Letter

# Scaling of von Neumann entropy in dissipative Liouvillian dynamics

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The universal dynamical phenomenon of von Neumann entropy plays an important role in understanding the nature of open many-body systems, as it uncovers general collective properties driven by correlation and quantum dynamics. Here, we reveal a universal time evolution behavior of bipartite von Neumann entropy in open quantum systems with dissipation into the surrounding environments. The coupling with the environment leads to a rapid growth of the entropy, resulting in an exotic short-time evolution law, $S(t) \sim -t \ln t$. Such a fast growth behavior is proved to be a general phenomenon for generic many-body systems in dissipative environments, as long as the systems exhibit well-defined fermionic quasiparticles. Interestingly, even for strongly interacting Luttinger liquids, which display vanishing fermionic but emergent bosonic quasiparticles, the same dynamical phenomenon is still found to persist. These results demonstrate a universal Liouvillian dynamical feature that has no counterpart in isolated Hamiltonian systems.

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*Introduction.* Von Neumann entropy plays a critical role in the study of quantum many-body systems. For closed systems at finite temperatures, it quantifies the amount of disorder or randomness in the thermal state. While at zero temperature, the bipartite von Neumann entropy captures the quantum entanglement of the pure state, which generally obeys the area law when only short-range interactions are present [1].

In isolated quantum systems, bipartite von Neumann entropy can exhibit universal dynamical features. For quantum states evolved under a quenched Hamiltonian, the subsystem entanglement entropy commonly witnesses a linear growth in the (generalized) thermalization processes [Fig. 1(a)]. Linear scaling with time is ubiquitously observed in many ergodic systems and integrable systems, where the subsystem is described by a (generalized) Gibbs ensemble that satisfies the entropy volume law as a result of (generalized) thermalization by the complement of the system [2–8]. This is in sharp contrast with systems with localization or frustration, whose subsystem entanglement entropy would grow logarithmically [9,10].

Another topical field under extensive recent study is the dynamics of open quantum systems [11–14]. When a system is coupled to a Markovian environment, the system density matrix evolves under a Liouvillian superoperator, obeying the Lindblad master equation [15,16], which is applied to, e.g., time-independent decoherence and weak measurement. In open systems, the Liouvillian superoperator acts on the density matrix. In stark contrast with the Hamiltonian description of an isolated system, this makes the system to be essentially a mixed state, which could display contrastingly different dynamical properties. Hence, it is intriguing to ask whether there could be any universal behaviors of the bipartite von Neumann entropy that are unique to the Liouvillian dynamics of open quantum systems? In addition, the bipartite von Neumann entropy for mixed states has important applications in many-body physics. For example, von Neumann entropy at finite temperatures can give an indication of certain ground state properties [17,18]. It can also be used as a powerful indicator to identify the phase transitions in correlated systems [19]. Therefore, its investigation will not only enrich existing knowledge of thermodynamics in open systems but also shed light on more potential applications.

In this Letter, we report a universal time-scaling law of the von Neumann entropy for dissipative open quantum systems, which has no counterpart in the isolated cases. We show that, for dissipative open systems with well-defined quasiparticles,

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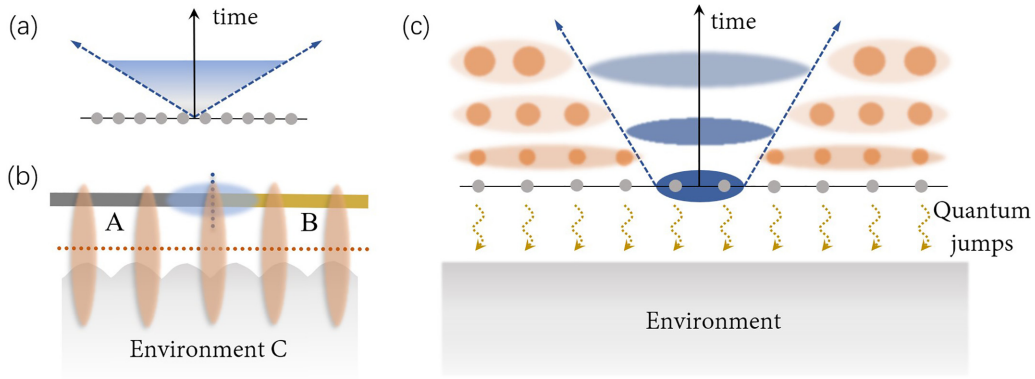



FIG. 1. (a) The linear light cone for information propagation in isolated Hamiltonian systems. (b) The open system $\mathcal{AB}$ interacting with environment $\mathcal{C}$. The von Neumann entropy $S_{\mathcal{A}}(t)$ is obtained by first tracing out $\mathcal{C}$ and then $\mathcal{B}$. (c) illustrates the short-time entropy growth for dissipative quantum systems. The orange region indicates the environment-induced rapid entropy growth, which has a different origin compared to the entanglement propagation denoted by the blue region.

i.e., the density matrix can be decomposed into quasiparticle subspaces, the bipartite von Neumann entropy of subsystem A always exhibits a universal time evolution in the short-time regime, $S_{\mathcal{A}}(t) \sim -t \ln t$. Different from the ballistic entanglement evolution in a closed system, this fast growth of $S_{\mathcal{A}}(t)$ is purely caused by the interaction with environments. Furthermore, in the long-time regime, we find that $S_{\mathcal{A}}(t)$ will finally decay either exponentially or in a power law, depending on the gapped or gapless nature of the Liouvillian spectrum, as a result of the dissipation.

The dynamical behavior found here is a general phenomenon. We prove that it always occurs for quantum systems (in dissipative environments) with fermionic quasiparticles. It even persists for strongly interacting systems whose low-energy quasiparticles become orthogonal to the original fermions, such as the Luttinger liquids (LLs). These results showcase the universal Liouvillian dynamics of von Neumann entropy. Since it efficiently reflects the quasiparticles' property of quantum states, different possibilities are opened to identify exotic non-Fermi liquids, thereby advancing the fundamental knowledge of correlated and open quantum systems [20–33].

*General theory.* We consider a general initial state with well-defined (fermionic) quasiparticles as $|\Psi\rangle = \prod_{i=1}^{N}[\sqrt{p_i}\gamma_{i,\mathcal{A}}^{\dagger} + \sqrt{1-p_i}\gamma_{i,\mathcal{B}}^{\dagger}]|\Omega\rangle$, where $\gamma_{i,\alpha}^{\dagger}$ is the quasiparticle creation operator with the quantum number $i = 1, 2, \ldots$, and $\alpha = \mathcal{A}, \mathcal{B}$ denotes the two partition sectors. $|\Omega\rangle$ is the many-body vacuum, which is the direct product of the local vacuum states, $|\Omega\rangle = \otimes_{i=1}^{N}|0\rangle_i$. $p_i$ is the probability of finding the state $|\phi_i\rangle \equiv \gamma_{i,\mathcal{A}}^{\dagger}|0\rangle$ in $\mathcal{A}$. The density matrix of the total system, $\rho_{\rm tot} = |\Psi\rangle\langle\Psi|$, can be accordingly written into the direct product of local density matrices, i.e., $\rho_{\rm tot} = \otimes_{i=1}^{N}\rho_i$.

For $t = 0$, the system-environment coupling is turned on, and the density matrix evolves as $\rho_{\rm tot}(t) = e^{\mathcal{L}t}\rho_{\rm tot}$, where $\mathcal{L}$ is the Liouvillian. In the quasiparticle basis, we assume $\mathcal{L}$ is decomposed into $\mathcal{L} = \otimes_{i=1}^{N}\mathcal{L}_i$ and $\rho_{\rm tot}(t) = \otimes_{i=1}^{N}e^{\mathcal{L}_i t}\rho_i$, where $\mathcal{L}_i$ is the Liouvillian acting on the local Hilbert space, which is applicable when $\mathcal{L}$ is diagonalized in the quasiparticle basis via third quantization [34]. Under dissipation, the quasiparticles are not conserved but have loss. Consequently, the probability $p_i$ of the state $\gamma_{i,\mathcal{A}}^{\dagger}|0\rangle$ usually decays as $p_i(t) = p_i e^{-r_i t}$, where $r_i$ is the decay rate of the $i$th quasiparticle. Meanwhile, the probability of observing a local vacuum state $|0\rangle_i$ is $1 - p_i e^{-r_i t}$. Thereby, the reduced density matrix is obtained after tracing out the $\mathcal{B}$ sector as $\rho_{\mathcal{A}}(t) = {\rm Tr}_{\mathcal{B}}\,\rho_{\rm tot}(t) = \otimes_{i=1}^{N}\rho_{\mathcal{A},i}(t)$, where $\rho_{\mathcal{A}_i}(t) = p_i e^{-r_i t}|\phi_i\rangle\langle\phi_i| + (1 - p_i e^{-r_i t})|0\rangle\langle 0|$. This leads to the entropy,

$$S_{\mathcal{A}}(t) = -\sum_{i=1}^{N} p_i(t)\ln p_i(t) + [1 - p_i(t)]\ln[1 - p_i(t)]. \tag{1}$$

Similarly, for $\mathcal{B}$, $\rho_{\mathcal{B}_i}(t) = (1 - p_i)e^{-r_i t}|\phi_i'\rangle\langle\phi_i'| + (1 + p_i e^{-r_i t} - e^{-r_i t})|0\rangle\langle 0|$. Define $p_i'(t) = (1 - p_i)e^{-r_i t}$, in striking contrast to the conservation of the particle number and energy in closed systems quenched by the Hamiltonian, $p_i'(t) + p_i(t) = e^{-r_i t} < 1$.

Taking $t \ll 1$ in Eq. (1), we can readily obtain $S_{\mathcal{AB}}(t) \sim -t \ln t$, since all $p_i = 1$. The time-dependent term in $S_{\mathcal{A}}(t)$ can be extracted as $S_{\mathcal{A}}(t) = -\Upsilon t \ln t + \cdots$ [similar for $S_{\mathcal{B}}(t)$] [35,36], where $\Upsilon = \int dq p_q r_q/2$ is a constant coefficient, and the sum of $i$ has been replaced by the integral over the continuous quantum number $q$ in the thermodynamic limit. The short-time behavior, $S \sim -t \ln t$, does not depend on whether the initial states are gapped or gapless and the detailed form of $\mathcal{L}$, as summarized in Table I of the Supplemental Material [35]. It generally occurs for dissipative quantum systems, as long as the system has fermionic quasiparticle excitations. Thus, we prove a "$-t \ln t$" evolution law for a generic dissipative open quantum system with particle loss into the environment, as along as the quasiparticles are well defined.

In addition, the long-time dynamics can be obtained by taking $t \gg 1$. The leading time dependence is derived as $S_{\mathcal{A}}(t) \sim \int dq p_q r_q t e^{-r_q t}$. For the case where a Liouvillian spectrum has a gap $\Lambda$ with $r(q) \sim \Lambda$, an exponential decay, $S_{\mathcal{A}}(t) \sim e^{-\Lambda t}$, is generally obtained [37], while for the $\mathcal{L}$-gapless case with the spectrum $\propto q^{\alpha}$, $S_{\mathcal{A}}(t) \sim t^{1/\alpha}$ is found.

*Example I: Free-fermion systems.* We now demonstrate the above general theory by a free-fermion model coupled to an environment. The dynamics of the system is captured by the

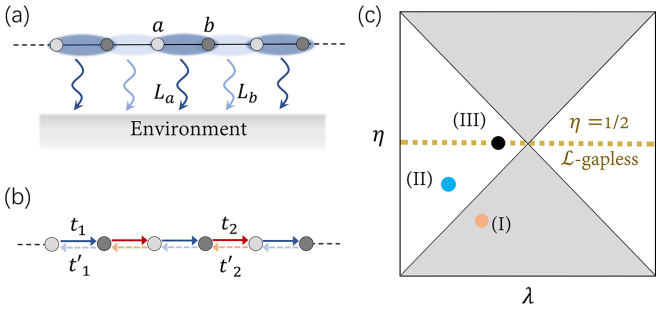



FIG. 2. (a) The dissipative SSH chain coupled to the environment. (b) The effective Hamiltonian $H_{\rm eff}$ describing a non-Hermitian SSH chain. (c) The phase diagram of $H_{\rm eff}$ with varying $\eta$ and $\lambda$.

Lindblad master equation [15,16],

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H_0, \rho] + \sum_n \left( L_n \rho L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho\} \right), \quad (2)$$

where $\mathcal{L}$ is the Liouvillian operator and $L_n$ the local jump term at site $n$. $H_0$ describes a Su-Schrieffer-Heeger (SSH) chain, with the intra- and intersublattice hopping, $t_{\rm intra}$ and $t_{\rm inter}$, as shown by Fig. 2(a). Dissipation (particle loss) into the environment is driven by two-site local quantum jumps, $L_{n,a} = \sqrt{\gamma_a}(c_{n,a} - c_{n,b})$ and $L_{n,b} = \sqrt{\gamma_b}(c_{n,b} - c_{n+1,a})$, with $a, b$ denoting sublattices. The insertion of $H_0$ and $L_{n,a/b}$ into Eq. (2) leads to

$$\mathcal{L}\rho = -\mathrm{i}[H_{\rm eff}, \rho]_{\rm nH} + \sum_{n,\tau=a,b} L_{n,\tau} \rho L_{n,\tau}^\dagger, \quad (3)$$

where $[X, Y]_{\rm nH} = XY - YX^\dagger$ is the non-Hermitian commutator. The Liouvillian spectrum $\lambda_\nu$ with $\nu = 1, 2, 3, \ldots$ can be obtained by the diagonalization of $\mathcal{L}$ using the third quantization [34]. $\lambda_\nu$ determines the long-time dynamics of the system, as is shown below. The short-time evolution is however governed by the effective Hamiltonian [38–40], $H_{\rm eff} = \sum_k \Psi_k^\dagger h_{\rm eff}(k) \Psi_k$ with $\Psi_k = [c_{k,a}, c_{k,b}]^{\rm T}$, mimicking a non-Hermitian closed system [41], where

$$h_{\rm eff}(k) = \begin{pmatrix} i\mu & t_2 e^{-ik} + t'_1 \\ t'_2 e^{ik} + t_1 & i\mu \end{pmatrix}. \quad (4)$$

We parametrize the model by two parameters, $\eta$ and $\lambda$, i.e., $t_{\rm intra} = \eta$, $t_{\rm inter} = (1-\eta)$, $\gamma_a = (1-2\lambda)(1-\eta)$, and $\gamma_b = (1-2\lambda)\eta$, with $\eta < 1$, $\lambda > 0$. Then, the parameters in Eq. (4) read as $t_1 = (1-\eta)(1-\lambda)$, $t_2 = \eta(1-\lambda)$, $t'_1 = \lambda(1-\eta)$, $t'_2 = \lambda\eta$, and $\mu = (2\lambda - 1)/2$.

The non-Hermitian model $H_{\rm eff}$ is illustrated by Fig. 2(b), whose phase diagram is shown in Fig. 2(c). The gray and white regions denote the gapped and gapless phases in terms of the real spectrum of $H_{\rm eff}$, respectively. The Liouvillian spectrum is gapless on the yellow dashed line ($\eta = 1/2$). In the following, we focus on three sets of parameters (away from the exceptional points) marked by the orange, blue, and black dot in Fig. 2(c), for which the Hamiltonian/Liouvillian spectrum is gapped/gapped (I), gapless/gapped (II), and gapless/gapless (III), respectively.

To calculate $S_{\mathcal{A}}(t)$, we adopt the correlation matrix [42,43], defined by

$$C_{mn}(t) = \mathrm{Tr}[c_m^\dagger c_n \rho(t)], \quad (5)$$

where $\rho(t)$ is the density matrix of the SSH chain. It can also be written as [44]

$$\rho(t) = Z_0^{-1} \exp\left[ -\sum_{m,n} H^E_{m,n}(t) c_m^\dagger c_n \right], \quad (6)$$

where the time-dependent entanglement Hamiltonian in the Fock space, $H^E_{m,n}(t)$, is defined, and $Z_0$ is the partition function at $t = 0$. The correlation matrix $C(t)$ can be proved to relate to $H^E(t)$ via [35]

$$\exp[H^E(t)] = \frac{1 - (Z_0/Z_t)C(t)}{(Z_0/Z_t)C(t)}, \quad (7)$$

where $Z_t = Z_0 \,\mathrm{Tr}[\rho(t)]$ is the partition function at a generic time $t > 0$, and $\mathrm{Tr}[\rho(t)] = \sum_{mn} C_{mn}(t) < 1$ [35].

The von Neumann entropy is obtained as $S_{\mathcal{A}}(t) = \mathrm{Tr}\{\exp[-H^E_{\mathcal{A}}(t)][\ln Z_{\mathcal{A},0} + H^E_{\mathcal{A}}(t)]\}/Z_{\mathcal{A},0}$, where $\rho_{\mathcal{A}}(t) = \mathrm{Tr}_{\mathcal{B}}\rho(t)$ is the reduced density matrix of sectors $\mathcal{A}$. Meanwhile, by restricting $m, n$ to the $\mathcal{A}$ sector in Eq. (5), we obtain the reduced correlation matrix, $C_{\mathcal{A},mn} = \mathrm{Tr}[c_m^\dagger c_n \rho(t)]$, $m, n \in \mathcal{A}$, and thus the entanglement Hamiltonian for sector $\mathcal{A}$, i.e., $H^E_{\mathcal{A}}(t)$. The insertion of Eq. (7) into $S_{\mathcal{A}}(t)$ leads to

$$S_{\mathcal{A}}(t) = -\sum_q \big\{ \mathfrak{c}_q(t) \ln \mathfrak{c}_q(t) - \mathfrak{c}_q(t) \ln[Z_t/Z_0 - \mathfrak{c}_q(t)] + Z_t/Z_0 \ln[1 - \mathfrak{c}_q(0)] \big\}, \quad (8)$$

where $\mathfrak{c}_q(t)$ is the $q$th eigenvalue of $C_{\mathcal{A}}(t)$. Note that $S_{\mathcal{A}}(t=0)$ is reduced to the entanglement entropy for the initial state of $H_0$.

To calculate $\mathfrak{c}_q(t)$, we consider the equation of motion, i.e., $\dot{C}_{mn}(t) = \mathrm{Tr}[c_m^\dagger c_n \dot{\rho}(t)]$. Using the Lindblad master equation in Eq. (2) and explicitly calculating the encountered commutators with the jump terms under purely loss or gain, we obtain [35]

$$C(t) = e^{i\mathcal{D}^\dagger t} C(0) e^{-i\mathcal{D}t}, \quad (9)$$

where $\mathcal{D} = H_0^{\rm T} - i\sum_n L_n^\dagger L_n/2$ is the damping operator [different from $H_{\rm eff}$ in Eq. (4)]. The eigenvalues $\lambda_{\mathcal{D}}$ of $\mathcal{D}$ are related to the Liouvillian spectrum $\lambda_\nu$ via $\lambda_\nu = -i\lambda_D$ under purely loss (or gain) [45]. The eigenvalues $\mathfrak{c}_q(t)$ can then be obtained from Eq. (9). Therefore, Eqs. (8) and (9) altogether promise an accurate evaluation of $S_{\mathcal{A}}(t)$ ranging from $t = 0$ to $t \to \infty$. We note that due to the non-Hermiticity of $\mathcal{D}$, the physical quantities calculated through $C(t)$ mimic those obtained in closed non-Hermitian systems. Interestingly, the latter have been recently shown to violate the speed limit for the propagation of correlations [46–48]. Thus, unusual dynamics of $S_{\mathcal{A}}(t)$ may be expected here.

For $t \leqslant 0$, without losing generality, we assume the initial state to be the ground state of $H_0$ at half filling. At $t = 0$, the quantum jumps are turned on, and the system undergoes dissipation induced by environment. Using Eqs. (8) and (9), for fixed length $L_{\mathcal{A}} = L_0$, we numerically calculate $S_{\mathcal{A}}(L_0, t)$, as shown in Fig. 3(a) for the three sets of parameters in Fig. 2(c). Interestingly, for all parameters, $S_{\mathcal{A}}(L_0, t)$ is found to first

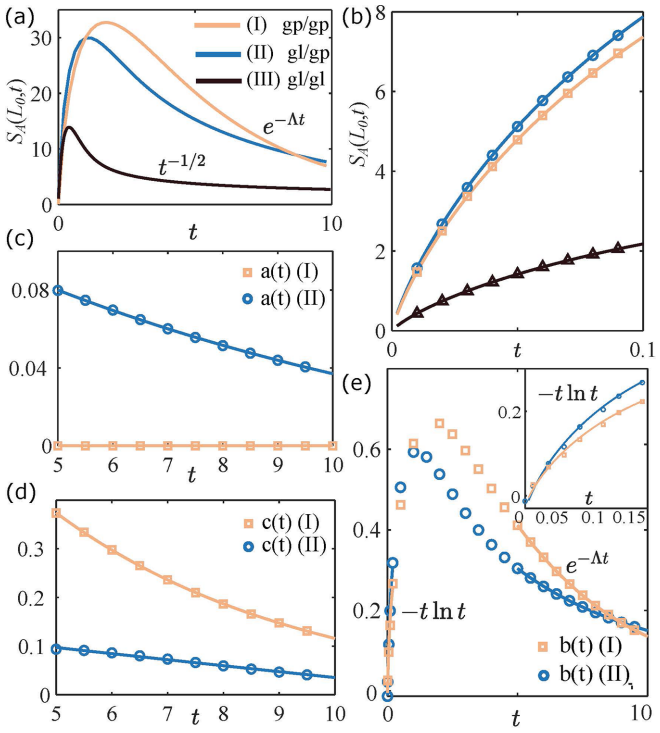



FIG. 3. (a) The calculated von Neumann entropy $S_{\mathcal{A}}(L_0,t)$ for the three sets of parameters denoted by Fig. 2(c). (b) The zoom-in data for $S_{\mathcal{A}}(L_0,t)$ in the short-time regime, which are well fitted by $-\gamma t\ln\gamma t$. (c)–(e) show the extracted values of $a(t)$, $b(t)$, and $c(t)$ in Eq. (10), for both the $H_{\text{eff}}$-gapped and -gapless cases. The inset to (e) shows the zoom-in data for $b(t)$ in the short-time regime.

grow and then decay with $t$. The resulting peak is a general feature that does not rely on whether the Hamiltonian ($H_{\text{eff}}$) or the Liouvillian ($\mathcal{L}$) is gapped or gapless. However, as shown by the long-time regime in Fig. 3(a), $S_{\mathcal{A}}(L_0,t)$ exhibits qualitatively different decaying behaviors between the $\mathcal{L}$-gapped and -gapless cases. The former decays exponentially while the latter in a power law.

We zoom in the short-time evolution of $S_{\mathcal{A}}(L_0,t)$. As shown in Fig. 3(b), $S_{\mathcal{A}}(L_0,t)$ increases rapidly for all three parameters. Remarkably, all the data can be accurately fit by $-\gamma t\ln(\gamma t)$, indicating that a universal dynamical phenomenon is emerging. Such dynamical behavior is in sharp contrast with those in closed systems, which either display a linear growth [$S(t)\sim t$] [49–53] or an algebraic growth [$S(t)\sim\ln t$] of entanglement [8,9,54]. We note that $S(t)$ diverges for $t\to 0$ due to the coupling with the environment, which has a nonentanglement origin.

To better understand the above numerics, we now analytically derive an asymptotic large $L_{\mathcal{A}}$ expression for $S_{\mathcal{A}}(t)$ [35]. We focus on the $\mathcal{L}$-gapped case with a Liouvillian gap $\Lambda$, and denote $S_{\mathcal{A}}(t)$ in Eq. (8) as $S_{\mathcal{A}}(t)=\sum_q S(\mathfrak{c}_q)=\oint\frac{d\lambda}{2i\pi}S(\lambda)\frac{d\ln\det[\lambda-C]}{d\lambda}$. Using Eq. (9), the correlation matrix is derived as $C_{nm}\simeq e^{-\Lambda t}\frac{\sin[k_F(n-m)]}{\pi(n-m)}$, which belongs to Toeplitz matrices. Using the Fisher-Hartwig conjecture for the Toeplitz matrix [55], the determinant, $\ln\det[C]$, can be obtained via the symbol of the matrix. The insertion of the determinant into $S_{\mathcal{A}}(t)$ leads to the following general form [35],

$$S_{\mathcal{A}}(L_{\mathcal{A}},t)=a(t)\ln L_{\mathcal{A}}+b(t)L_{\mathcal{A}}+c(t), \tag{10}$$

where $a(t)$, $b(t)$, and $c(t)$ are functions dependent on the model details. To extract these functions, we numerically calculate $S_{\mathcal{A}}(L_{\mathcal{A}},t)$ for fixed $t$ and a varying $L_{\mathcal{A}}$. For each fixed $t$, the fitting to Eq. (10) yields the values of $a$, $b$, $c$. With further scanning of $t$, $a(t)$, $b(t)$, and $c(t)$ are obtained, as shown in Figs. 3(c)–3(e).

As shown in Fig. 3(c), $a(t)$ is zero and nonzero for the $H_{\text{eff}}$-gapped and -gapless cases, respectively, whereas $c(t)$ remains finite for both cases, as indicated in Fig. 3(d). It is known that, in one dimension (1D), the entanglement entropy between $\mathcal{A}$ and $\mathcal{B}$ satisfies $S\sim\ln L_{\mathcal{A}}$ and $\sim$O(1) for the gapless and gapped states, respectively. Thus, Figs. 3(c) and 3(d) imply that the terms $a(t)\ln L_{\mathcal{A}}$ and $c(t)$ in Eq. (10) essentially describe such $\mathcal{A}$-$\mathcal{B}$ entanglement (or the mutual information $I_{\mathcal{A}-\mathcal{B}}$ [56,57]).

An additional extensive term, i.e., $S_{\text{env}}\equiv b(t)L$, is left in Eq. (10). This term is distinguished in its origin from $I_{\mathcal{A}-\mathcal{B}}$, as illustrated in Figs. 1(b) and 1(c). The entropy per site, $S_{\text{env}}/L=b(t)$, exhibits a rapid growth, $\sim -t\ln t$ in the short-time regime, independent of whether $H_{\text{eff}}$ and $\mathcal{L}$ are gapped or gapless, as shown in Fig. 3(e) and its inset. In addition, in the long-time regime, $b(t)$ decays exponentially (in a power law) when $\mathcal{L}$ is gapped (gapless), as a result of the dissipation. Here, we note that calculating $S_{\mathcal{AB}}$ also leads to $S_{\mathcal{AB}}(t)=b(t)L$, which displays the same universal behavior with that extracted from $S_{\mathcal{A}}(t)$. This indicates that the contribution here purely comes from the coupling with the environment.

*Example II: Strongly interacting Luttinger liquid.* We further step into strongly interacting systems, i.e., LLs exhibiting collective bosonic excitations. For $t\leqslant 0$, we consider 1D spinless fermions with a nearest-neighbor interaction $U$, i.e., $H_0=t\sum_{i=1}^{N}c_i^\dagger c_{i+1}+c_{i+1}^\dagger c_i+U(n_i-\frac{1}{2})(n_{i+1}-\frac{1}{2})$, which can be bosonized into

$$H_0=\sum_{q\neq 0}v|q|b_q^\dagger b_q+\frac{g_2|q|}{2}(b_q b_{-q}+b_q^\dagger b_{-q}^\dagger), \tag{11}$$

where the interaction $g_2$ and velocity $v$ are determined by $t$ and $U$ [35]. The operator $b_q^\dagger=i\sqrt{1/|q|}\sum_k:c_{k+q}^\dagger c_k:$ creates electron-hole pairs around the Fermi energy. For $t\geqslant 0$, the jump term capturing the particle loss into environment is turned on, with $L_i=\gamma(c_i-c_{i+1})$, similar to that of the noninteracting model above. After bosonization, the jump operator is transformed into an exponential function of boson linear combinations. Since no quartic boson terms are introduced, the LL remains quasifree such that the correlation matrix approach applies. In the following, we study the short-time dynamics based on Eq. (2). Interestingly, $H_{\text{eff}}$ derived from Eq. (3) displays the non-Hermitian skin effect with particle loss only for the right movers [35], as demonstrated by Fig. 4.

After a Bogoliubov-type transformation, $a_q=\alpha_q b_q+\beta_q b_{-q}^\dagger$ and $a_{-q}^\dagger=\beta_q b_q+\alpha_q b_{-q}^\dagger$ with $\alpha_q$, $\beta_q$ the transformation coefficients, Eq. (11) is diagonalized into $H_0=v\sum_{q\neq 0}w_q(a_q^\dagger a_q+\frac{1}{2})$. Here, the most convenient quantity that characterizes the time dependence of entanglement is the momentum-space von Neumann entanglement entropy $S$ [58]. The system is accordingly partitioned into $q>0$ ($\mathcal{A}$) and

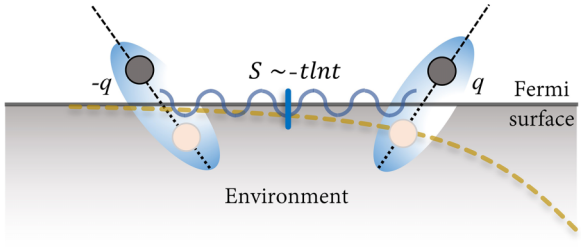



FIG. 4. The dissipative Luttinger liquid. Particle loss takes place only for the right mover for $t \ll 1$, a reflection of the non-Hermitian skin effect. The momentum-space von Neumann entropy shows the superballistic entanglement evolution.

$q < 0$ ($\mathcal{B}$) sectors. Similar to the noninteracting model above, the reduced correlation matrix for subspace $\mathcal{A}$ can be defined, with the eigenvalues, $\mathfrak{c}_q(t) = \mathrm{Tr}(a_q^\dagger a_q \rho_\mathcal{A})$, and the entanglement entropy after the partial trace satisfies [35]

$$S_\mathcal{A}(t) = -\sum_q \{\mathfrak{c}_q(t)\ln \mathfrak{c}_q(t) - \mathfrak{c}_q(t)\ln[\mathfrak{c}_q(t) + Z_t/Z_0] - Z_t/Z_0 \ln[\mathfrak{c}_q(0)+1]\}, \quad (12)$$

which is a bosonic counterpart of Eq. (8). Intuitively, $\mathfrak{c}_q(t)$, which approximately describes the number of bosons, is expected to increase with time for small $t$. This is because the particle loss lowers the Fermi energy of $H_0$, favoring particle-hole excitations in the dynamics. This is indeed verified by solving the equation of motion of the bosons, $a_{-q}(t)$, $a_q^\dagger(t)$, derived from Eq. (2), which yields [35,59] $\mathfrak{c}_q(t) \sim \gamma t$ for $t \ll 1/\gamma$. Then, Eq. (12) leads to $S_\mathcal{A}(t) \sim -t\ln t$.

*Conclusion and discussion.* In conclusion, we have revealed a universal Liouvillian dynamics of von Neumann entropy in dissipative quantum systems. The phenomenon takes place not only for weakly interacting systems with a finite quasiparticle weight of the original electrons, but also for strongly interacting cases with emergent bosonic excitations. This suggests that the found dynamics only relies on the *presence* of quasiparticles rather than their specific types. As a result, $S_\mathcal{A}(t)$ may offer a promising diagnostic for detecting non-Fermi-liquid behavior, hopefully in strange metals and unconventional quantum criticalities. It should be noted that although the rapid growth behavior is guaranteed by the non-Hermitian damping operator in Eq. (9), it is not in contradiction with the Lieb-Robison bound proved in nonunitary dynamics [60]. This is because the $S_\mathcal{A}(t)$ calculated here not does not reflect the propagation of information, but the entropy growth driven by the environment. Moreover, although 1D models are used for demonstration, our general proof is applicable to higher dimensions. Thus, the key conclusion still holds, as illustrated by the 2D case analyzed in the Supplemental Material [35]. Furthermore, it is also interesting to investigate the robustness of such Liouvillian dynamics in nonintegrable diffusive systems with disorders [61].

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[36] If loss and gain coexist, the $t$-leading term in $S_{\mathcal{A}}(t)$ would be different.
[37] As shown in Eq. (52) of the Supplemental Material [35], the specific expression of $S_{\mathcal{A}}$ in the long-time regime is $S_{\mathcal{A}}(t) = f(t)\exp(-\Lambda t)$, where $f(t) = a_0 + a_1 t + \mathrm{O}(t^2)$. Because of $a_0 \gg a_1$, $S_{\mathcal{A}}(t) \sim e^{-\Lambda t}$ usually dominates in the timescale where $S_{\mathcal{A}}(t)$ has observable finite values, although it asymptotically approaches zero obeying $\sim a_1 t e^{-\Lambda t}$ and the higher-order terms.
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