

Vibrationally coupled Rydberg atom-ion molecules

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We study the occurrence of Rydberg atom-ion molecules (RAIMs) in a hybrid atom-ion system with an ion crystal trapped in a Paul trap coupled to Rydberg atoms on its either ends. To assess the feasibility of such a system, we perform a detailed Floquet analysis of the effect of the Paul trap's rf potential on the RAIMs and provide a qualitative analysis of the survival probability based on scaling laws. We conclude that the RAIM survives for sufficiently weak and low-frequency traps. We then use this hybrid system and propose a scheme to utilize the common motional modes of the ion crystal to suppress (blockade) or enhance (antiblockade) the probability of forming two RAIMs at the ends of the chain, replacing the typical blockade radius by the length of the ion crystal.

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Recently, a new type of Rydberg molecule consisting of an ion and a Rydberg atom was proposed and observed in an ultracold cloud of rubidium atoms [1–4]. The binding mechanism is based on the flipping dipole induced in the Rydberg atom by the ion's electric field, which results in an atom-ion separation-dependent mixing of the different Rydberg states [5–12]. Molecular bound states are found in the potential wells located at the avoided crossings between the low orbital angular momentum Rydberg state (nS and nP) and the hydrogenic manifolds. These molecular ions can have remarkably large bond lengths on the order of a few μm and binding energies as low as a few MHz [1–4].

A natural question is whether we can extend the Rydberg atom-ion molecule to include more particles. In this letter, we study the possibility of replacing the single ion with a trapped ion crystal. We show that the Rydberg atom-ion molecule (RAIM) survives the effects of the Paul trap, provided the trap is sufficiently weak and low-frequency, and utilize the rich vibrational spectrum added by the additional ions to propose novel longer-range Rydberg blockade and antiblockade mechanisms.

The structure of the paper is as follows. We illustrate the system in Fig. 1(a)(i), where a two-ion chain interacts with Rydberg atoms placed at either end, to form two RAIMs. In Fig. 1(b), we plot the potential energy curves due to the Coulombic interaction between the ion and the Rydberg atom, assuming that the atomic core and the ion are stationary compared to the Rydberg electron [13]. The potential wells support RAIMs at specific values of the core-ion separation r_{ci} , one to three orders of magnitude smaller than the typical

ion-ion separation in the crystal. Next, we use Floquet formalism to study the effect of the Paul trap's rf drive on the RAIM, which can potentially induce transitions from the molecular state to unbound atomic states, as shown in Fig. 1(c). We use a simple qualitative model using scaling laws to understand the RAIM's survival probability and identify a broad range of Rydberg states and Paul trap drive frequencies where trap-induced losses will be negligible. Finally, we use the common motional modes of the trapped ion crystal to mediate interactions between the two Rydberg atoms. By resonantly driving specific transitions between phonon modes, we engineer an asymmetry in the level scheme, shown in Fig. 1(d), to realize either blockade or antiblockade over a range set by the ion crystal's length rather than the shorter typical blockade radius set by the Rydberg dipole-dipole interaction. This showcases the potential of hybrid atom-ion systems as an excellent platform for quantum technology.

Rydberg atom-ion molecule in a Paul trap. We consider the case where a singly charged ion of mass m_i is trapped in a Paul trap potential in the xy plane interacting with a Rydberg atom [as in Fig. 1(a)(ii)]. The position of the Rydberg atom's core defines the origin and the ion is at the Paul trap's rf null, $z = r_{\text{ci}}$. The Rydberg electron is at r_e and r_{ei} is the separation between the electron and the ion. Within the Born-Oppenheimer approximation, where the ion and the core are both considered stationary, the system is described by the time-dependent Hamiltonian [13–15]

$$\begin{aligned} H(r_{\text{ci}}, t) &= H_{\text{TI}}(r_{\text{ci}}) + H_{\text{P,rad}}(r_{\text{ci}}, t), \\ H_{\text{TI}}(r_{\text{ci}}) &= H_0 + H_{\text{ai}}(r_{\text{ci}}). \end{aligned} \quad (1)$$

Here, H_{TI} is the time-independent Hamiltonian with $H_0 = \frac{p_e^2}{2m_e} + V(r_e)$, $V(r_e)$ is the binding potential of the Rydberg atom, and H_{ai} is the Coulombic interaction between the Rydberg atom and the ion. We expand this interaction in the

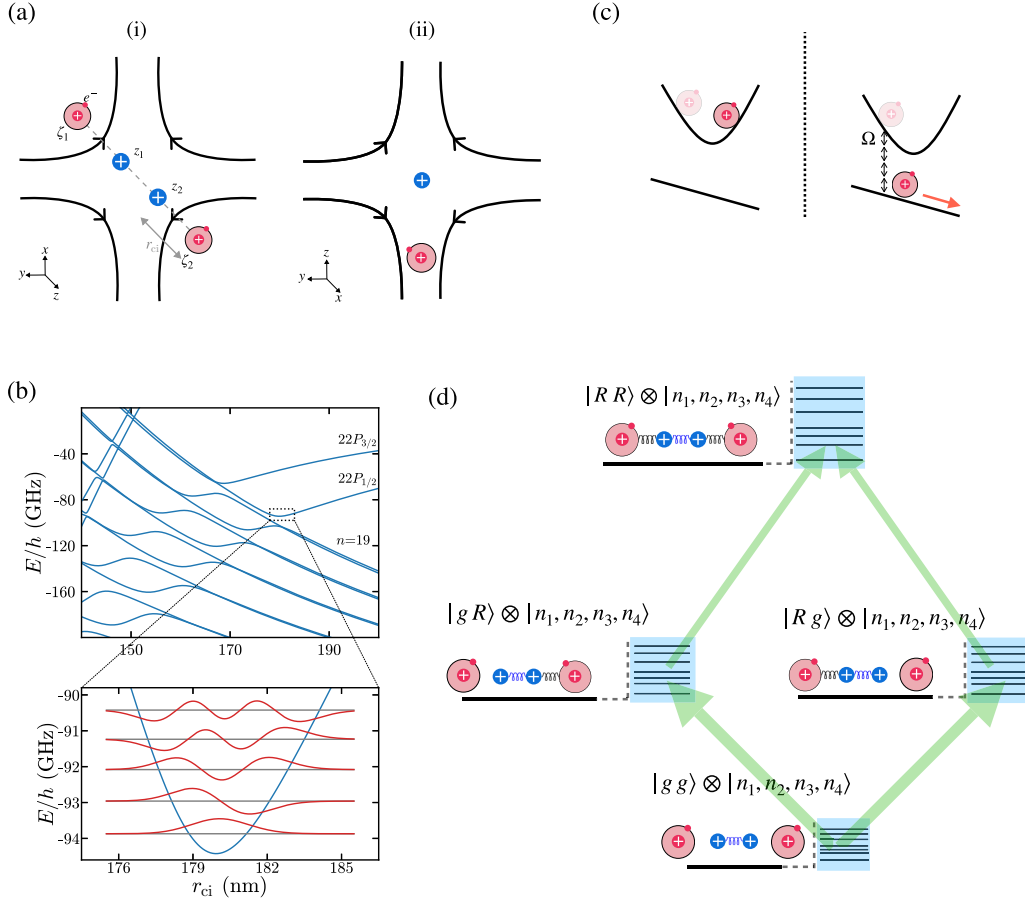


FIG. 1. (a) Two vibrationally coupled RAIMs aligned along the rf-null axis of a Paul trap potential in the xy plane (axial case) where z_1 , z_2 , ζ_1 , and ζ_2 represent the positions of the ions and the cores of the Rydberg atoms, respectively, is shown in (i). A RAIM aligned along the z axis of a Paul trap potential in the zy plane (radial case) is shown in (ii). (b) Potential energy curves of a $^{87}\text{Rb}^9\text{Be}^+$ RAIM for $|m_j| = 1/2$ in the vicinity of the $22P$ state is shown above and a magnified view of a part of the $22P_{1/2}$ potential well with its first five bound states is shown below. The energies are relative to the field-free atomic $22P_{3/2}$ state. (c) Schematic depicting a Rydberg atom escaping the well state by coupling with another state through a high-order photon process. (d) Energy levels of the four possible electronic states of the two atoms, each with its own ladder of motional states characterized by their respective phonon occupation numbers that can be exploited to mediate long-range Rydberg interactions.

multipole series truncated at the 64th pole ($l' = 6$)

$$H_{\text{ai}}(r_{\text{ci}}) = -\frac{e^2}{4\pi\epsilon_0} \sum_{l'=1}^6 \sqrt{\frac{4\pi}{2l'+1}} \frac{r_{\text{e}}^{l'}}{r_{\text{ci}}^{l'+1}} Y_{l'0}(\theta_{\text{e}}, \phi_{\text{e}}), \quad (2)$$

where e is the elementary charge, ϵ_0 is the vacuum permittivity, and $Y_{l'0}$ correspond to the spherical harmonics of degree l' and order 0. We numerically diagonalize H_{TI} at a given value of r_{ci} to obtain the eigenvalues $\{E_{\alpha}(r_{\text{ci}})\}$ and their respective eigenvectors $\{|\psi_{\alpha}(r_{\text{ci}})\rangle\}$ [13]. Figure 1(b) shows a typical spectrum analogous to the potential energy curves obtained in Ref. [2]. These curves can be interpreted as atom-ion molecular potentials, with the radial bound states of each of the wells in the spectrum corresponding to vibrational eigenstates of the atom-ion system at a particular angular momentum. In the following we will focus on states of one of these bound-state wells and use $|\psi_{\text{RAIM}}(r_{\text{ci}})\rangle$ to indicate its eigenstate at a given value of r_{ci} .

Next, we characterize the trap drive's effect on the energy spectrum of H_{TI} and quantify the survival probability of the RAIM in the trap. The Hamiltonian $H_{\text{P,rad}}(r_{\text{ci}}, t)$ describes the

Rydberg electron, Rydberg core, and the ion in the Paul trap potential which is given by [13]

$$H_{\text{P,rad}}(r_{\text{ci}}, t) = \frac{m_{\text{i}} q \Omega_{\text{rf}}^2}{4} \left(\frac{r_{\text{e}}^2 \sin^2 \theta_{\text{e}}}{2} - \frac{r_{\text{e}}^2 \sin^2 \theta_{\text{e}} \cos 2\phi_{\text{e}}}{2} - r_{\text{e}}^2 \cos^2 \theta_{\text{e}} + 2r_{\text{ci}} r_{\text{e}} \cos \theta_{\text{e}} \right) \cos(\Omega_{\text{rf}} t), \quad (3)$$

where q ($= |q_z| = |q_y|$) and Ω_{rf} are the dynamic stability parameter (measure of the electric field gradient in the trap) [16] and the trap drive frequency, respectively.

We use Floquet formalism to analyze the full Hamiltonian. The Floquet Hamiltonian at each r_{ci} is given by $\mathcal{H}_{\mathcal{F}} = \frac{i\hbar}{T} \ln(\mathcal{F}_0)$ where $\mathcal{F}_0 = \mathcal{T} \exp[\frac{-i}{\hbar} \int_0^T H(r_{\text{ci}}, \tau) d\tau]$ is the Floquet propagator ($\mathcal{T}$ denotes time ordering) and $T = 2\pi/\Omega_{\text{rf}}$. The eigenvalues $\{\epsilon_{\alpha}(r_{\text{ci}})\}$ and eigenvectors $\{|\phi_{\alpha}(r_{\text{ci}})\rangle\}$ of $\mathcal{H}_{\mathcal{F}}$ are the quasienergies and Floquet modes respectively. The rf-drive frequency sets the width of the band of Floquet quasienergies.

We restrict ourselves to a subspace of the Rydberg atom's internal states characterized by n , s , l , j , and fix m_j to one

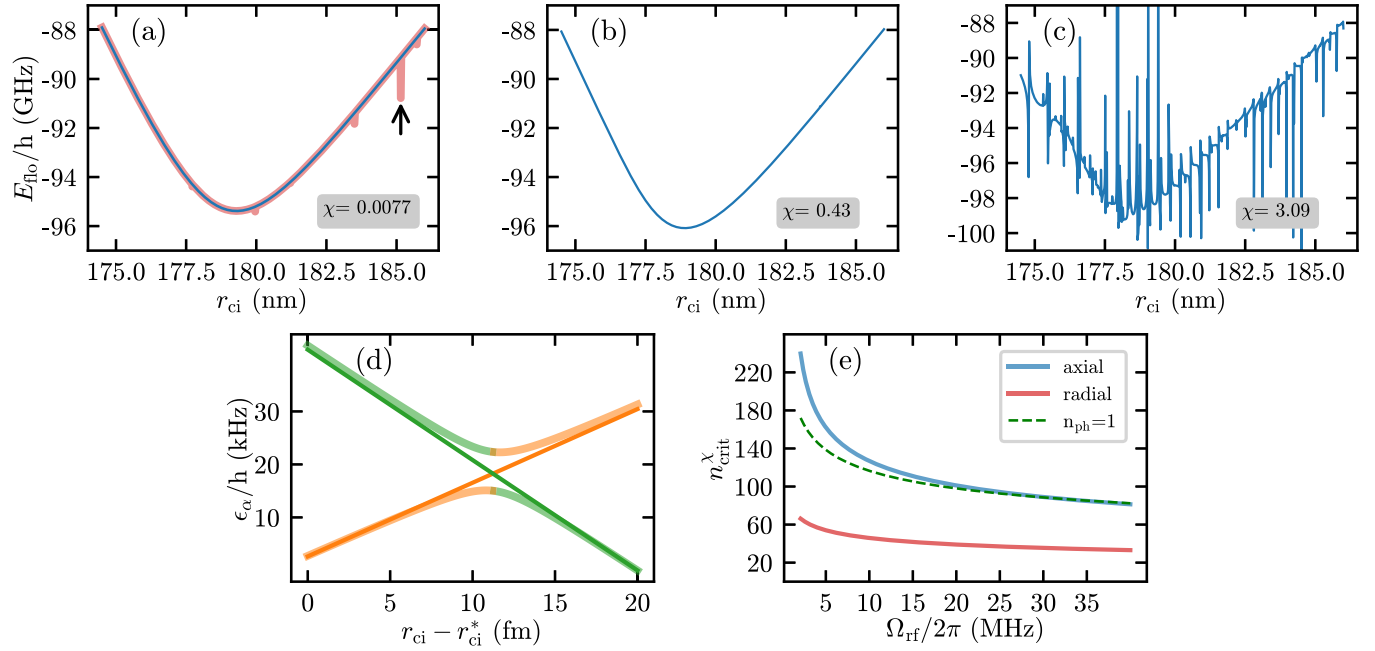


FIG. 2. Energy expectation value of the static Hamiltonian H_{TI} for the Floquet state $|\phi_{\text{RAIM}}\rangle$ (E_{flo}) that corresponds to the $22P_{1/2}$ molecular well state of a $^{87}\text{Rb}^9\text{Be}^+$ RAIM oriented along one of the radial axes of a sinusoidal Paul trap which traps $^9\text{Be}^+$ ions with $q = 0.1$ driven at (a) $2\pi \times 20$ MHz [100 (red transparent) and 8000 (blue) Trotter steps], (b) $2\pi \times 150$ MHz (8000 Trotter steps) and (c) $2\pi \times 400$ MHz (8000 Trotter steps). Energies are relative to the field-free atomic $22P_{3/2}$ state. The scan over r_{ci} was done at a resolution of 10 pm. (d) Avoided crossing between the quasienergy of the Floquet state that corresponds to the well state (orange) and to the other eigenstate (green) it couples with, at the core-ion distance (r_{ci}) marked by the black arrow ($r_{\text{ci}}^* = 185.15$ nm) in Fig. 2(a) for a sinusoidal trap. The apparent avoided crossing for 100 Trotter steps (thick transparent lines) disappears for 8000 Trotter steps (thin solid lines). (e) n_{crit}^{χ} plotted as a function of Ω_{rf} for the RAIM oriented along the radial (red) and rf-null (blue) axis. The area below the green dashed curve indicates the region of parameter space that needs a multiphoton process for a LZ transition.

of the two values $\pm 1/2$. We note that the Hamiltonian $H_{\text{p,rad}}$, with the exception of the second term which for $n = 22$ is a factor of ~ 1000 times smaller than the dominant term $2r_{\text{ci}}r_{\text{e}} \cos \theta_{\text{e}}$, does not couple sectors with different m_j values.

In general, if the Paul trap is a perturbation on the static terms, we can find one Floquet state with significant overlap with each eigenvector in $\{|\psi_{\alpha}(r_{\text{ci}})\rangle\}$, which also includes $|\psi_{\text{RAIM}}(r_{\text{ci}})\rangle$. We use this procedure to scan over a range of r_{ci} and at each point identify the Floquet state $|\phi_{\alpha}(r_{\text{ci}})\rangle$ with significant overlap with $|\psi_{\text{RAIM}}(r_{\text{ci}})\rangle$, which we call $|\phi_{\text{RAIM}}(r_{\text{ci}})\rangle$. Due to the relatively small values of Ω_{rf} compared to the Rydberg energy scales, the energy spectrum is highly compressed in the quasienergy spectrum, inhibiting a clear physical interpretation of their values. To circumvent this issue, we evaluate $E_{\text{flo}} = \langle \phi_{\text{RAIM}} | H_{\text{TI}} | \phi_{\text{RAIM}} \rangle$, which provides a useful picture for visualizing the role of couplings to other Rydberg states due to the Paul trap, see Fig. 2.

In Figs. 2(a)–2(c), we numerically calculate E_{flo} using first-order Trotter decomposition, in which we approximate the sinusoidal drive by small time steps of constant amplitude. The emergences of spikes in the energy herald the presence of avoided crossings in the quasienergies. Physically, these avoided crossings correspond to the absorption and emission of n_{ph} photons of frequency Ω_{rf} , which may cause the RAIM to dissociate [illustrated schematically in Fig. 1(c)]. As we increase the number of Trotter steps, the vast majority of these avoided crossings disappear like the example in Fig. 2(d),

which we associate with fictitious frequency components in the drive that emerge due to Trotterization.

While we expect the presence of the avoided crossings to reduce the RAIM's lifetime, we need to quantify how the increasing number of crossings with the trap drive [as seen in Fig. 2(a)–2(c)], which reduces the number of photons (n_{ph}) to couple to a nearby state and thus enhances its strength, affects the survival probability of the RAIM. Note that a fully numerical analysis becomes impractical for larger principal quantum number n , as the coupling between Rydberg states with different m_j values start becoming significant requiring a prohibitively large Hilbert space.

Instead, we identify parameter regimes where the RAIM survives, using an approach based on Landau-Zener-Stückleberg interferometry, which studies the effect of an oscillating field (e.g., the Paul trap here) on Landau-Zener (LZ) processes [17], aided by scaling arguments. To identify the critical parameters at which the RAIM no longer survives in the Paul trap, we consider an effective two-level treatment. This is well-justified by the fact that for relevant parameter regimes, the corresponding excitations involve multiphoton resonances, in which case the onset of LZ transitions will be dominated by the nearest state (in energy) which requires the least number of photons. This is illustrated by the fact that most of the resonances in Fig. 2(c) are equally spaced, corresponding to different numbers of photons for the same level.

The energy gap between the well state and the nearest (in energy) Rydberg state K at the location of the well minima is $\Delta E \approx 0.28E_H/n^4$, where E_H is the Hartree energy [13]. The extra power of n compared to the usual Rydberg energy separations ($\sim E_H/n^3$) comes from the conservation of m_j and the fact that each Stark fan contains n different angular momentum (l) states [18], while the prefactor is largely determined by the Rydberg atom's quantum defects. Since each photon has energy $\hbar\Omega_{\text{rf}}$, the minimum number of photons needed for a LZ transition is $n_{\text{ph}} \equiv \Delta E/\hbar\Omega_{\text{rf}}$. Thus, we find a principal quantum number $n = 0.727(E_H/(n_{\text{ph}}\hbar\Omega_{\text{rf}}))^{1/4}$ which can be used to indicate regions of parameter space (n, Ω_{rf}) which need more than a single photon for a LZ transition, as in Fig. 2(e).

When $n_{\text{ph}} \gtrsim 1$, single-photon processes become less important, and the probability of a LZ transition in each period is increasingly unlikely when $g \lesssim \Delta E$ [17], so we take $\chi = g/\Delta E \approx 1$ to define critical parameter values below which the RAIM is likely to survive. However, we note that parameters close to these critical values (particularly for few-photon processes) may warrant a more careful analysis to precisely determine the loss rates. While destructive interference for $\chi \gtrsim 1$ or resonances which occur at noninteger numbers of photons can reduce the probability of LZ transitions, the wave function of the RAIM spans many transition frequencies, so there will always be important LZ transitions when $\chi \gtrsim 1$.

When the RAIM is formed along one of the trap's radial axes [as in Fig. 1(a)(ii)], the coupling strength between the well state and the atomic state K is dominated by the last term in Eq. (3) and is given by $g_{K,\text{rad}} = m_i q \Omega_{\text{rf}}^2 d z_{e,K}/2$. Here, d represents the core-ion distance (r_{ci}) at the minimum of the $nP_{1/2}$ molecular well state of the RAIM that scales like $d \approx 1.85a_0 n^{5/2}$ [13] as a consequence of the so-called Inglis-Teller limit [18], where a_0 is the Bohr radius. We use the numerically obtained solutions at different n to extract the scaling law for the transition dipole moment $z_{e,K} \approx 0.6a_0 n^2$. Thus, we find $g_{K,\text{rad}} = 0.55m_i q \Omega_{\text{rf}}^2 a_0^2 n^{9/2}$, which gives a critical principal quantum number $n_{\text{crit,rad}}^{\chi} = 0.923(E_H/(m_i q \Omega_{\text{rf}}^2 a_0^2))^{2/17}$. For Figs. 2(b) and 2(c), $\chi_{\text{rad}} = 0.43$ and 3.09 , respectively, so they fall on either side of the critical Ω_{rf} . Since the former has negligible crossings and the latter has significant crossings, the Floquet analysis is consistent with our scaling argument.

When we consider the case where the RAIM is formed along the Paul trap's rf-null axis [as in Fig. 1(a)(i)], the coupling strength is given by $g_{K,\text{ax}} = m_i q \Omega_{\text{rf}}^2 \rho_{e,K}/4$, where $\rho_{e,K} = y_{e,K}^2 - x_{e,K}^2$ scales as $4.8a_0^2 n^{2.6}$. Thus, $g_{K,\text{ax}} = 1.2m_i q \Omega_{\text{rf}}^2 a_0^2 n^{2.6}$ which gives $n_{\text{crit,ax}}^{\chi} = 0.8(E_H/(m_i q \Omega_{\text{rf}}^2 a_0^2))^{0.15}$. In this case, the coupling g to the closest state is relatively small, so we consider the coupling due to the next-nearest states, which require several times as many photons for resonant excitations [13], although we continue to use the smallest ΔE as a more stringent condition on the survivability.

Phonon-mediated (anti) blockade. Next, we show how we can use the dependence of the ion crystal's collective motional modes on whether there are zero, one, or two RAIMs at its ends to extend the range of Rydberg (anti) blockade to the length of the ion crystal. We demonstrate our results using a crystal of two singly charged ions of mass m_i at z_1 and z_2

with two atoms of mass m_a placed at the crystal's opposite ends at ζ_1 and ζ_2 , respectively, such that $|\zeta_l - z_l| \approx d$ with $l = 1, 2$ indexing the ion-Rydberg pair forming the RAIM [see Fig. 1(a)(i)]. Although we focus on the two-ion case, our results can be generalized to longer ion crystals. We assume that the atoms are held in place using optical tweezers with a trap frequency ω_t . We approximate the RAIM potential near its minimum as harmonic with frequency ω_M . The ions are in a harmonic trap with tight transverse confinement, such that we may neglect the motion in this direction, and an axial trap frequency ω_i . Upon the application of a Rydberg excitation pulse using laser beams propagating along the trap's axis, the system can be in one of the four possible electronic states with zero, one, or two RAIMs. The potential energy of the crystal in the different electronic states is given by

$$V_{|o\rangle} = \frac{1}{2}m_i\omega_i^2(z_1^2 + z_2^2) - \frac{e^2}{4\pi\epsilon_0|z_1 - z_2|} + U_{|o\rangle}, \quad (4)$$

where $o \in \{o_1, o_2, o_3, o_4\} = \{gg, gR, Rg, RR\}$, $U_{|gg\rangle} = 0$, $U_{|gR\rangle} = U_{|Rg\rangle} = \frac{1}{2}\mu\omega_M^2(|z_l - \zeta_l| - d)^2$, and $U_{|RR\rangle} = U_{|gR\rangle} + U_{|Rg\rangle}$. We expect the RAIM's lateral motion to be too slow in comparison to the time scales in which we are interested ($\sim 10\mu\text{s}$) to have any significant impact, and we neglect it in this work. We find the normal modes $\mathbf{v}^{(o)}$ and corresponding mode frequencies $\omega^{(o)}$ for each of the four electronic states, as shown in Fig. 1(d) [13,19,20]. For a given electronic state, the motional state is characterized by $N = (n_1, n_2, n_3, n_4)$ which represents the occupation numbers of the four modes in descending order of their mode frequencies.

As an example, we consider a system with a crystal of two ${}^9\text{Be}^+$ ions with an axial trap frequency $\omega_i = 2\pi \times 1\text{ MHz}$ and two ${}^{87}\text{Rb}$ atoms placed at either ends at a distance $\sim d_{50P_{1/2}} (\approx 1.735\mu\text{m})$ from the ion crystal using optical tweezers with trap frequency $\omega_t = 2\pi \times 0.2\text{ MHz}$. We use a Rydberg pulse to excite to the $50P_{1/2}$ molecular well state of the ${}^{87}\text{Rb} {}^9\text{Be}^+$ RAIM, which has $\omega_M = 2\pi \times 36\text{ MHz}$.

We prepare the system in the motional ground state of the trap, $|\psi(0)\rangle = |gg, 0000\rangle$. For blockade, we resonantly drive transitions from $|gg, 0000\rangle$ to $|gR, 0010\rangle$ using a laser at a frequency ω_1 , as shown schematically in Fig. 1(d). Although there are a multitude of levels in the $|RR\rangle$ manifold that could potentially resonantly couple to $|gR, 0010\rangle$, their overlaps are small, and thus $|RR\rangle$ remains unpopulated. In this way, we exploit both the existence and long-range nature of the motional modes $|gR, N\rangle$ to realize an effective asymmetry in the level spectrum. In Fig. 3(a) we demonstrate this blockade mechanism by simulating the dynamics under

$$\begin{aligned} H &= H_d + H_{\text{nd}}, \\ H_{\text{nd}} &= \frac{\hbar}{2} \sum_{N, N'} \Omega_L (\hat{\Sigma}_{1, NN'} + \hat{\Sigma}_{2, NN'}) + \text{H.c.}, \\ H_d &= \hbar \sum_{o, N} \omega_{o, N} |o, N\rangle \langle o, N|, \end{aligned} \quad (5)$$

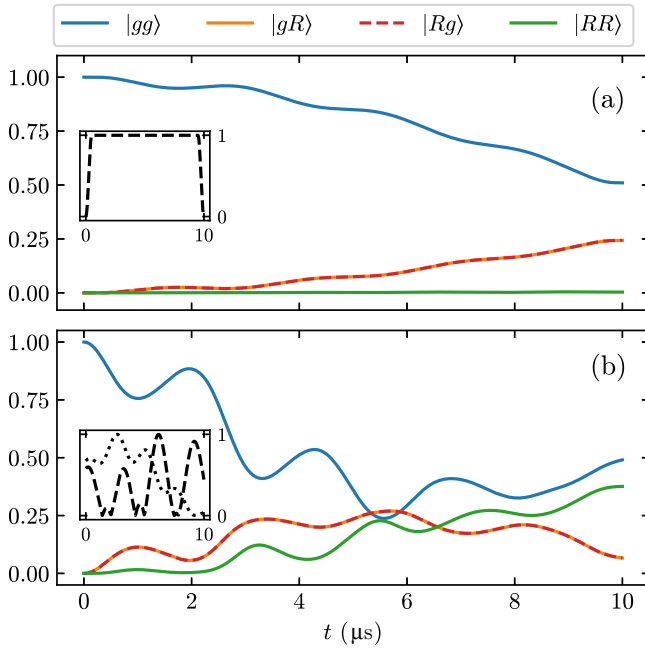


FIG. 3. Population dynamics of $|gg\rangle$ (blue), $|gR\rangle$ (orange), $|Rg\rangle$ (dashed red), and $|RR\rangle$ (green). (a) We demonstrate blockade by using only a single driving field at ω_1 with a square pulse shape (inset). (b) Dynamics of the antiblockade protocol which uses two drive fields with their optimized pulse shapes (inset) at frequencies ω_1 (dotted) and ω_2 (dashed).

where $\Omega_L = \Omega_1 e^{i\omega_1 t}$ and the operators describing the transitions to Rydberg states are given by

$$\begin{aligned}\hat{\Sigma}_{1,NN'} &= S_{NN'}^{o_1 o_2} |gg, N\rangle \langle gR, N'| + S_{NN'}^{o_2 o_3} |gg, N\rangle \langle Rg, N'|, \\ \hat{\Sigma}_{2,NN'} &= S_{NN'}^{o_1 o_4} |gR, N\rangle \langle RR, N'| + S_{NN'}^{o_3 o_4} |Rg, N\rangle \langle RR, N'|.\end{aligned}$$

Here, the prefactors represent the wavefunction overlap integral between the Fock state N of the electronic state o_a and the Fock state N' of the electronic state o_b when the system is subjected to a Rydberg excitation pulse using a laser beam propagating along the system's axis, i.e., $S_{NN'}^{o_a o_b} = \langle o_a, N | e^{ik\hat{z}_i} | o_b, N' \rangle$, where k is the (effective) wave number of the Rydberg laser. We set a Rabi frequency $\Omega_1 = 2\pi \times 0.057$ MHz and simulate evolution under a square pulse. The Rabi frequency was chosen based on the wavefunction overlap integrals to complete the state preparation in $10 \mu\text{s}$ which is well within the lifetime of the RAIM $\tau \approx 100 \mu\text{s}$ (the radiative lifetime of $50P$ state at 300 K) [1]. At $t_f = 10 \mu\text{s}$, our protocol prepares the entangled state $\psi(t_f) \approx 1/\sqrt{2}|gg\rangle + 1/2(|gR\rangle + |Rg\rangle)$ with the maximum population of $|RR\rangle$ during the protocol $\max_{0 \leq t \leq t_f} [|\langle RR | \psi(t) \rangle|^2] = 0.004$. We therefore realize Rydberg blockade with a longer-range set by the ion crystal's length.

To realize antiblockade, we use an additional laser at frequency ω_2 with strength Ω_2 , i.e., $\Omega_L = \Omega_1 f_1(t) e^{i\omega_1 t} + \Omega_2 f_2(t) e^{i\omega_2 t}$ [$f_{1,2}(t)$ are the pulse shapes], to resonantly drive the transition between $|gR, 0010\rangle$ (and $|Rg, 0010\rangle$) and $|RR, 0001\rangle$. We choose to drive to $|RR, 0001\rangle$ due to its high overlap with $|gR, 0010\rangle$ and $|Rg, 0010\rangle$. We use Rabi frequencies $\Omega_1 = 2\pi \times 0.153$ MHz and $\Omega_2 = 2\pi \times 0.14$ MHz. To minimize the impact of off-resonant coupling to levels in the $|gR\rangle$, $|Rg\rangle$ manifolds while still using a sufficiently large Rabi frequency to reduce the protocol time t_f , we numerically optimize the pulse shapes $f_i(t)$ of the driving fields [13,21]. In Fig. 3(b), we plot the dynamics (colors) evolving under Eq. (5) and the optimized pulse shapes (black lines). At $t_f \approx 10 \mu\text{s}$, we prepare an entangled state of $|gg\rangle$ and $|RR\rangle$ such that the populations $|\langle gR | \psi(t_f) \rangle|^2 = |\langle Rg | \psi(t_f) \rangle|^2 = 0.07$ are small.

In summary, we showed that the recently observed molecular bound state between a Rydberg atom and an ion can survive in the presence of the rf drive of a Paul trap. We presented an extensive numerical study, utilizing the Floquet formalism, to demonstrate that while strong trap drives may cause the RAIM to dissociate, the molecule remains stable for an extended region of parameter space. Furthermore, we presented an analytic scaling relation which can be applied as a rule of thumb to a variety of systems to determine their stability regimes. Finally, we devised a protocol utilizing the modification to the collective motion of the trapped ion crystal due to the formation of the RAIM on the ends of an arbitrary-length crystal to generate phonon-mediated, long-ranged, Rydberg (anti)blockade. Our analysis presents a new exploratory direction for hybrid Rydberg-ion quantum simulation platforms, where the scalability of currently available neutral atom platforms and the long-range connectivity of ion chains can be combined. The ion chain effectively acts as a bus that transports quantum information between two (ensembles of) neutral atoms, and it could be viewed as a viable alternative to the tweezer shuttling techniques used for long range connectivity in neutral atom platforms. Moreover, the quantized motion in the collective motional modes can be used to simulate quantum many-body systems involving bosonic degrees of freedom, such as spin-boson models.

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