

Measuring a localization phase diagram controlled by the interplay of disorder and driving

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The interplay of various localizing mechanisms is a central topic of modern condensed matter physics. In this work we experimentally explore the interplay between quasiperiodic disorder and periodic driving, each of which in isolation is capable of driving a metal-insulator phase transition. Using a 1D quasiperiodic cold-atom chain we measure transport across the full phase diagram varying both drive strength and quasidisorder strength. We observe lobes of metallic phases bounded by quantum phase transitions which depend on both drive and disorder. While these observations are broadly consistent with expectations from a high-drive-frequency theoretical model, we also observe clear departures from the predictions of this model, including anomalous changes in localization behavior at lower drive frequency. We demonstrate experimentally and theoretically that understanding the full measured phase diagram requires an extension to commonly used approximate theories of Floquet matter.

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The interplay of localizing phenomena is a topic perennially at the forefront of the study of quantum condensed matter. In this work, we experimentally and theoretically investigate the interplay of two independent localizing influences: periodic driving and quasiperiodic pseudodisorder. A time-periodic driving force applied at specific amplitudes can cause dynamic localization by flattening out a Floquet band [1]. Separately, quasiperiodic pseudodisorder can drive a localization-delocalization phase transition via Anderson-insulator-like interference effects, as expressed in the 1D Aubry-André-Harper (AAH) model [2–6]. This work aims to explore open questions concerning the simultaneous application of both of these localizing influences to a quantum system: how does the driving affect the Anderson-like insulator? Does quasidisorder disrupt dynamic localization? What phase diagram arises from the interplay of the two effects? How does the phase diagram depend on drive frequency? Such questions are important not only for applications in novel Floquet engineering but also for expanding our understanding of light-induced dynamical control of solids.

Previous work has theoretically explored transport in a driven disordered lattice [7–10] and experimentally investigated the effects of disorder in the vicinity of dynamic localization in waveguides [11]. These studies were generally restricted to the high-frequency limit.

The experiments begin by loading a Bose-Einstein condensate of 8×10^4 ^{84}Sr atoms into a pair of superimposed 1D optical lattices with different spatial periods aligned along an

axis perpendicular to gravity. The primary (secondary) lattice is formed by two counterpropagating beams of wavelength $\lambda_p = 1064$ nm ($\lambda_s = 874.61$ nm $\equiv \lambda_p/\beta$). All four beams pass through individual acousto-optic modulators. This allows for control of the lattice depths, denoted V_p and V_s for the primary and secondary lattices, and for controlled translation of the primary (secondary) lattice by adjusting the frequency difference $\Delta\nu_p$ ($\Delta\nu_s$) between the two beams forming the lattice, which both have frequency ν_p (ν_s) otherwise [Fig. 1(a)]. The resulting ability to rapidly tune the depth and spatial phase of both lattice potentials enables the simultaneous experimental realization of dynamic and quasidisorder-induced localization. More experimental details appear in [12].

Synchronized sinusoidal translation of both lattices is conveniently treated by transforming to the comoving frame, in which the modulation appears as a time-varying electric field. We focus on the case where the secondary lattice is at rest in the comoving frame of the primary lattice, implemented by fixing $\Delta\nu_s = \beta\Delta\nu_p$. This admits a description by the tight-binding Hamiltonian [9]

$$H = \sum_l -J(|l+1\rangle\langle l| + \text{H.c.}) + \Delta \cos(2\pi\beta l - \delta) |l\rangle\langle l| + \hbar\omega K_0 \cos(\omega t) l |l\rangle\langle l|, \quad (1)$$

where J is the tunneling energy, Δ is the quasidisorder strength, K_0 is the dimensionless drive strength, ω is the angular drive frequency, $\beta = \lambda_p/\lambda_s = 1064/874.61$, and the phason parameter δ is constant. The tight-binding approximation is justified when V_p and V_p/V_s are sufficiently large that only nearest-neighbor tunneling contributes significantly, as is the case in our experiments [13,14]. V_p is chosen to be $9E_{R,p}$, leading to $J = 0.024E_{R,p} = \hbar \times 50$ Hz, where $E_{R,p} = 2\pi^2\hbar^2/m\lambda_p^2$ is the recoil energy of the primary lattice. We explore a range of V_s values, from 0 up to $0.59E_{R,p}$. Confinement transverse to the lattice axis allows the state to be well

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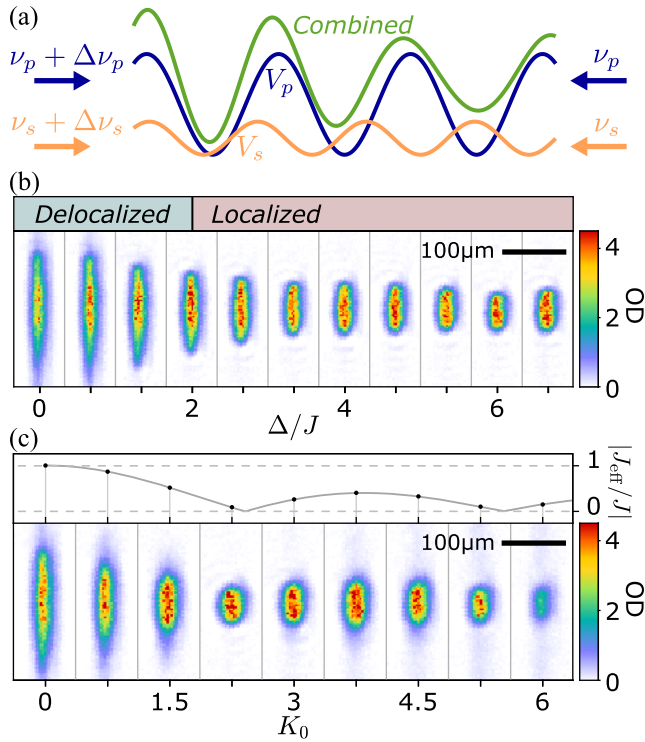


FIG. 1. Observing two kinds of localization. (a) Primary (blue) and secondary (orange) lattices combine to create a quasiperiodic potential (green) which can be translated via frequency modulation. (b) Atoms imaged after 500 ms hold time for different secondary lattice strengths demonstrate AAH localization at $\Delta/J > 2$. (c) Atoms imaged after 500 ms hold time for different drive strengths K_0 demonstrate dynamic localization. Top panel indicates theoretically predicted $|J_{\text{eff}}|$. The lattice wave vector is in the vertical direction in (b) and (c).

approximated as a product of a 1D component obeying Eq. (1) and a static 2D transverse state [12].

Two special cases of Hamiltonian (1) correspond to the two localization mechanisms previously mentioned. When $K_0 = 0$, the Hamiltonian becomes the static AAH model. For irrational β [15], all eigenstates of this Hamiltonian exhibit a localization-delocalization quantum phase transition at $\Delta/J = 2$. Figure 1(b) experimentally demonstrates the phase transition of the AAH model, showing *in situ* absorption images taken after 500 ms of expansion in the bichromatic lattice potential for $K_0 = 0$ with Δ/J values ranging from 0 to 6.7. As in Ref. [3], the phase transition is observed in the suppression of expansion for values of $\Delta/J > 2$. We note that the existence of a mean-field interaction causes subdiffusion [16,17], signaling an insulating phase in the noninteracting limit. If instead $\Delta = 0$ in Hamiltonian (1), dynamic localization can occur [18–21]. Applying Floquet theory [22,23] results in a modified tight-binding Hamiltonian with effective tunneling energy $J_{\text{eff}} = J\mathcal{J}_0(K_0)$ [18]. Crucially, hopping is entirely suppressed for dimensionless drive amplitudes corresponding to zeros of the Bessel function $\mathcal{J}_0$, for example at $K_0 \approx 2.405$ and 5.520 . Figure 1(c) shows *in situ* absorption images taken after 500 ms of expansion for varying drive amplitudes at $\Delta = 0$ and $\omega = 2\pi \times (1 \text{ kHz}) = 20J/\hbar$.

Expansion is initiated by sudden removal of a confining optical dipole trap, simultaneous with the beginning of sinusoidal phase modulation. These data clearly demonstrate the suppression of transport due to dynamic localization for K_0 close to the zeros of $\mathcal{J}_0(K_0)$.

Having verified that the experimental system can exhibit both quasidisorder-induced localization and drive-induced localization, we proceed to investigate the main topic of this work, which is the interplay of these two phenomena. We measured the width of the atomic density distribution after 500 ms of expansion, with quasidisorder strength Δ and drive amplitude K_0 independently varied to map out a 2D parameter space. Figures 2(a) and 2(b) present experimental measurements of the localization phase diagram for a drive frequency of 1 kHz ($\hbar\omega = 20J$) and 200 Hz ($\hbar\omega = 3.9J$), respectively. For comparison, Figs. 2(c) and 2(d) present numerically determined average inverse participation ratio (IPR), calculated by averaging the IPR values of the Floquet eigenstates as detailed in [12].

The basic structure of Figs. 2(a) and 2(c) can be understood by applying the unitary transformation $\hat{U} = \exp\{-iK_0 \sin(\omega t) \sum_l |l\rangle\langle l|\}$ to Hamiltonian (1). This converts the oscillating linear force to an oscillating phase in the tunneling strength [18,24,25]:

$$H(t) \rightarrow H'(t) = \sum_l -J(e^{iK_0 \sin(\omega t)} |l+1\rangle\langle l| + \text{H.c.}) + \Delta \cos(2\pi\beta l - \delta) |l\rangle\langle l|. \quad (2)$$

With the high-frequency approximation ($\hbar\omega \gg 4J, 2\Delta$), the dynamics can be well captured by the time-averaged Hamiltonian:

$$H_{\text{eff}} \approx \sum_l -J_{\text{eff}}(|l+1\rangle\langle l| + \text{H.c.}) + \Delta \cos(2\pi\beta l - \delta) |l\rangle\langle l|. \quad (3)$$

Equation (2) is the AAH model with a Bessel-renormalized hopping amplitude J_{eff} . Thus, the localization phase transition occurs at $\Delta_c = 2|J_{\text{eff}}| = 2|J\mathcal{J}_0(K_0)|$. This theoretically predicted phase boundary is indicated by dashed curves in Fig. 2.

The first main result of this work is the experimental observation of the predicted delocalized lobes in the region $|\Delta/J_{\text{eff}}| < 2$. The data clearly exhibit a drive-amplitude-dependent localization-delocalization phase transition, featuring metallic lobes separated by fingers of an insulating phase touching the $\Delta = 0$ axis at the zeros of the Bessel function $\mathcal{J}_0(K_0)$ where $J_{\text{eff}} = 0$. The analytic $\Delta/|J_{\text{eff}}| = 2$ curves correspond very well to the transition observed both in experimental expansion data [Figs. 2(a) and 2(b)] and in numerically calculated (IPR) [Figs. 2(c) and 2(d)]. The transition between phases is intrinsically somewhat less sharply seen in measured change in width $\sigma - \sigma_0$ than (IPR) because $\sigma - \sigma_0$ scales with the effective hopping time $\hbar/|J_{\text{eff}}|$ even in delocalized regions. The form of the measured high-frequency phase diagram of Fig. 2(a) agrees with both the numerical theory of Fig. 2(b) and the analytic approximation of Eq. (3), indicating that these data constitute observation of the phase boundaries arising from the interplay between quasidisorder and driving.

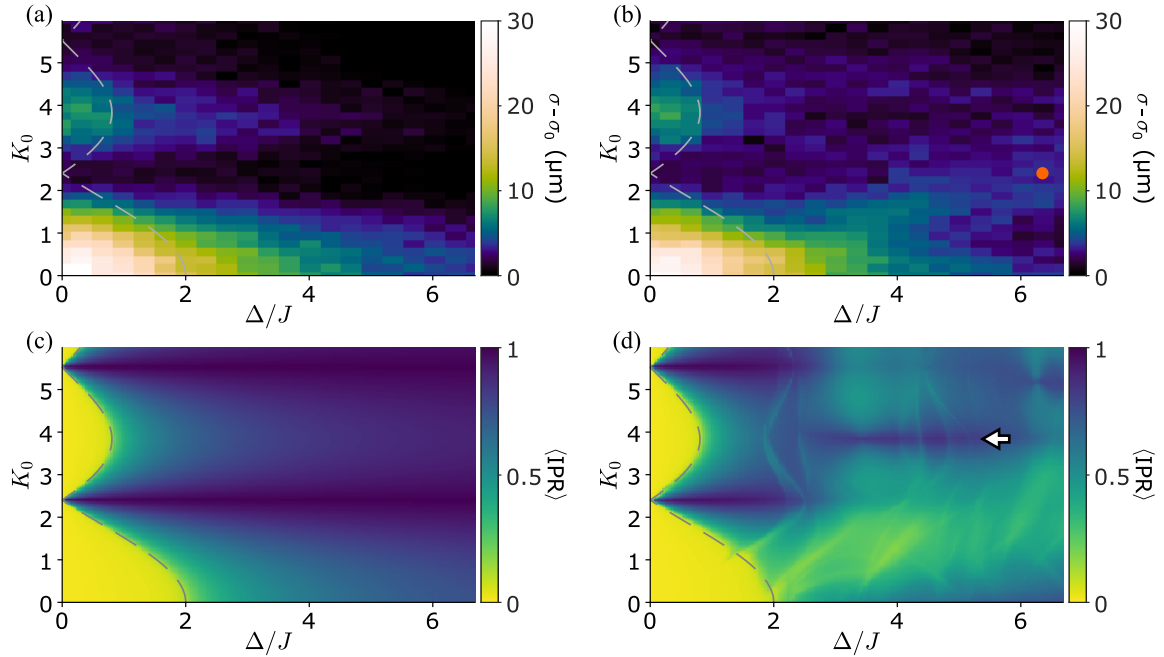


FIG. 2. Interplay of dynamic localization and quasidisorder-induced localization in two different drive frequency regimes. (a) Measured width σ (standard deviation of a Gaussian fit) of the atomic density distribution after a 500 ms hold minus its initial value $\sigma_0 = 10 \mu\text{m}$, for a high drive frequency $\omega = 2\pi \times (1 \text{ kHz}) = 20J/\hbar$. (b) Same quantity for a lower drive frequency $\omega = 2\pi \times (200 \text{ Hz}) = 3.93J/\hbar$, near the width of the ground band. The orange dot marks the experimental condition in Fig. 4. (c) and (d) show numerically calculated average Floquet eigenstate IPR for the 1 kHz drive and the 200 Hz drive, respectively. In all panels, dashed lines indicate phase boundaries predicted by the approximate model of Eq. (3). In the region $\Delta/J > 2$, we observe clear differences between (a) and (b) and between (c) and (d), indicating differing Floquet eigenstate properties in the two regimes of drive frequency. $V_p = 9E_{R,p}$ for all panels. The white arrow in (d) indicates the observed region of enhanced localization, and corresponds to the arrow in Fig. 3(a).

For lower drive frequencies, comparable to the bandwidth, new features appear in both the experimental and numerical phase diagrams which are not captured by the model of Eq. (3). The observation and characterization of these new features constitutes the second main result of this work. These differences are apparent in Figs. 2(b) and 2(d). We visualize the differences more starkly in Fig. 3(a), which shows the ratio between the measured low-frequency and high-frequency widths, denoted $\sigma_{200\text{Hz}}/\sigma_{1\text{kHz}}$. We observe the dominant deviation from the high-frequency behavior to occur when $\Delta/J \gtrsim 2$. We note in particular two features, which appear in both the experimental data of Figs. 2(b) and 3(a) and the numerical results of Fig. 2(d): a narrow band of anomalously low expansion near the line $K_0 \approx 3.8$ for larger values of Δ/J , and a broad area of anomalously high expansion in the lower-right of the parameter space. We interpret these features as a signature of nontrivial phenomena not captured by the simple high-frequency approximation.

To further investigate the drive frequency dependence of these features of the phase diagram, we took similar measurements of the expansion with a drive frequency of 300 Hz ($\hbar\omega = 5.90J$), but otherwise identical experimental conditions, presented in [12]. The ratio between the measured widths at 300 Hz and the 1 kHz data set, denoted $\sigma_{300\text{Hz}}/\sigma_{1\text{kHz}}$, appears in Fig. 3(b). We observe the same general features at this slightly higher drive frequency, but the extent and magnitude of the deviation is reduced, indicating the approach toward the high-frequency limit.

The first anomalous feature of the low-drive-frequency phase diagram, the observed enhanced localization near $K_0 \approx 3.8$, corresponds closely to a zero of the first Bessel function $\mathcal{J}_1(K_0)$. In the high-frequency limit, the nonzero-frequency Bessel terms average away over many cycles. However, when $\hbar\omega \approx 4J$, higher-order terms from the Jacobi-Anger expansion contribute,

$$e^{iK_0 \sin(\omega t)} = \sum_{m=-\infty}^{\infty} \mathcal{J}_m(K_0) e^{im\omega t},$$

with the $\mathcal{J}_1(K_0)$ term the most relevant term past leading order. We thus interpret the observed localized stripe near $K_0 \approx 3.8$, indistinguishable within experimental error from the first zero of the $\mathcal{J}_1$ Bessel function near 3.8317, as the effect of a beyond-leading-order localization process not captured in the commonly used high-frequency approximate description of dynamic localization.

The second anomalous feature, the apparent decrease in localization in the lower-right quadrant of the low-drive-frequency phase diagram, requires a separate explanation. This feature could in principle arise from true delocalization of the Floquet eigenstates or from the existence of an extended critical phase. However, as discussed below and in the Supplemental Material [12], additional measurements of transport dynamics and numerical IPR calculations indicate that this region remains a localized phase in the presence of driving, but with an increased localization length.

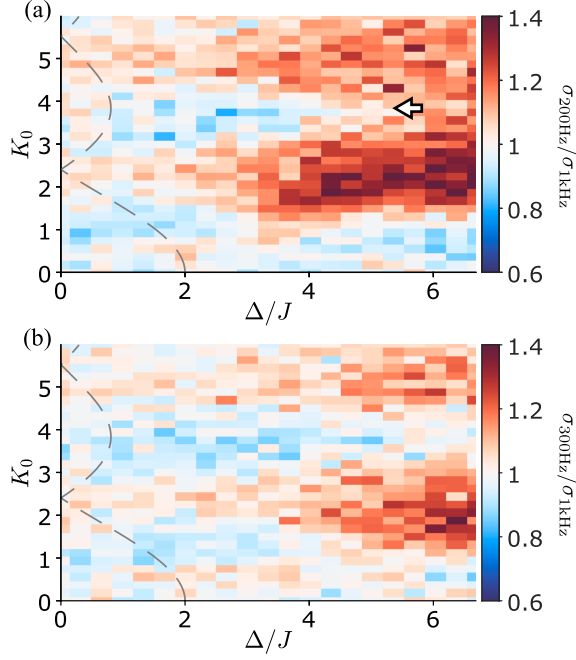


FIG. 3. Measured ratio between high and low-frequency phase diagrams. (a) Ratio between the expansion with drive frequency $\omega = 2\pi \times (200 \text{ Hz}) = 3.93J/\hbar$ and $\omega = 2\pi \times (1 \text{ kHz}) = 20J/\hbar$. The data used to generate (a) is the same as that presented in Fig. 2. (b) Ratio between data taken at drive frequency $\omega = 2\pi \times (300 \text{ Hz}) = 5.90J/\hbar$ and the same 1 kHz data. The white arrow in (a) is the same as that in Fig. 2(d) and indicates the observed region of enhanced localization.

To explore the nature of this region of the phase diagrams, we choose the representative point $\Delta/J \approx 6.35$ (a strong quasiperiodic potential) and $K_0 \approx 2.4046$. The considered potential alone in the absence of driving and interactions is well in the localized phase of the AAH model; the considered drive alone essentially produces dynamic localization ($\mathcal{J}_0(K_0) = 1.3 \times 10^{-4}$) in the absence of quasiperiodicity. To experimentally investigate the interplay between quasidisorder-induced and dynamic localization at various drive frequencies, we conduct transport measurements observing the evolution of the width of the condensate for up to two seconds ($\approx 639.4\hbar/J$). The results are shown in Fig. 4, along with measurements of the typical fast expansion in a static periodic lattice ($\Delta = 0$), shown with diamond markers in Fig. 4(a) as a point of comparison.

We observe in the data of Fig. 4(b) that when the driving frequency is low, as for $\omega = 2\pi \times 200 \text{ Hz}$, the measured evolution follows that of the static quasiperiodic system (denoted as $\omega = 0$). The expansion slows further as the driving frequency increases. Notably, the expansion is totally arrested for the duration of the experiment in the high-frequency case ($\omega = 2\pi \times 1 \text{ kHz}$). In all cases reported, the measured expansion is slower than for the localized phase of the static quasiperiodic case. We extract a transport exponent γ for each drive frequency from the data in Fig. 4 and present it in Table I. We find that the expansion is subdiffusive as in the static quasiperiodic case, and is reduced at high drive frequencies. These observations suggest that this anomalous region is still a localized phase.

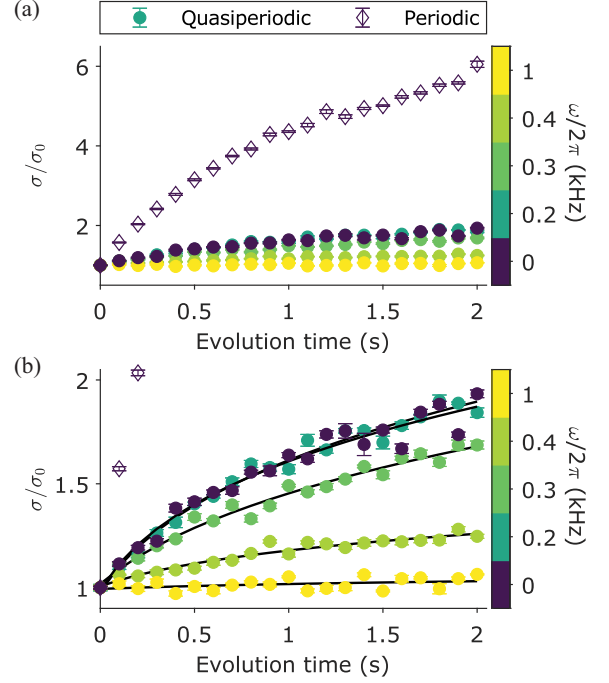


FIG. 4. Measured expansion versus time in various regimes. (a) Expansion in a purely periodic lattice (diamond markers) is much more rapid than that in the quasiperiodic lattice in the localized phase (circular markers). (b) Same data as in (a) with the y axis expanded to show variation among the expansions in quasiperiodic potentials. Different driving frequencies are represented by the color of the marker. Each data point is averaged from five experimental realizations. Here $\Delta/J = 6.35$ and $K_0 \approx 2.4046$. Error bars show standard error of the mean.

The experimental results are supported by calculations of the scaling of IPR versus system size, which indicate the phase to be fully localized. Numerical calculations of the IPR indicate that this anomalous region is distinguished by larger localization lengths which are more strongly dependent on quasienergy than in the high-frequency case. This is in stark contrast to the static AAH model or the high-frequency approximation of the driven case, in which the localization lengths of the eigenstates, and thus their IPR, are independent of the (quasi)energy. Further details on the experimental and numerical analysis are provided in the Supplemental Material [12]. Therefore, although the expansion of the condensate in the low-frequency regime resembles that of the static quasiperiodic lattice, their mechanisms are completely different.

In summary, we have experimentally and theoretically investigated the interplay of dynamic and quasidisorder-induced localization. In the high-frequency regime, the measured

TABLE I. Transport exponents γ extracted by fitting the expansion of the condensate in a quasiperiodic lattice under various driving frequencies. Error bars represent 95% confidence interval.

$\omega/2\pi$ (kHz)	0	0.2	0.3	0.4	1
γ	0.24(5)	0.26(5)	0.26(7)	0.12(6)	0.03(21)

phase diagram is consistent with both numerical and time-averaged analytic theory, featuring metallic Bessel lobes separated from insulating regions by a phase boundary which depends on both drive strength and quasidisorder strength. For lower frequency driving, comparable to the bandwidth, we observe localization behavior not captured by the simple time-averaged analytic model, including areas of both enhanced and suppressed transport, and show that it can be understood via extensions to this simple high-frequency model. This work touches on some of the rich structures that can arise when quantum systems are driven at frequencies near their natural energy scales, which may prove relevant for both broadening the applicability of Floquet engineering techniques and for understanding results of optically driven solid-state experiments. A natural direction for future work is the addition of tunable

interactions, enabling experimental probes of the interplay among three distinct fundamental localizing mechanisms of many-body quantum matter: interactions, driving, and disorder [26].

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