

Quantum geometric tensor and wavepacket dynamics in two-dimensional non-Hermitian systems

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The quantum geometric tensor (QGT) characterizes the local geometry of quantum states, and its components directly account for the dynamical effects observed, e.g., in condensed-matter systems. In this work, we address the problem of extending the QGT formalism to non-Hermitian systems with gain and loss. In particular, we investigate a wavepacket dynamics in two-band non-Hermitian systems to elucidate how non-Hermiticity affects the definition of QGT. We employ first-order perturbation theory to account for nonadiabatic corrections due to interband mixing. Our results suggest that two different generalizations of the QGT, one defined using only the right eigenstates and the other one using both the left and right eigenstates, both play a significant role in wavepacket dynamics. We then determine the accuracy of the perturbative approach by simulating a wavepacket dynamics in a well-studied physical non-Hermitian system—exciton polaritons in a semiconductor microcavity. Our work aids deeper understanding of quantum geometry and dynamical behavior in non-Hermitian systems.

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Introduction. The complex-valued quantum geometric tensor (QGT) was first proposed by Provost and Vallee in 1980 to quantify the distance (in parameter space) between two quantum states [1]. Its real (symmetric) part is the quantum metric tensor (QMT) describing the overlap between two quantum states and its imaginary (antisymmetric) part is the Berry curvature describing the topology of the system's energy bands. In periodic systems, e.g., solid-state lattices, the integral of the Berry curvature over a Brillouin zone is a topological invariant known as the Chern number [1–9]. The Berry curvature acts on the wavepacket center of mass (e.g., an electron) as an effective magnetic field in momentum space, and results in an anomalous velocity transverse to the direction of the applied force [10–17]. The QMT appears in the wavepacket equation of motions as perturbative corrections to the Berry curvature and describes the field-induced positional shift [12,13] and nonadiabatic effects [14]. It also plays an important role in flat-band superconductivity [18–20] and can also be associated with fidelity susceptibility that has been used to detect criticality, which plays an important role in quantum information theory [21–25].

Generalizing the QGT to systems effectively described by non-Hermitian Hamiltonians [26–33] is of fundamental importance for understanding the dynamical effects in systems with gain and loss. Non-Hermitian systems have attracted a great amount of interest because they can feature novel topological invariants [34], topological phases [34,35], and topological edge states [36–44]. Non-Hermitian effects have also been experimentally studied in a variety of physical

systems including cold atoms [45,46], photonics [47–51], mechanical systems [52], electric circuits [53], magnons [54,55], spin liquid [56], and exciton polaritons [57,58].

The challenge in generalizing the QGT lies in the fact that non-Hermitian Hamiltonians have distinct left and right eigenstates [30–33]. Some works therefore define the QGT using only the right eigenstates, which we denote the right-right (RR) formalism [59–64], while others use both the left and right eigenstates, which we denote the left-right (LR) formalism [65–70]. Previous studies proposed that it is the RR version of the Berry curvature that describes the anomalous velocity in the dynamics of a wavepacket center of mass [71–73]. However, there are also non-Hermitian corrections arising from the Berry connections defined in both the RR and the LR formalism [71–73]. At the same time, while Refs. [59,64] suggests that the RR QMT plays a significant role in the wavepacket dynamics, Ref. [70] states that the RR QMT is ill defined due to the nonorthogonality of the right eigenstates. So far, it remains unclear which version of the QMT correctly represents the physical (measurable) quantities of non-Hermitian systems and how well the QMT describes the dynamical effects in non-Hermitian systems.

In this work, we address this ambiguity in the QGT definition by investigating the wavepacket dynamics of a two-band non-Hermitian system using the first-order perturbative theory. We derive a semiclassical equation of the center-of-mass motion of a wavepacket, which explicitly shows the contributions of both the LR and RR components of the QGT. We then compare the analytical results with a numerical simulation using a model describing an experimentally accessible non-Hermitian system. The model describes two-dimensional hybrid light-matter particles in a solid state—microcavity exciton polaritons [74]. Both real and imaginary parts of the eigenenergies [58] as well as QGT components [15,75] can be measured in this system, in principle allowing for experimental verification of theoretical predictions.

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Theory. For an n th level quantum state $|u_n\rangle$ that is smoothly dependent on momentum, the QGT can be defined as [1]

$$Q_{n,ij} = \langle \partial_{k_i} u_n | \partial_{k_j} u_n \rangle - \langle \partial_{k_i} u_n | u_n \rangle \langle u_n | \partial_{k_j} u_n \rangle, \quad (1)$$

where k_i, k_j denote the indices in momentum space. The QGT can also be written in terms of interband Berry connections $A_{nm} = \langle u_n | i \partial_{\mathbf{k}} u_m \rangle$, in the form of $Q_{n,ij} = \sum_{m \neq n} A_{nm,i} A_{mn,j}$ [14,76]. The QMT $g_{n,ij} = \text{Re}[Q_{n,ij}]$ and the Berry curvature $\Omega_{n,ij} = -2 \text{Im}[Q_{n,ij}]$ correspond to the real and imaginary parts of the QGT, respectively.

In non-Hermitian systems, where the Hamiltonian $H \neq H^\dagger$, the left and right eigenstates defined as

$$H |u_n^R\rangle = E_n |u_n^R\rangle, \quad \langle u_n^L | H = \langle u_n^L | E_n \quad (2)$$

are not equal but are biorthogonal, satisfying $\langle u_n^L | u_m^R \rangle = 0$ for $m \neq n$ [31,32]. This distinction between the left and right eigenstates leads to different ways of defining the QGT. The QGT can be defined using only the right eigenstates, like in Refs. [59–63]. In this formalism, the inner product is defined as $\langle \cdot \rangle = \langle u_n^R | \cdot | u_n^R \rangle$ and the QGT and its components can be written as

$$\begin{aligned} Q_{n,ij}^{RR} &= \langle \partial_{k_i} u_n^R | \partial_{k_j} u_n^R \rangle - \langle \partial_{k_i} u_n^R | u_n^R \rangle \langle u_n^R | \partial_{k_j} u_n^R \rangle, \\ g_{n,ij}^{RR} &= \text{Re}[Q_{n,ij}^{RR}], \quad \Omega_{n,ij}^{RR} = -2 \text{Im}[Q_{n,ij}^{RR}]. \end{aligned} \quad (3)$$

In what follows, we will denote the components of the QGT defined this way as the right-right (RR) generalization. Similarly to the Hermitian case, the RR QMT and the RR Berry curvature correspond to the real and imaginary parts of the RR QGT, and both quantities are real valued.

Alternatively, the QGT can also be defined using both the left and the right eigenstates, like in Refs. [66–70], where the inner product is defined using biorthogonality, $\langle \cdot \rangle = \langle u_n^L | \cdot | u_n^R \rangle$. The corresponding QGT and its components are

$$\begin{aligned} Q_{n,ij}^{LR} &= \langle \partial_{k_i} u_n^L | \partial_{k_j} u_n^R \rangle - \langle \partial_{k_i} u_n^L | u_n^R \rangle \langle u_n^L | \partial_{k_j} u_n^R \rangle, \\ g_{n,ij}^{LR} &= \frac{1}{2} (Q_{n,ij}^{LR} + Q_{n,ji}^{LR}), \\ \Omega_{n,ij}^{LR} &= i(Q_{n,ij}^{LR} - Q_{n,ji}^{LR}). \end{aligned} \quad (4)$$

We refer to this formalism as the left-right (LR) generalization. Unlike the Hermitian case, the LR QMT and the LR Berry curvature can be complex-valued, and correspond to the symmetric and the antisymmetric parts of the LR QGT. Note that this is not the only way to define a LR generalization of the QMT. Reference [70] proposed two other possible generalizations such as defining the LR QMT as the real part of the LR QGT or the real and symmetric part of the LR QGT. Here, we use Eqs. (4) to maintain the symmetric properties in the Hermitian case and our results support this definition, as will be shown below. Note that, in both formalisms, we define the eigenstates such that $\langle u_n^L | u_n^R \rangle = \langle u_n^R | u_n^R \rangle = 1$ to ensure that both $Q_{n,ij}^{RR}$ and $Q_{n,ij}^{LR}$ are gauge invariant.

The relevance of the components of the QGT to the dynamics has been investigated in both Hermitian and non-Hermitian systems. In the Hermitian case, the semiclassical equations of motion of a narrow wavepacket in a single band can be written

as [12–14]

$$\begin{aligned} \dot{\mathbf{k}}_c &= \mathbf{F}/\hbar, \\ \dot{\mathbf{r}}_c &= \partial_{\mathbf{k}} \epsilon_0 - \mathbf{F} \times \left(\boldsymbol{\Omega}_0 + \nabla_{\mathbf{k}} \times \frac{2g_0 \cdot \mathbf{F}}{\epsilon_1 - \epsilon_0} \right), \end{aligned} \quad (5)$$

where $\mathbf{k}_c$ and $\mathbf{r}_c$ are the center-of-mass (COM) momentum and position, respectively, and $\mathbf{F}$ is an external force. The first term, the gradient of the energy ϵ_0 of the band, is the group velocity. The second term is a field-induced correction due to the (vector) Berry curvature $\boldsymbol{\Omega}_0$, resulting in the anomalous Hall drift [9,14,15,76,77] perpendicular to the field $\mathbf{F}$. The last term is the result of first-order perturbation theory accounting for nonadiabatic corrections due to a neighboring band with energy ϵ_1 . This term is second-order in $\mathbf{F}$ and is proportional to the QMT g_0 .

In the non-Hermitian case, only the first-order field-induced corrections were previously worked out [71–73] with the COM position written as

$$\dot{\mathbf{r}}_c = \partial_{\mathbf{k}} \text{Re}[\epsilon_0 + \mathbf{F} \cdot (\mathbf{A}_{00}^{RR} - \mathbf{A}_{00}^{LR})] - \mathbf{F} \times \boldsymbol{\Omega}_0^{RR}. \quad (6)$$

Here, the real part of the complex-valued eigenenergy ϵ_0 determines the group velocity and the RR Berry curvature determines the anomalous Hall drift. The non-Hermitian contribution is due to the real part of the difference of the RR and LR intraband Berry connections of the band A_{00}^{RR}, A_{00}^{LR} defined as

$$\mathbf{A}_{nn}^{RR} = \langle u_n^R | i \partial_{\mathbf{k}} u_n^R \rangle, \quad \mathbf{A}_{nn}^{LR} = \langle u_n^L | i \partial_{\mathbf{k}} u_n^R \rangle. \quad (7)$$

Note that $\mathbf{A}_{nn}^{LR}$ is generally complex valued, but only the real part is relevant in Eq. (6). The Berry connection $\mathbf{A}_{nn}^{RR}$ is real valued when the right eigenstates are normalized as $\langle u_n^R | u_n^R \rangle = 1$ [71,72,75].

The non-Hermitian anomalous Berry connection [71,73] $(\mathbf{A}_{nn}^{RR} - \mathbf{A}_{nn}^{LR})$ results in a field-induced correction to the energy [72]. In the Hermitian limit, this term in the equation of motion vanishes and $\boldsymbol{\Omega}^{RR} \rightarrow \boldsymbol{\Omega}$, reducing Eq. (6) to the Hermitian case [10], Eq. (5), without the adiabatic correction.

In this work, we derive the second-order field-induced corrections to find out how the non-Hermitian QGT components affect the wavepacket dynamics by including interband mixing.

Results. We consider a perturbation to a two-dimensional (2D) Hamiltonian in the form of

$$\tilde{H} = H - \mathbf{F} \cdot \mathbf{r}, \quad (8)$$

where $\mathbf{F}$ represents an external force and $\mathbf{r}$ is the position operator. We then consider a Gaussian wavepacket in the state

$$|W\rangle = \int w(\mathbf{k}, t) e^{i\mathbf{k} \cdot \mathbf{r}} |\tilde{u}_0^R(\mathbf{k})\rangle d^2\mathbf{k}, \quad (9)$$

where $|\tilde{u}_0^R\rangle$ is a right eigenstate of the Hamiltonian $\tilde{H}$ that includes the external force.

In this work, we focus on two-band systems with gain and loss and denote $|\tilde{u}_0^R\rangle$ to be the eigenstate with the larger growth (or the smaller decay) rate. The wavepacket will eventually evolve into the eigenstate with the largest $\text{Im} E_n$ due to the loss or gain [72,78]. Therefore, in order to employ a single-band approximation, we need to ensure that the wavepacket stays

in the eigenstate that has the larger $\text{Im } E_n$, i.e., larger gain (or smaller loss).

We then apply the perturbation theory [79] by expanding the eigenstate to the first-order correction in $\mathbf{F}$ as

$$|\tilde{u}_0^R(\mathbf{k})\rangle = |u_0^R(\mathbf{k})\rangle - \frac{\mathbf{F} \cdot \mathbf{A}_{10}^{LR}}{\epsilon_0 - \epsilon_1} |u_1^R(\mathbf{k})\rangle, \quad (10)$$

where $|u_1^R(\mathbf{k})\rangle$ is the other eigenstate (with larger loss), $\mathbf{A}_{10}^{LR} = \langle u_1^L | i\partial_{\mathbf{k}} u_0^R \rangle$ is the LR interband Berry connection, and $\epsilon_{0,1}$ are the complex-valued energy eigenvalues of the unperturbed Hamiltonian H . We emphasize that the interband Berry connection in the perturbative correction has to be defined in the LR instead of the conventional RR formalism due to biorthogonality in non-Hermitian systems [79].

Assuming that the wavepacket is narrowly centered at its center of mass in momentum space $|w(\mathbf{k}, t)|^2 \approx \delta(\mathbf{k} - \mathbf{k}_c)$, we then follow the formalism presented in Refs. [71,73] and derive the equation of motion of the wavepacket COM in position space up to the first-order perturbative correction (see Supplemental Material (SM) [80] for details) as

$$\hbar \dot{\mathbf{r}}_c = \partial_{\mathbf{k}} \text{Re} [\epsilon_0 + \mathbf{F} \cdot (\tilde{\mathbf{A}}_{00}^{RR} - \tilde{\mathbf{A}}_{00}^{LR})] - \mathbf{F} \times \tilde{\boldsymbol{\Omega}}_0^{RR}, \quad (11)$$

where

$$\tilde{\mathbf{A}}_{00}^{RR} = \mathbf{A}_{00}^{RR} + \text{Re} \left[\frac{2Q_0^{RR} \cdot \mathbf{F}}{\epsilon_1 - \epsilon_0} \right] - \text{Im} \left[\partial_{\mathbf{k}} \left(\frac{\mathbf{F} \cdot (\mathbf{A}_{00}^{RR} - \mathbf{A}_{00}^{LR})}{\epsilon_1 - \epsilon_0} \right) \right], \quad (12)$$

$$\tilde{\mathbf{A}}_{00}^{LR} = \mathbf{A}_{00}^{LR} + \frac{2g_0^{LR} \cdot \mathbf{F}}{\epsilon_1 - \epsilon_0}, \quad (13)$$

$$\tilde{\boldsymbol{\Omega}}_0^{RR} = \boldsymbol{\Omega}_0^{RR} + \nabla_{\mathbf{k}} \times \text{Re} \left[\frac{2Q_0^{RR} \cdot \mathbf{F}}{\epsilon_1 - \epsilon_0} \right]. \quad (14)$$

These expressions form the main result of our work: Eq. (11) is the non-Hermitian generalization of Eqs. (5) and (12)–(14) explicitly show the first-order perturbation corrections to Eq. (6). In the following, we will show that the second-order (in $\mathbf{F}$) corrections have contributions from the QMT, Berry curvature, and the anomalous non-Hermitian Berry connection.

Let us first examine the correction to the Berry curvature $\tilde{\boldsymbol{\Omega}}_0^{RR}$ by comparing Eq. (11) to Eq. (5). The second-order field-induced correction clearly transforms as $g_0/(\epsilon_1 - \epsilon_0) \rightarrow \text{Re}[Q_0^{RR}/(\epsilon_1 - \epsilon_0)]$. Note, however, that this is not simply $g_0 \rightarrow g_0^{RR}$ due to the complex denominator. In fact, there is an additional non-Hermitian correction which comes from the RR Berry curvature itself, as shown below:

$$\text{Re} \left[\frac{2Q_0^{RR} \cdot \mathbf{F}}{\Delta\epsilon} \right] = \text{Re} \left[\frac{1}{\Delta\epsilon} \right] 2g_0^{RR} \cdot \mathbf{F} - \text{Im} \left[\frac{1}{\Delta\epsilon} \right] \mathbf{F} \times \boldsymbol{\Omega}_0^{RR}. \quad (15)$$

This new term depends on the imaginary part of the complex energy gap, which naturally vanishes in the Hermitian limit or along the imaginary Fermi arc, where $\text{Im } \Delta\epsilon = 0$ [58,78].

The second-order field-induced corrections to the anomalous non-Hermitian Berry connection have two components, i.e., $\text{Re}[\mathbf{F} \cdot (\tilde{\mathbf{A}}_{00}^{RR} - \tilde{\mathbf{A}}_{00}^{LR})] = I_0 + I_1 + I_2$, where $I_0 = \mathbf{F} \cdot (\mathbf{A}_{00}^{RR} - \mathbf{A}_{00}^{LR})$ is the first-order term. The first correction is proportional to the difference of the RR and LR QMTs and has the form

$$I_1 = 2\mathbf{F} \cdot \text{Re} \left[\frac{g_0^{RR} - g_0^{LR}}{\Delta\epsilon} \right] \cdot \mathbf{F}, \quad (16)$$

while the second term is proportional to the anomalous Berry connection and has the form

$$I_2 = -\mathbf{F} \cdot \text{Im} \left[\partial_{\mathbf{k}} \left(\frac{\mathbf{F} \cdot (\mathbf{A}_{00}^{RR} - \mathbf{A}_{00}^{LR})}{\Delta\epsilon} \right) \right]. \quad (17)$$

Note that both the real and imaginary parts of the LR QMT and Berry connections contribute to the corrections due to the complex denominator (see SM [80] for the explicit forms of I_1 and I_2). This novel non-Hermitian effect shows that the LR QMT should indeed be defined as a complex-valued quantity [66,67,70]. This is in contrast to the corrections to the Berry curvature, Eq. (14), which originate from the real-valued RR QMT and Berry curvature.

To examine the corrections closely, we integrate Eq. (11) to get (see SM [80] for the derivation)

$$\mathbf{r}_c = \left[-\partial_{\mathbf{k}} \varphi + \tilde{\mathbf{A}}_{00}^{RR} \right]_{\mathbf{k}=\mathbf{k}_c}, \quad (18)$$

$$\varphi = -\frac{1}{\hbar} \int_{t_0}^t \text{Re} [\epsilon_0 - \mathbf{F} \cdot \tilde{\mathbf{A}}_{00}^{LR}]_{\mathbf{k}=\mathbf{k}_c} dt'. \quad (19)$$

The positional shift [12] of the COM, given by Eq. (18), is due to the gradient of the phase φ and the corrected RR Berry connection. The latter, which is given by Eq. (12), has two field-induced corrections. The first depends on the RR QGT, Q_0^{RR} , which is the same term that appears in field-induced correction to the Berry curvature $\tilde{\boldsymbol{\Omega}}_0^{RR}$ in Eq. (14). The second correction depends on the anomalous non-Hermitian Berry connection $\mathbf{A}_{00}^{RR} - \mathbf{A}_{00}^{LR}$, which also appears in the field-induced correction to the energy in Eq. (6). There is also a field-induced correction to the geometric (Berry) phase [2,71,73], Eq. (19), which is proportional to the LR QMT g_0^{LR} .

In the Hermitian limit, the eigenenergies become real-valued, $\mathbf{A}_{00}^{LR} = \mathbf{A}_{00}^{RR}$, and $Q_0^{RR} = Q_0^{LR}$ become Hermitian as well. Therefore, the correction to the LR and RR Berry connections both take the forms of $\mathbf{A}_{00} \rightarrow \mathbf{A}_{00} + 2(g \cdot \mathbf{F})/(\Delta\epsilon)$, and we recover the previous results given by Eq. (5) [12–14].

To demonstrate the relevance of the QGT components to the description of dynamical effects in a two-band non-Hermitian system, we simulate the wavepacket dynamics in an exciton-polariton system formed by strong coupling of an optically active, confined exciton to a photon confined in an optical microcavity. It is a 2D (two-dimensional) system featuring rich non-Hermitian effects due to inherent loss and gain [81]. The non-Hermiticity of the system arises from the decay of polaritons, mainly due to the photons leaking out through the imperfect mirrors of the microcavity. Due to a pseudospin (polarization) degree of freedom, this system can also feature nontrivial bands with exceptional points in momentum space [58,75,78]. In principle, exciton-polaritons

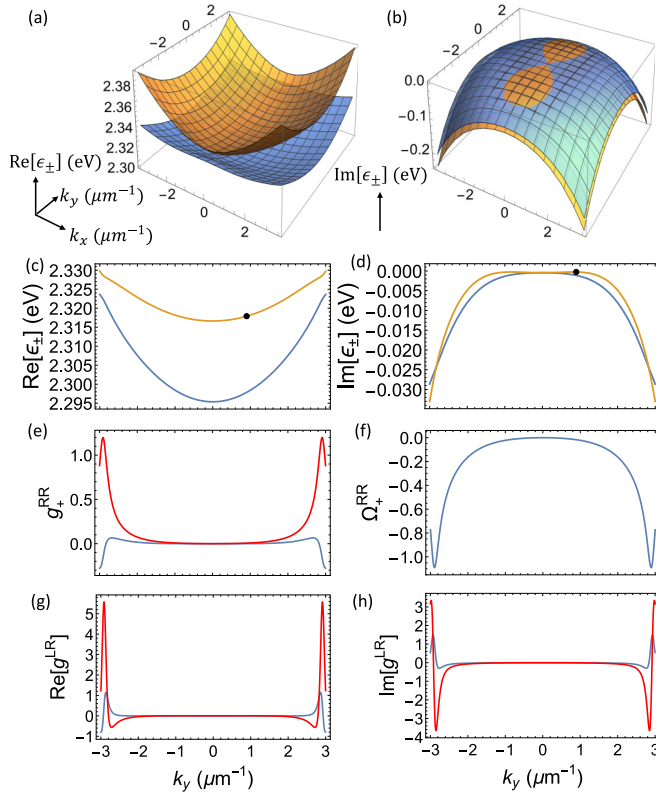


FIG. 1. The (a) real and (b) imaginary parts of the eigenenergies of the exciton polaritons in a gapped phase with $\Delta_z = 0.007$ eV. (c), (d) The cross-sections of the real and imaginary parts of the eigenenergies along the k_y axis, respectively. The orange line and surface represent the upper band, the blue line and surface represent the lower band, and the black dots represent the initial momentum of the wavepacket center of mass. (e), (f) The RR quantum metric and Berry curvature. (g), (h) The real and imaginary parts of the LR quantum metric. The blue and red lines in (e), (g), (h) show the xy components and the yy components, respectively, since these are the only components that appears in the dynamics when the force is in the y direction.

can provide an experimental platform for verifying the results presented here since the components of both the QGT [9,15] and its non-Hermitian generalization [75] can be measured in this system.

We employ split-step methods [82,83] to verify that the first-order perturbation theory developed here improves the accuracy of the semiclassical equation of motion. We consider a two-band non-Hermitian effective Hamiltonian for exciton polaritons in a form of a complex-valued effective magnetic field $\vec{G} = [G_x, G_y, G_z]$ acting on the exciton-polariton pseudospins described by the 2×2 Pauli matrices $\vec{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$. The Hamiltonian has the form [58,78]

$$\begin{aligned} H(\mathbf{k}) &= H_0(\mathbf{k}) + \vec{G}(\mathbf{k}) \cdot \vec{\sigma}, \\ H_0(\mathbf{k}) &= E_0 + \frac{\hbar^2 k^2}{2m} - i\gamma_0 - i\gamma_2 k^2 - i\gamma_4 k^4, \\ \vec{G}(\mathbf{k}) &= [\tilde{\alpha} + \tilde{\beta}(k_x^2 - k_y^2), 2k_x k_y \tilde{\beta}, \Delta_z], \end{aligned} \quad (20)$$

where m is the effective exciton-polariton mass and γ represent the mean exciton-polariton linewidth expanded to the fourth order in k , which arises from the experimentally observed nonparabolic dependence of the linewidth on momentum [58]. Here, $\tilde{\alpha} = \alpha - ia$ and $\tilde{\beta} = \beta - ib$, where α denotes the splitting of the linearly polarized modes due to the cavity anisotropy, β describes the TE-TM splitting as a result of the photonic spin-orbit coupling, while a, b represent the splitting of the exciton-polariton linewidths due to the polarization-dependent photonic losses, and Δ_z is the Zeeman splitting which arises from an external magnetic field and splits the circularly polarized modes [58,84]. In the simulations [85], we set the values of the parameters to be $E_0 - i\gamma_0 = (2.31 - 4.5 \times 10^{-4}i)$ eV, $\hbar^2/(2m) = 2.3 \times 10^{-3}$ eV μm^2 , $\gamma_2 = 0$ eV μm^2 , $\gamma_4 = 3.75 \times 10^{-4}$ eV μm^4 , $\tilde{\alpha} = (8 \times 10^{-3} - 10^{-5}i)$ eV, $\tilde{\beta} = (10^{-3} - 7.5 \times 10^{-4}i)$ eV μm^2 , which corresponds to the experimentally relevant values. Figures 1(a)–1(d) show the complex eigenenergies as a function of the momentum $\mathbf{k}$ (see also SM [80]).

We initialized the wavepacket at $\mathbf{k} = (0, 0.9)\mu\text{m}^{-1}$ in the upper eigenstate which has the larger $\text{Im } E$ (or less loss) [see the dot in Figs. 1(c) and 1(d)]. The two bands are separated at this point in momentum space and $\partial_{k_x} \text{Re } \epsilon_0 = 0$. Experimentally, the wavepacket can be prepared using resonant optical excitation, where the exciting laser wavelength is adjusted to match the energy and the excitation angle is tuned to match the momentum of the target state [15].

An external force $\mathbf{F} = (0, 5 \times 10^{-4})$ eV/ μm is applied accelerating the wavepacket along the $+k_y$ axis towards smaller (real) energy spacing. In an optical microcavity, this force naturally arises from the small variation or tilt in the cavity length introduced by the sample fabrication process, which translates to a linear energy gradient experienced by polaritons [86]. As the bands get closer, the mixing between the two eigenstates becomes more profound and can result in oscillation due to the Zitterbewegung effect [77,87]. However, since the wavepacket evolves into a less dissipative eigenstate, the interband mixing will gradually disappear and so will the oscillation. We also ensure that the wavepacket does not cross the imaginary Fermi arc [58], where the imaginary parts of the eigenenergies cross, otherwise the single-band approximation will fail [78] as the loss rates of the eigenstates will switch. The numerical results are presented as dots in Figs. 2.

In this simple case, Eq. (11) becomes

$$v_x = \frac{1}{\hbar} \partial_{k_x} \text{Re} [F_y (\tilde{A}_{00,y}^{RR} - \tilde{A}_{00,y}^{LR})] - \frac{F_y}{\hbar} \tilde{\Omega}_{0,xy}^{RR}, \quad (21)$$

$$v_y = \frac{1}{\hbar} \partial_{k_y} \text{Re} [\epsilon_0 + F_y (\tilde{A}_{00,y}^{RR} - \tilde{A}_{00,y}^{LR})]. \quad (22)$$

The explicit forms of $\tilde{A}_{00,y}^{RR} - \tilde{A}_{00,y}^{LR}$ and $\tilde{\Omega}_{0,xy}^{RR}$ are presented in the SM [80].

The solution of Eq. (11) gives a better fit to the numerical results than the solution of Eq. (6). This is apparent from the discrepancy between the numerical and analytical solutions plotted in Figs. 2(c) and 2(d). Although the nonadiabatic correction is rather small in our example, a larger force or a smaller energy gap will result in a much larger correction, meaning that the previously derived equation of motion without perturbative corrections Eq. (6) will no longer be adequately describing the dynamics. In this case, however, we

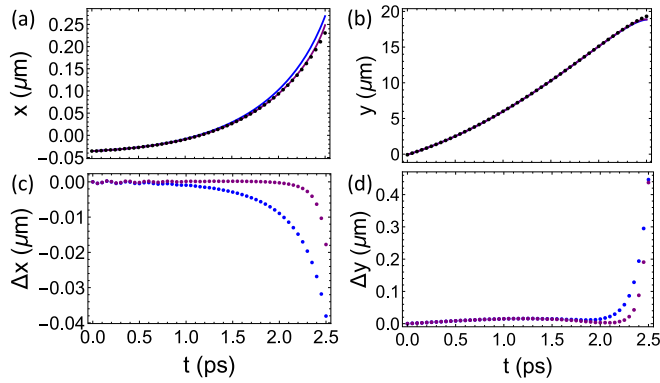


FIG. 2. (a), (d) The real-space trajectory of exciton-polariton wavepackets initially placed in the upper eigenstate in 3 different gapped phases with $\Delta_z = 0.007$ eV. Shown are the simulated results (black dots), the solution of semiclassical equation of motion derived previously, Eq. (6) (blue lines), and the solution of the semiclassical equation of motion derived here, Eq. (11) (purple lines). (c), (d) The difference between the analytical results and the numerical simulation with the same color scheme as in (a), (b).

might also need to extend our treatment beyond the first-order perturbation theory and consider higher-order corrections. On the other hand, if the wavepacket propagates for a longer time, it would cross the imaginary Fermi arc, and the semiclassical equations of motion will no longer be valid [78].

Conclusion. We derive the first-order perturbative corrections to the semiclassical equation of motion describing the center-of-mass motion of a wavepacket of a non-Hermitian two-band system. Our results show that both the left-right (LR) and right-right (RR) quantum geometric tensor (QGT) play a role in the dynamics. In particular, the RR QGT describes the non-Hermitian generalization of the field-induced

positional shift, while the LR QGT describes the field-induced correction to the Berry phase. Interestingly, our results also suggest novel second-order anomalous Hall drift correction arising from the RR Berry curvature, and the positional-shift correction arising from the difference between the RR and LR QMT and the anomalous Berry connection. More importantly, our theoretical results show that the imaginary part of the LR QMT and Berry connections play a direct role in the second-order field-induced corrections, unlike the first-order case. Our results settle the dispute on whether the LR or the RR QGT should be used to describe the dynamics of non-Hermitian systems, and instead suggest that both LR and RR QGTs represent physical quantities and play a significant role in the dynamics of non-Hermitian systems.

We further confirmed the accuracy of our results by simulating the center-of-mass dynamics of a non-Hermitian exciton-polariton system with a gapped spectrum, which can be realized in microcavity exciton polaritons exposed to a strong out-of-plane magnetic field [88,89] or hosted in liquid-crystal cavities [90].

Our work opens up many possibilities for further investigation in the QGT in non-Hermitian systems. In particular, our results should be relevant for the physics of second-harmonic generation [91], nonlinear response theory [92,93], quantum information [21,22], and flat-band superconductivity [18–20], where the quantum metric tensor plays a significant role.

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Data availability. The data that support the findings of this article are openly available [85].

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