

Exponential optimization of adiabatic quantum-state preparation

Davide Cugini¹, Davide Nigro¹, Mattia Bruno², and Dario Gerace¹¹Dipartimento di Fisica, *Università di Pavia*, via Bassi 6, 27100 Pavia, Italy²Dipartimento di Fisica, *Università degli Studi di Milano–Bicocca*, and INFN, sezione di Milano–Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy

(Received 30 May 2024; revised 1 October 2024; accepted 4 February 2025; published 19 March 2025)

The preparation of a given quantum state on a quantum computing register is a typically demanding operation, requiring a number of elementary gates that scales exponentially with the size of the problem. Using the adiabatic theorem for state preparation, whose error decreases exponentially as a function of the preparation time, we derive an explicit analytic expression for the dependence of the characteristic time on the Hamiltonian used in the adiabatic evolution. Exploiting this knowledge, we then design a preconditioning term that modifies the adiabatic preparation, thus reducing its characteristic time and hence giving an exponential advantage in state preparation. We prove the efficiency of our method with extensive numerical experiments on prototypical spin models, which gives a promising strategy to perform quantum simulations of manybody models via Trotter evolution on near-term quantum processors.

DOI: [10.1103/PhysRevResearch.7.L012074](https://doi.org/10.1103/PhysRevResearch.7.L012074)

Introduction. We are currently living the so-called noisy intermediate-scale quantum (NISQ) era [1], in which actual quantum processing devices are made of a limited number of noisy qubits available in a number of different technological platforms. Restricting to the realm of digital quantum processors satisfying the DiVincenzo criteria [2], typically 30 all-to-all connected qubits are available in trapped ions quantum processors [3] and more than 100 in superconducting quantum circuits with next-neighbor coupling [4]. The physics of manybody quantum systems may thus be efficiently simulated through the direct application of such devices [5,6]. However, a proven advantage in quantum simulations is yet to be presented. In fact, while we may expect NISQ hardware to overcome known issues in simulating quantum systems on classical devices, such as the sign problem for fermionic systems [7,8], a substantial limitation resides in the severely restricted hardware resources nowadays. Nevertheless, first proof-of-concept quantum simulations of chemical properties have been reported [9–11], as well as the time dynamics of manybody Hamiltonians [12–14] and quantum field theories [15–19].

In quantum simulations on NISQ devices, one of the most pressing targets is the efficient generation of a desired quantum state of a given model Hamiltonian on a quantum register, which is essential, e.g., to directly compute expectation values of the observables under study. This is the focus of the present work. We define H_1 as the target Hamiltonian, properly mapped on a quantum computer, and denote with $\{|\Omega_n(1)\rangle\}$ the set of its eigenstates. Using a sequence of single- and two-qubit gates [20], a unitary operator U_n acting on a register

of L qubits such that $|\Omega_n(1)\rangle = U_n|0\rangle^{\otimes L}$ may always be found, and the number of gates required to exactly implement U_n scales exponentially with L , i.e., with the size of the system. Currently, the noisy character of state-of-the-art qubits and quantum gates available in NISQ devices limits the number of operations that can be reliably executed on actual hardware. Hence, significant effort has been devoted to propose efficient algorithms allowing to obtain a well approximated U_n within limited quantum computing resources. For state preparation, variational quantum eigensolvers have been proposed as a valid solution, consisting of hybrid quantum/classical algorithms in which quantum circuits with parametrized operations are classically optimized to achieve the desired state [21–24]. Despite their usefulness for simple models, a general strategy to define such circuits for arbitrary systems is still not available; moreover, they may involve demanding classical optimizations in high-dimensional spaces [25]. An alternative approach has been recently proposed [26], based on imaginary time evolution, which might prove more effective than variational eigensolvers on NISQ devices, despite its intrinsic limitations in the preparation of quantum states with long correlation lengths.

In this work, we provide an original strategy to apply well-established methods developed in the context of adiabatic preparation (AP) [27] as a viable approach allowing to practically achieve exponential advantage in quantum state preparation on quantum hardware. The latter allow to generate any eigenstate of a given Hamiltonian, in the following referenced as the target Hamiltonian H_1 , with high fidelity. Specifically, the key idea is to start from a Hamiltonian H_0 , different from H_1 , whose eigenstates are analytically known and trivial to prepare on a quantum register. Then, its n th eigenstate $|\Omega_n(0)\rangle$ is evolved in time from $t = 0$ to $t = \tau$, according to the auxiliary Hamiltonian

$$H(s) = (1 - f(s))H_0 + f(s)H_1, \quad s = t/\tau, \quad (1)$$

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

which is parametrized in terms of a dimensionless variable $s \in [0, 1]$. Essentially, $H(s)$ interpolates between H_0 and H_1 through the time-dependent holomorphic function $f(s)$ such that $f(0) = 0$ and $f(1) = 1$. In what follows we denote the time evolution operator associated with $H(s)$ in the time interval $[0, \tau]$ as U_τ . Its application to the n th eigenstate of H_0 produces the final state $U_\tau |\Omega_n(0)\rangle$, which is an approximation of the actual eigenstate $|\Omega_n(1)\rangle$ of H_1 , with a discrepancy quantified by the infidelity measure, defined as

$$\mathcal{I}(\tau) \equiv 1 - |\langle \Omega_n(1) | U_\tau | \Omega_n(0) \rangle|^2. \quad (2)$$

In the limit $\tau \rightarrow \infty$, and provided that no level crossing occurs during the adiabatic evolution, the AP procedure becomes exact, i.e., $\mathcal{I}(\tau) \rightarrow 0$, and the final result is independent from the initial choice on H_0 . In practice, this is far from being the case, and the overall output infidelity strongly depends not only on the amount of finite available computational time τ , but also on the specific choice of H_0 and the peculiar features of the the interpolating function, $f(s)$. In this respect, significant effort from the community went in the understanding of how different constraints on such quantities give rise to different upper bounds on the output infidelity [28]. Starting from different assumptions, several alternative bounds have been derived, most of which showcase a powerlike dependence of the infidelity with the inverse of the characteristic adiabatic timescale τ [29–31]. Among these seminal theoretical works, in Ref. [32] it is proven that the overall infidelity is bounded by an expression with an exponential rather than power-law decay as a function of τ , for a broad class of interpolating functions. The latter, despite representing a theoretical milestone in the field of adiabatic processes, does not explicitly reveal the interplay between the infidelity's bound and the two arbitrary quantities H_0 and $f(s)$, thereby preventing its usage in a practical strategy for improving the performance of AP protocols.

The main goal of our work is to fill this gap. By revising the results provided in [32], here we are able to determine the explicit form for an upper bound to the infidelity, which naturally accounts for the dependence on the spectral properties of the initial Hamiltonian H_0 and the interpolating function $f(s)$. Our result provides a practical protocol for selecting the H_0 and $f(s)$ that minimize the bound expression, thus leading to a potentially significant improvement of the convergence performances of a standard AP algorithm run on actual NISQ hardware. In the following, we support the effectiveness of this protocol in choosing the initial H_0 , by actually applying the AP approach to the determination of the ground-state configuration of two prototypical spin models, namely the Heisenberg and Ising models in either one or two dimensions. We thus perform numerical experiments at fixed quantum circuit depth. Our numerical results demonstrate that our proposed scheme leads to a pronounced error suppression when compared to a standard AP protocol. These results are considerably relevant for prospective quantum simulations of manybody models on NISQ hardware with unprecedented system size, possibly toward quantum advantage.

Exponential scaling and relevant parameters. In the previous section we outlined AP ideas by focusing on the base case of determining a single eigenstate. We hereby pay attention

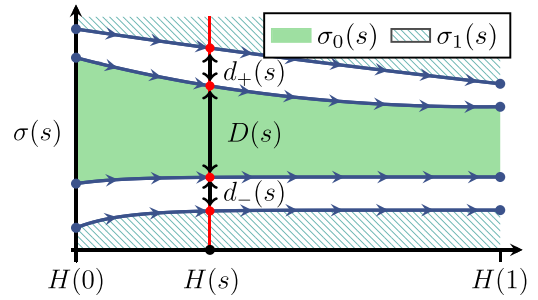


FIG. 1. Schematic illustration of the adiabatic evolution of the spectral features related to a time-dependent Hamiltonian $H(s)$, in which $\sigma(s)$ is the full spectrum, partitioned into the subsets of eigenvalues $\sigma_0(s)$ and $\sigma_1(s)$. Here, $D(s)$ denotes the bandwidth of $\sigma_0(s)$, while $d(s) = \min[d_+(s), d_-(s)]$ defines the minimal gap between the two subsets of eigenvalues.

to a more general scenario, and discuss our protocol in the case where the target of AP is a subspace, that is $\bar{V}$, of the entire Hilbert space $\mathcal{H}$. Specifically, let $\sigma_0(s)$ be a subset of the full spectrum $\sigma(s)$ of the interpolating Hamiltonian $H(s)$ defined in Eq. (1). Furthermore, let us denote with $V(s)$ the subspace spanned by the eigenstates of $H(s)$ with eigenvalues in $\sigma_0(s)$. In particular, we assume that in our formalism $\bar{V} \equiv V(1)$. With this in mind, the AP task of targeting $\bar{V}$ is equivalent to devise a strategy for evolving $V(0)$ into $V(1)$ efficiently. In this respect, a proper figure of merit is given by the adiabatic error

$$\epsilon_{\text{AP}}(\tau) \equiv \|(\mathbb{1} - P(1))U_\tau P(0)\|, \quad (3)$$

where $\|A\| = \sup_{|\Psi\rangle \in \mathcal{H}} \frac{\|A|\Psi\rangle\|}{\| |\Psi\rangle \|}$ denotes the sup(remum)-norm of the operator A , and with $\mathbb{1}$ and $P(s)$ ($s \in [0, 1]$) being the identity and the projector on $V(s)$, respectively. It has been shown in [32] that under quite general conditions there exist finite and positive C and $g(s) \in \mathbb{R}$ such that, for τ large enough, the adiabatic error decays exponentially, that is

$$\epsilon_{\text{AP}}(\tau) \leq C \int_0^1 ds \exp(-\lfloor \tau/g(s) \rfloor), \quad (4)$$

in which $\lfloor \cdot \rfloor$ is the floor operation. This is the case provided that $H(s)$ is holomorphic, that the gap $d(s)$ between $\sigma_0(s)$ and $\sigma_1(s) = \sigma(s) \setminus \sigma_0(s)$ is positive, and that the bandwidth $D(s)$ of $\sigma_0(s)$ is finite for any $s \in [0, 1]$ (see, e.g., the scheme displayed in Fig. 1).

By defining the characteristic decay time as

$$\tilde{g} \geq \sup_{s \in [0, 1]} g(s), \quad (5)$$

the exponential bound

$$\epsilon_{\text{AP}} \leq C \exp(-\lfloor \tau/\tilde{g} \rfloor) \quad (6)$$

is straightforwardly obtained, which clearly shows that a reduction of $\tilde{g}$ determines an exponential reduction of ϵ_{AP} . In this work we extend the results of [32] where only the existence of $g(s)$ is proven, by finding an explicit analytic expression for $g(s)$ that relates it to the metrics $\{\|\Delta\| \equiv \|H_0 - H_1\|, D(s), d(s)\}$, thus providing a concrete way to control $\tilde{g}$ and therefore to define an efficient optimization

strategy. In addition, it is relevant to notice that these metrics can be efficiently estimated by classical means for systems that are not too large, e.g., via the Lanczos algorithm [33,34], with a computational cost that is $\mathcal{O}(\dim \mathcal{H})$. In the Supplemental Material (SM) [35] we report a possible approach to tackle the reduction of $\tilde{g}$ also for large systems.

As we explicitly derive in the SM [35], a possible choice for $g(s)$ reads

$$g(s) = 4 \frac{W(d(s), D(s))}{\varrho\left(s, \frac{d(s)}{\|\Delta\|}\right)}, \quad (7)$$

where $W(x, y) = (\frac{4}{x})(1 + \frac{2}{\pi} \frac{y}{x})^5 [2^8 (\frac{4\pi^2}{3} + 1)]^{5/2}$ and with $\varrho(x, y) = \sup_{\rho \in \mathbb{C}} [|\rho| \text{ such that } |f(x + \rho) - f(x)| < \frac{y}{4}]$. A direct inspection of Eq. (7) reveals that $g(s)$ is a nonincreasing function of $d(s)$ and $1/D(s)$. As a consequence, one obtains that

$$\tilde{g} = 4 \frac{W(d_{\min}, D_{\max})}{\inf_{s \in [0, 1]} \varrho\left(s, \frac{d_{\min}}{\|\Delta\|}\right)}, \quad (8)$$

in which we defined the minimum of $d(s)$ and the maximum of $D(s)$ as $d_{\min} = \inf_{s \in [0, 1]} d(s)$ and $D_{\max} = \sup_{s \in [0, 1]} D(s)$, respectively. For the commonly employed linear holomorphic function $f(s) = s$, the expression above reduces to

$$\tilde{g} = 16 \frac{\|\Delta\|}{d_{\min}} W(d_{\min}, D_{\max}). \quad (9)$$

Notice that a dependence of the bound on $\|\Delta\|$ has already been shown in the literature [30]. However, in all previous results the bounds resulted in an inverse power dependence on τ . Not only does our result confirm that a smaller $\|\Delta\|$ is preferable, but it also ensures that its reduction provides an exponential advantage.

Selecting H_0 . In the context of AP, given a target Hamiltonian H_1 , the naive choice for H_0 is the diagonal part of H_1 (see, e.g., Refs. [36,37]), such that its eigenstates are manifest. In what follows, we present a universal procedure to select a more convenient auxiliary Hamiltonian, H_0 , in the context of digital quantum simulations, where U_τ is constructed using the Trotter-Suzuki approximation [38–40]. Our proposal may not lead to the absolute best choice for specific problems, but it guarantees an exponential advantage, in the quantum state preparation of a target Hamiltonian H_1 , over the naively picked H_0 . The most generic starting Hamiltonian can be expressed in terms of its matrix elements with respect to the computational basis as

$$\langle i | H_0 | j \rangle \equiv \delta_{ij} \langle i | H_1 | j \rangle + \langle i | M | j \rangle, \quad (10)$$

in which M denotes a parametrized Hermitian operator (called the preconditioner) to be optimized in order to minimize $\tilde{g}$. We impose three conditions on M : (i) H_0 should be diagonal, such that its eigenstates are the elements of the computational basis and it is then trivial to initialize them on a quantum computing register; (ii) the additional computational cost for the implementation of $\mathcal{U}_M(\theta) = \exp(-i\theta M)$ at each Trotter step (where $\theta \in \mathbf{R}$) should not frustrate the cost reduction stemming from the minimization of $\tilde{g}$; finally, (iii) the number of real parameters needed to describe M should not increase

exponentially with the number of qubits, L . The first condition is satisfied by any diagonal operator. We hereby propose the extremely “cheap” form

$$\langle i | M | i \rangle = \sum_{j=0}^{L-1} \alpha_j \text{bin}_j(i), \quad (11)$$

in which $|i\rangle$ is the i th element of the computational basis, and $\text{bin}_j(i)$ is the j th digit of i once expressed in binary notation. The key idea behind the choice of Eq. (11) is detailed in the SM [35]. Most importantly, this choice allows to fulfill both conditions (ii) and (iii). In fact, the number of free parameters is L in this case, thus increasing linearly with the system size. Moreover, the corresponding time evolution operator is

$$\mathcal{U}_M(\theta) = \bigotimes_{j=0}^{L-1} P_j(\alpha_j \theta), \quad (12)$$

in which P_i is the single-qubit phase gate, applied to the i th qubit. Notice that the quantum circuit depth of $\mathcal{U}_M(\theta)$ corresponds to a single qubit gate, independently from the size of the system. Essentially, we propose to interpolate between H_0 [Eqs. (10) and (11)] and the targeted H_1 by AP, after classically minimizing $\tilde{g}$ through a scanning of the free parameters $\{\alpha_j\}$ in H_0 . This simple protocol will guarantee an exponentially faster quantum state preparation. Furthermore, since the approach we propose is independent from the realization of the Trotter evolution on the quantum register, it can be fully combined with other optimization techniques such as those in Refs. [36,41,42].

Numerical results. Here we apply the AP to the preparation of the ground state for the Heisenberg model in either one- or two-dimensional (1D, 2D) square lattices with periodic boundary conditions. The purpose is to numerically confirm the previous theoretical result in model cases of practical relevance for quantum simulations. The corresponding quantum circuits are implemented and executed on the Qiskit qasm simulator [43].

To focus only on ϵ_{AP} , we use sufficiently short Trotter steps, to suppress the systematic error coming from the Trotter-Suzuki approximation. In the following, the state preparation is performed by applying three different approaches, all with the same adiabatic function $f(s) = s$. The first is the standard AP, where M is set to zero in Eq. (10). Since the calculation of $\|\Delta\|$ comes at the computational cost of a single application of the Lanczos algorithm (or an equivalent one), and it is independent from the chosen target subspace $V(s)$, we consider a second approach in which the parameters α_j are chosen to minimize $\|\Delta\|$ in Eq. (9). Notice that this does not guarantee that $\tilde{g}$ is minimized. In fact, we finally consider a third approach consisting in the full classical evaluation of $\tilde{g}$ in Eq. (9), involving the estimation $d_{\min}$ and $D_{\max}$. Despite yielding a significantly better scaling for our case studies, for more complicated models it may be less practical, since this approach requires the knowledge of the spectrum at several intermediate values of s , each requiring a Lanczos algorithm application.

The XZ Heisenberg model is a prototypical manybody model describing systems of spin-1/2 particles on a lattice,

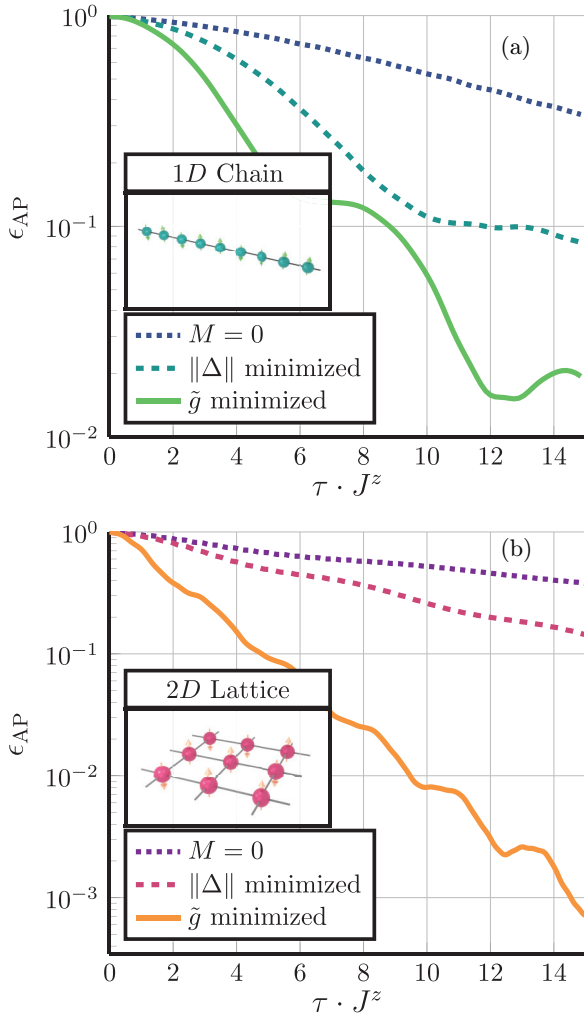


FIG. 2. Ground state preparation for the nine site 1D and 2D Heisenberg models with periodic boundary conditions, schematically illustrated in panels (a) and (b), respectively, where the error dependence is plotted as a function of preparation timescale, τ , for $J^x = 5J^z$.

defined by the interacting Hamiltonian

$$H_1 = -\frac{1}{2} \sum_{\langle i,j \rangle} (J^z Z_i Z_j + J^x X_i X_j), \quad (13)$$

where the sum runs over next-neighboring sites, while X_i and Z_i are the Pauli matrices acting on the i th qubit, conventionally defined as

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (14)$$

In this work we consider real and positive coupling constants, J^z and J^x . The AP process is then described by the operator

$$H(s) = -\frac{1}{2} \sum_{\langle i,j \rangle} (J^z Z_i Z_j + s J^x X_i X_j) + (1-s)M. \quad (15)$$

At $s = 0$ the Hamiltonian is diagonal and, in the unpreconditioned case ($M = 0$), has a degenerate ground state. The corresponding eigenstates, $|0\rangle^{\otimes L}$ and $|1\rangle^{\otimes L}$, are orthonormal and invariant under translations. It is possible to preserve the

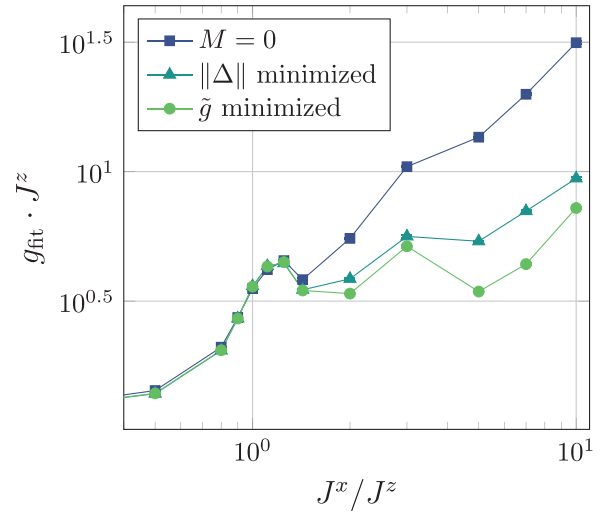


FIG. 3. Characteristic times g_{fit} obtained from fitting the numerical curves $\epsilon_{\text{AP}}(\tau)$ (as the ones in Fig. 2) with a single exponential (see text), for different values of the ratio J^x/J^z of the 1D Heisenberg model. Standard deviations from the fit are plotted as error bars (whose size is sometimes smaller than the markers).

translational invariance of the AP process by imposing the condition $\alpha_i = \alpha$ ($\forall i$) in Eq. (12). This choice, albeit not unique, allows to reduce the number of parameters to a single one. The effect of the preconditioner M on the physical system is equivalent to the introduction of an external uniform magnetic field along the z axis, with field intensity α . The results for the ground state preparation of this model are reported in Fig. 2, for either the 1D or the 2D model. The outcome is quite striking: independently on the dimensionality of the model or the parameters regime, the AP provides a way of determining the ground state of the XZ Heisenberg model with an error that scales almost exponentially with the preparation timescale. In addition, by comparing the three different approaches above, we notice a significant reduction in the required preparation timescale to achieve a given error, with the third (most optimal) approach allowing to obtain the shortest τ . The performance differences between the different approaches become more and more evident on increasing J^x/J^z , as shown in Fig. 3. Here, the characteristic time obtained by fitting the natural logarithm of the error, i.e. “ $\ln(\epsilon_{\text{AP}}(\tau))$ ”, with the function “ $C_{\text{fit}} - \tau/g_{\text{fit}}$ ” is reported as a function of J^x/J^z . In particular, the g_{fit} values obtained with the three approaches are nearly indistinguishable when $J^x \ll J^z$, while a clear advantage is obtained when applying our preconditioning strategy for $J^x \gg J^z$, i.e., when the target Hamiltonian is significantly different from the starting one. The peak in the plot occurring around $J^x/J^z \simeq 1$ corresponds to the point where, for an infinite number of spins, the model exhibits a phase transition. In that region the preparation of the ground state is still a rather expensive task and our proposal is less effective. Similar results have been obtained for the Ising model in a transverse magnetic field, in both one and two dimensions, as reported in the SM [35] for completeness. In practice, the AP of excited states would be more involved, due to the increased dimension of the subspaces involved in the time evolution. However, different techniques could be exploited to this purpose, which we aim to explore and communicate in a

future work. As a final comment, we notice that, as far as these very specific spin models are concerned, the actual improvement due to the preconditioning term only occurs for an odd number of sites with periodic boundary conditions, for which a detailed explanation is given in the SM [35].

Conclusions. We have applied the theory of adiabatic preparation (AP) to the framework of quantum simulations on NISQ devices, specifically focusing on the efficient preparation of a targeted quantum state. Starting from the seminal result of Nenciu in Ref. [32], we have analytically derived an exponential bound for the AP error, thus obtaining an explicit expression for its characteristic time, in terms of quantities that can be classically estimated. In future works, it would be beneficial to examine the tightness of such a bound with analytical evidence or numerical tests. Using this improved analytical control, we propose a universal preconditioning term, denoted as M and tailored for NISQ computing [44], which can be effectively optimized using the aforementioned formulation. Upon incorporation into the Hamiltonian model to be simulated on quantum hardware, M allows to exponentially suppress the AP error. Finally, we have tested our protocol by numerical simulations performed for the representative Heisenberg and Ising models, in

either one or two dimensions, respectively. Our results demonstrate an exponential improvement, particularly notable under more strenuous computational scenarios. Despite performing numerical tests on simple spin models, we expect similar behaviors to occur for more complicated systems. We envision that our work will significantly enhance the efficiency of quantum state preparation, thereby boosting the prospective achievement of quantum utility in state-of-the-art digital quantum devices.

Acknowledgments. Financial support from the Italian Ministry of University and Research (MUR), funded by the European Union–NextGenerationEU through the National Recovery and Resilience Plan (NRRP), is acknowledged as follows: D.N. under Project No. PE0000023-NQSTI, D.G. under Project No. CN00000013-HPC. M.B. acknowledges funding from MUR through the program for young researchers “Rita Levi Montalcini,” and ICSC-Centro Nazionale di Ricerca in High Performance Computing, Big Data, and Quantum Computing, funded by the European Union–NextGenerationEU. The authors warmly acknowledge G. Guarnieri, C. Macchiavello, F. Scala, and F. Tacchino for several scientific discussions, suggestions, and support.

-
- [1] J. Preskill, Quantum computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).
 - [2] D. P. DiVincenzo, The physical implementation of quantum computation, *Fortschr. Phys.* **48**, 771 (2000).
 - [3] S. A. Moses *et al.*, A race-track trapped-ion quantum processor, *Phys. Rev. X* **13**, 041052 (2023).
 - [4] Y. Kim *et al.*, Evidence for the utility of quantum computing before fault tolerance, *Nature (London)* **618**, 500 (2023).
 - [5] F. Tacchino, A. Chiesa, S. Carretta, and D. Gerace, Quantum computers as universal quantum simulators: State-of-the-art and perspectives, *Adv. Quantum Technol.* **3**, 1900052 (2020).
 - [6] A. J. Daley *et al.*, Practical quantum advantage in quantum simulation, *Nature (London)* **607**, 667 (2022).
 - [7] D. S. Abrams and S. Lloyd, Simulation of many-body Fermi systems on a universal quantum computer, *Phys. Rev. Lett.* **79**, 2586 (1997).
 - [8] M. Troyer and U.-J. Wiese, Computational complexity and fundamental limitations to fermionic quantum Monte Carlo simulations, *Phys. Rev. Lett.* **94**, 170201 (2005).
 - [9] A. Kandala *et al.*, Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets, *Nature (London)* **549**, 242 (2017).
 - [10] A. Kandala, K. Temme, A. D. Córcoles, A. Mezzacapo, J. M. Chow, and J. M. Gambetta, Error mitigation extends the computational reach of a noisy quantum processor, *Nature (London)* **567**, 491 (2019).
 - [11] M. Motta and J. E. Rice, Emerging quantum computing algorithms for quantum chemistry, *WIREs Computat. Mol. Sci.* **12**, e1580 (2022).
 - [12] A. Chiesa, F. Tacchino, M. Grossi, P. Santini, I. Tavernelli, D. Gerace, and S. Carretta, Quantum hardware simulating four-dimensional inelastic neutron scattering, *Nat. Phys.* **15**, 455 (2019).
 - [13] F. Arute *et al.*, Observation of separated dynamics of charge and spin in the Fermi-Hubbard model, [arXiv:2010.07965](https://arxiv.org/abs/2010.07965).
 - [14] K. Klymko *et al.*, Real-time evolution for ultracompact Hamiltonian eigenstates on quantum hardware, *PRX Quantum* **3**, 020323 (2022).
 - [15] E. A. Martinez *et al.*, Real-time dynamics of lattice gauge theories with a few-qubit quantum computer, *Nature (London)* **534**, 516 (2016).
 - [16] N. Klcio, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, Quantum-classical computation of Schwinger model dynamics using quantum computers, *Phys. Rev. A* **98**, 032331 (2018).
 - [17] W. A. de Jong, K. Lee, J. Mulligan, M. Ploskon, F. Ringer, and X. Yao, Quantum simulation of nonequilibrium dynamics and thermalization in the Schwinger model, *Phys. Rev. D* **106**, 054508 (2022).
 - [18] B. Chakraborty, M. Honda, T. Izubuchi, Y. Kikuchi, and A. Tomiya, Classically emulated digital quantum simulation of the Schwinger model with a topological term via adiabatic state preparation, *Phys. Rev. D* **105**, 094503 (2022).
 - [19] N. H. Nguyen, M. C. Tran, Y. Zhu, A. M. Green, C. H. Alderete, Z. Davoudi, and N. M. Linke, Digital quantum simulation of the Schwinger model and symmetry protection with trapped ions, *PRX Quantum* **3**, 020324 (2022).
 - [20] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
 - [21] A. Peruzzo *et al.*, A variational eigenvalue solver on a photonic quantum processor, *Nat. Commun.* **5**, 4213 (2014).
 - [22] J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, The theory of variational hybrid quantum-classical algorithms, *New J. Phys.* **18**, 023023 (2016).

- [23] N. Moll *et al.*, Quantum optimization using variational algorithms on near-term quantum devices, *Quantum Sci. Technol.* **3**, 030503 (2018).
- [24] A. Anand *et al.*, A quantum computing view on unitary coupled cluster theory, *Chem. Soc. Rev.* **51**, 1659 (2022).
- [25] J. R. McClean *et al.*, Barren plateaus in quantum neural network training landscapes, *Nat. Commun.* **9**, 4812 (2018).
- [26] M. Motta *et al.*, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution, *Nat. Phys.* **16**, 205 (2020).
- [27] M. Born and V. Fock, Beweis des adiabatensatzes, *Z. Phys.* **51**, 165 (1928).
- [28] T. Albash and D. A. Lidar, Adiabatic quantum computation, *Rev. Mod. Phys.* **90**, 015002 (2018).
- [29] A. Ambainis and O. Regev, An elementary proof of the quantum adiabatic theorem, [arXiv:quant-ph/0411152](https://arxiv.org/abs/quant-ph/0411152).
- [30] S. Jansen, M.-B. Ruskai, and R. Seiler, Bounds for the adiabatic approximation with applications to quantum computation, *J. Math. Phys.* **48**, 102111 (2007).
- [31] A. Elgart and G. A. Hagedorn, A note on the switching adiabatic theorem, *J. Math. Phys.* **53**, 102202 (2012).
- [32] G. Nenciu, Linear adiabatic theory exponential estimates, *Commun. Math. Phys.* **152**, 479 (1993).
- [33] C. Lanczos, An iteration method for the solution of the eigenvalue problem of linear differential and integral operators, *J. Res. Natl. Bur. Stand. B* **45**, 255 (1950).
- [34] P. Virtanen *et al.*, SciPy 1.0: Fundamental algorithms for scientific computing in python, *Nat. Meth.* **17**, 261 (2020).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevResearch.7.L012074> for details concerning the calculations and additional data on which the main text is based. Specifically, the Supplemental Material file contains the following sections: Exponential behaviour of the adiabatic error; Quantum circuit design; Numerical results the Ising model in a transverse field; Periodic boundary conditions and site parity; Reduction of $\tilde{g}$ for large systems; Comparing exponential and power-like bounds.
- [36] C. Mc Keever and M. Lubasch, Towards adiabatic quantum computing using compressed quantum circuits, *PRX Quantum* **5**, 020362 (2024).
- [37] E. Granet and H. Dreyer, Continuous Hamiltonian dynamics on noisy digital quantum computers without Trotter error, *npj Quantum Inf.* **10**, 82 (2024).
- [38] H. F. Trotter, On the product of semi-groups of operators, *Proc. Am. Math. Soc.* **10**, 545 (1959).
- [39] C. F. Gauss, Methodus nova integralium valores per approximationem inveniendi, in *Werke*, Cambridge Library Collection—Mathematics (Cambridge University Press, Cambridge, 2011), pp. 165–196.
- [40] A. Quarteroni *et al.*, *Scientific Computing with MATLAB and Octave* (Springer, Berlin, 2006), Vol. 3.
- [41] L. K. Kovalsky, F. A. Calderon-Vargas, M. D. Grace, A. B. Magann, J. B. Larsen, A. D. Baczewski, and M. Sarovar, Self-healing of Trotter error in digital adiabatic state preparation, *Phys. Rev. Lett.* **131**, 060602 (2023).
- [42] E. A. C. Pérez, J. Bonitati, D. Lee, S. Quaglioni, and K. A. Wendt, Quantum state preparation by adiabatic evolution with customized gates, *Phys. Rev. A* **105**, 032403 (2022).
- [43] Qiskit contributors, Qiskit An open-source framework for quantum computing, Zenodo (2023), doi:[10.5281/zenodo.2562110](https://doi.org/10.5281/zenodo.2562110).
- [44] D. C. McKay, C. J. Wood, S. Sheldon, J. M. Chow, and J. M. Gambetta, Efficient Z gates for quantum computing, *Phys. Rev. A* **96**, 022330 (2017).