

Dynamics of Onsager vortex clustering in decaying turbulent polariton quantum fluids

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We investigate the turbulent properties of a confined driven-dissipative polariton quantum fluid after a pulsed excitation. Using numerical simulations, we provide insight into the vortex clustering processes that emerge during the relaxation dynamics of the initially injected vortex cloud in a compressible quantum fluid. A confrontation between conservative and nonconservative dynamics reveals that the onset of clusterization strongly depends on the interplay between the different characteristic system length scales and timescales at stake, with an additional timescale due to dissipation in the nonconservative case. Quantification of the clustering observables allows us to numerically characterize the optimal conditions for observing Onsager condensation in decaying polariton systems, demonstrating its experimental reachability under pulse excitation. These findings hold significance for exploring the onset of turbulent dynamics in open and compressible systems, spanning both classical and quantum domains.

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Quantum turbulence [1,2], characterized by the reorganization of chaotic motion of topological defects and waves in quantum fluids, has been widely explored in many systems, such as Bose-Einstein condensates of ultracold atoms, superfluid helium He^3 and He^4 [3–5]. In two dimensions (2D), nonequilibrium turbulent phenomena in quantum systems have sparked renewed interest over the past decade, both experimentally [6–10] and theoretically [11–17]. Photonic systems are emerging as an alternative platform for the investigation of 2D turbulence, allowing the direct measurement of the phase of the quantum fluid with interferometric techniques [18–21]. In particular, the hybrid light-matter nature of exciton polaritons in semiconductor microcavities enables effective photon-photon interactions and a light mass, which lead to the observation of superfluidity [22–25] and polariton condensation [26,27]. While extensive investigations have been performed on the dynamics of quantum vortices [28–30], which can be formed during the nonlinear dynamics of the polariton quantum fluid [31–34] or externally imprinted by optical means [35,36], their clustering dynamics was addressed only recently [37,38].

A generalized picture of two-dimensional confined quantum systems dynamics is given by means of statistical tools, similar to those employed in the study of classical chaos [39]. In a pioneering paper [40], Onsager was able to justify

the formation of vortex clusters—directly connected with the inverse cascade of the system’s energy from small to large length scales—using only equilibrium arguments. The theory predicts a transition from a configuration of randomly distributed vortices to more energetic—and less-entropic—configurations of giant clouds of defects, the so-called Onsager vortices [41]. Previously investigated in numerous numerical works [16,42–44], the emergence of Onsager order has been reported in ultracold gases [9,10]. The spontaneous emergence of order from chaos, even in the absence of a constant energy injection (decaying turbulence), has been explained by means of an evaporative-heating mechanism: The number of vortices decreases, due to vortex-pair annihilation and vortex losses at the edges of the trap, but the total kinetic energy of the system is conserved, leading to an increase of the mean energy per vortex [45,46]. This leads the randomly distributed vortex cloud to eventually evolve to a negative-temperature, low-entropic state.

In the aforementioned studies, this phenomenon is discussed for systems where the particle number is conserved. Notably, the formation of vortex clusters is generally not guaranteed in a nonconservative system [47], where the decaying energy is expected to preclude the formation of negative-temperature states [43]. How dissipation affects the turbulent properties of a quantum fluid and their vortex dynamics represents a complex problem, encompassing both ultracold atomic gases with energetic dissipation [48] and systems where particle loss plays a significant role, such as polaritons. Weakly dissipative systems—characterized by either minimal energy loss in mass-conserving fluids or weak particle loss in non-conserving systems—are often the most intriguing [49]. These systems find diverse applications, ranging from optomechanics [50] to atomic Bose-Einstein condensates (BECs) [51],

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and in contexts such as optical cavities [52], trapped ions [53], or Rydberg atoms [54]. The problem is particularly relevant to fluids in light systems, which possess intrinsic dissipation, such as microcavity polaritons, polaritons in liquid crystals [55], and hot atomic vapors [20]. It is therefore relevant to investigate the conditions required for the emergence of Onsager clustering in a paradigmatic driven-dissipative system such as exciton-polariton quantum fluids [56]. The question of (i) whether a decaying vortex cloud in a polariton condensate formed after an initial pulsed excitation can still exhibit clustering of same-sign vortices toward Onsager order, and (ii) what are the underlying competing mechanisms that regulate their dynamics, and their importance, remain therefore unanswered and of high relevance for a comprehensive understanding of turbulent regimes in lossy quantum fluids.

In this Letter, we numerically investigate vortex clustering in the decaying turbulence regime of a dissipative polariton condensate and extract the optimal conditions for its observation in realistic parameter ranges. We demonstrate that the onset of order depends on the interplay of different spatial and temporal scales, and it is eventually the result of the competition between opposite tendencies driven by particle dissipation and vortex interactions, respectively. We prove that, despite dissipations, the observation of Onsager condensation in decaying polariton fluids is experimentally achievable within realistic length scales and timescales.

Theoretical model and system parameters. We solve the generalized equation of motion for the 2D polariton field ψ for the lower-polariton branch, a function of the position $\mathbf{r} = (x, y)$, which reads ($\hbar = 1$) [57,58]

$$id\psi = dt \left[-\frac{\nabla^2}{2m} + g|\psi|^2 + V(\mathbf{r}) - \frac{i}{2}\gamma_{LP} \right] \psi + dW. \quad (1)$$

Here, m represents the polariton mass, g the polariton-polariton interaction strength, γ_{LP} the polariton loss rate $\gamma_{LP} = 1/\tau_{LP}$, with τ_{LP} the polariton lifetime. We consider a simplified case where the complex relaxation processes [58,59] are disregarded, and where the exciton reservoir is treated as adiabatically following the condensate dynamics [60]. Equation (1) can be derived from both truncated Wigner (TW) and Keldysh field theories [56,61]. The renormalized density $|\psi|_-^2 \equiv |\psi|^2 - 1/2dV$ includes the subtraction of the Wigner commutator contribution (where $dV = a^2$ is the volume element of our 2D grid of spacing a). The zero-mean white Wiener noise dW , which accounts for quantum fluctuations, fulfills $\langle dW(\mathbf{r}, t)dW(\mathbf{r}', t) \rangle = 0$, $\langle dW^*(\mathbf{r}, t)dW(\mathbf{r}', t) \rangle = (\gamma_{LP}/2)\delta_{\mathbf{r},\mathbf{r}'}dt$, as required by the TW approach. The light fluid is confined in a radially symmetric potential $V(\mathbf{r})$ with diameter D which, for simplicity, is modeled as a hard-bounded box. A more elaborated description as well as further details of the modeling are reported in the Supplemental Material (SM) [62].

Modeling the vortex dynamics. In order to simulate the turbulent processes of a decaying driven-dissipative system after an initial pulsed excitation, we solve Eq. (1) starting from a homogeneous condensate with imprinted a random configuration of $N_v^0 = 40(\text{vortices}) + 40(\text{antivortices})$ (see SM [62] for

details). The particular choice of initial condition allows us to initiate the dynamics in an entropy-dominated vortex gas state [41]. In the absence of the vortex cloud, the homogeneous but arbitrary phase would emulate a quasisonant laser excitation [62]. Topological defects are numerically detected at each snapshot of the evolution as in Refs. [58,63]. Evaluation of the position, vorticity, and charge of each vortex allows us to track the evolution of the average distance between the vortex cores $\ell_v = DN_v^{-1/2}$ [62], and to extract the relevant clustering observables: the dipole moment $d_m = |\mathbf{d}_m| = |\sum_{i=1}^N \kappa_i \mathbf{r}_i|$ and the correlation function $C = \frac{1}{N} \sum_{i=1}^N c_i$ [10]. Here, N is the total vortex number, κ_i the circulation sign, $\mathbf{r}_i$ the position, and c_i the product of the circulations between the i th vortex and its nearest neighbor. As such, increasing values of C correspond to higher energetic states of the vortex gas. In the Onsager model, $C = -1$ corresponds to the smallest positive temperature, which is associated with a gas of vortex-antivortex dipoles. On the other extreme, $C = +1$ is associated with the smallest negative temperature (Onsager condensation), where vortices of the same sign group together into large clusters. $C = 0$ instead corresponds to the configuration of maximum entropy, with randomly distributed vortices. Temperature tends to infinity as $C \rightarrow 0^-$, however, if energy is increased beyond this point, the entropy starts to decrease, leading to negative temperatures for $C > 0$, i.e., larger population of high-energy states than low-energy states [39,41,43]. Since we are interested in the early stages of the dynamics, in all our simulations we probe the polaritons up to 1 ns (five times the value of the polariton lifetime in typical high-quality samples [36]).

Clustering processes strongly depend on the interplay of different system lengths and timescales of the physical processes: the healing length (proportional to the size of the vortex core) $\xi(t) = 1/\sqrt{2mg|\psi(t)|^2}$, the circular trapping diameter D , and the average intervortex length $\ell_v(t)$ [64]. Typically, $\xi(t) < \ell_v(t) < D$. In our numerical experiments, we impose the initial polariton density n_0 , which sets the initial healing length of the system ξ_0 , and the number of initial vortices N_v^0 . Fixing N_v^0 allows us to use the disk diameter D as a parameter controlling the initial intravortex length $\ell_v^0 = D/\sqrt{N_v^0}$.

Onsager condensation in the conservative case. First, we investigate vortex clustering in the conservative case ($\gamma_{LP} = 0$), where the total number of particles and angular momentum are conserved in time [$\xi(t) = \xi_0$], and compute the dynamics for different initial intervortex lengths ℓ_v^0 .

Figures 2(a)–2(c) shows the evolution of the averaged vortex number $\langle N_v \rangle$ [Fig. 2(a), dashed lines], intervortex length ℓ_v [Fig. 2(a), solid lines], dipole moment $\langle d_m \rangle$ [Fig. 2(b)], and clustering correlation function $\langle C \rangle$ [Fig. 2(c)]. A typical example of conservative vortex dynamics, characterized by growing ℓ_v , $\langle d_m \rangle$, and positive $\langle C \rangle$, is given by the case depicted in pink color ($\ell_0 = 28 \mu\text{m}$). Single-realization real-space density distributions sampled at different times are reported in Figs. 1(a)–1(c); the whole vortex dynamics is illustrated in the Supplemental Movie 1 [62]. The system evolves as follows: At the beginning, a large number of vortex-antivortex pairs annihilate, followed by the consequent emission of sound waves. In agreement with previous litera-

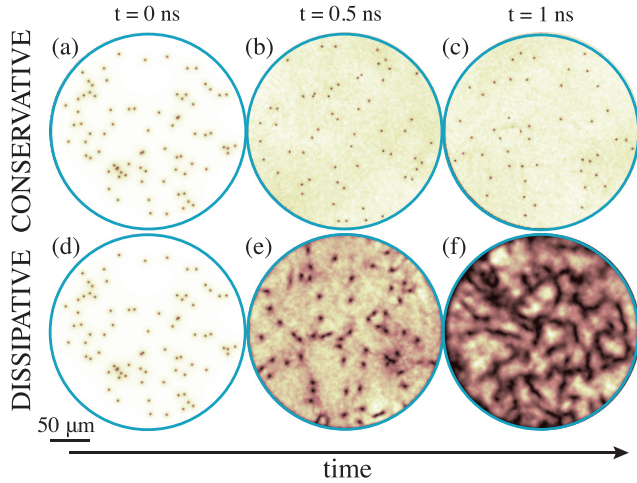


FIG. 1. Conservative and dissipative temporal evolution. The vortex gas dynamics is depicted for the (a)–(c) conservative and (d)–(f) dissipative cases. Snapshots are taken at a time $t = 0$ ns [(a), (d)], $t = 0.5$ ns (central panels), and $t = 1$ ns (right panels). While in the conservative dynamics, the polariton density and vortex healing length are constant in time, and in the dissipative case the density decays while the vortex healing length increases, modifying the evaporative heating mechanism. The top row illustrates the conservative case with an initial healing length $\xi_0 = 0.7 \mu\text{m}$ (corresponding to $n_0 = 400 \mu\text{m}^{-2}$) and initial intervortex length $\ell_v^0 = 28 \mu\text{m}$ (corresponding to $D = 250 \mu\text{m}$), respective to the pink solid line in Figs. 2(a)–2(c). The bottom row shows the dissipative case. Typical experimental parameters for the polariton system are used [36,37]: $\tau_{\text{LP}} = 0.2$ ns, $m = 0.22 \text{ ps}^2 \text{ meV } \mu\text{m}^{-2} = 3.52 \times 10^{-35} \text{ kg}$, $g = 5 \times 10^{-3} \text{ meV } \mu\text{m}^2$. We solve Eq. (1) using an eighth-order Runge-Kutta method on a 512^2 numerical grid.

ture [10,41], as the vortex cloud decays, evaporative heating processes take place followed by the tendency of the system to form same-sign vortex clusters of different sizes. The decay towards a higher-temperature state is verified by the growth of the clustering quantities $\langle d_m \rangle$ and $\langle C \rangle$ towards larger positive values.

Let us discuss how clustering behaves under the change of the intervortex length ℓ_v^0 . Figure 2(a) shows that decreasing ℓ_v^0 , the number of vortices exhibits a faster decay in time, especially in the first transients of the evolution; this shows that annihilation processes become more efficient as we increase the initial density of the vortex gas [65]. To tentatively quantify this behavior, we extract $\langle C \rangle$ at time $T = 1$ ns.

In the inset of Fig. 2(c) we plot $C(T)$ as a function of the ℓ_v^0 . The data verify the inverse proportionality between the two quantities, but up to a threshold value for ℓ_v^0 ($\sim 14 \mu\text{m}$ for the set of parameters chosen) below which $\langle C \rangle^{\text{max}}$ changes behavior and decreases. Figure 2(c) shows that this change of trend can be explained by a quenching of the clustering processes taking place in the very early stage of dynamics (0–20 ps for the case $\ell_v^0 \sim 8 \mu\text{m}$, blue curves). Inspection of the density profiles reveals that the quenching effect is originated by the fast vortex-antivortex annihilation events which, given the more dense gas of vortex-antivortex pairs, take place at shorter timescales and length scales than the clustering processes. As

a result, in the diluted (therefore less interacting) vortex gas, the onset of clustering is delayed. Only when the density of the vortex gas (due to the abrupt decay of the number of vortices) reaches sufficiently low values to slow down the vortex interactions and trigger the clusterization processes, does the “evaporative cooling” observed at early times transform to “evaporative heating,” allowing $\langle C \rangle$ to grow towards positive values. In the meantime, a large amount of incompressible energy has been already transformed to compressible energy, resulting in a total clusterization process being slowed down in comparison to cases with larger ℓ_v^0 .

Onsager condensation in the dissipative case. We proceed by investigating how the vortex clustering is affected when accounting for a finite lifetime. The dissipative Hamiltonian of Eq. (1) does not preserve the total average number of particles, and the polariton density exponentially decays in time as $|\psi_{\text{LP}}|^2 \propto n_0 \exp(-t/\tau_{\text{LP}})$. Consequently, dissipation introduces a temporal dependence to the healing length, which is redefined as $\xi(t) = 1/\sqrt{2mgn_0 \exp(-t/\tau_{\text{LP}})}$. This is expected to bear important consequences on the vortex dynamics, and it is still unclear if (and in which range of realistic system parameters) vortex clustering occurs.

In Figs. 2(d)–2(f), we repropose the same analysis as in Figs. 2(a)–2(c) for different values of the particle lifetime τ_{LP} , in a system with fixed geometry D (i.e., fixed ℓ_v^0) and fixed initial density n_0 . We choose τ_{LP} from the range of typical experimental polariton lifetimes $\in [0.05, 2]$ ns. The first important result we infer from Fig. 2(f) is that, despite the presence of dissipation, the clustering processes are still able to take place at early-stage dynamics. This is verified by a growing $\langle C \rangle$. We note, however, that the onset of clustering takes place only up to a certain time, after which $\langle C \rangle$ starts decreasing due to the effects of dissipation. This results in the presence of a maximum in the dynamical curve of the clustering correlation function. Figure 2(f) shows that both the maximum value of $\langle C \rangle$ and the timescale measured along the clustering process increase as a function of τ_{LP} .

The condensate profiles depicted in Figs. 1(d)–1(f) show that this behavior is intrinsically linked to the time dependence of the size of a single-vortex core (the vortex dynamics of a single realization in the dissipative case is also provided in the Supplemental Movie 2 [62]). The density profiles show that, as the density decays, the vortex cores expand, slowing down the vortex gas dynamics. The increased dissipation results in a deceleration of individual vortex motion (due to a decrease of the single-vortex energy—more details are provided in SM [62]) and overall vortex dynamics, which corresponds to a slower rate of vortex gas clustering.

Furthermore, we analyze the relative contributions of vortex escape and vortex-antivortex annihilation to vortex loss in both conservative and dissipative regimes depicted in Fig. 1, finding that annihilation dominates in both cases but is slightly reduced in the presence of dissipation. This suggests that dissipation enhances vortex loss at the boundaries, though the dominant mechanism may vary under different initial conditions, as suggested in Ref. [46].

Optimal condition for Onsager condensation. We proceed to study the onset of Onsager condensation as a function of both the confinement size D (i.e., ℓ_v^0) and n_0 . Given the

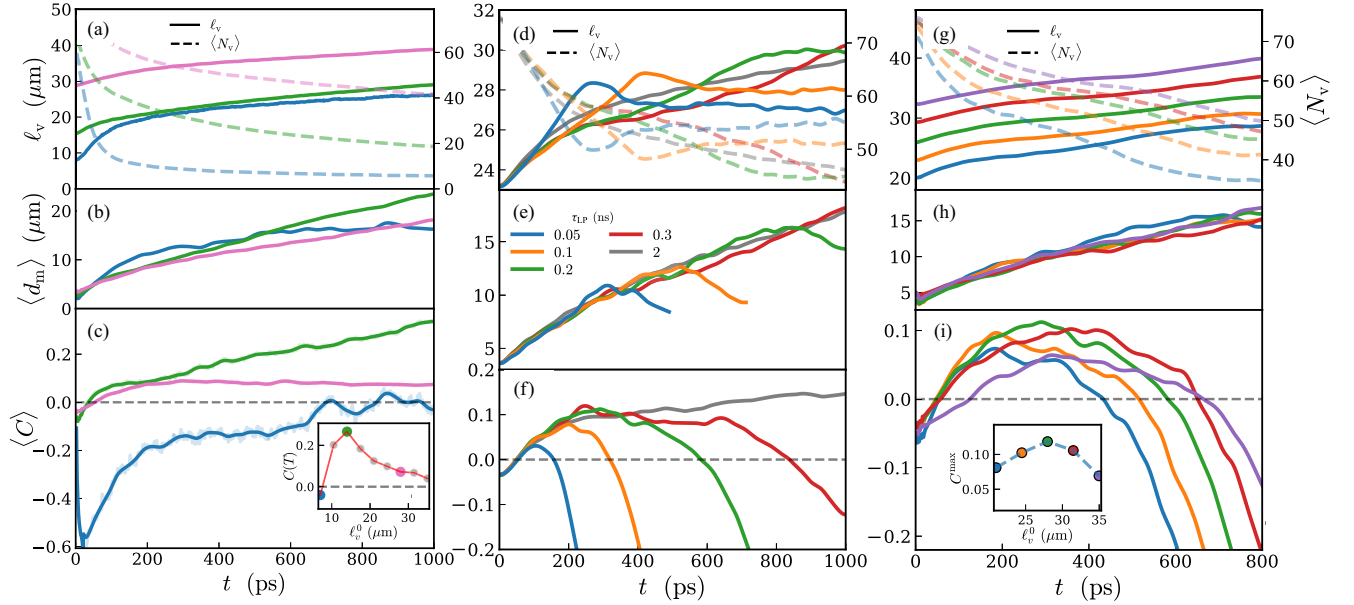


FIG. 2. (a)–(c) Clustering in the conservative dynamics. (a) The average intervortex length ℓ_v and total number of vortices (top), (b) averaged dipole moment $\langle d_m \rangle$ (center), and (c) clustering correlation function $\langle C \rangle$ (bottom) are used to trace clustering of vortices along the system dynamics. The panels show the behavior of the different observables for different initial conditions of ℓ_v^0 , at a fixed initial density (corresponding to healing length $\xi_0 = 0.7 \mu\text{m}$). The inset of the bottom panel shows the value of $\langle C \rangle$ calculated at a time $T = 1 \text{ ns}$. The interplay between early-time annihilation and long-time clustering mechanism leads to a maximum for $C(T)$ at $\ell_v^0 = 14 \mu\text{m}$. The faint colored data underneath the solid curves show the data averaged over $\mathcal{N} = 10^2$ realizations. (d)–(f) Clustering in the presence of dissipation. Clustering observables calculated in the case of a nonvanishing dissipative term of Eq. (1), for different particle lifetimes and constant trap sizes ($\ell_v^0 = 28 \mu\text{m}$). For the system’s parameters investigated, the curves show that the clustering mechanism is quenched as the polariton lifetime is decreased. (g)–(i) Clustering in the dissipative dynamics. As in (a)–(c) but for the dissipative case ($\gamma_{LP} = 0.2 \text{ ns}$). The green curve corresponds to the green curve in (d)–(f). The initial healing length reads $\xi_0 = 0.7 \mu\text{m}$. When the initial intervortex length ℓ_v^0 is varied, the interplay between the different dynamical mechanisms leads to a maximum in the clustering correlation function, found at $\ell_v^0 = 28 \mu\text{m}$, as shown in the inset of (c).

competition of the different length scales and timescales of the relevant processes discussed in the previous sections, understanding the vortex dynamics and clustering becomes a nontrivial problem.

First, we solve the polariton dynamics for the dissipative case with $\gamma_{LP} = 0.2 \text{ ns}$, with constant n_0 but varying the initial intravortex length: Results are shown in Figs. 2(g)–2(i). The evolution of $\langle C \rangle$ depicted in Fig. 2(i) shows that there exists an optimal curve which maximizes the onset of vortex clustering. This is in analogy with the conservative case, but the optimal intervortex length is in general larger in the dissipative case. Then, we extend our analysis by investigating the influence of the initial density n_0 .

Figure 3(a) depicts a carpet plot of $\langle C \rangle^{\text{max}}$ as a function of the two relevant lengths of the system, namely ℓ_v^0 (proportional to the trap size D) and the initial healing length ξ_0 (function of the initial density n_0 , at $t = 0$). The plot presents a maximum at $\xi_0 = 0.7 \mu\text{m}$ and $\ell_v^0 = 28 \mu\text{m}$, which corresponds to the optimal condition for vortex clustering with experimentally available system parameters. The maximum is reached at $t = 300 \text{ ps}$. Single-realization real-space distributions of the vortex gas at the initial and at the optimal point, are reported in Figs. 3(b) and 3(c), respectively, for comparison. Figure 3(a) also shows that for any fixed ℓ_v^0 , by decreasing ξ_0 from large values, the clustering processes are enhanced. This is an expected behavior, as the energy per vortex would

increase as its healing length decreases. Interestingly, we note that there exists an optimal condition also for ξ_0 (namely $\xi_0 = 0.7 \mu\text{m}$), below which C decreases. The evolution of the density and phase shows that this is due to the large amount of sound produced by the annihilation of vortex-antivortex pair (which is a stronger effect at larger densities), sound which eventually hampers vortex clustering over the whole evolution.

Discussion and conclusions. In this Letter, we give a qualitative and quantitative analysis of vortex clustering for dissipative condensates in the decaying turbulence scenario. Our investigation gives a clearer qualitative picture of the role and interplay of the different physical processes involved. Notably, the fluid’s compressibility and the role of sound waves are consistently accounted for. This significantly impacts the clustering dynamics, providing optimal conditions—both in conservative and dissipative cases—in terms of intervortex spacing and the system’s sound speed. In the conservative case, we demonstrate that while a denser and more interacting vortex cloud speeds up the clustering processes, there exists a threshold above which the annihilation prevails over clustering, diluting the gas too fast and retarding the evaporative heating mechanisms. A similar scenario is also found modulating the initial density of the system: While larger densities speed up vortex clustering, there exists a threshold above which the sound emitted by the

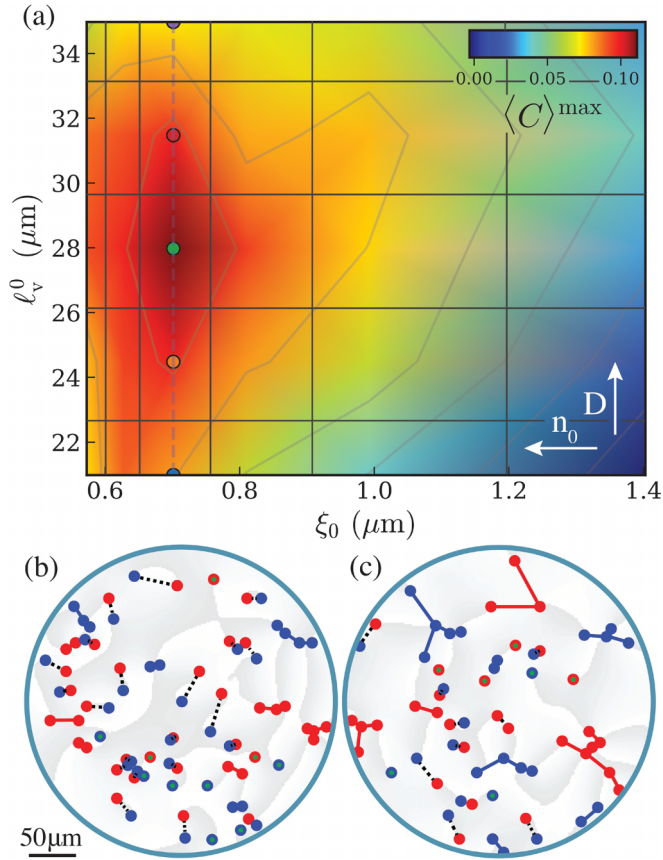


FIG. 3. Clustering phase diagram. (a) The maximum of the clustering correlation function ($\langle C \rangle^{\max}$), calculated over 1 ns of the polariton evolution, is numerically extracted and plotted as a function of the initial intervortex length ℓ_v^0 and initial healing length ξ_0 . The vertical dashed blue line and points at $\xi_0 = 0.7 \mu\text{m}$ correspond to those depicted in the inset of Fig. 2(i). The maximum of the resulting two-dimensional surface, identified as a green dot, corresponds to the optimal condition for observing the onset of Onsager condensation [which also corresponds to the green curve in Figs. 2(d)–2(f) and the green dot in the inset of Figs. 2(g)–2(i)]. Single-realization snapshots of the phase are reported for $t = 0$ ps (b), and for the optimal case at $t = 300$ ps (c). Positive (negative) circulation vortices are reported as blue (red) dots, and classified [9] into single vortices (dots with a green center), dipoles with opposite sign (dots connected with a black dashed line), and same-sign dipoles and clusters (dots connected with a line with the same color of the polarization).

frequent vortex annihilations would slow down the motion of vortices.

As expected, when accounting for dissipation, evaporative heating is compromised. Our calculations reveal that, with respect to the conservative case, the optimal intervortex length is larger and the maximum value of the clustering correlation function is limited by the dissipation rate. However, we show that, for experimentally accessible parameters, the onset of vortex clustering can be observed in decaying polariton systems. Additionally, the persistence of a global dipole orientation over the nearest-neighbor correlation $\langle C \rangle$ motivates further research and deeper insights on the mechanisms underlying the onset of turbulence in driven-dissipative condensates. In this context, the possibility to inject a large number of vortices and to precisely measure higher-order spatial correlations can be an asset of optical systems. Finally, these findings point towards the intriguing possibility of tuning the density and phase of exciton-polariton quantum fluids to access different regimes of 2D quantum turbulence, ranging from superfluid turbulence to weak wave turbulence [66–68].

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