

Bloch-band structure of cavity optomechanical oscillations

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Cavity optomechanical oscillations (OMOs) have been extensively studied for their rich physics and various practical applications. However, due to the highly nonlinear nature of the dynamical process, the exact sideband structure of an optomechanically oscillating optical cavity field remains unknown, although it is essential to a comprehensive understanding and accurate manipulation of such systems. Here, we establish a correspondence between the Bloch-band structure and the coupled sideband dynamics of OMOs, thus providing a theoretical framework for unveiling the detailed structure of cavity optical modes in terms of their resemblance to the well-known Wannier-Stark states and ladders. Surprisingly, the locations of these ladders are irrelevant to pump frequency or power but only depend on the resonant frequency of the optical mode and mechanical-mode frequency. By an energy transfer picture, we build up a connection between the highly nonlinear OMOs and a Bloch-band structure that can be solved linearly. Quantitatively, this picture uncovers the underlying mechanism of the optimization of the pump detuning and optical decay rate, as well as the determination of the minimum input pump power, for sustaining a cavity OMO.

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Introduction. Owing to its rich underlying physics and numerous opportunities for technological innovation, cavity optomechanics [1–5] has aroused tremendous interest over the past 20 years. One notable aspect of this field is the realization of optomechanical oscillations (OMOs) achieved by employing a slightly blue-detuned input pump laser in a cavity optomechanical system [6–14]. These OMOs have been harnessed to develop low-noise radio-frequency (rf) oscillators [15–17], high-precision sensors [4,18–20], photonic rf down-converters [21], and optical pulse train generators [22]. In the past, considerable efforts have been made to understand the small oscillatory perturbations near the equilibria of some relevant systems, known as “Floquet modes” by a linearization of system dynamics [23–28]. However, the creation of a genuine OMO is a highly nonlinear dynamic process, and the study on such nonlinear optomechanics is much less developed. To the best of our knowledge, although mechanical motion in OMOs is well known [29], the exact optical sideband structure in an optomechanically oscillating cavity had not been revealed in the past, except for a mathematical form that gives the individual sideband amplitudes in terms of the pump field intensity and induced mechanical oscillation amplitude [29,30]. This defect hin-

ders not only an in-depth comprehension of OMOs but also the further progress to explore the rich nonlinear dynamical phenomena in multiple-optomechanical-oscillator systems [31–33].

Another less explored application of OMOs is the generation of optical frequency combs (OFCs). Nowadays, extensive research is being conducted on OFCs due to their crucial role in various applications such as optical metrology, precision spectroscopy, optical clocks, and rf photonics [34–36]. So far, researchers have successfully utilized electro-optic (EO) modulation or intrinsic parametric nonlinearities of materials (such as Kerr media), commonly exhibiting high repetition rates, to demonstrate the integrated optical microcombs [36–40]. On the other hand, OFCs with low repetition rates (typically below 1 GHz) are better suited for other applications, such as high-resolution spectroscopy [41,42], mode-locked lasers with ultranarrow spectral widths [43], integrated pulse sources for quantum optics [44], and low-noise microwave frequency comb generations [45,46]. Unlike Kerr- and EO-based microcombs, the repetition rates of the optomechanically generated OFCs (OMOFCs) hinge on the mechanical oscillation frequency rather than the free spectral range of the adopted cavity [30,47–49], facilitating micron-scale, coherent low-repetition-rate OFCs. However, without any exact quantitative relation known about the concerned nonlinear dynamic process, the optimization of the system parameters (the pump driving frequency, optical coupling rate, and others) for the generation of OMOFCs under a limited pump power remained to be a challenge. The previous estimations [50,51] predicted limited comb lines, lacking practical relevance to the ongoing experimental pursuits.

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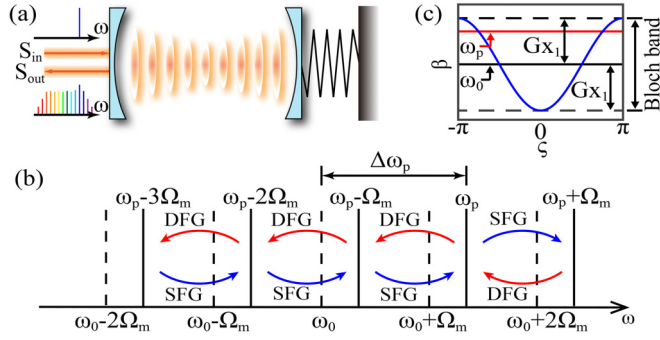


FIG. 1. Analogous Bloch bands in OMOFC generation. (a) Exemplary OMOFC generation setup. (b) Mechanism of OMOFC generation: Wannier-Stark ladders (vertical dashed lines) represent oscillating cavity eigenfrequencies sequentially located at $\omega_0 + l\Omega_m$ ($l = 0, \pm 1, \pm 2, \dots$), while corresponding optical sidebands (vertical solid lines) radiate at frequencies $\Delta\omega_p + l'\Omega_m$ ($l' = 0, \pm 1, \pm 2, \dots$). In the case of exciting the first mechanical mode, photons primarily interact with adjacent sidebands through a series of difference- or sum-frequency generation (DFG or SFG), mediated by phonon emission or absorption. (c) Bloch-like structure of OMOFC in the reduced Brillouin zone $\zeta \in [-\pi, \pi]$.

Here, we introduce a theory to tackle the aforementioned challenge. Our theory draws on an analog between the sideband structures of cavity OMOs and the well-known tight-binding model governing the motion of individual electrons within lattices. This insight enables one to derive the effective eigenstates of the OMO mirroring the Wannier-Stark (WS) states in a tight-binding framework. Intriguingly, the eigenfrequencies of these WS states are only determined by the resonant frequency of the optical mode and mechanical oscillatory frequency involved in the OMOs, irrelevant to the highly nonlinear nature of the cavity optomechanics. To find the connection between the highly nonlinear dynamics to the linear Bloch approximation, we study the detailed energy transfer dynamics of the optomechanical process. As a result, it reveals the underlying mechanism to expand the OMOFC spectral span by various parameter conditions. Finally, we demonstrate that our theory can also serve as a convenient tool to obtain the sideband structures of coupled multiple optomechanical oscillators.

Basic model. As schematically shown in Fig. 1(a), our focused cavity optomechanical setup involves the radiation pressure of a single cavity mode interacting with a mechanical oscillator, inducing optical field modulation via dynamical backaction [1,7,52]. Once the mechanical motion stabilizes in a sinusoidal behavior [29], the motional displacement of the oscillator $x(t) \approx x_1 \cos(\Omega_m t) = b_1 e^{i\Omega_m t} + \text{c.c.}$ [Ω_m represents the oscillatory frequency of the mechanical mode; x_1 ($b_1 = \frac{x_1}{2}$) denotes the amplitude of mechanical oscillation; c.c. means the complex conjugate], the optical field inside the cavity takes a form of the summation of harmonic components, $a(t) = \sum_k a_k e^{ik\Omega_m t}$ with a_k ($k = 0, \pm 1, \pm 2, \dots$) being the k th harmonic amplitude. This ensemble of harmonic sidebands showcases an equally distanced and fixed spacing of Ω_m . These oscillations will lead to the following equations of the coupled mechanical and optical modes [see more details in the

Supplemental Material (SM) [53]],

$$\frac{da_k}{dt} = -i\left(\Delta\omega_p + k\Omega_m - i\frac{\kappa}{2}\right)a_k - iGb_1 a_{k-1} - iGb_1^* a_{k+1} + \delta_{0,k} \sqrt{\kappa_e} s_{\text{in}}, \quad (1)$$

$$\frac{db_1}{dt} = -i\left(\Delta\Omega_m - i\frac{\Gamma_m}{2}\right)b_1 - i\frac{G}{2\Omega_0 m_0 \omega_0} \sum_k a_k a_{k-1}^*, \quad (2)$$

where $\Delta\Omega_m = \Omega_m - \Omega_0$ [2] with Ω_0 being the resonant frequency of mechanical mode and $\Delta\omega_p = \omega_p - \omega_0$ [54] with ω_0 being the optical resonant frequency. Here, κ and Γ_m represent the decay rates of optical and mechanical frequencies, respectively, m_0 characterizes the effective mass of the cavity, and $G = \partial\omega_0/\partial r$ measures the coupling between the optical and mechanical modes for the cavity with a radius r . Finally, s_{in} and κ_e give the amplitude of the input pump field and the optical external coupling rate, respectively. According to these coupling equations, the OMOFC generation can be intuitively understood as photons primarily engaging with those situated in neighboring sidebands, propelled by a sequence of successive difference-frequency generations (DFGs) and sum-frequency generations (SFGs), as illustrated in Fig. 1(b). Within each DFG (SFG) process, a photon residing in a higher (lower) indexed optical sideband transits to an adjacent lower (higher) sideband through the emission (absorption) of a fundamental-mode phonon. The interplay intensity amid neighboring sidebands correlates with Gb_1 , the product of the coupling coefficient G and the mechanical oscillation amplitude b_1 .

Bloch-band-like structure. Equations (1) and (2) are highly nonlinear. However, once the system has been stabilized, a constant mechanical amplitude b_1 will help to clarify the sideband structure. To see this, we extract the concerned features in Eq. (1) by the following equation (the damping and drive terms are removed here for a clearer illustration but will be fully considered in dealing with the real dynamical processes):

$$\frac{da_k}{dt} = -i(\Delta\omega_p + k\Omega_m)a_k - iGb_1 a_{k-1} - iGb_1 a_{k+1}. \quad (3)$$

Note that b_1 is real under the assumption $x(t) \approx x_1 \cos(\Omega_m t)$ [29]. Now, its underlying physics is apparent: When the mechanical oscillation stabilizes with a constant b_1 , Eq. (3) resembles a tight-binding model [55]. This model is used to analyze uniform-accelerating electrical wave packets traversing periodic potential wells under a static electric field [56,57]. In Eq. (3), the coefficients Gb_1 and Ω_m are analogous to the coupling coefficient between adjacent wells and the static-electric-field amplitude in such a tight-binding context, respectively. Our approach differs from the previous one-dimensional frequency crystal constructions [58,59] in that we achieve the comparable dynamical effects using only a single cavity-field mode instead of multiple optical modes. Equation (3) can be solved via the Bloch theory (e.g., Refs. [55,56]) to yield the eigenvalues (see the details in SM [53]),

$$\beta_l = l\Omega_m - \Delta\omega_p, \quad (4)$$

uniformly spaced by Ω_m and redshifted by $-\Delta\omega_p$ (relative to ω_p) for all components $l = 0, \pm 1, \pm 2, \dots$. Essentially, these equidistant eigenvalues represent eigenfrequencies peaking at

$\omega_0 + l\Omega_m$, forming a discrete ladder spectrum similar to the renowned WS ladders [60] seen in Fig. 1(b). Note that the locations of these eigenfrequencies are only determined by ω_0 and Ω_m , independent of the amplitude of mechanical motion x_1 . The corresponding eigenstates assume the form

$$a_k^l(t) = J_{k+l} \left(-\frac{Gx_1}{\Omega_m} \right) e^{i\beta_l t}, \quad (5)$$

with $J_n(x)$ being the n th order Bessel function of the first kind. Given $J_n(x)$'s rapid decay for $|n| > |x|$, Eq. (5) implies that the l th WS-like state carries optical sidebands with significant amplitudes only within the frequency range between $-Gx_1 - l\Omega_m + \omega_p$ and $Gx_1 - l\Omega_m + \omega_p$. Under an external pump field, Eq. (5) indicates that it efficiently excites the WS states at resonance (when $\Delta\omega_p \approx l\Omega_m$). As a result, the excited optical sidebands are confined to the frequency range $(-Gx_1 + \omega_0, Gx_1 + \omega_0)$ as the WS states.

From the analysis of Eq. (3) (see details in SM [53]), we obtain the spectrum

$$\beta = \omega_0 - Gx_1 \cos \varsigma \quad (6)$$

as an effective eigenvalue involving all optical sidebands a_k . This Bloch-band-like configuration, depicted in Fig. 1(c), centers at ω_0 and encompasses a bandwidth of $2Gx_1$. Spanning the reduced Brillouin-like zone $(-\pi \leq \varsigma \leq \pi)$, this structure mirrors those in solid state physics [55,56]. Accordingly, any sideband of the excited OMOFC must align within this Bloch-band configuration. To effectively excite the WS states, the injected pump frequency ω_p should therefore align within the Bloch band, as the red line in Fig. 1(c), to sustain the cavity-OMO-induced OFC with its spectral extent matching that of the Bloch band.

Interpretation of comb expansion. So far, we have illustrated the full sideband structure of the cavity OMOs, including the effective eigenstates, the bandwidth, as well as the pump condition. These results offers an indispensable insight into the optimal span of an OMOFC under a constant pump laser power $P = |s_{\text{in}}|^2$. To this end, we introduce an effective radiation pressure from Eq. (2),

$$F_{\text{eff}} = \frac{2G}{\omega_0} \text{Im} \left(\sum_k a_k a_{k-1}^* \right), \quad (7)$$

quantifying the mechanical gain driven by optical radiation pressure. The stabilized mechanical oscillation amplitude $x_1 = F_{\text{eff}}/\Gamma_m \Omega_0 m_0$ exhibits proportionality to F_{eff} . Within the parameter space $(Gx_1/\Omega_0, \Delta\omega_p/\Omega_0)$, the distribution of F_{eff} obtained numerically from Eqs. (1) and (2) reveals asymmetry concerning ω_0 ; refer to Fig. 2(a). With the Bloch-band theory, we infer the Bloch-band edges (represented by two dashed black lines in Fig. 2) at $\Delta\omega_p = \pm Gx_1$. Inside these boundaries ($|\Delta\omega_p| < Gx_1$), F_{eff} demonstrates ascending (the red shaded regions) and descending (the blue shaded regions) trends from zero. This behavior underscores the optical radiation pressure's pivotal role in the oscillator dynamics. The F_{eff} suddenly dwindles to zero beyond the Bloch-band edges (i.e., $|\Delta\omega_p| > Gx_1$), indicating rapid dampings once ω_p goes outside the Bloch band. The most substantial F_{eff} exists near the positive edge $\Delta\omega_p = +Gx_1$ of the Bloch band.

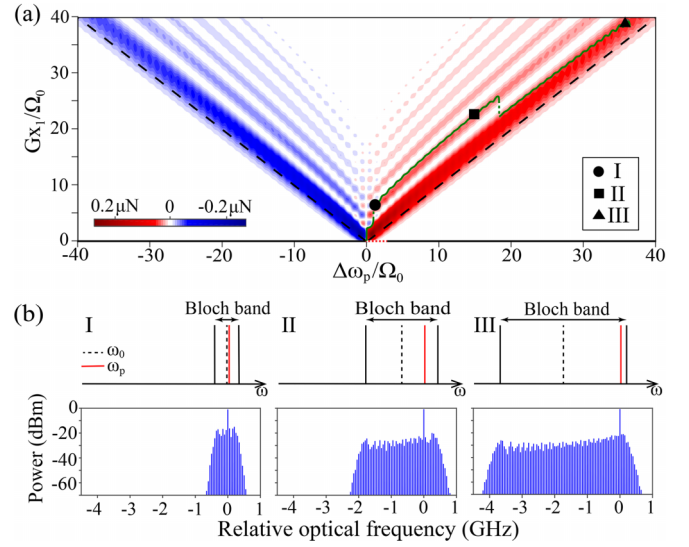


FIG. 2. Effective radiation pressure and OMOFC analysis. (a) F_{eff} mapping in parameter space $(Gx_1/\Omega_0, \Delta\omega_p/\Omega_0)$. The solid green curve represents stable OMO states achieved by increasing pump detuning, while dashed portions signify transitions between OMO-state branches. The dashed black lines denote analogous Bloch-band edges. The red dashed line highlights the range of $\Delta\omega_p$ above OMO threshold on the $\Delta\omega_p/\Omega_0$ axis. Simulation parameters: $\Omega_0 = 2\pi \times 50$ MHz, $\Gamma_m = 2\pi \times 10$ kHz, $m_0 = 15$ ng, $\omega_0 = 2\pi \times 194$ THz, $\kappa_0 = 2\pi \times 2$ MHz, $\kappa_e = 2\pi \times 50$ MHz, $r = 15$ μm , and $P = 1$ mW. (b) Exact correspondence of comb spectra and analogous Bloch bandwidths for OMO states I, II, and III in (a). Here, the comb spectra are shifted by the pump frequency. Numerical simulations utilize pump detunings $\Delta\omega_p$ corresponding to states I, II, and III: Ω_0 , $15\Omega_0$, and $36.1\Omega_0$, respectively.

To validate the consistency between the Bloch theory and actual system dynamics, we employ the fourth-order Runge-Kutta numerical integration to solve Eqs. (S1) and (S2) of SM [53]. The resultant trajectory of stabilized x_1 , impacted by continuous detuning $\Delta\omega_p$, is illustrated by the continuous green curve (solid and dashed) in Fig. 2(a). Encompassing robust F_{eff} near the Bloch band's peak results in substantial x_1 amplification during the process. Illustrated as I, II, and III in Fig. 2, these three selected states exemplify the phenomenon. Remarkably, state III exhibits an approximate sixfold increase in x_1 compared to state I.

The spectral widening of the OMOFC's Bloch-band-like structure along the green curve in Fig. 2(a) is specified further by a direct correlation between varied $\Delta\omega_p$ and the reachable extent of the OMOFC's spectral coverage, as shown in Fig. 2(b). Then, a clear trend emerges: The augmented Bloch bandwidth leads to a broader comb span. Additionally, with pump laser frequency ω_p near the Bloch band's right edge, the pump field concentrates on the comb spectrum's right wing [Fig. 2(b)]. The obtained Bloch-band-like configuration thus clarifies the sideband structure in an optomechanically oscillating cavity, going beyond a previous theoretical description only concerning the associated mechanical motion [29].

Connection between nonlinear dynamics and Bloch-band-like structure. To validate the Bloch-band calculations further, we will employ another theoretical tool involving the full

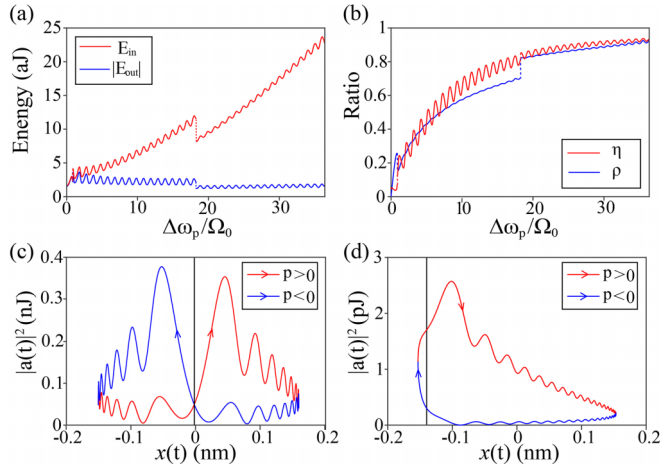


FIG. 3. Analysis of mechanical energy flow in OMO generation. (a) Example of E_{in} and E_{out} . (b) Trends of ratios η and ρ with pump detuning $\Delta\omega_p$. (c) and (d) Phase contours of intracavity optical energy $|a(t)|^2$ and mechanical oscillator displacement $x(t)$, with red and blue curves representing $p(t) > 0$ and $p(t) < 0$ branches, respectively. Arrows indicate OMO cycling directions, and black vertical lines mark the positions when cavity resonance coincides with input pump frequency.

nonlinear dynamics as a complement. Since the Bloch-like structure is well associated to mechanical oscillation, we examine the energy transfer dynamics in and out of the mechanical oscillator over an oscillation cycle. It is through evaluating the energy current $L(t) = \frac{G}{m_0\omega_0}|a(t)|^2 p(t)$ [7], where $p(t) = m_0 \frac{dx}{dt}$ is the instantaneous momentum of the mechanical oscillator. Mechanical energy increases solely when $L(t) > 0$ [manifested when the mechanical oscillator advances with $p(t) > 0$], given the concordance between radiation pressure and motion direction. Starting with a mechanical oscillation $x(t) = x_0 + x_1 \cos(\Omega_m t)$ and the corresponding $p(t) = -m_0\Omega_m x_1 \sin(\Omega_m t)$, we compute energy inflow (E_{in}) and outflow (E_{out}) for $p(t) > 0$ [and $p(t) < 0$] over each oscillation period ($2\pi/\Omega_m$) through integration of $L(t)$, yielding $E_{\text{in}} = \int_{\pi/\Omega_m}^{2\pi/\Omega_m} L(t) dt$ [and $E_{\text{out}} = \int_0^{\pi/\Omega_m} L(t) dt$]. Cumulatively, these energy flows contribute to the net energy $E_{\text{net}} = E_{\text{in}} + E_{\text{out}}$ absorbed by the motional oscillator within an oscillation cycle.

We comprehensively investigate the energy fluxes, especially focusing on those tied to the OMOs depicted along the green curve in Fig. 2(a). Figure 3(a) demonstrates a substantial E_{in} growth with increasing $\Delta\omega_p$, while E_{out} remains relatively stable. Amplified energy inflow signifies larger-amplitude OMO realized through increased energy transfer to the oscillator. Pump field efficiency, defined by $\eta = E_{\text{net}}/E_{\text{in}}$, is displayed in the blue curve of Fig. 3(b). For the self-sustained OMOs at a significant blue pump detuning, η asymptotically approaches 0.9, implying a nearly complete utilization of absorbed optical energy to maintain oscillation (barring minor losses). Concurrently, we introduce $\rho = \Delta\omega_p/Gx_1$, a relative deviation measure of input pump frequency to Bloch-band edge. As shown in Fig. 3(b), both η and ρ converge to the same value as input pump frequency approaches the upper Bloch-band edge. This convergence signifies an ultimate limit for the excited OMO, achievable by any fixed-power pump laser,

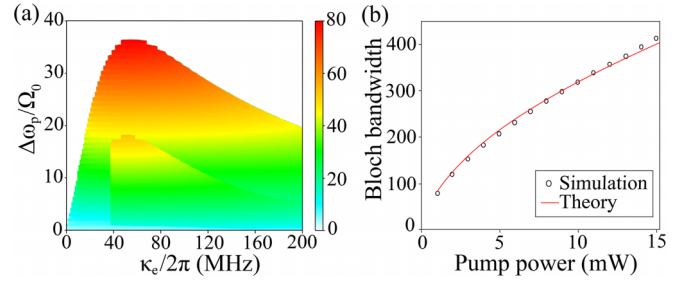


FIG. 4. Impact of cavity decay rate and pump power. (a) Distribution of analogous Bloch bandwidth in $(\kappa_e/2\pi, \Delta\omega_p/\Omega_0)$ space, illustrating transitions between different OMO branches with optical decay rate (indicated by color changes) and with pump detuning $\Delta\omega_p/\Omega_0$ as in Fig. 2. (b) Maximal achievable Bloch bandwidth (in Ω_0 units) vs input pump power. Numerical results shown as circles, while red solid curve is derived from the analytical theory in SM [53].

owing to the optimal energy transfer at the upper Bloch-band edge.

Upon closer examination, we attribute the discrepancy in η to the different intracavity optical field buildups during each oscillation cycle. This complexity is clarified by Figs. 3(c) and 3(d), the phase diagrams depicting intracavity optical energy $|a(t)|^2$ proportional to radiation pressure on the cavity wall. Within each OMO period, $|a(t)|^2$ accumulates swiftly around the resonance at $\omega_{\text{res}}(t) = \omega_p$ (black solid vertical lines), followed by its decay as the oscillator departs the equilibrium point. For each oscillator cycle, $|a(t)|^2$ intersects with the black vertical lines at two distinct moments: the forward motion trajectory $p(t) > 0$ (red curve) and the backward motion $p(t) < 0$ (blue curve) in the phase space. In Fig. 3(c) with a slightly blue-detuned pump, these instances lie near the midpoints of trajectory branches, so that the optical energy peaks on both trajectories—the right side of the solid black line for the red curve and the left side for the blue curve. For a mechanical oscillation of $Gx_1/\Omega_0 = 40.5$, e.g., the energy inflow (outflow) in each of such a mechanical cycle is $E_{\text{in}} \approx 2.20 \times 10^{-15}$ J ($E_{\text{out}} \approx -2.18 \times 10^{-15}$ J), as in OMO state I of Fig. 2 (the input pump power is $P = 0.9$ W). The resultant η is only 0.01. Conversely, by tuning ω_p to near Bloch band's top, the trajectories of $|a(t)|^2$ will become those in Fig. 3(d), which are totally asymmetric to have the OMO corresponding to state III in Fig. 2. Then, the intracavity field imparts substantial pushing force during the oscillator's initial forward motion (red curve). The mechanical energy loss starts with a backward motion (blue curve), during which the stored intracavity energy decreases to have a negligible pushing force on the oscillator. For the OMO depicted in Fig. 3(d), a mere input power of around $P = 1$ mW supports a similar relative amplitude $Gx_1/\Omega_0 = 39.5$, but yielding a significantly high efficiency $\eta = 0.92$ with $E_{\text{in}} \approx 2.37 \times 10^{-17}$ J and $E_{\text{out}} \approx -1.83 \times 10^{-19}$ J.

The energy flow picture can also explain the influence of the optical decay rate on the excited OMOs [53], which suggests that a near-equal optical decay rate ($\kappa_e = 2\pi \times 55$ MHz) to the mechanical resonant frequency ($\Omega_0 = 50$ MHz) achieves a deep overcoupling for efficient generations [Fig. 4(a)]. More interestingly, this picture leads to an

analytical relation [53] between the optimal Bloch bandwidth and input pump power [Fig. 4(b)].

Beyond single optomechanical oscillators. Our theory can be conveniently generalized to study the OMOs emerging in multiple optomechanical oscillators [31–33], which play a key role in optomechanical networks. As a brief example (see the details in SM [53]), we demonstrate how to utilize the Bloch theory to reveal the sideband structure of coupled optomechanical oscillators [32]. Moreover, the acquired effective eigenfrequencies are evidenced by the computed probe transmission spectrum (Fig. S2 of SM [53]), which exhibits two interlaced WS ladders. These calculations display the power of our theory in dealing with the OMOs beyond a single cavity.

Conclusion. We establish the analogy between the effective OMO eigenstates and the WS states and ladders in condensed matter physics. Consistent with our prior experimental data [48], the theory guides toward generating broadband OMOFCs capable of low-repetition-rate microcombs spacing from megahertz to gigahertz. These microcombs revolutionize spectroscopy with enhanced resolution, facilitate integrated

quantum optics with coherent pulse trains, and enable compact microwave frequency generators. Beyond simulating Bloch oscillations [58,61], our framework links the cavity OMOs to many other pioneering explorations such as the construction of photonic frequency crystals [59,62] without external modulation. Due to the identical form of system dynamics, we also expect that our theory would promote the investigations on magnonic-frequency-comb generations in magnomechanical resonators [63,64].

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