturbations ϵ in two-photon detuning. A reducible case of the matrix (4), as a *counterexample*, shows that only an EP2 exists (see the Supplemental Material [52]). Further extending this system by adding an atomic ensemble undergoing an SU(1,1)-type interaction with the cavity mode elevates the order of EP to 4 with the condition $\delta'_1 \neq \delta'_2 \neq -\delta'_3$ [see Figs. 2(c) and 2(d)]. At this EP4, the eigenvalues exhibit biquadratic-root responses to the perturbations ϵ in two-photon detuning.

EP3-based sensor. The higher-order response at the HOEP suggests dramatic changes in the system's dynamics, indicating enhanced performance of the HOEP-based sensor. For demonstration, we develop an EP3-based sensor in the system described by the dynamical matrix (4).

The system's dynamics on both sides of EP3 exhibit different behaviors: parametric oscillation when $g_1 < \kappa_1$ and parametric amplification when $g_1 > \kappa_1$. During the parametric-amplification phase, atomic excitation grows exponentially with time, inducing extra higher-order nonlinear effects (such as $\hat{b}_1^\dagger \hat{b}_1 \hat{b}_1 \hat{a} + \hat{b}_2^\dagger \hat{b}_2 \hat{b}_2 \hat{a} + \text{H.c.}$) in atomic ensembles that disrupt the non-Hermitian symmetries. Therefore, we focus on the EP3-based sensor operating in the parametric-oscillation phase.

For a perfect optical cavity, the time evolution of the magnons can be obtained from Eqs. (3) and (4):

$$\hat{b}_1(t) = A_1 \hat{b}_1(0) - iB \hat{a}^\dagger(0) + C \hat{b}_2^\dagger(0), \quad (5)$$

$$\hat{b}_2(t) = -C \hat{b}_1^\dagger(0) - iD \hat{a}(0) + A_2 \hat{b}_2(0). \quad (6)$$

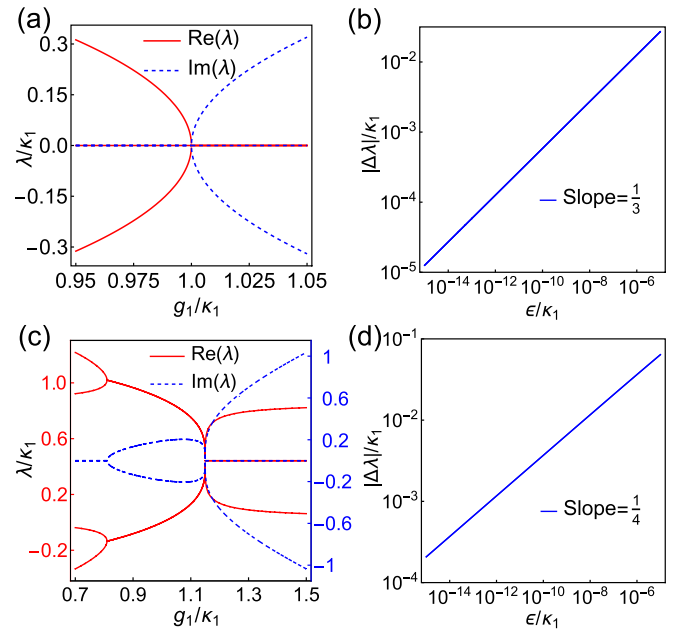


FIG. 2. EP3 and EP4 realized in the Hermitian atom-cavity system. (a) The eigenvalues of $H_D^{(3,1)}$. Red solid (blue dashed) lines represent the real (imaginary) parts. Parameters used are $\delta_1 = \delta_2 = 0$. (b) Logarithmic scale plot of the absolute eigenvalue splitting at EP3 ($g_1 = \kappa_1$) as a function of perturbation ϵ with slope = 1/3. (c) The eigenvalues of $H_D^{(4,2)}$. Parameters used are $\delta_1 = 0.8845\kappa_1$, $\delta_2 = 0.0340\kappa_1$, $\delta_3 = -0.8505\kappa_1$, and $g_2 = 0.2\kappa_1$. (d) Logarithmic scale plot of the absolute eigenvalue splitting at EP4 ($g_1 = 1.1499\kappa_1$) as a function of perturbation ϵ with slope = 1/4.

Here, $\{A_1, A_2, B, C, D\}$ are the propagation coefficients depending on $\{g_1, \kappa_1, \epsilon, t\}$ which determine the evolution of magnons. By constructing a biorthogonal complete eigenbasis of the dynamical matrix and applying non-Hermitian perturbation theory [53], we obtain the first-order perturbation approximation for these propagation coefficients that matches well with the numerical results when $\epsilon \ll (\kappa_1^2 - g_1^2)^{3/2}$ [52].

We can estimate the external perturbations ϵ by performing homodyne measurements on the magnon modes. This can be implemented by leveraging an SU(2)-type Raman interaction to convert the magnons into photons, which are then detected through standard homodyne measurements [52].

We consider a coherent initial state for magnon modes $|\alpha_1\rangle_{b_1} |\alpha_2\rangle_{b_2}$. For convenience, we relabel $g_1 = g$, $\kappa_1 = \kappa$. Without loss of generality, we further set $\alpha_1 = -\alpha_2 \equiv i\alpha$ ($\alpha \in \mathbb{R}$), and then only the quadratures $\hat{X}_1$ and $\hat{X}_2$ are sensitive to the phase perturbation ϵ . Here, $\hat{X}_i = (\hat{b}_i + \hat{b}_i^\dagger)/\sqrt{2}$ is the quadrature operator of magnon mode i . First, we consider the observable $\hat{O} = \hat{X}_1 - \hat{X}_2$ to measure the system's change. The system's change is quantified by the susceptibility $\mathcal{S} = |\partial_\epsilon \langle \hat{O} \rangle|$, which is maximized near $\epsilon = 0$, as shown in Fig. 3(a). From the first-order perturbation theory, we have the susceptibility [52]

$$\mathcal{S}|_{\epsilon=0} = \frac{\sqrt{2}\alpha(\kappa + g)^2\xi}{2\chi^5}, \quad (7)$$

where $\xi = (\kappa^2 + g^2)\chi t[2 + \cos(\chi t)] + (g^2 - 8g\kappa + \kappa^2)\sin(\chi t)$ and $\chi = \sqrt{\kappa^2 - g^2}$. The local maxima

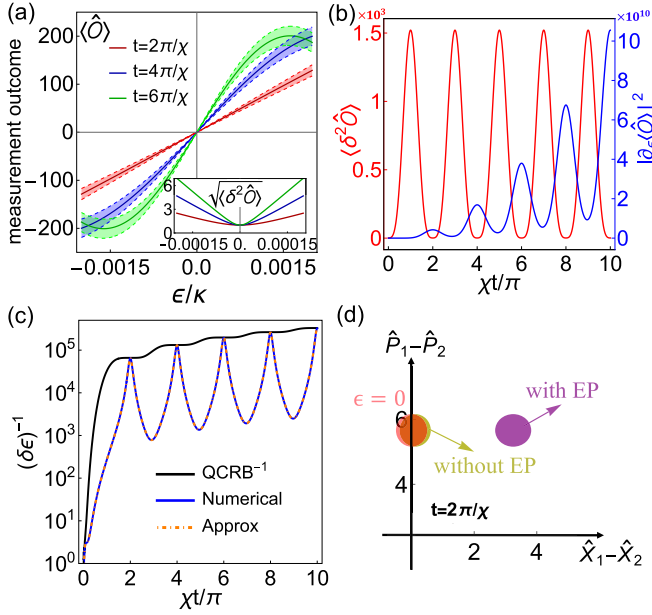


FIG. 3. (a) The mean value $\langle \hat{O} \rangle$ and the standard deviation $\sqrt{\langle \delta^2 \hat{O} \rangle}$ (the inset) as a function of ϵ at different working points. The shaded areas represent the measurement errors. (b) The variance $\langle \delta^2 \hat{O} \rangle$ (red) and susceptibility $|\partial_\epsilon \langle \hat{O} \rangle|$ (blue) of observable $\hat{O} = \hat{X}_1 - \hat{X}_2$ as a function of evolution time when $\epsilon = 0$. (c) The reciprocal of sensitivity $(\delta\epsilon)^{-1}$ as a function of evolution time when $\epsilon = 0$. The black solid curve represents the QCRB [52]. In (a)–(c), we set $g/\kappa = 0.95$, $\kappa = 1$, and $\alpha = 2$. (d) The error contours of the magnon state at $\epsilon = 0$ (red circle) and at $\epsilon = 5 \times 10^{-5}\kappa$ (purple circle). The result is compared with the case without EP (yellow circle) using the same magnon number $N \sim \chi^{-4}$. In all plots, we set $g/\kappa = 0.95$, $\kappa = 1$, and $\alpha = 2$.

$S_{\max} = 3\sqrt{2}\alpha(\kappa^2 + g^2)(\kappa + g)^2 q\pi/\chi^5 \sim \chi^{-5}$ can be reached at $t = 2q\pi/\chi$ with $q = 1, 2, \dots$ [see the blue curve in Fig. 3(b)]. Near EP3 ($\chi \rightarrow 0$), the susceptibility exhibits a divergent feature, induced by EP and representing EP-enhanced response. The noise of this sensor is the variance of the observable $\langle \delta^2 \hat{O} \rangle = [\kappa - g\cos(\chi t)]^2/(\kappa - g)^2$. As shown by the red curve in Fig. 3(b), the local minimum of noise $\langle \delta^2 \hat{O} \rangle_{\min} = 1$ is reached at $t = 2q\pi/\chi$. In such a case, the system evolves back to its initial coherent state. Thus at the working point $\chi t = 2q\pi$, the optimal sensitivity of the system is $\delta\epsilon_{\text{opt}} = \sqrt{\langle \delta^2 \hat{O} \rangle_{\min}}/S_{\max} \sim \chi^5$, which saturates the quantum Cramér-Rao bound (QCRB), as shown in Fig. 3(c). With time evolution, the magnon number oscillates and is bounded by $N \sim \chi^{-4}$ [52]. The time for reaching the working point is $t \sim \chi^{-1}$. Therefore, the optimal sensitivity $\delta\epsilon_{\text{opt}} \sim \chi^5$ scales at the Heisenberg limit $N^{-1}t^{-1}$.

As shown in Figs. 3(a) and 3(d), at the working point $\chi t = 2q\pi$, a small ϵ causes an amplified displacement of the magnon state in phase space, leading to a dramatic change in the measurement outcome $\langle \hat{O} \rangle$, while the change of the standard deviation $\sqrt{\langle \delta^2 \hat{O} \rangle}$ via ϵ is negligible. This indicates that the sensitivity enhancement originates from the signal amplification through HOEP, rather than from the reduction of noise through quantum squeezing/entanglement. This conclusion is further confirmed by the Gaussian decomposition [52,54].

It should be emphasized that entanglement is generated between two atomic ensembles [55,56], which exhibit the antisqueezing feature of the observable $\hat{O} = \hat{X}_1 - \hat{X}_2$ or $\hat{P}_1 + \hat{P}_2$ and the squeezing feature of the alternative observable $\hat{X}_1 + \hat{X}_2$ or $\hat{P}_1 - \hat{P}_2$ off the working point $\chi t \neq 2q\pi$. During the first half of the oscillation period ($\chi t \in [0, \pi]$), the magnon state is squeezed due to the parametric interaction, while the detuning ϵ induces a displacement in the X -direction. In the second half ($\chi t \in [\pi, 2\pi]$), the accumulation of detuning leads to a phase reversal of the parametric interaction. As a result, the displacement is amplified, while the noise evolves back to the initial coherent level [57–61]. Such noiseless amplification intensifies as the system approaches the HOEP. One can also utilize quantum squeezing to enhance sensitivity by measuring $\hat{X}_1 + \hat{X}_2$, which saturates the QCRB at $\chi t = (2q + 1)\pi$ [52]. However, the squeezing-based strategy has an intrinsic disadvantage over the EP-based one when there exist losses in the system and detection [62]. In general, the losses of the system and detection themselves and their introduced extra noises degrade the sensitivity. The squeezing-based strategy relies on quantum squeezing, which is fragile due to the loss-induced decoherence. In contrast, the EP-based strategy employs noiseless signal amplification, which inherently holds loss tolerance and noise resistance [58,63–66]. A detailed analysis demonstrates the loss dependence of both strategies in [52]. The results indicate that the EP-based strategy is evidently a preferred candidate for sensitivity enhancement in the atom-cavity system.

Higher-order EP enhancement. For comparison, we consider a quantum sensor based on Langevin noise-free EP2 [33], with its optimal sensitivity scales at χ^3 . This is evidently lower than that of our EP3-based sensor (scales at χ^5), though both of them scale at the Heisenberg limit. The advantage of the EP3 arises from the additional SU(2)-type interaction, which significantly increases the sensing resources and enhances the sensitivity [65,66].

We further analyze a general EP n -based sensor with initial coherent states and homodyne detection. For the parametric-oscillation phase with oscillation period $2\pi/\chi \sim \epsilon^{-1/n}$, through evaluating the derivative of propagation coefficients $\delta\epsilon_{\text{opt}} \sim (\partial_\epsilon \mathbf{K}_{jk})^{-1} \sim \epsilon^{2-1/n}$, we can deduce a general scaling law for its optimal sensitivity $\delta\epsilon_{\text{opt}} \sim \chi^{2n-1}$ [52]. This is attributed to the higher amplification rate of atomic (bosonic) excitations near HOEP, enhancing the sensitivity.

Experimental feasibility. Here, we provide a possible experimental condition. We assume $N_a \sim 10^6$ ^{87}Rb atoms, which are initially prepared in spin coherent states with $\alpha \sim 10^2$. We select atomic transition coefficients of $g_{pa,bs}/2\pi \sim 50$ kHz and detuning of $\Delta_{pa,bs}/2\pi \sim 250$ MHz [67–69]. With pump beams of approximately 100 μW and waists around 600 μm , the resulting Rabi frequencies are $\Omega_{pa,bs}/2\pi \sim 2.6$ MHz. This setup can yield magnon-photon interaction strengths of $g/2\pi, \kappa/2\pi \sim 500$ kHz. Focusing on the EP3-based sensor, we can fine-tune g and κ by adjusting the intensity of the pump light. Within the allowed precision of adjustment, $g/\kappa = 0.995$ is achievable, corresponding to $\chi \sim 50$ kHz. Given these parameters, this ^{87}Rb atom-based sensor is capable of detecting perturbations ϵ , with sensitivity $\delta\epsilon \sim 5.27 \times 10^{-6} \text{ Hz Hz}^{-1/2}$, corresponding to, e.g., a sensitivity $\sim 0.75 \text{ fT Hz}^{-1/2}$ for magnetic field measurement

at the working point $t = 2\pi/\chi$. With the experimentally achievable cavity decay rate $\gamma = 0.1\kappa \sim 2\pi \times 50$ kHz and magnon decay rate $\Gamma = 0.01\kappa \sim 2\pi \times 5$ kHz [67–69], this sensor can still surpass the standard quantum limit by over 10 dB.

Conclusion. In summary, we develop a magnon-cavity sensor that allows Langevin noise-free HOEPs to achieve high sensitivity. Notably, this sensor demonstrates a sensitivity far surpassing that of EP2-based sensors. Due to the versatility of atoms, the external perturbation ϵ can be induced by different interactions of atoms with various measured fields, including the Stark effect from electric fields [40], the Zeeman effect from magnetic fields [41], the ac Stark effect from optical fields [42], and even exotic interactions from axion-like particles [43]. Furthermore, compared with other platforms such as superconducting circuits, atom-based ones offer advantages in experimental simplicity without cryogenic systems [70] and the capability to operate in both optical and microwave frequencies. The crucial contribution of Langevin noise-free

HOEPs in enhancing the atom-cavity system's response to various measured fields represents a significant advance in precision measurement, indicating a new direction in sensing with quantum technology.

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