

Single atomic layer quantum holography

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(Received 8 February 2023; accepted 31 January 2025; published 11 April 2025)

Quantum metasurfaces composed of subwavelength two-dimensional atomic arrays hold great promise as a practical and scalable atom-photon interface. However, these quantum metasurfaces are limited to amplitude control of photonic states thereby restricting the quantum information processing task they can implement. In this work, we present a method that enables precise control over the phase distribution of quantum photonic states. We achieve this by developing a quantum holography technique, which involves preparing a superposition of holograms and leveraging both electromagnetically induced transparency and long-range Rydberg interactions. Our method enables encoding of arbitrary recoverable images that are entangled with the atomic array degrees of freedom rapidly on the order of microseconds. Furthermore, we extend our analysis to include many-body atom-photon entangled states by establishing a quantitative mapping between the atomic and photonic states. Finally, we discuss the experimental realization of our approach, potential error mechanisms, and practical quantum optics applications.

DOI: [10.1103/PhysRevResearch.7.L022014](https://doi.org/10.1103/PhysRevResearch.7.L022014)

Introduction. A crucial source for quantum information processing is entangled states of many qubits. Emerging quantum optical systems implement quantum operations on photons via strong interaction with atomic qubits, for example in the mainframe of cavity quantum electrodynamics [1]. In conventional systems the atomic qubits are usually spatially localized and control the quantum photonic state of a confined spatial waveform. Following pioneering work [2–4], recently the enhanced response of subwavelength two-dimensional atomic arrays to light have been analyzed [5–9]. This led to the introduction of quantum metasurfaces [10]: a new tool which offers coherent control of light’s quantum state by scattering off a single atomic layer while employing long-range Rydberg interaction. This analysis of quantum metasurfaces was soon followed by thorough theoretical and experimental studies on Rydberg-photon interactions [11–13], including nonlinear scenarios [14]. Very recently, pioneering experiments implemented quantum metasurfaces in ultracold atomic arrays [15–19]. However, the work so far is limited in the number of qubits (up to tens) and lacks control over the phase of the photonic wavefunction [10,17].

In a different domain, classical optical holograms provide complete control over the spatial and phase profiles of far-field beams [20,21]. Their applications span multiple disciplines [22,23], including light shaping, computer-generated

holograms (CGH) [24–32], and both conventional and dynamic metasurfaces [6,33,34]. Can holograms be extended to a system with quantum coherence? It would be intriguing to implement quantum holograms with two-dimensional subwavelength atom arrays. These can be potentially utilized to control atom-photon quantum states with high resolution and large-scale spatial information.

In this work, we develop single atomic layer quantum holograms which control the phase of a many-body photonic state. These quantum holograms are based on the long-range interaction of highly excited Rydberg states and holography techniques we develop for precise photonic phase control by a single atomic layer. We show that this system is capable of generating atomic and photonic entangled states with many spatial degrees of freedom enabling arbitrary amplitude and phase control. We further discuss the feasibility of the system architecture in recent experimental systems [17,19] and analyze an example in which the quantum holography method presented enables many-body entanglement. Finally, we discuss potential applications such as nondestructive measurements for probing many-body atomic states.

The article is organized as follows. In the first section we describe the two-dimensional subwavelength array architecture. In the second section we introduce the quantum holography method we developed for precise phase control over the scattered photon state. We then present a mapping between the atomic and the photonic states, generalizing to atom-photon many-body states. We finally discuss practical experimental systems and fidelity analysis for imperfections that affect our architecture. We conclude with an overview of possible applications to quantum optics.

Phase control with quantum metasurface. Our system is composed of a two-dimensional uniform lattice of atoms. The atoms that make up the array are a three-level system

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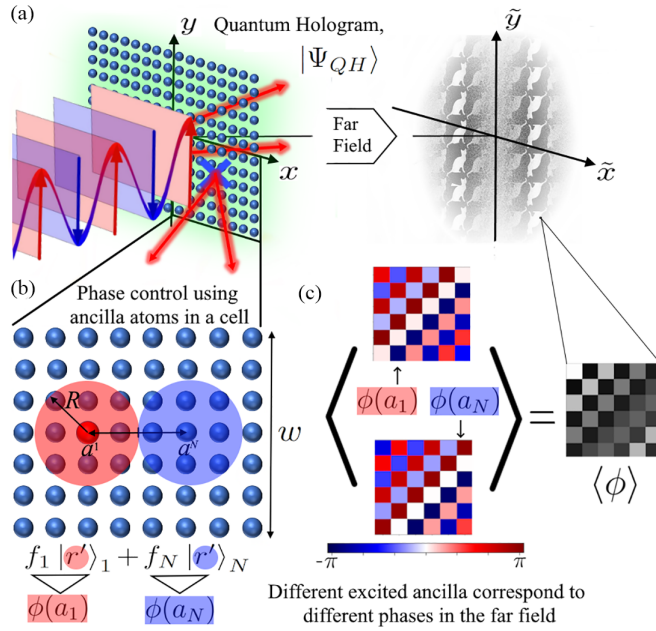


FIG. 1. Phase control with a quantum metasurface. (a) A plane wave is scattered off an atom array prepared in the state $|\Psi_{QM}\rangle$, Eq. (2), that consists of reflective and transparent regions. The scattering results in a photonic-atomic entangled state—the quantum hologram $|\Psi_{QH}\rangle$, Eq. (3). Measurements of the photonic state's expectation value in the far-field results in the target classical image (cats). (b) Example of a cell prepared in a spin-wave excitation shared by two ancilla atoms at positions a_1 and a_N . Each ancilla that is excited to the Rydberg state creates a circularlike reflective region with radius R . (c) Ancillas in different locations impart different phases $\phi(a_j)$ in the far field. Phase control by a cell is displayed by the expectation value of the phase in grayscale (for $f_1 = f_N = \frac{1}{\sqrt{2}}$). Notice that by choosing to excite these specific ancillas the phases exactly cancel in the upper left corner.

with ground $|g\rangle$, excited $|e\rangle$, and highly excited Rydberg $|r\rangle$ states. This lattice is constructed from equally sized square subatomic arrays which we refer to as cells. As detailed below, each cell encodes a phase pattern onto the scattered light. In order to control the phase of scattered light within a cell, we utilize the level structure of specific control atoms, which we call ancillas. Each ancilla is a two-level system with ground ($|g'\rangle$) and Rydberg ($|r'\rangle$) states. These ancillas are the N atoms located on the vertically centered horizontal axis, where a_j is ancilla j 's distance from the cell center as schematically presented in Fig. 1(b).

We consider the situation of applying a weak probe field and a strong classical control field that are resonant with $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |r\rangle$ transitions, respectively. If all ancilla atoms are in the $|g'\rangle$ state, then the array is transparent to light of wavelength λ_{ge} due to electromagnetically induced transparency (EIT). Whereas, if a single ancilla is excited to the Rydberg state $|r'\rangle$ it shifts the level $|r\rangle \rightarrow |r + V\rangle$ of the surrounding atoms which in turn breaks the EIT condition [10]. Due to the cooperative resonance [35,36] the area affected by the shift $|r\rangle \rightarrow |r + V\rangle$ will form a circular region of radius R that reflects the probe light. Diffraction calculations for an incoming plane wave at oblique incident angle θ return the following expression for complex amplitude in the far

field:

$$A_H = -\pi R^2 e^{ik_0 \sin(\theta) a_j} e^{-i(k_{\tilde{x}} x_m + k_{\tilde{y}} y_n)}. \quad (1)$$

As is evident from Eq. (1), the ancilla's location a_j controls the phase of light scattered off the cell: $\phi(a_j) = k_0 \sin(\theta) a_j$ (where $k_0 = 2\pi\lambda_{ge}^{-1}$). If a single ancilla j is excited within a cell we write the cell's state as $|r'\rangle_j = |r'\rangle_j \otimes \prod_{i \neq j} |g'\rangle_i$. This is a product state for which ancilla j is excited to the Rydberg state $|r'\rangle$ and the other $N - 1$ ancillas are in the ground state $|g'\rangle$. The limitation on discrete ancilla locations, that generates the circular reflective regions, poses a limitation on the scattered light phase resolution. (See Fig. 1, and Supplemental Material [39] for detailed derivations. The variables x_m, y_n reflect the position of the cell's center and $k_{\tilde{x}}, k_{\tilde{y}}$ the coordinates of a region in the far field. See Supplemental Material [39] for details.)

This motivates us to consider a spin-wave excitation for precise phase control. A spin wave is a superposition of single ancilla excitations $|r'\rangle_j$, where each state has a probability amplitude f_j . In Figs. 1(b) and 1(c) we present an example of the precise phase control of scattered light by a spin-wave excitation of the a_1 and a_N ancillas where they are both equally excited by the spin wave such that $f_1 = f_N = \frac{1}{\sqrt{2}}$. We note that we work in the weak limit [37] of a single excitation to the Rydberg level, which does not include states of multiple excited ancillas (e.g., states like $|r'_1 r'_2 g'_3\rangle$ are not allowed).

Quantum control in our quantum holography method includes an initial superposition state of all ancillas in the ground state and the spin-wave excitation, expressed as $|\psi\rangle = f_0 |g'\rangle + \sum_j f_j |r'\rangle_j$ where f_0, f_j are complex numbers. $|f_0|^2$ is the probability of all N ancilla atoms in the ground state ($|g'\rangle = \prod_{i=1}^N |g'\rangle_i$) and f_j incorporates the specific spin wave excitation (where $|f_0|^2 + \sum_j |f_j|^2 = 1$).

Quantum holography with super atom array. A super-array of cells as described above allows full control of the phase structure of the reflected and transmitted light conditioned on the state of the ancillas. This is the principle we now utilize for analyzing quantum control over the phase of many photonics degrees of freedom. We consider a product state of $\mathcal{M}$ cells of width w organized in a $n \times m$ grid [Fig. 1(a)]

$$|\Psi_{QM}\rangle = \prod_{n,m} \left(f_0 |g'\rangle + \sum_j f_j |r'\rangle_j \right)_{nm}, \quad (2)$$

which we refer to as the quantum mask state. This defines an array profile of $n \times m$ circular reflective regions. A right-propagating ($|0\rangle$) wave (coherent state) scatters off the quantum mask with a specific phase ϕ determined by each cell's atomic state, Eq. (1), and entangles with the quantum mask's degrees of freedom. This results in the entangled atom-photon state which is our quantum hologram:

$$|\Psi_{QH}\rangle = \prod_{n,m} \left(f_0 |g'\rangle |0\rangle + \sum_j f_j |r'\rangle_j (\mathcal{T}|0\rangle + \mathcal{R}|1\rangle)_j \right)_{nm} \quad (3)$$

where $|0\rangle$ marks the transmitted-light state and $|1\rangle$ the reflected-light state. When no ancilla is excited, EIT results in

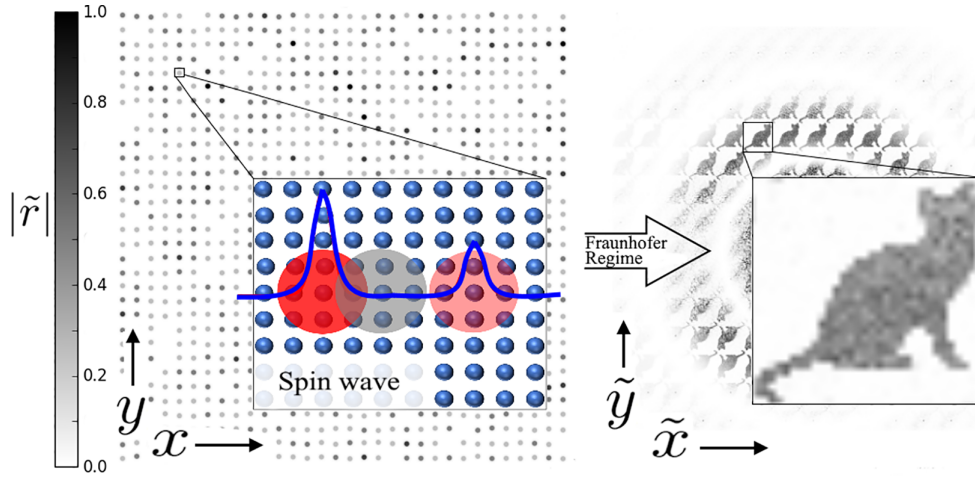


FIG. 2. To simulate our control of a 32×32 cell quantum mask we encode an atomic state that produces a photonic image of a cat after diffraction (note that multiple diffraction orders will result in repeats of our target image). Using Eq. (1) we calculate the required ancilla excitations that correspond to the image and encode them into the ancillas' states using Eq. (2). After light scatters, the expectation value of the quantum hologram indeed produces the targeted cat array image in the Fourier plane, $(\tilde{x}, \tilde{y})$. The grayscale indicates the cells' effective reflectivity resultant from $|\Psi_{QH}\rangle$. Inset: a single cell in which a spin-wave excitation (blue line) involving two ancillas (red circles), implementing the cell's effective reflectivity and phase.

full transmission. When the ancilla is in the $|r'\rangle$ state, the cells' transmission ($\mathcal{T}$) and reflection ($\mathcal{R}$) amplitudes are determined by the reflectivity spatial profile of the cell (originated from the dipole-dipole interaction analyzed quantitatively below). After propagation into the Fraunhofer regime, we express the photonic phase and amplitude distribution in the Fourier plane by rewriting the second term of Eq. (3) as $\sum_j^N f_j(\mathcal{T}|r', 0; \phi)_j + \mathcal{R}|r', 1; \phi)_j$ which defines the quantum hologram's image, where ϕ is the light's accumulated phase expressed in Eq. (1) (see Sec. A in the Supplemental Material for diffraction calculation, [38,39]). The scattering generates correlations between the far-field photonic distribution and the ancilla state. Due to the quantum nature of the light-atom interaction, the system is entangled and coherence exist until a measurement of either the atomic or photonic degrees of freedom collapses the atomic-photonic state.

Considering solely the reflected light projects the system into the subspace,

$$|1\rangle\langle 1|\Psi_{QH}\rangle = \prod_{n,m} \left(\mathcal{R} \sum_j^N f_j |r', 1; \phi)_j \right)_{nm}. \quad (4)$$

Hence, measuring the reflected light collapses the light-atom superposition state, after which there is a single corresponding atomic and photonic state that results in an (classical) image in the far field. If one repeats the experiment multiple times, the differing probabilities f_j project the system into different images within this subspace. Then each cell contributes an effective classical amplitude $|\tilde{r}\mathcal{R}|$ in the imaging plane [example in Fig. 1(a)] where $\tilde{r}$ is given by the expectation value of the reflected light, $\tilde{r} = \sum_j^N |f_j|^2$. Similarly, the effective phase is $\tilde{\phi} = \sum_j^N |f_j|^2 \phi_j$. By controlling the atomic state in each cell (f_j) we control the amplitude and phase of the projected images which allow for encoding of arbitrary classical images as displayed in Fig. 2 (and explored further in Sec. H in the

Supplemental Material [39]). Note that all atoms are at fixed locations at all times. Modification of ancilla's location, and hence the reflection region, is implemented solely by modification of the spin-wave excitation by control lasers [40].

We consider now the exact reflection spatial profile as it affects precise phase and is crucial for our holography method (as opposed to previous work that considered merely the total amplitude). We conduct a thorough analysis by calculating the reflectivity's explicit dependence on the dipole-dipole shift $V(\rho)$ which is inhomogeneous throughout the array and depends strongly on the distance ρ from the excited ancilla. We find the reflectivity obeys $|\xi(\rho)| = 1/\sqrt{1 + (\frac{\rho}{R_c})^{12}}$ (see Sec. B in the Supplemental Material [39]) where R_c is a scale factor serving a similar purpose as R in defining the area over which the array is reflective (typically between $1 - 5 \mu\text{m}$, e.g., $4.6 \mu\text{m}$ in [17]). The effect of this inhomogeneity is reflected in the results presented in the figures. In addition, errors in ancilla locations will reduce the phase fidelity as discussed in the fidelity section and presented in Figs. 3 and 5.

Next, we consider the many-body and generalize the state to include entanglement between ancillas in different cells. For this aim we consider a single ancilla within a cell that is prepared in a superposition state of $|g'\rangle$ and $|r'\rangle$ (which is a specific case of the spin wave analyzed above).

Many-body quantum hologram implementation. In order to scale our system architecture for quantum information, we now include entanglement between ancillas in different cells (e.g., Rydberg interaction) [41] enabling atomic many-body entangled states. Previously, ancillas in one cell would have no relation to ancillas in other cells, whereas now we consider correlations between ancillas in different cells. We aim to find a systematic method of converting between ancillas' state and photonic far-field state. We will endeavor to create a transfer matrix for which the atomic state (the state of all the ancillas) is its input and the far-field photonic state is its output. This transfer matrix should be invertible, i.e., transforming a

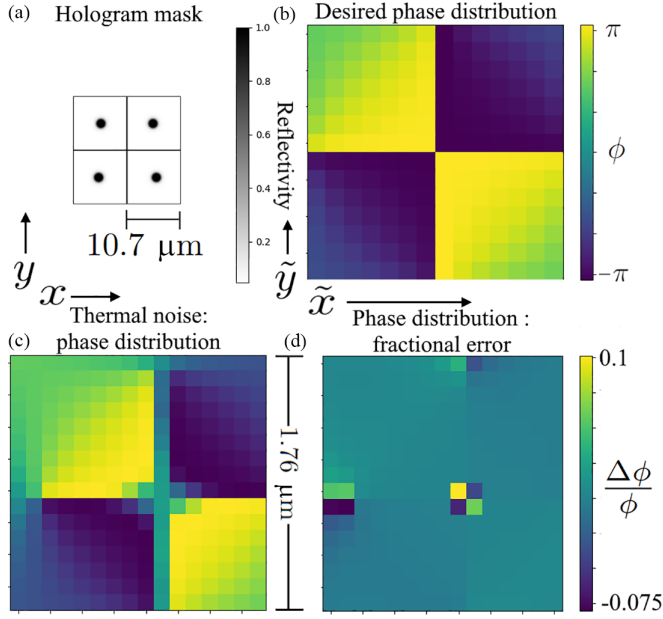


FIG. 3. Example of error in phase control due to thermal noise. The cell width is given by $w = \frac{2\lambda}{\sin(\theta)}$ and we examine the phase distribution in a region in the far field where the holography assumptions of the detour phase technique hold. Specifically within this region the $k_{\tilde{x}}a \ll 1$ and $k_{\tilde{y}}R \ll 1$ limits hold (see Sec. A in the Supplemental Material [39] for further discussion). (a) A toy model quantum mask of four excited ancillas produces a phase distribution pattern in the far field $(\tilde{x}, \tilde{y})$ as seen in (b). (c) Simulation of 100 experiments where we introduce random thermal error of $\approx 10\%$, with respect to lattice spacing, in ancilla location. The average phase is depicted showing that the largest error occurs at the border between the different phases. (d) Phase error extracted from (c) after removal of the high derivatives areas in (c) reveals that the majority of the phase distribution in the far field is resistant to thermal heating.

photonic distribution in the far field to an atomic state. While the atomic state is more or less clear in its construction, we must be more precise in regards to the photonic state. While previously we did not go down to the single-photon level, in this section we will do exactly that. We consider a photonic Gaussian wave-packet incident on a cell. In the limit of large Gaussian width, this reproduces the plane wave behavior analyzed previously. However, in our intermediary approach, we work in the limit that the wave-packet distributes solely over a single cell (while neglecting the overlap between wave packets incident upon different cells that is analyzed in the fidelity section below). Thus, we can employ our plane wave analysis [i.e., Eq. (1)] for each cell separately for a set number of photons.

As an example, let us consider an atomic state with $\mathcal{M} = 4$ cells and $N = 2$ ancillas per cell. For simplicity let us assume only two photons are incident with the diagonal cells. We can probe the far-field distribution through correlation measurements of scattered photons in order to readout the phase distribution. In our example this results in a two-photon two-slit interference pattern which we treat explicitly by our far-field calculations [i.e., Eqs. (1)–(4)]. In Fig. 4 we demonstrate the patterns generated by this scenario and how modifying the coherent phase of the ancillas state influences

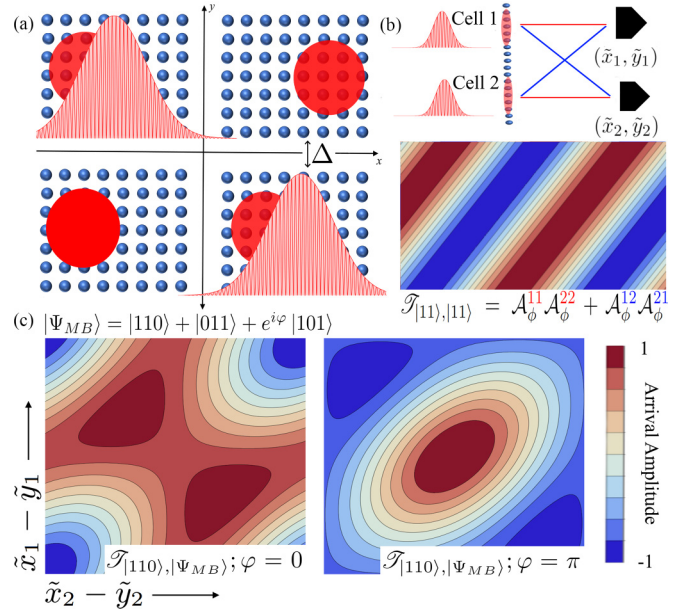


FIG. 4. Probing a many-body atomic-photonic state as a quantum hologram. (a) Two Gaussian wavepackets are incident upon a 2×2 many-body mask, $|\Psi_{MB}\rangle = |1211\rangle + |0010\rangle$, such that each photon interacts with a single cell, thus reducing the atomic state to $|11\rangle + |00\rangle$. (b) Two-photon arrival amplitude $\mathcal{T}_{|11\rangle,|11\rangle}$ calculated by Eq. (5) where all possible scattering scenarios are considered. $\mathcal{A}_{\phi}^{11}$ refers to the amplitude of the path of a photon from cell one to $\tilde{x}_1$ while $\mathcal{A}_{\phi}^{12}$ refers to the amplitude of a photon from cell one to $\tilde{x}_2$ where the colors in the superscript correlate to the paths drawn in the figure. The result is plotted as a function of $\tilde{x}_1 - \tilde{y}_1$, vertical axis, and $\tilde{x}_2 - \tilde{y}_2$, horizontal axis (Sec. E in the Supplemental Material [39]). (c) Extension to a 3×3 many-body initial atomic state ($|\Psi_{MB}\rangle$) where a photon is incident on each main diagonal cell. The two-photon arrival amplitude of the photonic state $|110\rangle$ is displayed demonstrating the effect of the phase ϕ .

these patterns (see Sec. E in the Supplemental Material [39] for detailed calculations).

To formalize our atomic-photonic state correspondence with a transfer matrix we must adopt a new formalism for our states as the quantum mask state is no longer a tensor product of separate ancilla states in their respective cells. Thus, we make an adjustment to our presentation of the atomic state in this many-body scenario. We consider a single ancilla within a cell that is prepared in a superposition state of $|g'\rangle$ and $|r'\rangle$ (which is a specific case of the spin wave analyzed above). We express the atomic states $|\beta\rangle$ that span the many-body mask state $|\Psi_{MB}\rangle$ in the $\mathcal{M}N$ -level qudit basis where each element's index corresponds to a cell and its value to the excited ancilla within that cell [e.g., if there are $\mathcal{M} = 2$ cells and $N = 4$ ancillas per cell then the state $|23\rangle$ indicates that the second (third) ancilla is excited in the first (second) cell]. When light scatters off the mask, it generates a many-body atomic-photonic entangled state equivalent to a superposition of quantum holograms $|\Psi_{QH}\rangle$, Eq. (3). If there are P incoming photons we define the photonic state $|\alpha\rangle$ as the arrival of up to P photons at up to P locations in the imaging plane (far field). These locations, $(\tilde{x}_i, \tilde{y}_i)$, are 2D in nature, but we will let the $\tilde{x}_i$ component stand in for a total location and leave the

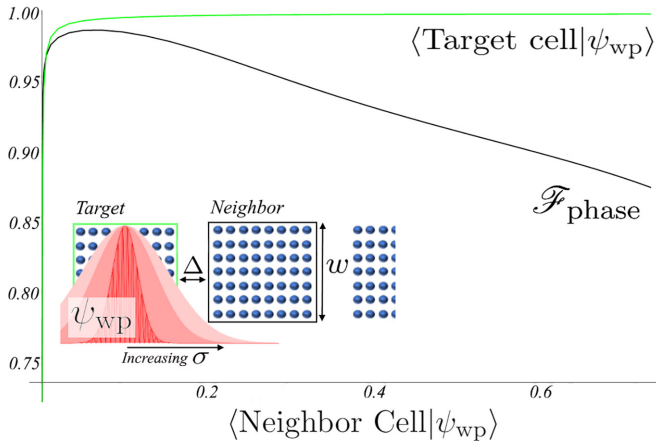


FIG. 5. Experimental parameters influence on a quantum many-body hologram's fidelity. To maximize the phase fidelity, $\mathcal{F}(\sigma, \Delta, w)_{\text{phase}} = 1 - |\frac{\phi_{\text{wp}} - \phi_{\text{pw}}}{\phi_{\text{pw}}}|$, for a many-body system as a function of Δ , w and the wavepacket's Gaussian width σ , ψ_{wp} 's fractional overlap with the target cell, green line, must be large enough such that it appears plane wavelike. However, a balance must be found such that ψ_{wp} does not interact with the quantum state of off target cells. The black line is a representative phase fidelity in the imaging plane (Sec. F in the Supplemental Material [39]). As expected for increasing overlap with the target cell, $\mathcal{F}_{\text{phase}}$ grows, but the increasing overlap with neighboring cells, horizontal axis, becomes detrimental. In this example we simulate a target cell with two neighbors.

$\tilde{y}_i$ component implicit for ease of notation. Each element of the photonic state $|\alpha\rangle$ corresponds to a photonic mode with a specified location in the imaging plane where a value of one indicates a single-photon population at that location and zero indicates no photons (e.g., for $P = 4$ photons $|\alpha\rangle = |0101\rangle$ implies detection of a photons only at $\tilde{x}_2$ and $\tilde{x}_4$ in the far field. Notice that the y component is left implicit. The values $\tilde{x}_i$ are the detector locations). We thus construct a transformation matrix $\mathcal{T}$ where each entry $\mathcal{T}_{\alpha\beta}$ gives the complex amplitude of the scattered photonic state $|\alpha\rangle$ due to the atomic product state $|\beta\rangle$:

$$\mathcal{T}_{\alpha\beta} = \sum_{\text{allowed diagrams}} \left(\prod_{\substack{\text{single cell} \\ \text{photonic} \\ \text{amplitude}}} \mathcal{A}^{\mu\nu} e^{2\pi a_{\mu} \sin(\theta)/\lambda_{ge}} \right)_{\alpha_v \neq 0, \beta_{\mu} \neq 0}, \quad (5)$$

where μ (ν) iterates over the cells (photons) excluding indices where that element in the atomic or photonic state is zero (corresponding respectively to no excited ancilla or no photon population, Sec. D in the Supplemental Material [39]). $\mathcal{A}_{\phi}^{\mu\nu} = \mathcal{A}^{\mu\nu} e^{2\pi a_{\mu} \sin(\theta)/\lambda_{ge}}$ gives the amplitude of a photon scattering from cell μ to far-field location $\tilde{x}_\nu$ where a_{μ} is the ancilla shift in cell μ . The value in the exponent is the explicit dependence of the encoded phase on experimental parameters [i.e., Eq. (1), cell width, wavelength, etc.]. Multiple photon scattering processes are built with products of these amplitudes. One must consider all the allowed configurations for the scattering process [e.g., see Fig. 4(b) where if a photon is detected at both $\tilde{x}_1$ and $\tilde{x}_2$ then we must consider both possible

scattering scenarios: the photon from cell 1 was detected at $\tilde{x}_1$ and the photon from cell 2 at $\tilde{x}_2$ or the photons were detected by opposite detectors. These are the two possible scattering configurations]. Summing over all indistinguishable configurations gives $|\alpha\rangle$'s overall photonic amplitude in the Fourier plane due to a cell in atomic state $|\beta\rangle$.

In Fig. 4 we use the transformation matrix Eq. (5) to calculate the photonic state in the Fourier plane for a specific case consisting of two photons that scatter off a four cell many-body atomic state. We calculate the arrival amplitude distribution of both a single photon and two simultaneous photons (Secs. D and E in the Supplemental Material [39]). Correlations between arrival locations as well as phases between the quantum holograms [Fig. 4(c)] that make up the many-body hologram can be extracted from the transformation matrix by considering a range of locations. This allows us to extract a photonic or atomic state from a prepared or measured atomic or photonic state, respectively. Hence, using our analysis, any desired photonic state with precise phase distribution can be generated by manipulating the many-body atomic state.

Photonic state fidelity in feasible experimental implementations. In this section we will address sources of error during experimental implementation of our system, including thermal shift, spatial dependence, aperture size, and overlap with off-target cells. Errors may affect the phase and amplitude of the photonic states. We define the phase as the quantity of merit when discussing image fidelity. We therefore define a phase fidelity $\mathcal{F}_{\text{phase}} = 1 - |\frac{\phi_{\text{target}} - \phi_{\text{experimental}}}{\phi_{\text{target}}}|$ to track its decoherence (if $\phi_{\text{target}} = 0$ then use $\mathcal{F}_{\text{phase}} = |\frac{\pi - \phi_{\text{experimental}}}{\pi}|$). Importantly, this fidelity criterion is independent of hologram amplitude and is applicable to single-photon scenarios [42]. The target phase to which we benchmark our results is the result from plane wave incidence (ϕ_{pw}). An expected source of decoherence is an undesired shift, δ_a , in the location a of the ancilla. Recent work demonstrates the ability to determine the ancilla location in experiments [17,40]. However, the thermal movements of the ancilla atom will be of consequence. In experimental realizations, thermal fluctuations shift the ancilla $\pm 10\%$ with respect to the lattice spacing [15] which effect the phase in Fig. 3. While the specific parameters of the cell depend on the experiment, we can lay out certain guidelines. The control light which excites the $|g\rangle$ to $|r\rangle$ transition, has a resolution fundamentally limited to $\frac{1}{2}\lambda_{g'r'}$. Therefore, the necessity to excite deterministically a specific atom sets a lower bound on the lattice spacing. Next, in order to set minimum limits on cell size, it is necessary to consider the size of the reflective area R and the relation $\phi_j = 2\pi a_j \sin(\theta)/\lambda_{ge}$. Solving for a_j for $|\phi_j| = \pi$ (the maximum shift necessary to encode all phases) implies $a_{\pi} = \lambda_{ge}/(2 \sin(\theta))$. Thus, given the minimum resolution cited above, the number of accessible phases N is $2a_{\pi}/\lambda_{g'r'}$. In addition to a dimension greater than a_{π} , clearly the cell width should satisfy $w > 2R$ at a minimum. From this calculation we estimate the minimum cell width to be roughly $18 \mu\text{m}$ for the experimental parameters presented in [17] which will allow precision control of more than 15 distinct phases. For smaller R and λ , smaller cells can be constructed. To give an example including practical experimental parameters, we present in Sec. G in the Supplemental Material

[39] a quantum hologram of a classical image. The original 16×16 pixel image is mapped onto a lattice of 16×16 cells, each square width $10.6 \mu\text{m}$. We assume a practical Rydberg radius of $3 \mu\text{m}$ and light wavelength of 532 nm , and lattice spacing of 380 nm . Thus, a cell consists of 784 atoms where only one atom needs to be singly addressed (one ancilla) while over the total array 256 ancilla atoms should be singly addressable (via a direct laser excitation). This number of atoms are well within practical parameters of state-of-the-art experiments [17,43,44].

Experimentally R_c may deviate from theoretical predictions. This discrepancy, δ_c , will contribute an extraneous phase and magnitude in the Fourier plane. Numerically we determine the phase error is negligible for $\delta_c/R_c \leq 0.3$ and amplitude errors scale quadratically (Sec. B in the Supplemental Material [39]).

In the many-body scenario, we developed $\mathcal{T}$ approximating incident Gaussian wave packets, ψ_{wp} , as spatially smeared such that they appear as plane waves wholly overlapping their target cell and negligibly affected by other cells. In Fig. 5 we plot the phase fidelity achieved with a practical Gaussian wave packet of width σ as a function of the wave packet overlap with the target and neighboring cells. Initially, as the Gaussian overlap with the target cell grows, so does the agreement between the wave packets phase, ϕ_{wp} , and the plane wave phase, ϕ_{pw} , and thus $\mathcal{F}_{\text{phase}}$ increases. After hitting an ideal regime, further increasing σ causes overlaps with neighboring cells which contribute extraneous phases lowering the phase fidelity (Sec. F in the Supplemental Material [39]). For complementary error analyses, see Ref. [10].

Immediate and soon to be feasible implementation and applications. The system and techniques presented are conducive toward many interesting applications including state readout, dynamic holography, and qudit implementation. It also offers a few advantages. First, the photons do not interact directly with the ancilla, thus minimizing decoherence [45]. Similarly, since to probe our atomic state we do not need to directly address the atom, our system is resistant to heating and thus does not suffer from array atom loss over the course of many measurements [17]. Second, our system offers both temporal (sequential photons) and spatial (number

of cells) scalability. Third, our detection schema benefits from many scattering events within single ancilla coherence time. Specifically, the spontaneous emission rate of the array atoms, $\gamma \approx \mathcal{O}(10)$ s of MHz [10,15], determines the maximum rate at which one can probe the array with sequential photons. For example, $T_2 \approx \mathcal{O}(10)$ s of μs for entangled Rydberg blockade states [46], gates on the order of μs [47], and μs lifetimes of $N = 20$ cluster Rydberg GHZ states [41] dictates order $\approx \mathcal{O}(10)$ sequential imaging photons per operation. State readout benefits from all these elucidated points since it often entails the destruction of a key element of the system, thus making nondestructive measurement techniques in photonic and atomic systems paramount [45,48–50].

As an example, consider an array in a superposition of two spin wave excitations with momentums' k_1 and k_2 . In the imaging plane, photons will be detected at specific far-field transverse locations which will indicate the specific spin-wave momentum by which they were scattered. Holography has also been studied in order to image arrays and hidden objects [51–54] while here we are imaging quantum states. Finally, automated phase recovery may be highly effective and greatly speed up transformation matrix calculations [55].

Another implementation of this system is dynamic holography [6] due to the arrays highly programmable nature in terms of operational frequency, size, and single-ancilla control on the order of μs [3,45,47] (up to nanosecond dynamics as in [56,57]). Thus, fully dynamic holography and real-time spatial light modulation, including desirably the visible regime, is immediately evident as a possible application. Finally, our system also suggests a possible qudit implementation [58,59]. Considering each cell as our unit of quantum information, each ancilla then serves as a level within the cell. As in Eq. (2), $f_j|r'\rangle_j$ determines j unique states of a cell that can be placed in superpositions of each other. This state can then be measured in the Fourier plane using the transformation matrix.

Acknowledgments. This research was funded by Grant No. 2021775 from the U.S.-Israel Binational Science Foundation (BSF) and by Grant No. 2207972. from the United States National Science Foundation (NSF) and Israel Science Foundation Grant No. 2402/22.

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