Letter

Adaptive strategy automation with large language models in paradoxical games

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Finding an ideal sequence in games is crucial for maximizing gains in various scenarios. This process requires extensive investigation, which is both time consuming and demands significant domain expertise. The emergence of large language models (LLMs) represents a significant turning point in addressing this issue, attributable to their potent analytical capabilities. In this context, LLMs can serve to substantially alleviate the human labor needed to manage these complexities. Through comprehensive simulations of coin-tossing games, we have demonstrated that the adaptive switching strategies formulated by LLMs surpass predefined sequences in profitability when applied to certain paradoxical games. Furthermore, our experimental findings indicate that the proposed method not only automates the identification of effective strategies but also provides adaptability to various forms of these paradoxical games.

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Introduction. Inspired by the flashing Brownian ratchet, Parrondo's paradox demonstrates how combining two losing strategies can yield a winning outcome [1], a phenomenon that has garnered considerable attention across fields such as economics [2], quantum game theory [3–6], biology [7,8], sociophysics [9–11], and decision analysis [12]. For instance, chaotic switching in quantum coin tosses has potential applications in cryptography [13]. Similarly, the alternation between competitive and noncompetitive growth strategies among cells could potentially delay the onset of cancer [14]. In medical epidemiology, while both strict lockdowns and open communities individually accrue high daily costs during a pandemic, strategically switching between these approaches can significantly reduce overall expenses in managing COVID-19, as modeled by Parrondo's paradox [15]. The key to leveraging this paradox effectively lies in the strategic sequencing of the games. Optimizing this sequence is critical for maximizing outcomes [16]. Prior studies have shown that even a simple sequence such as "ABABB" achieves the highest profit for the periodic capital-dependent Parrondo's game, combining theoretical insights with empirical data [17]. However, this optimality holds only under the condition of periodic sequence, which limits its utility in dynamically changing game environments. Consequently, the quest for an ideal sequence in arbitrary order has led researchers to employ various methodologies, including genetic algorithms [18,19], known for their robust capability in solving complex combinatorial problems [20,21]. These algorithms, along with other approaches, have significantly contributed to a deeper understanding of the complex relationship between game sequence and outcomes within Parrondo's paradox [15,22].

Despite these advancements, identifying adaptable sequences that respond effectively to changing game environments remains a pivotal challenge due to the inherent complexity of Parrondo's paradox. Fixed sequences may not effectively adapt to changes within the gameplay or to the evolving conditions of the game [23]. Recognizing this limitation, some researchers have turned to adaptive strategies that recalibrate based on the current state of the game, thereby maximizing profits under changing conditions [24–26]. However, the implementation of these adaptive strategies can be labor intensive, requiring continuous updates and substantial domain expertise, which may limit their accessibility and practical use. This challenge could become even more pronounced as the complexity of the game increases. Therefore, this gap presents an opportunity for the application of advanced computational technologies such as ChatGPT, a large language model that excels in data analysis and pattern recognition. A recent study has revealed that large language models (LLMs) can achieve high accuracy in predicting human behavior with sentiment analysis based on outcome-based utility [27]. Research exploring large language models in various games suggests that their behavior often aligns with human behavior [28,29], although other findings indicate that LLMs tend to overestimate human altruism [30]. These findings have significant implications for policymaking, as discussed in Ref. [31].

Due to the strength of LLMs, we propose to automate the analysis of game strategies with effective prompt engineering. Our contributions can be summarized as follows.

(1) The capability of large language models, trained on extensive data sets, provides a unique advantage in identifying adaptive strategies for complex scenarios such as Parrondo's

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games. In this Letter, we employ LLMs to determine an ideal strategic sequence, bypassing the need for dynamic adjustments during gameplay. We wish to highlight that this pioneering approach may allow us to analyze other complex systems, by leveraging the LLMs' advanced pattern recognition and predictive analytics.

(2) Our research explores the robustness of using LLMs to analyze different strategic configurations in Parrondo's games. The experimental results affirm that LLMs can adapt effectively to various game settings by modifying parameters in prompt to LLMs. Importantly, our approach extends beyond the typical capital-dependent games and has proven effective in the history-dependent scenario and cooperative scenario as well. This versatility demonstrates its potential application across diverse strategic contexts.

Preliminaries. There are several variants of Parrondo's paradox. The first variant is capital dependent, involving two games [1]. In game A, a player tosses a coin and wins if it lands on heads, with a $1/2 - \epsilon$ probability, and loses if it lands on tails, with a $1/2 + \epsilon$ probability. In game B, players choose between two coins based on their current capital and a parameter M. Here, coin 2 offers a $1/10 - \epsilon$ probability of landing on heads, and coin 3 provides a $3/4 - \epsilon$ probability of heads. If the player's current capital is a multiple of M, coin 2 is chosen; otherwise, coin 3 is selected. In this study, M is set to 2 and 3, respectively, for comparison's sake.

Another classic example of Parrondo's paradox is history dependent, where game A is identical to the one in the capital-dependent version. However, in game B, the choice of coin is determined by the outcomes of the two previous games. With each game resulting in either a win or loss, four possible outcome combinations occur: {lose, lose}, {lose, win}, {win, lose}, and {win, win}. Each combination is associated with a different coin, offering winning probabilities of $1/10 - \epsilon$, $1/4 - \epsilon$, $1/4 - \epsilon$, and $7/10 - \epsilon$, respectively, where ϵ is also a small constant that is set to 0.005 in this work. Note that the above parameter settings are consistent with previous studies [20,24] for comparison.

The third example is the cooperative Parrondo's paradox [32], where there is a group of N players. For each player, game A is the same as in the capital-dependent and history-dependent games, and game B assumes that the winning probabilities at any given time for player i are subject to the current states of the adjacent players:

(a) p_1 if both neighboring players (player $i-1$ and player $i+1$) are winners;

(b) p_2 if player $i-1$ is a winner and player $i+1$ is a loser;

(c) p_3 if player $i-1$ is a loser and player $i+1$ is a winner; and

(d) p_4 if both neighboring players are losers.

The probabilities follow the typical setting [32], where $p_1 = 1.0$, $p_2 = p_3 = 0.16$, and $p_4 = 0.7$.

For the aforementioned three variants of Parrondo's paradox, the player (or players if in the cooperative game) starts with an initial capital of zero, gaining one unit for a win and losing one for a loss. Letting $X(t)$ be the mean capital at round t, the mean capital for the next round in game A is

$$X(t+1) = X(t) + 2p_{\mathrm{win_A}}(t) - 1, \tag{1}$$

where $p_{\mathrm{win_A}}$ indicates the probability of winning in game A. Similarly, if game B is played at round t, the mean capital for the next round is

$$X(t+1) = X(t) + 2p_{\mathrm{win_B}}(t) - 1. \tag{2}$$

Methods. As a proof of principles on the effectiveness of LLMs in our proposed paradoxical games, this study relaxes the fixed and periodic sequential rules of Parrondo's paradox in the experimental setup. Our prompt mainly comprises three components, including the following:

(1) Task description: The prompt requests the development of a strategy for playing two distinct games alternatively to achieve the highest profit, emphasizing strategic decision-making based on game outcomes.

(2) Game introduction: Two games are introduced, each with unique mechanics: Game A involves a single coin toss with defined probabilities for winning and losing. Game B is varied with the type of games (including capital-dependent, history-dependent, and cooperative Parrondo's games).

(3) Initial conditions: The games start with a player's capital at zero, with each game outcome influencing the capital by either increasing or decreasing it based on whether the player wins or loses.

The full prompt to analyze the best strategies for capital-dependent, history-dependent, and cooperative Parrondo's games is given in the Appendix.

The applicability of our proposed method is demonstrated via a comparison with the genetic algorithm, theoretical strategy, and reinforcement learning, thereby identifying the best sequence of Parrondo's games under various settings. The comparative benchmarks include the following:

Random sequence employs a randomly generated sequence of games, alternating between game A and game B without any strategic influence or predetermined bias. This sequence serves as a baseline to evaluate the effectiveness of our proposed method.

Fixed sequence requires players to alternate two games with a given rule. For $M = 3$, the sequence ABABB, determined through theoretical analysis, is used as a benchmark [17]. For $M = 2$ and $M = 4$, the sequences AB and AABB are empirically utilized for comparison.

Genetic algorithm (GA) is incorporated as a method to explore the best game sequence, following the approach outlined in Ref. [20].

Reinforcement learning (RL) is a decision-making framework in which an agent learns to make the most advantageous decisions by interacting with an environment and receiving feedback through rewards. It has shown state-of-the-art performance in playing paradoxical games in Ref. [26] and is included in our comparison.

The strategy suggested by LLMs, specifically ChatGPT-4, for different games, is outlined below. For the capital-dependent game, $M = 3$ is given as an illustration. For the full response, please refer to the Appendix.

Start with game A: Since it begins with a capital of 0, which is divisible by 3, playing game B immediately would force to use coin 2, the unfavorable option.

Switch to game B when the capital is not a multiple of 3: This ensures coin 3 will be used. For instance, if the capital after playing game A is either 1 or 2, switch to game B.

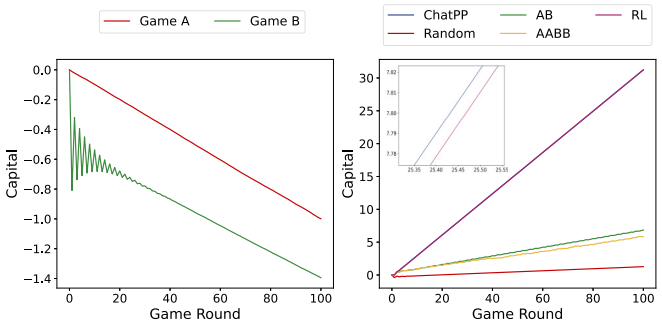


FIG. 1. Simulation results for the capital-dependent Parrondo's games ($M = 3$).

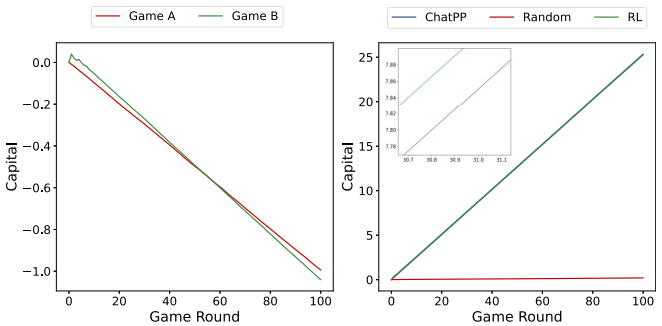


FIG. 3. Simulation results for the history-dependent Parrondo's games.

Observe the results and adapt: If the capital after playing game B (using coin 3) becomes a multiple of 3, switch back to game A to adjust your capital. If it remains nondivisible by 3, continue with game B using coin 3.

Repeat the process: Continue this process of alternating between games to manipulate the capital.

Due to the stochasticity of LLMs, the structure of the suggested strategy is not always the same. The strategy of playing the history-dependent game suggested by ChatGPT-4 is as follows:

Initial phase: Start with game A, given no history to leverage the adaptive nature of game B. Continue playing game A until there are two consecutive results (either win or lose). The reason is that game A is less dependent on previous outcomes and provides a neutral starting point.

Switching to game B: If we start with two consecutive losses in game A, switch to game B. The high probability of winning with {lose, lose} (0.895) can help turn the situation around. If we start with two consecutive wins in game A, continue in game B for at least one game because the {win, win} probability of winning is still favorable (0.695).

Adaptive play: Monitor the outcomes in sequences of two for game B. If one gets {lose, lose} or {win, win}, keep playing game B. If the outcome sequence becomes mixed (either {lose, win} or {win, lose}), switch back to game A until two similar results occur (either two wins or two losses).

Furthermore, the strategy for the cooperative case suggested by ChatGPT-4 is as follows:

Initial state: Each player starts with zero capital. Initially, the game outcomes for game B are not predictable, so starting with game A might be a reasonable choice for the first few rounds until a pattern or trend can be established.

Observing patterns: As the play progresses, each player should observe the outcomes of their neighbors:

If both neighbors lose in a round, the player should switch to game B in the next round. . ..

If the outcome is mixed (one win and one loss among neighbors), sticking to game A may be safer given the low win probability in game B under these conditions.

If both neighbors win, playing game B could be beneficial....

Results and discussion. The strategies suggested by LLMs may not always be viable due to the LLM hallucinations [33,34]. However, the capital-dependent case is easier to manage than the history-dependent case, possibly because fewer coins are involved in the former scenario. In this work, the strategy suggested by ChatGPT-4 to deal with Parrondo's paradox is denoted as "ChatPP." Each strategy will be evaluated over 100 game rounds with 10^6 repetitions of the simulations in the experiments.

Figure 1 displays the results from playing Parrondo's games under a capital-dependent setting with $M = 3$ (which is the most well known). In the left graph, game A shows a rapid initial loss followed by a steady decline in capital, whereas game B starts with a fluctuating pattern. This result implies that, although game B initially offers a more dynamic performance, both games are fundamentally losing games over the long term, with game B's early fluctuations not enough to counteract the overall downward trend. The right graph compares the various strategies: ChatPP, random

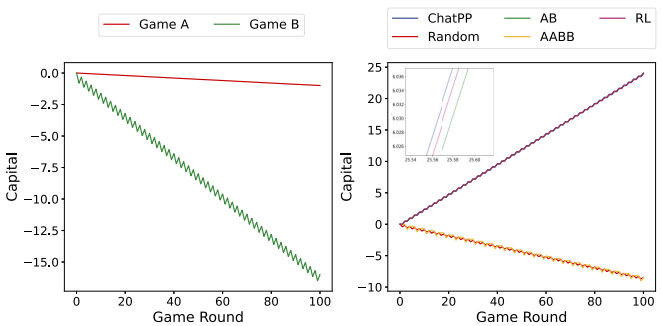


FIG. 2. Simulation results for the capital-dependent Parrondo's games ($M = 2$).

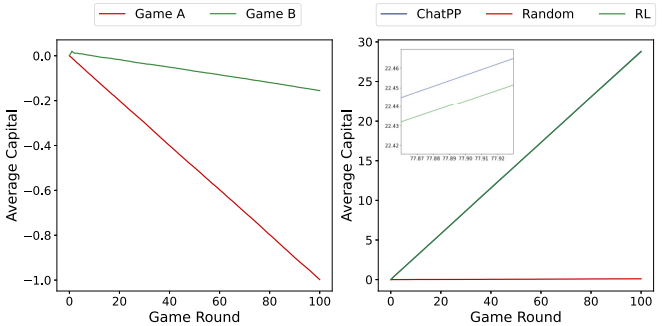


FIG. 4. Simulation results (average capital per player) for the cooperative Parrondo's games.

sequence, ABBAB, GA, and RL. It can be found that the sequences found by the genetic algorithm (GA) and the periodic ABABB achieve better results than the results obtained by the random sequence that achieves only minimal gains. The ChatPP strategy achieves the highest capital of up to 30, suggesting that LLMs effectively leverage the capital-dependent rules to maximize gains. Reinforcement learning (RL) employs sophisticated methods that deliver performance levels comparable to ChatPP. However, the design cost and required expertise for ChatPP are significantly lower than that for RL, highlighting the applicability and feasibility of the LLM-based approach. This result indicates that strategic sequence planning, especially one that dynamically adapts to the changing conditions of the player's capital as in the ChatPP method, can transform losing scenarios into winning outcomes.

As a benchmark, we choose the sequence AB as it is empirically known to be the most effective strategy for Parrondo's games of $M = 2$. The choice of the particular sequences AABB is primarily driven by their structural simplicity and common use in exploratory analyses of Parrondo's paradox. Figure 2 shows results from Parrondo's games with $M = 2$. Similar to the previous scenario with $M = 3$, the left graph presents the performance of game A and game B individually, with game A maintaining a consistent loss, while game B displays a more gradual but consistent decline over 100 rounds. The loss trajectory for game B is slightly steeper compared to the $M = 3$ setting, indicating a more pronounced negative impact at a lower M value. In the right graph, four strategies are compared with ChatPP. Here, the ChatPP strategy and RL demonstrate a similar performance and are the most successful, exhibiting a robust, linear increase in capital. The sequence AB is exactly the same as the suggested strategy by LLMs, leading to overlapped plots (green and blue). On the contrary, the outcomes obtained by the AABB sequence and random strategy are losing.

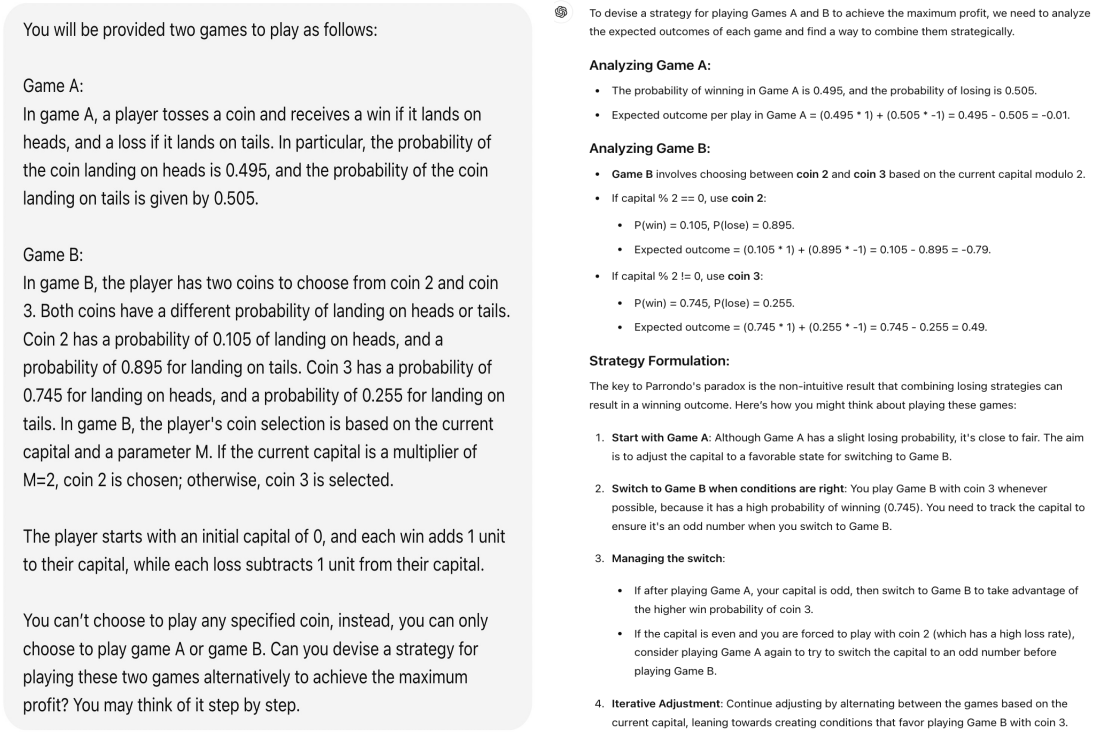
Comparing these results to the $M = 3$ setting, it is evident that a lower M value in this scenario ($M = 2$) results in generally more significant losses in game B. This particular M value seems to amplify the difficulty of achieving gains through nonoptimized sequences. However, the effectiveness of the ChatPP method remains relatively constant across different M values, showcasing its robustness in exploiting the capital-dependent rules of Parrondo's games to achieve maximum capital gain. This consistency suggests that the ChatPP strategy is effectively tailored to capitalize on the specifics of the game dynamics, regardless of the value of M, highlighting its potential as a powerful tool for strategic decision-making in stochastic environments.

Furthermore, Fig. 3 illustrates the outcomes of playing Parrondo's games for the history-dependent case. In the left graph, both game A and game B result in a steady decline in capital, indicating that each game individually is a losing game. The trajectories of both games almost overlap, suggesting that their losing rates are very similar when played in isolation. In contrast, the right graph demonstrates a stark difference in outcomes when a strategic sequence is employed. The ChatPP strategy shows a significant increase in capital, converting a mix of two losing games into a profitable one. Random strategy, however, can only achieve a small profit in the long run. Moreover, Fig. 4 presents the result of playing cooperative Parrondo's games with different strategies where the number of players is empirically set to 4. As seen in the left graph, playing game A or game B individually will lead to a losing outcome. However, the strategy of ChatPP can achieve much more profit than a random sequence, as demonstrated by the right graph. The in-depth analysis of Parrondo's games, facilitated by ChatPP, has showcased the significant potential of intelligent sequence selection to influence outcomes in probabilistic systems. The adaptability of ChatPP to various game modes and parameter settings highlight its significant capacity to analyze complex systems, including those modeled after physical phenomena. The success of this LLM-based approach illustrates its effectiveness in overcoming inherent disadvantages within probabilistic systems, thereby demonstrating the practical utility of LLMs in improving decision-making processes in complex environments.

Conclusion. Overall, the integration of LLMs in studying Parrondo's paradox within our current context not only advances our understanding of strategic optimization in both theoretical and applied domains but also enhances problem solving in models derived from physical phenomena. Our findings demonstrate how AI can bridge the gap between theoretical research and practical applications, paving the way for future innovations across diverse fields.

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Appendix. The complete prompt for analyzing optimal strategies in capital-dependent, history-dependent, and cooperative Parrondo's games is presented in Appendix Figs. 5–8. The prompt of each case includes a task description and a game introduction, which is on the left side. The response of ChatGPT-4 is on the right side, including the analysis of the game as well as the suggested strategy to play.



You will be provided two games to play as follows:

Game A:
In game A, a player tosses a coin and receives a win if it lands on heads, and a loss if it lands on tails. In particular, the probability of the coin landing on heads is 0.495, and the probability of the coin landing on tails is given by 0.505.

Game B:
In game B, the player has two coins to choose from coin 2 and coin 3. Both coins have a different probability of landing on heads or tails. Coin 2 has a probability of 0.105 of landing on heads, and a probability of 0.895 for landing on tails. Coin 3 has a probability of 0.745 for landing on heads, and a probability of 0.255 for landing on tails. In game B, the player's coin selection is based on the current capital and a parameter M. If the current capital is a multiplier of M=2, coin 2 is chosen; otherwise, coin 3 is selected.

The player starts with an initial capital of 0, and each win adds 1 unit to their capital, while each loss subtracts 1 unit from their capital.

You can't choose to play any specified coin, instead, you can only choose to play game A or game B. Can you devise a strategy for playing these two games alternatively to achieve the maximum profit? You may think of it step by step.

To devise a strategy for playing Games A and B to achieve the maximum profit, we need to analyze the expected outcomes of each game and find a way to combine them strategically.

Analyzing Game A:

- The probability of winning in Game A is 0.495, and the probability of losing is 0.505.
- Expected outcome per play in Game A = (0.495 * 1) + (0.505 * -1) = 0.495 - 0.505 = -0.01.

Analyzing Game B:

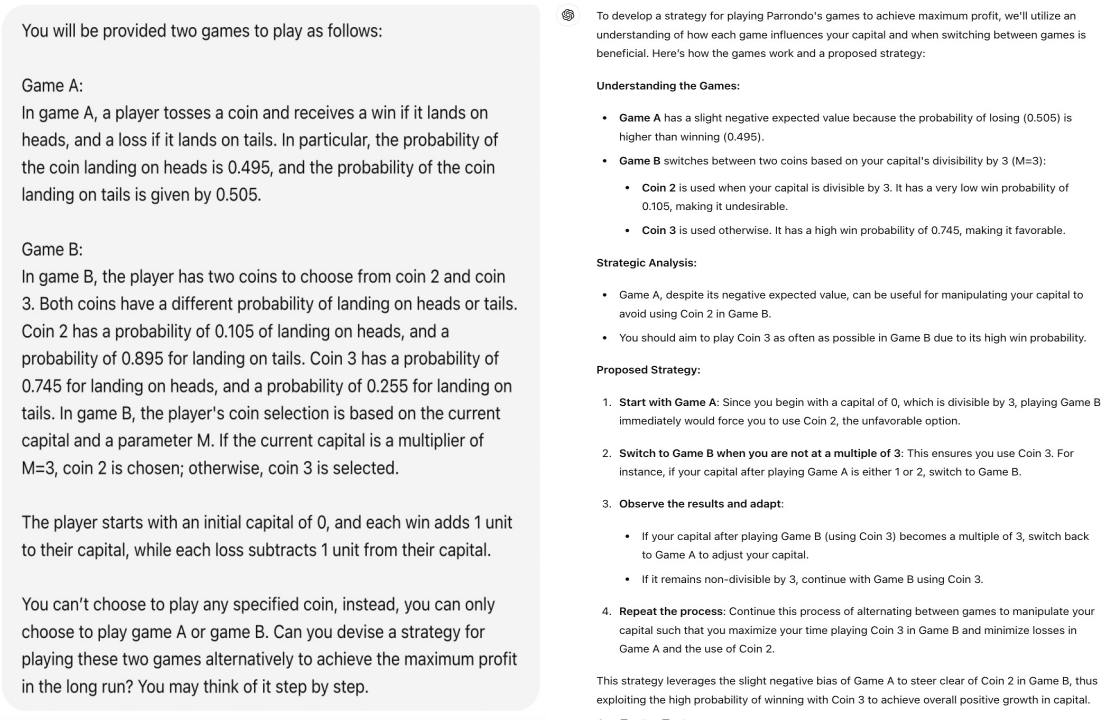
- **Game B** involves choosing between **coin 2** and **coin 3** based on the current capital modulo 2.
- If capital % 2 == 0, use **coin 2**:
  - P(win) = 0.105, P(lose) = 0.895.
  - Expected outcome = (0.105 * 1) + (0.895 * -1) = 0.105 - 0.895 = -0.79.
- If capital % 2 != 0, use **coin 3**:
  - P(win) = 0.745, P(lose) = 0.255.
  - Expected outcome = (0.745 * 1) + (0.255 * -1) = 0.745 - 0.255 = 0.49.

Strategy Formulation:

The key to Parrondo's paradox is the non-intuitive result that combining losing strategies can result in a winning outcome. Here's how you might think about playing these games:

1. **Start with Game A**: Although Game A has a slight losing probability, it's close to fair. The aim is to adjust the capital to a favorable state for switching to Game B.
2. **Switch to Game B when conditions are right**: You play Game B with coin 3 whenever possible, because it has a high probability of winning (0.745). You need to track the capital to ensure it's an odd number when you switch to Game B.
3. **Managing the switch**:
   - If after playing Game A, your capital is odd, then switch to Game B to take advantage of the higher win probability of coin 3.
   - If the capital is even and you are forced to play with coin 2 (which has a high loss rate), consider playing Game A again to try to switch the capital to an odd number before playing Game B.
4. **Iterative Adjustment**: Continue adjusting by alternating between the games based on the current capital, leaning towards creating conditions that favor playing Game B with coin 3.

FIG. 5. The prompt to find an ideal strategy in the capital-dependent case ($M = 2$).



You will be provided two games to play as follows:

Game A:
In game A, a player tosses a coin and receives a win if it lands on heads, and a loss if it lands on tails. In particular, the probability of the coin landing on heads is 0.495, and the probability of the coin landing on tails is given by 0.505.

Game B:
In game B, the player has two coins to choose from coin 2 and coin 3. Both coins have a different probability of landing on heads or tails. Coin 2 has a probability of 0.105 of landing on heads, and a probability of 0.895 for landing on tails. Coin 3 has a probability of 0.745 for landing on heads, and a probability of 0.255 for landing on tails. In game B, the player's coin selection is based on the current capital and a parameter M. If the current capital is a multiplier of M=3, coin 2 is chosen; otherwise, coin 3 is selected.

The player starts with an initial capital of 0, and each win adds 1 unit to their capital, while each loss subtracts 1 unit from their capital.

You can't choose to play any specified coin, instead, you can only choose to play game A or game B. Can you devise a strategy for playing these two games alternatively to achieve the maximum profit in the long run? You may think of it step by step.

To develop a strategy for playing Parrondo's games to achieve maximum profit, we'll utilize an understanding of how each game influences your capital and when switching between games is beneficial. Here's how the games work and a proposed strategy:

Understanding the Games:

- **Game A** has a slight negative expected value because the probability of losing (0.505) is higher than winning (0.495).
- **Game B** switches between two coins based on your capital's divisibility by 3 (M=3):
  - **Coin 2** is used when your capital is divisible by 3. It has a very low win probability of 0.105, making it undesirable.
  - **Coin 3** is used otherwise. It has a high win probability of 0.745, making it favorable.

Strategic Analysis:

- Game A, despite its negative expected value, can be useful for manipulating your capital to avoid using Coin 2 in Game B.
- You should aim to play Coin 3 as often as possible in Game B due to its high win probability.

Proposed Strategy:

1. **Start with Game A**: Since you begin with a capital of 0, which is divisible by 3, playing Game B immediately would force you to use Coin 2, the unfavorable option.
2. **Switch to Game B when you are not at a multiple of 3**: This ensures you use Coin 3. For instance, if your capital after playing Game A is either 1 or 2, switch to Game B.
3. **Observe the results and adapt**:
   - If your capital after playing Game B (using Coin 3) becomes a multiple of 3, switch back to Game A to adjust your capital.
   - If it remains non-divisible by 3, continue with Game B using Coin 3.
4. **Repeat the process**: Continue this process of alternating between games to manipulate your capital such that you maximize your time playing Coin 3 in Game B and minimize losses in Game A and the use of Coin 2.

This strategy leverages the slight negative bias of Game A to steer clear of Coin 2 in Game B, thus exploiting the high probability of winning with Coin 3 to achieve overall positive growth in capital.

FIG. 6. The prompt to find an ideal strategy in the capital-dependent case ($M = 3$).

You will be provided two games to play as follows:

Game A:
In game A, a player tosses a coin and receives a win if it lands on heads, and a loss if it lands on tails. In particular, the probability of the coin landing on heads is 0.495, and the probability of the coin landing on tails is given by 0.505.

Game B:
In game B, the determination of the coin to be played hinges upon the sequential results of the preceding two games. As each game can culminate in either a win or a loss, a total of four distinct combinations emerge from the possible outcomes of the previous two games: {lose, lose}, {lose, win}, {win, lose} and {win, win}. These four combinations, in turn, correspond to distinct coin options that can be chosen. Therefore, game B involves 4 coins, each with probabilities of winning of 0.895, 0.245, 0.245 and 0.695, respectively.

The player starts with an initial capital of 0, and each win adds 1 unit to their capital, while each loss subtracts 1 unit from their capital. It is assumed that the first two outcomes are given.

You can't choose to play any specific coin, rather, you can only choose to play game A or game B. Can you devise a strategy for playing these two games alternatively to achieve the maximum profit? You may think of it step by step.

To develop a strategy for maximizing profit in the described scenario involving games A and B, we should consider both the probabilities of winning each game and the effects of the previous outcomes when choosing to play game B. Since game B's outcome and the coin used depend on the results of the two previous games, it creates an opportunity to explore a mixed strategy between games A and B. Here's a step-by-step approach to formulating a strategy:

Understanding the Games

1. **Game A:** This is a straightforward game with a slight disadvantage as the probability of winning is slightly less than losing (0.495 winning vs. 0.505 losing).
2. **Game B:** This game is more complex as it has variable probabilities based on the outcomes of the last two games:
   - **{lose, lose}:** Probability of winning = 0.895
   - **{lose, win} and {win, lose}:** Probability of winning = 0.245
   - **{win, win}:** Probability of winning = 0.695

Strategy Formulation

To utilize the probabilities effectively, consider the following strategy:

- **Initial Rounds:** Start by playing Game A for the first two rounds since the outcomes of these initial rounds determine the starting point for Game B's probabilities.
- **Subsequent Rounds:** Choose between Game A and Game B based on the results of the previous two games:
  - If the previous outcomes were **{lose, lose}** or **{win, win}**, play Game B to take advantage of the higher winning probabilities (0.895 and 0.695, respectively).
  - If the previous outcomes were **{lose, win}** or **{win, lose}**, play Game A because Game B has a very low probability of winning in these scenarios (0.245).

Practical Application

Implement this strategy by tracking the outcomes of each round and making decisions accordingly. The key is to adaptively switch between Game A and Game B based on the recent outcomes to exploit the advantageous probabilities of Game B whenever possible, while reverting to Game A when the odds are less favorable.

FIG. 7. The prompt to find an ideal strategy in the history-dependent case.

Consider an ensemble of 4 players, each with his/hers own capital $C_i(t)$, $i = 1, 2, \ldots, 4$. As in the cooperative parrondo paradox, we consider two games, A and B. Player i can play either game A or B according to the following rules.

Game A: In game A, a player tosses a coin and receives a win if it lands on heads, and a loss if it lands on tails. In particular, the probability of the coin landing on heads is 0.495, and the probability of the coin landing on tails is given by 0.505.

Game B: In game B, we only consider the case when the probabilities of winning at time t, depend only on the present state of the neighbors, hence the probabilities are given by:

0.7 if player $i - 1$ is a winner and player $i + 1$ is a winner
0.16 if player $i - 1$ is a winner and player $i + 1$ is a loser
0.16 if player $i - 1$ is a loser and player $i + 1$ is a winner
1.0 if player $i - 1$ is a loser and player $i + 1$ is a loser

Each player starts with an initial capital of 0, and each win adds 1 unit to their capital, while each loss subtracts 1 unit from their capital. They can't choose to play any specified coin, instead, they can only choose to play game A or game B.

Can you devise a strategy for each player to play these two games alternatively to achieve the maximum profit? You may think of it step by step.

To devise a strategy for the ensemble of players to maximize their profit in this cooperative Parrondo paradox involving games A and B, let's consider the rules and probabilities involved. We will develop a strategy based on player cooperation and alternating game choices, informed by the stated win probabilities of games A and B and their dependency on neighboring players' previous game outcomes.

Analysis of Game Probabilities:

1. **Game A** is quite straightforward, with a slightly lower than even chance of winning (0.495). Over the long term, this leads to an expected slight loss.
2. **Game B** is more complex, as it involves the states of neighboring players:
   - 0.7 chance of winning if both neighbors were winners in the previous round.
   - 0.16 chance of winning if one neighbor was a winner and the other a loser.
   - 1.0 (certain win) if both neighbors were losers.

Strategic Considerations:

1. **Game B's strong dependence on neighbors' states** makes cooperative strategies crucial. Players should aim to influence each other's states to optimize Game B's conditions, specifically aiming for both neighbors to lose simultaneously when possible.

Step-by-Step Strategy Development:

1. **Initial State:** Each player starts with zero capital. Initially, the game outcomes for Game B are not predictable, so starting with Game A might be a reasonable choice for the first few rounds until a pattern or trend can be established.
2. **Observing Patterns:** As play progresses, each player should observe the outcomes of their neighbors:
   - If both neighbors lose in a round, the player should switch to Game B in the next round for a guaranteed win.
   - If the outcome is mixed (one win and one loss among neighbors), sticking to Game A may be safer given the low win probability (0.16) in Game B under these conditions.
   - If both neighbors win, playing Game B could be beneficial (0.7 probability of winning), but this is riskier than the guaranteed win after two losses.
3. **Coordination for Guaranteed Wins:** If possible, players should try to synchronize their plays such that they can force two consecutive losses for themselves, setting up a guaranteed win in Game B in the following round. This can be strategically timed based on observed outcomes and coordination.
4. **Switching Based on Results:** Players should be flexible in their strategy, switching between games based on the most recent results and aiming to maximize the advantageous conditions for Game B.
5. **Simulation and Adjustment:** Simulating a few rounds of these strategies can provide insights into effectiveness and allow for adjustments. Keeping track of overall capital changes and adapting strategies accordingly is crucial.

FIG. 8. The prompt to find an ideal strategy in the cooperative case.

[1] G. P. Harmer and D. Abbott, Losing strategies can win by Parrondo's paradox, Nature (London) **402**, 864 (1999).

[2] T. Wen, J. W. Lai, and K. H. Cheong, Switching between two losing stocks may enable paradoxical win: An empirical analysis, Fractals **31**, 2340001 (2023).

[3] D. K. Panda, B. V. Govind, and C. Benjamin, Generating highly entangled states via discrete-time quantum walks with Parrondo sequences, Physica A **608**, 128256 (2022).

[4] J. Rajendran and C. Benjamin, Implementing Parrondo's paradox with two-coin quantum walks, R. Soc. Open Sci. **5**, 171599 (2018).

[5] Z. Walczak and J. H. Bauer, Noise-induced Parrondo's paradox in discrete-time quantum walks, Phys. Rev. E **108**, 044212 (2023).

[6] Z. Walczak and J. H. Bauer, Parrondo's paradox in quantum walks with three coins, Phys. Rev. E **105**, 064211 (2022).

[7] T. Wen and K. H. Cheong, Parrondo's paradox reveals counterintuitive wins in biology and decision making in society, Phys. Life Rev. **51**, 33 (2024).

[8] K. H. Cheong, T. Wen, S. Benler, J. M. Koh, and E. V. Koonin, Alternating lysis and lysogeny is a winning strategy in bacteriophages due to Parrondo's paradox, Proc. Natl. Acad. Sci. USA **119**, e2115145119 (2022).

[9] J. M. Koh and K. H. Cheong, New doubly-anomalous Parrondo's games suggest emergent sustainability and inequality, Nonlinear Dyn. **96**, 257 (2019).

[10] Y. Ye, Z.-Y. Zhai, X.-R. Hang, and N.-G. Xie, Influence analysis of network evolution on Parrondo effect, Biosystems **236**, 105124 (2024).

[11] N.-G. Xie, J.-Y. Guo, Y. Ye, C. Wang, and L. Wang, The paradox of group behaviors based on Parrondo's games, Physica A **391**, 6146 (2012).

[12] K. H. Cheong, D. B. Saakian, and R. Zadourian, Allison mixture and the two-envelope problem, Phys. Rev. E **96**, 062303 (2017).

[13] J. W. Lai and K. H. Cheong, Chaotic switching for quantum coin Parrondo's games with application to encryption, Phys. Rev. Res. **3**, L022019 (2021).

[14] T. Wen, K. H. Cheong, J. W. Lai, J. M. Koh, and E. V. Koonin, Extending the lifespan of multicellular organisms via periodic and stochastic intercellular competition, Phys. Rev. Lett. **128**, 218101 (2022).

[15] K. H. Cheong, T. Wen, and J. W. Lai, Relieving cost of epidemic by Parrondo's paradox: A COVID-19 case study, Adv. Sci. **7**, 2002324 (2020).

[16] L. Dinis and J. M. R. Parrondo, Optimal strategies in collective Parrondo games, Europhys. Lett. **63**, 319 (2003).

[17] L. Dinis, Optimal sequence for Parrondo games, Phys. Rev. E **77**, 021124 (2008).

[18] J. Zhao, Z. Wang, J. Cao, and K. H. Cheong, A self-adaptive evolutionary deception framework for community structure, IEEE Trans. Syst. Man Cybern., Syst. **53**, 4954 (2023).

[19] J. Zhao and K. H. Cheong, Obfuscating community structure in complex network with evolutionary divide-and-conquer strategy, IEEE Trans. Evol. Comput. **27**, 1926 (2023).

[20] D. Wu and K. Y. Szeto, Applications of genetic algorithm on optimal sequence for Parrondo games, in *Proceedings of the International Conference on Evolutionary Computation Theory and Applications* (ACM, New York, 2014), p. 30.

[21] D. Wu and K. Y. Szeto, Sequence analysis with motif-preserving genetic algorithm for iterated Parrondo games, in *Computational Intelligence: International Joint Conference, IJCCI 2014*, edited by J. J. Merelo *et al.*, Studies in Computational Intelligence Vol. 620 (Springer, Berlin, 2016), pp. 33–48.

[22] T. Wen, E. V. Koonin, and K. H. Cheong, How flexible parasites can outsmart their hosts for evolutionary dominance, Phys. Rev. Res. **6**, 023104 (2024).

[23] X. Molinero and C. Mégnien, Markov chains applied to Parrondo's paradox: the coin tossing problem. in *12th International Conference on Advanced Information Technologies and Applications* (ICAITA, 2023).

[24] K. W. Cheung, H. F. Ma, D. Wu, G. C. Lui, and K. Y. Szeto, Winning in sequential Parrondo games by players with short-term memory, J. Stat. Mech. (2016) 054042.

[25] S. Rahmann, Optimal adaptive strategies for games of the Parrondo type, Science Direct Working Paper No. S1574-0358(04)70285-3 (2002), https://ssrn.com/abstract=3137703.

[26] K. H. Cheong and J. Zhao, Adaptive strategy optimization in game-theoretic paradigm using reinforcement learning, Phys. Rev. Res. **6**, L032009 (2024).

[27] V. Capraro, R. Di Paolo, M. Perc, and V. Pizziol, Language-based game theory in the age of artificial intelligence, J. R. Soc. Interface **21**, 20230720 (2024).

[28] Y. Chen, T. X. Liu, Y. Shan, and S. Zhong, The emergence of economic rationality of GPT, Proc. Natl. Acad. Sci. USA. **120**, e2316205120 (2023).

[29] Q. Mei, Y. Xie, W. Yuan, and M. O. Jackson, A Turing test of whether AI chatbots are behaviorally similar to humans, Proc. Natl. Acad. Sci. USA **121**, e2313925121 (2024).

[30] V. Capraro, R. Di Paolo, and V. Pizziol, Assessing large language models' ability to predict how humans balance self-interest and the interest of others, arXiv:2307.12776.

[31] V. Capraro, A. Lentsch, D. Acemoglu, S. Akgun, A. Akhmedova, E. Bilancini, J.-F. Bonnefon, P. Brañas-Garza, L. Butera, K. M. Douglas *et al.*, The impact of generative artificial intelligence on socioeconomic inequalities and policy making, PNAS Nexus **3**, 191 (2024).

[32] Raúl Toral, Cooperative Parrondo's games, Fluctuation Noise Lett. **01**, L7 (2001).

[33] J.-Y. Yao, K.-P. Ning, Z.-H. Liu, M.-N. Ning, and L. Yuan, LLM lies: Hallucinations are not bugs, but features as adversarial examples, arXiv:2310.01469.

[34] H. Duan, Y. Yang, and K. Y. Tam, Do LLMs know about hallucination? An empirical investigation of LLM's hidden states, arXiv:2402.09733.