

Toward resolving the discrepancy in helium-3 and helium-4 nuclear charge radii

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The discrepancy in the squared nuclear charge radius difference, ΔR^2 , between ^3He and ^4He , as determined from electronic and muonic atom energy levels, presents an unresolved puzzle. This paper shows that accounting for off-diagonal hyperfine mixing effects can substantially reduce this discrepancy. We find that hyperfine mixing with the n^3S and n^1S states ($n > 2$) in ^3He introduces a correction of -1.37 kHz to the isotope shift of the $2^1S - 2^3S$ transition, a factor of seven times larger than the current uncertainty. This correction modifies ΔR^2 by -0.0064 fm², shifting it from $1.0757(15)$ fm² to $1.0693(15)$ fm², as initially reported by Werf *et al.* [arXiv:2306.02333]. This brings ΔR^2 closer to the value of $1.0636(31)$ fm² obtained from muonic helium μHe^+ by Schuhmann *et al.* [arXiv:2305.11679], narrowing the existing discrepancy from 3.6σ to 1.7σ . The adjusted value ΔR^2 also agrees well with the result of $1.069(3)$ fm² derived from the helium $2^3S - 2^3P$ transitions, as analyzed by Patkoš *et al.* [Phys. Rev. A **94**, 052508 (2016)]. Our results provide key insights for resolving the discrepancy in ΔR^2 .

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Introduction. The proton size puzzle has attracted a tremendous amount of attention in the physics community (see Refs. [1–3] and references therein) and has stimulated experimental searches for similar anomalies in other light atomic nuclei such as D, ^3He , and ^4He [4–8]. For $^3,4\text{He}$ nuclei, the puzzle arises from an apparent discrepancy between the values derived from electronic energy levels in pure helium atoms and those derived from the energy levels of muonic helium ions. For instance, the difference of squared nuclear charge radii of $^3,4\text{He}$ is measured as $1.0757(15)$ fm² from electronic $^3,4\text{He}$ energy levels [5,6], while the value obtained from muonic $\mu^{3,4}\text{He}^+$ [7] is $1.0636(31)$ fm², showing a 3.6σ discrepancy. This discrepancy is puzzling, and Schuhmann *et al.* [7] have suggested that it could potentially be attributed to physics beyond the Standard Model. Recently, Muli *et al.* [9] performed additional calculations of nuclear structure corrections in an effort to resolve this discrepancy; however, the puzzle remains unsolved, and the discrepancy has increased from 3.6σ to 4σ .

Currently, precise spectroscopic measurements represent a primary approach for obtaining detailed information about

nuclear structure [10–16]. These measurements can provide data on key nuclear properties such as the nuclear charge radius, Zemach radius, electric quadrupole moment, and related characteristics. While the accuracy of current experimental techniques is sufficient to reveal nuclear size effects in $^3,4\text{He}$ [17–19], direct determination of their nuclear charge radii is hindered by limitations in theoretical precision [20–28]. Thus, calculating the isotope shifts in transition frequencies has emerged as a powerful method for precisely determining the difference in the squared nuclear charge radii between these isotopes, as it effectively cancels out nuclear mass-independent contributions and their associated uncertainties.

Pachucki *et al.* [29] conducted a comprehensive analysis of the isotope shifts and the nuclear charge radius difference, ΔR^2 , between ^3He and ^4He . In 2016 and 2017, higher-order recoil corrections of $O(\alpha^6 m^2/M)$ were calculated for both the triplet and singlet states of helium [30,31], leading to updated values of ΔR^2 . These updated values are $1.061(3)$ fm² from Ref. [32] and $1.069(3)$ fm² from Ref. [11] for the $2^3S - 2^3P$ transition, and $1.027(11)$ fm² from Ref. [17] for the $2^1S - 2^3S$ transition. Notably, an unexpected discrepancy exists between the extracted ΔR^2 values obtained from these two distinct electronic transitions.

Aiming at resolving the unexplained discrepancy in ΔR^2 of $^3,4\text{He}$, numerous high-precision spectroscopic measurements have been undertaken in recent years [5–8,33,34]. Zheng *et al.* [33] updated ΔR^2 to $1.028(2)$ fm² by measuring the $2^3S - 2^3P$ transition frequency. Rengelink *et al.* [6] and Werf *et al.* [5] measured the $2^1S - 2^3S$ transition frequency,

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resulting in updated ΔR^2 of $1.041(7) \text{ fm}^2$ and $1.0757(15) \text{ fm}^2$, respectively. Recently, the absolute nuclear charge radii for ^3He and ^4He have been precisely extracted as $1.97007(94) \text{ fm}$ [7] and $1.6786(12) \text{ fm}$ [8], respectively, by measuring the $2S - 2P$ transition in $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$, yielding a value of $1.0636(31) \text{ fm}^2$ for ΔR^2 .

On the scattering side, Ottermann *et al.* [35] determined ΔR^2 to be $1.112(75) \text{ fm}^2$ from electron scattering experiments on ^3He and ^4He . Sick [36] conducted a comprehensive analysis of global data on elastic electron-helium scattering, yielding a more accurate charge radius for ^4He and updating ΔR^2 to $1.079(61) \text{ fm}^2$. Later, Sick [37] revised this value to $1.066(60) \text{ fm}^2$. However, there have been no substantial developments in improving the accuracy of electron- ^3He scattering measurements.

This paper aims to address several unresolved discrepancies in the nuclear charge radii of the stable helium isotopes, with a primary focus on the significant 3.6σ discrepancy in ΔR^2 extracted from electronic and muonic helium spectra. By rigorously analyzing the influence of fine and hyperfine mixing effects on isotope shifts factors that may have been overlooked or underestimated in previous theoretical calculations this study seeks to clarify the origins of these discrepancies and contribute to a more accurate determination of helium nuclear charge radii. An early version of our work was posted on arXiv in September 2024 [38] and has since received significant recognition [39–42]. Notably, Pachucki *et al.* independently validated our findings [39]. Additionally, the Merkt group incorporated our results into their recent study on the ionization energy of the 2^3S_1 state of ^4He [40], while Potvliege referenced our work in his latest manuscript on physics beyond the Standard Model [41].

Fine and hyperfine mixing. Fine and hyperfine mixing effects can influence isotope shifts, typically at the order of $m\alpha^6$. Accurately calculating these effects has long been a challenging task. Currently, the primary approach estimates the contributions of fine and hyperfine mixing effects by using second-order perturbation theory for dominant states, such as the mixing of $2^3S - 2^1S$ and $2^1P - 2^3P$. However, as experimental accuracy improves, these approximations may no longer suffice for the required theoretical precision. A more precise analysis of mixing effects involves two key aspects: (i) accurately calculating contributions from additional mixed intermediate states that were previously neglected, and (ii) evaluating whether higher-order perturbative corrections affect the current precision level. A comparative analysis between second-order perturbation (method 1) and exact diagonalization (method 2) provides an effective way to assess higher-order perturbation contributions [43]. Here, we use the 2^1P and 2^3P states as an example to illustrate these methods. In method 1, second-order perturbation theory is applied to compute the contribution of the $2^1P - 2^3P$ mixing effect to the 2^3P manifold [29]. Method 2 expands the 2^3P manifold to include the 2^1P state, followed by an exact diagonalization of the extended matrix [12,43].

In method 1, the contribution of mixing effects consists of two parts, fine structure (fs) and hyperfine structure (hfs):

$$E_{\text{mix}} = E_{\text{mix,fs}} + E_{\text{mix,hfs}}, \quad (1)$$

where

$$E_{\text{mix,fs}}(2^3P) = \sum_n \frac{|\langle 2^3\bar{P} | H_{\text{fs}}^{(4)} | n^1\bar{P} \rangle|^2}{E(2^3P) - E(n^1P)}, \quad (2)$$

and

$$E_{\text{mix,hfs}}(2^i\chi) = \sum_n \frac{|\langle 2^i\bar{\chi} | H_{\text{hfs}}^{(4)} | n^j\bar{\chi} \rangle|^2}{E(2^i\chi) - E(n^j\chi)}, \quad (3)$$

where $\chi \in \{S, P\}$, $\{i, j\} \in \{1, 3\}$, and $i \neq j$. In the above, the initial state is on the left, while the intermediate state is on the right. Here, $H_{\text{fs}}^{(4)}$ represents the spin-dependent component of Eq. (17) in the Supplemental Material [44], and $H_{\text{hfs}}^{(4)}$ accounts for the leading-order hyperfine splitting [29,45]. Following the method outlined in Ref. [45], the relevant formulas for calculating the impact of hfs mixing on the isotope shift are as follows:

$$E_{\text{mix,hfs}}(2^3S) = C_{56}^2 \times \frac{1}{4} \sum_n \frac{|\langle 2^3S | Q_A | n^1S \rangle|^2}{E(2^3S) - E(n^1S)}, \quad (4)$$

$$E_{\text{mix,hfs}}(2^1S) = C_{56}^2 \times \frac{3}{4} \sum_n \frac{|\langle 2^1S | Q_A | n^3S \rangle|^2}{E(2^1S) - E(n^3S)}, \quad (5)$$

and

$$E_{\text{mix,hfs}}(2^3P) = C_{56}^2 \times \sum_n \frac{1}{E(2^3P) - E(n^1P)} \times \left[\frac{1}{4} |\langle 2^3P | Q_A | n^1P \rangle|^2 + \frac{1}{20} |\langle 2^3P | \hat{Q}_A | n^1P \rangle|^2 \right], \quad (6)$$

where $Q_A = \frac{4\pi Z}{3} [\delta^3(r_1) - \delta^3(r_2)]$ and $\hat{Q}_A = \frac{Z}{2} [\frac{1}{r_1^3} (\delta^{ij} - 3\frac{r_1^i r_1^j}{r_1^2}) - \frac{1}{r_2^3} (\delta^{ij} - 3\frac{r_2^i r_2^j}{r_2^2})]$ are antisymmetric electronic operators, with r_1 and r_2 representing the radial coordinates of the two electrons. The angular-momentum operators are evaluated first, as reflected in the coefficients of Eqs. (4)–(6). The coefficient C_{xz}^y here is defined as $\mu^x [(1 + \kappa)/(mM)]^y \alpha^z$, where κ represents the anomalous magnetic moment of the nucleus, M and m are the masses of the nucleus and the electron, respectively, μ is the reduced mass, Z denotes the nuclear charge, and α is the fine structure constant. It is important to note that the second-order perturbation calculation of the delta function does not converge, which necessitates the process of renormalization. For details, see Ref. [45] and the Supplemental Material [44]. It is noted that since the contribution of the $\hat{Q}_A$ term in the second part of Eq. (6) is relatively small, only the dominant $2^1P - 2^3P$ mixing is considered in the $\hat{Q}_A$ part.

For method 2, we use the following 2×2 matrix to illustrate the contribution of the mixing effects:

$$\begin{bmatrix} E(n'^{2S'+1}\chi'_{J'}) & E_{\text{off}} \\ E_{\text{off}} & E(n^{2S+1}\chi_J) \end{bmatrix}, \quad (7)$$

where S and J represent the electronic spin and total angular momentum of the state, respectively, while E_{off} denotes the off-diagonal matrix element for hyperfine mixing. Further details and general explanations are provided in the Supplemental Material [44]. By calculating the correction

TABLE I. Isotope shifts between ^3He and ^4He , assuming pointlike nuclei, in kHz. The estimated numerical uncertainty is typically below one in the last digit unless otherwise indicated.

	$2\ ^1S - 2\ ^3S$		$2\ ^3S - 2\ ^3P$	
	This work	Pachucki <i>et al.</i> [29]	This work	Pachucki <i>et al.</i> [29]
$\mu\alpha^2$	-8632567.86	-8632567.86	12412458.1	12412458.1
$\mu\alpha^2(\mu/M)$	608175.58	608175.58	21243041.3	21243041.3
$\mu\alpha^2(\mu/M)^2$	-7319.80	-7319.80	13874.6	13874.6
$\mu\alpha^2(\mu/M)^3$	0.30	0.30	4.6	4.6
$\mu\alpha^4$	-8954.22	-8954.22	17872.8	17872.8
$\mu\alpha^4(\mu/M)$	6458.22	6458.23	-20082.3	-20082.4
$\mu\alpha^4(\mu/M)^2$	1.84	1.84	-3.0	-3.0
$m\alpha^5(m/M)$	56.59	56.61	-60.6	-60.7
$[\sim m\alpha^6(m/M)]$	2.33(59)	2.75(69)	-13.1(3.3)	-15.5(3.9)
$E_{\text{mix,hfs}}^{n=2}$	80.76	80.72	54.6	54.6
Nuclear polarization	0.20(2)	0.20(2)	-1.1(1)	-1.1(1)
$m\alpha^6(m/M)$	2.73 ^a		-9.4 ^a	
$\sim m\alpha^6(m/M)^2$	0.00(15) ^a			
$\sim m\alpha^7(m/M)$	-0.21(11) ^a		0.0(0.9) ^a	
Subtotal	-8034065.87(19)	-8034065.91(19) ^a	33667149.6(0.9)	33667149.3(0.9) ^a
$\delta E_{\text{mix,hfs}} = E_{\text{mix,hfs}} - E_{\text{mix,hfs}}^{n=2}$	-1.37		0.09	
Fine structure mixing			-0.43	
Isotope shift (final)	-8034067.24(19)		33667149.3(0.9)	
Ref. [46]	-8034067.8(1.1)		33667146.2(7)	

^aThe values are cited from Refs. [30,31].

$\sim$ Values obtained through approximate evaluation.

$[\sim m\alpha^6(m/M)]$ indicates that this value is not included in the final result.

$\delta E(n^{2S+1}\chi_J^F)$ for each energy level through diagonalization, the corresponding contribution to the isotope shift can be determined as

$$E_{\text{mix}} = \frac{\sum_{J,F} (2F+1) \delta E(n^{2S+1}\chi_J^F)}{(2I+1)(2S+1)(2L+1)}, \quad (8)$$

where L and I are, respectively, the electronic orbital angular momentum and the nuclear spin.

Numerical results. In this section, we first calculate isotope shifts following the method used by Pachucki *et al.* [29] to independently verify their results for the $2\ ^1S - 2\ ^3S$ and $2\ ^3S - 2\ ^3P$ transitions in ^3He - ^4He . Our calculations employed a variational approach with the Hylleraas basis set [12,14], incorporating various relativistic and QED corrections (see the Supplemental Material [44]). Furthermore, method 1 was used to evaluate the mixing effects in the $2\ ^1S - 2\ ^3S$ and $2\ ^1P - 2\ ^3P$ transitions. The nuclear parameters used are as follows: the nuclear masses are $M(^3\text{He}) = 5495.88528007(24)m_e$ for ^3He and $M(^4\text{He}) = 7294.29954142(24)m_e$ for ^4He , with the nuclear anomalous magnetic moment of ^3He given by $\kappa = -4.18415368(3)$ [47]. The results are summarized in Table I. Our calculated values for terms of order $\sim m\alpha^6(m/M)$ differ from those of Pachucki *et al.* [29], primarily due to an error in the formula they used, specifically in Eq. (27) of Ref. [29]. This minor discrepancy has since been corrected in their subsequent work on comprehensive $m\alpha^6(m/M)$ calculations [30,31].

Subsequently, we performed a thorough analysis of the contributions from fine and hyperfine mixing effects, employing both methods 1 and 2. The numerical results for the

operators used in the calculations are provided in Tables III and IV in the Supplemental Material [44]. Table II summarizes the effects of mixing on isotope shifts, specifically for the mixing of the $2\ ^1S$ state with $n\ ^3S$, the $2\ ^3S$ state with $n\ ^1S$, and the $2\ ^3P$ state with $n\ ^1P$. For M2, consider the example of $n = 3$: The value was determined by subtracting the diagonalization result of the $n = 2$ matrix from that of the matrix involving $n = 2, 3$. No difference was found between the two methods, demonstrating the effectiveness of M1. Consequently, there is no need to extend the calculation of M2 for $n > 3$.

From the table, it is clear that the mixing contributions from the $3\ ^3S$ and $4\ ^3S$ states are significantly larger than those from the $3\ ^1S$ and $3\ ^1P$ states. Our results also indicate that,

TABLE II. Assessing the hyperfine mixing effect on the ^3He - ^4He isotope shifts for the $2\ ^1S$, $2\ ^3S$, and $2\ ^3P$ states, in kHz. Here, $E_{\text{mix,hfs}}^{n=i}$ denotes the contribution from the i th intermediate state, while $E_{\text{mix,hfs}}$ indicates the total contribution from all intermediate states. M1 and M2 refer to method 1 and method 2, respectively.

	$n\ ^3S$ to $2\ ^1S$		$n\ ^1S$ to $2\ ^3S$		$n\ ^1P$ to $2\ ^3P$	
	M1	M2	M1	M2	M1	M2
$E_{\text{mix,hfs}}^{n=2}$	60.57	60.57	-20.19	-20.19	-74.80	-74.73
$E_{\text{mix,hfs}}^{n=3}$	-1.12	-1.12	-0.07	-0.07	-0.04	-0.04
$E_{\text{mix,hfs}}^{n=4}$	-0.17					
$E_{\text{mix,hfs}}^{n=5}$	-0.06					
Sum	59.22		-20.26		-74.84	
$E_{\text{mix,hfs}}$	59.46		-19.93		-74.64	

TABLE III. Difference in squared nuclear charge radii between ^3He and ^4He (in fm^2), along with the nuclear charge radius of ^3He (in fm). For the $2^1S - 2^3S$ transition, the corrected values are defined as $\Delta R^2 = \Delta R_0^2 + \delta\Delta R^2$ and $R(^3\text{He}) = R_0(^3\text{He}) + \delta R(^3\text{He})$, where $\delta\Delta R^2 = -0.0064 \text{ fm}^2$ and $\delta R(^3\text{He}) = -0.0017 \text{ fm}$. The corrections for other transitions are negligible. The nuclear charge radius of ^4He is $R(^4\text{He}) = 1.6786(12) \text{ fm}$, as reported in Ref. [7]. After our work appeared on arXiv [38], the value of ΔR^2 in Ref. [33] was recently updated to $1.0733(21) \text{ fm}^2$ [48].

	ΔR_0^2	ΔR^2	$R_0(^3\text{He})$	$R(^3\text{He})$	Reference
$2^1S - 2^3S$	1.027(11)	1.021(11)	1.9608(30)	1.9591(30)	Theo. [29–31]; Expt. [17]
	1.041(7)	1.035(7)	1.9644(21)	1.9627(21)	Theo. [29–31]; Expt. [6,17]
	1.0757(15)	1.0693(15)	1.9732(11)	1.9715(11)	Theo. [29–31]; Expt. [5,6]
$2^3S - 2^3P$		1.069(3)		1.9715(13)	Theo. [29–31]; Expt. [10,11]
		1.061(3)		1.9694(13)	Theo. [29–31]; Expt. [32]
		1.028(2)		1.9610(11)	Theo. [29–31]; Expt. [11,33]
$2^1S - 3^1D$		1.059(25)		1.9689(64)	Theo. [46]; Expt. [34]
$\mu^3\text{He}^+ : 2S - 2P$		1.0636(31)		1.97007(94)	Theo. [49]; Expt. [7,8]
Electron scattering (ES)		1.066(60)		1.973(14)	Expt. [35–37]

at the present level of precision, there is no discernible difference between methods 1 and 2, implying that high-order corrections, such as cross terms between fine and hyperfine structures, can be neglected. However, as measurement precision improves, the exact diagonalization approach (method 2) may become essential. Furthermore, by employing the renormalized delta function (see the Supplemental Material [44]), we calculated the total contributions of the $n^3S - 2^1S$, $n^1S - 2^3S$, and $n^1P - 2^3P$ mixing effects using method 1, as shown by $E_{\text{mix,hfs}}$ in Table II. Notably, the mixing effect from the n^3S and n^1S states has a substantial impact on the $2^1S - 2^3S$ transition, resulting in a -1.37 kHz shift in the isotope shift approximately seven times larger than the current theoretical uncertainty, as presented in Table I.

Below, we analyze the contributions of $\delta E_{\text{mix,hfs}}$ to the difference in squared nuclear charge radii ΔR^2 . The isotope shift E_{IS} is composed of the mass shift E_{MS} and the field shift E_{FS} : $E_{\text{IS}} = E_{\text{MS}} + E_{\text{FS}}$, where $E_{\text{FS}} = C\Delta R^2$, and the field shift coefficient $C = C_X - C_Y$ for two involved atomic states X and Y , and $C_{X,Y} = \frac{2\pi Z}{3} \langle \sum_i \delta^3(r_i) \rangle_{X,Y}$. The values of coefficient C for the $2^1S - 2^3S$ and $2^3S - 2^3P$ transitions are calculated to be $C(2^1S - 2^3S) = -214.45 \text{ kHz/fm}^2$ and $C(2^3S - 2^3P) = 1210.5 \text{ kHz/fm}^2$, respectively. The impact of the $\delta E_{\text{mix,hfs}}$ on ΔR^2 can be assessed according to

$$\delta\Delta R^2 = -\frac{\delta E_{\text{mix,hfs}}}{C(2^1S - 2^3S)} = -0.0064 \text{ fm}^2. \quad (9)$$

It can be observed that the inclusion of $\delta E_{\text{mix,hfs}}$ causes a change of -0.0064 fm^2 in ΔR^2 for the $2^1S - 2^3S$ transition. This, in turn, results in an alteration of -0.0017 fm in the absolute nuclear charge radius of ^3He . The change in ΔR^2 determined by the $2^3S - 2^3P$ transition is only 0.0004 fm^2 , which falls within the current margin of error, so we will disregard it here. The updated results for ΔR^2 and $R(^3\text{He})$ are presented in Table III, while Fig. 1 displays only the results for ΔR^2 .

From Table III and Fig. 1, we see that our updated results adjust the difference in squared nuclear charge radii, as determined from the latest measured $2^1S - 2^3S$ transition in ^3He , from $1.0757(15) \text{ fm}^2$ to $1.0693(15) \text{ fm}^2$ [5,6]. This revision significantly reduces the discrepancy with the value of $1.0636(31) \text{ fm}^2$ [7,8] obtained from μHe^+ , narrowing the

initial difference from 3.6σ to 1.7σ . Notably, the revised result also closely matches the value of $1.069(3) \text{ fm}^2$ [10,11] obtained from the $2^3S - 2^3P$ transition, sharing the same central value. Additionally, the mixing effect with the n^3S and n^1S states leads to a slight adjustment in the absolute nuclear

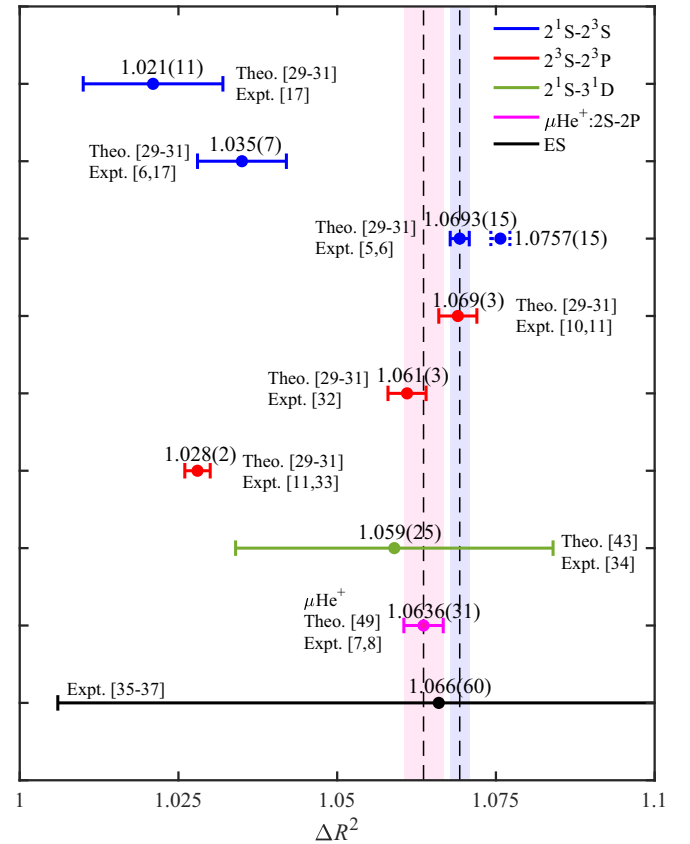


FIG. 1. Difference in squared nuclear charge radii for ^3He , in fm^2 . Similar colors denote results derived from the same transition. The blue dashed line represents data prior to the update, while the blue solid line shows data following the update. Note that only updated values are shown for the first two blue results in the upper portion of the figure, as they are not the focus of our current discussion.

charge radius of ^3He , from 1.9732(11) fm to 1.9715(11) fm, which aligns well with the $\mu^3\text{He}^+$ value of 1.97007(94) fm, reducing the discrepancy from 2σ to 1σ .

Very recently, substantial progress has been made by Pachucki *et al.* [39] with a complete treatment of second-order hyperfine corrections. In their work, they verified the correctness of our calculated singlet-triplet mixing contribution of -1.37 kHz [38], as shown in Table I, which constitutes a major part of their updated value of -1.77 kHz. This adjustment further reduces the remaining discrepancy between the electronic-helium and muonic-helium determinations of the squared nuclear charge radii difference from 1.7σ to 1.3σ . With this progress, we can now conclude that the initial 3.6σ discrepancy has been largely resolved.

Summary. This study investigates the impact of fine and hyperfine mixing effects on isotope shifts, utilizing both perturbation theory and an exact diagonalization method. Initially, we independently verified the isotope shift results of Pachucki *et al.* for the $2^1S - 2^3S$ and $2^3S - 2^3P$ transitions in ^3He - ^4He through precise numerical calculations. We then examined the mixing effects on the $n^3S - 2^1S$, $n^1S - 2^3S$, and $n^1P - 2^3P$ transitions.

Our findings indicate that, at the current level of accuracy, there is no significant difference between the results obtained using the perturbation approach and the exact diagonalization method. Notably, hyperfine mixing effects involving the n^3S and n^1S states have a substantial impact on the $2^1S - 2^3S$ transition, causing a shift of -1.37 kHz in the isotope shift. This shift, in turn, leads to a change of -0.0064 fm² in the calculated ΔR^2 , helping to resolve the previously

observed discrepancy in the squared difference of nuclear charge radii.

Furthermore, our results demonstrate that extracting ΔR^2 from atomic isotope shifts is a reliable method and can be extended to other heliumlike systems. The revised nuclear charge radius of ^3He now aligns well with the value obtained from $\mu^3\text{He}^+$ measurements, offering an essential basis for precise evaluations of the ^3He nucleus size and structure.

In light of these findings, along with the confirmation of our results by Pachucki *et al.* [39], we speculate that the current discrepancy in helium nuclear charge radii does not arise from physics beyond the Standard Model, supporting the validity of existing theoretical frameworks for both electronic and muonic atoms at current precision levels. We strongly encourage further high-precision isotope shift measurements and electron scattering experiments, particularly involving ^3He , to refine nuclear charge radii of both ^3He and ^4He and to provide stringent tests of nuclear structure models.

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