

Anomalous heat transfer in nonequilibrium quantum systems

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Anomalous heat transfer (AHT), a process by which heat spontaneously flows from a cold system into a hot one, superficially contradicts the Clausius statement of the second law of thermodynamics. Here we provide a full classification of mechanisms of AHT in nonequilibrium quantum systems from a quantum-information perspective. For initial states in local equilibrium, we find that AHT can arise from three resources: initial correlation, intrasystem interaction, and intrasystem temperature inhomogeneity. In particular, for qubit systems, we prove that initial quantum coherence is necessary for AHT if the intersystem interactions are limited to the two-body type. We explicitly show AHT dominated by each of the mechanisms in a three-qubit system. Our classification scheme may offer a guideline for developing high-efficiency quantum autonomous thermal machines.

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Introduction. The second law of thermodynamics, as a fundamental physical law of nature, establishes the direction of thermodynamic processes. Its Clausius statement claims that heat can only spontaneously flow from a hot object into a cold one [1], and entropy, a state function characterizing the randomness of the isolated joint system, must be non-decreasing in the process. This thermodynamic irreversibility has been popularly termed “the arrow of time” [2]. However, in an absolutely isolated and finite-sized quantum system, the second law holds only under certain initial conditions, since the system evolution is unitary and reversible [3,4].

One commonly discussed restriction is that the hot and cold systems should be weakly correlated [4–8]. This weak-correlation condition faithfully describes the usual circumstance between macroscopic objects in daily life [9,10], as Boltzmann suggested for classical systems [11,12]. However, correlation effects can be rather significant in generic quantum systems, and in particular, there exist quantum correlations such as quantum entanglement [13,14] and quantum discord [15–17] having no classical counterparts, which play an

important role in energy flow [18–20]. It has been proposed that when two quantum systems are strongly correlated, “anomalous” heat transfer (AHT), that is, heat spontaneously flows from the cold system into the hot one, can occur, at the cost of consuming the intersystem correlation [5–7,21]. More recently, correlation-induced AHT has been experimentally observed in nuclear spin systems [22] and further investigated in a wide range of systems from trapped ions [23] to wormholes [24]. In addition, the AHT phenomenon has also been discussed in the context of symmetry-protected edge/surface states [25], frustrated classical spin systems [26], and quenching dynamics of hot quantum gases [27].

In spite of enormous progress, a general and systematic framework for analyzing AHT in quantum systems is still lacking. In this Letter, we generically define the heat transfer problem, and develop a quantum-information classification of the AHT mechanisms based on the notion of local-equilibrium states [28–31]. We find that, in addition to the intersystem correlation, the nature of interactions, as well as the intrasystem correlation and temperature inhomogeneity, can give rise to AHT. We further investigate qubit systems in which the forms of interactions are limited by the so-called heat transfer condition (HTC). In particular, we prove that classical correlation is not sufficient to induce AHT if the intersystem interactions are of two-body type. We calculate the heat transfer for a three-qubit model and show explicitly that each of the mechanisms can independently realize AHT.

Heat transfer Hamiltonian. We consider heat transfer between two quantum systems A and B , while the joint system

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is isolated. In general, the time-independent Hamiltonian of the joint system reads

$$H = H_A + H_B + H_I, \quad (1)$$

where H_X with $X \in \{A, B\}$ is the Hamiltonian of the system X and H_I describes the intersystem interactions. In Eq. (1) we adopt the HTC, $[H_I, H_A + H_B] = 0$; that is, the total energy of A and B is conserved, which ensures that the energy loss of A equals the energy gain of B during the evolution of the joint system [32]. Therefore, as the joint system evolving from initial ($t = 0$) state ρ to final state ρ' , one can unambiguously define the heat transfer $\mathcal{Q} \equiv \Delta\langle H_B \rangle = -\Delta\langle H_A \rangle$, where $\Delta\langle O \rangle \equiv \text{tr}[O(\rho' - \rho)]$ denotes the variation of the expectation value of an operator O . The two states ρ and ρ' are related by the evolution operation $\rho' = e^{-iHt}\rho e^{iHt}$, where we have taken $\hbar = 1$. The HTC has been widely exploited in the study of thermal operations in the context of the resource theory [33–36], and it tremendously constrains the parameters of the Hamiltonian. We note that two joint systems, described by Hamiltonians $H_{1,2}$ and initial states $\rho_{1,2}$, exhibit identical heat transfer processes, if $UH_1 = H_2U$ and $U\rho_1 = \rho_2U$ with U a time-independent unitary transformation.

An instantaneous state $\varrho(t)$ satisfies the Liouville–von Neumann equation $d\varrho/dt = i[\varrho, H]$. In consequence, the n th time derivative of the heat transfer can be written as

$$\frac{d^n \mathcal{Q}}{dt^n} = i^n \langle [H_I, H_B]_n \rangle_{\varrho}, \quad (2)$$

where the operator $[H_I, H_B]_n$ is defined by the iteration relation $[H_I, H_B]_n = [H_I, [H_I, H_B]_{n-1}]$ with $[H_I, H_B]_{n=1} \equiv [H_I, H_B]$, and $\langle O \rangle_{\varrho} \equiv \text{tr}(\varrho O)$ (see Supplemental Material (SM) [37]). Equation (2) characterizes the instantaneous heat transfer process of the model (1). Especially, the sign and amplitude of the leading-order nonvanishing derivative indicates the instantaneous direction and speed of the heat flow, respectively. We note that the heat flow $\mathcal{Q}(t)$ is in general a nonanalytic smooth function of time. Therefore, two flow functions $\mathcal{Q}_{1,2}(t)$ are not necessarily identical, even if their instantaneous characteristics at a special time t_0 are identical; that is, $d^n \mathcal{Q}_1/dt^n|_{t=t_0} = d^n \mathcal{Q}_2/dt^n|_{t=t_0}$ for any $n \geq 0$.

For revealing the effect of the intrasystem nonequilibrium, we assume that the system A is composed of N interacting subsystems $A_{1,2,\dots,N}$ [see Fig. 1(a)]. The Hamiltonian reads $H_A = H_{A0} + H_{AI}$, where $H_{A0} = \sum_{k=1}^N H_{A_k}$ with H_{A_k} the Hamiltonian of the subsystem A_k , H_{AI} describes the intra-subsystem interactions, and H_{A0} and H_{AI} do not need to commute.

Local-equilibrium states. We employ the concept of the local-equilibrium (LE) states to characterize the initial hotness/coldness of the systems, in which “temperatures” are locally defined parameters. For the Hamiltonian in Eq. (1), a LE state ρ requires that the reduced density matrices of the subsystems $A_{k=1,2,\dots,N}$ and B take the form of Gibbs states [see Fig. 1(a)],

$$\rho_{A_k} = e^{-\beta_{A_k} H_{A_k}} / \mathcal{Z}_{A_k}, \quad \rho_B = e^{-\beta_B H_B} / \mathcal{Z}_B, \quad (3)$$

where $1/\beta_u \equiv T_u$ and $\mathcal{Z}_u = \text{tr}[\exp(-\beta_u H_u)]$ are the effective temperature and the partition function of the subsystem $u \in \{A_{1,2,\dots,N}, B\}$, respectively, and we have taken the Boltzmann constant $k_B = 1$. We denote the final state of the subsystem

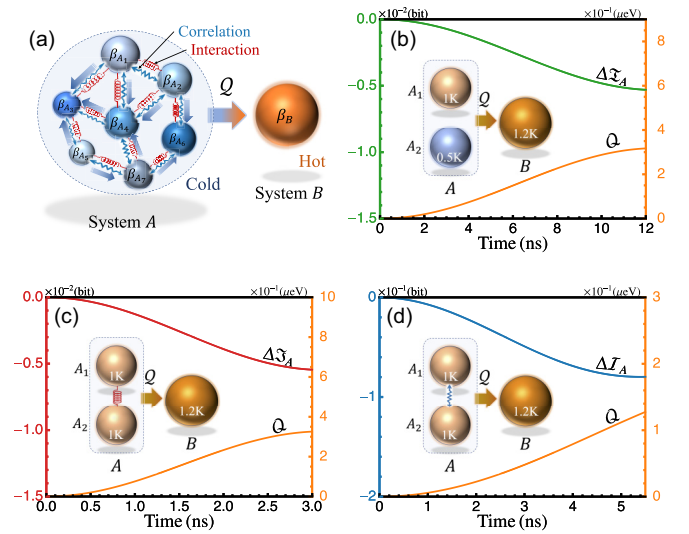


FIG. 1. AHT between two quantum systems that are initially in local equilibrium. (a) Schematic diagram for the Hamiltonian in Eq. (1) and the initial LE state in Eq. (3). The initial nonequilibrium of the system A is represented by temperature inhomogeneity ($\{\beta_{A_k}\}$), interactions (strings), and correlations (arrowed zigzags). We assume that A and B systems are the cold and the hot ones, respectively. (b)–(d) AHT in a three-qubit system. (b) Temperature-inhomogeneity-dominated AHT. We take the initial temperatures $T_{A1} = 1$ K, $T_{A2} = 0.5$ K, and $T_B = 1.2$ K, and the qubit frequencies $\omega_{A2} = \omega_B = \omega_{A1}/2 = \omega_0$ with $\omega_0/2\pi \equiv 4.0$ GHz. (c) Interaction-dominated AHT. (d) Correlation-dominated AHT. In (c) and (d), we take $T_{A1} = T_{A2} = 1$ K, $T_B = 1.2$ K, and $\omega_{A1} = \omega_{A2} = \omega_B = \omega_0$.

u by ρ'_u . In practice, the LE states can be prepared by thermalizing the subsystems with independent baths. For later discussion, it is convenient to introduce the product states

$$\rho_{A,0} \equiv \otimes_{k=1}^N \rho_{A_k}, \quad \rho_0 \equiv \rho_{A,0} \otimes \rho_B. \quad (4)$$

The initial LE state ρ is unnecessarily the product state ρ_0 , which releases a crucial assumption for deducing entropy production [38] and Landauer’s principle [39]. Without loss of generality, we assume that A and B systems are the cold and the hot ones, respectively; that is, $\beta_A > \beta_B$ where $\beta_A \equiv \min\{\beta_{A_k}\}_{k=1}^N$.

Classification of AHT mechanisms. We are able to express the heat transfer $\mathcal{Q}$ in terms of quantum relative entropy (QRE) and obtain the heat transfer equation

$$(\beta_B - \beta_A)\mathcal{Q} = \Delta\mathcal{S} + \Delta\mathcal{I}_{AB} + \Delta\mathcal{I}_A + \Delta\mathcal{I}_B. \quad (5)$$

The derivation of Eq. (5) and the connection between Eq. (2) and Eq. (5) are presented in the SM [37]. In Eq. (5), the left-hand side describes the effective entropy flux flowing from A to B , and $(\beta_B - \beta_A)\mathcal{Q} < 0$ defines the AHT regime. The right-hand side characterizes various origins of the entropy flux: (I) $\Delta\mathcal{S} \equiv \sum_{k=1}^N S(\rho'_{A_k} || \rho_{A_k}) + S(\rho'_B || \rho_B)$, in which each term describes the QRE between the final and initial states of an individual subsystem. Here $S(\varrho || \varsigma) \equiv -\text{tr}(\varrho \ln \varsigma) - S(\varrho)$ is the QRE between states ϱ and ς , with $S(\varrho) \equiv -\text{tr}(\varrho \ln \varrho)$ the von Neumann entropy of the state ϱ [40]. (II) $\Delta\mathcal{I}_{AB} \equiv \mathcal{I}_{AB}(\rho') - \mathcal{I}_{AB}(\rho)$, in which $\mathcal{I}_{AB}(\rho) \equiv S(\rho || \rho_0)$ is the multipartite mutual information (MI) among all the subsystems

[41,42], describes the variation of the subsystem correlation. (III) $\Delta\mathcal{T}_A \equiv \sum_{k=1}^N (\beta_A - \beta_{A_k}) \Delta\langle H_{A_k} \rangle$ describes the entropy flux induced by the initial temperature inhomogeneity within the system A . (IV) $\Delta\mathcal{J}_A \equiv \beta_A \Delta\langle H_{AI} \rangle$ characterizes the entropy flux generated by the interaction energy variation of the system A . We note that $S(\rho'_{A_k,B} || \rho_{A_k,B}) \geq 0$ by definition [40], while the signs of $\Delta\mathcal{I}_{AB}$, $\Delta\mathcal{T}_A$, and $\Delta\mathcal{J}_A$ are conditional. For example, $\Delta\mathcal{I}_{AB} < 0$ requires initial-state correlations. In addition, we point out Eq. (5) can be extended to the situations where the HTC is broken (refer to the SM [37] for detailed discussion).

An immediate observation for Eq. (5) is that the heat transfer is normal, i.e., $(\beta_B - \beta_A)Q \geq 0$, and the Clausius statement of the second law is recovered, when A is interaction-free ($\Delta\mathcal{J}_A = 0$), and, initially, A is temperature-homogenous ($\Delta\mathcal{T}_A = 0$) and quantum correlation is absent ($\Delta\mathcal{I}_{AB} = 0$).

Most importantly, Eq. (5) implies that AHT occurs only if $\Delta\mathcal{I}_{AB} + \Delta\mathcal{T}_A + \Delta\mathcal{J}_A < 0$ and at least one of three mechanisms induces negative entropy fluxes: (I) *Initial-state correlations* giving $\Delta\mathcal{I}_{AB} < 0$. Since the evolution of the joint system is unitary, the MI variation $\Delta\mathcal{I}_{AB}$ can be further decomposed into the intra- and intersystem components: $\Delta\mathcal{I}_{AB} = \Delta\mathcal{I}_A + \Delta\mathcal{I}_{A:B}$. Here $\Delta\mathcal{I}_A \equiv \mathcal{I}_A(\rho'_A) - \mathcal{I}_A(\rho_A)$, with $\mathcal{I}_A(\rho_A) \equiv S(\rho_A || \rho_{A,0})$ the multipartite MI within A , gives the MI variation in A , and $\Delta\mathcal{I}_{A:B} \equiv \mathcal{I}_{A:B}(\rho') - \mathcal{I}_{A:B}(\rho)$, with $\mathcal{I}_{A:B}(\rho) \equiv S(\rho_A) + S(\rho_B) - S(\rho)$ the bipartite MI [40], captures the variation of the total correlation between A and B [17]. (II) *Temperature inhomogeneity* giving $\Delta\mathcal{T}_A < 0$. (III) *Intrasystem interactions* giving $\Delta\mathcal{J}_A < 0$. We note that $\Delta\mathcal{I}_{AB} + \Delta\mathcal{T}_A + \Delta\mathcal{J}_A < 0$ is insufficient for AHT. For example, even if $\Delta\mathcal{I}_{A:B} < 0$, the entropy production $\Delta\mathcal{I}_{A:B} + S(\rho'_B || \rho_B)$ can still be positive due to large $S(\rho'_B || \rho_B)$, which likely produces a normal heat flow [38,43,44].

The AHT dominated by only one of the mechanisms, e.g., $\Delta\mathcal{I}_{AB} < 0$ and all the other entropy flux terms being non-negative, is of particular interest. The AHT induced by consuming the initial intersystem MI, i.e., $\Delta\mathcal{I}_{A:B} < 0$, has been widely discussed in previous studies [5,6,8,21–23,45]; consuming the intrasystem correlation, i.e., $\Delta\mathcal{I}_A < 0$, can also give rise to AHT [46]. In this work, we give exhaustive examples of single-mechanism-dominated AHT in qubit systems.

Heat transfer in multiqubit systems. We develop heat transfer models in multiqubit systems under constraint of the HTC, and identify the AHT mechanisms in these models. Consider the system $X \in \{A, B\}$ is composed of N_X qubits. The interaction-free Hamiltonian reads

$$H_{X0} = \sum_{i=1}^{N_X} H_{X_i}, \quad H_{X_i} = -\frac{1}{2} \omega_{X_i} \sigma_{X_i}^z, \quad (6)$$

where $\omega_{X_i} \geq 0$ and $\sigma_{X_i}^{x,y,z}$ are the frequency and the Pauli operators of the qubit X_i , respectively. A qubit model with arbitrary Zeeman fields is identical to that in Eq. (6) up to local qubit rotations. The interaction Hamiltonian between A and B takes the general form

$$H_I = H_{AB}^I + H_{AAB}^I + H_{ABB}^I + \dots, \quad (7)$$

where H_{AB}^I , $H_{AAB(ABB)}^I$, and $\dots$ denote two-body (2B), three-body (3B), and higher-order many-body interactions,

respectively. Here we focus on 2B and 3B interactions. The detailed derivation of the Hamiltonians is presented in the SM [37].

(I) *2B intersystem interactions.* Specifying the intersystem interactions of the 2B form $H_{AB}^I = \sum_{k,l} \sum_{a,b} c_{kl}^{ab} \sigma_{A_k}^a \sigma_{B_l}^b$ and the intrasystem interactions of the general multibody form $H_{XI} = \sum_{i,j,\dots,k} \sum_{a,b,\dots,c} J_{X,ij\dots k}^{ab\dots c} \sigma_{X_i}^a \sigma_{X_j}^b \dots \sigma_{X_k}^c$, we find that the HTC leads to two independent commutation relations $[H_{A0} + H_{B0}, H_{AB}^I] = 0$ and $[H_{XI}, H_{AB}^I] = 0$. The former relation reduces the intersystem interactions to

$$H_{AB}^I = \sum_{k,l} (c_{A_k B_l} \sigma_{A_k}^- \sigma_{B_l}^+ + \text{H.c.}), \quad (8)$$

where $\sigma_{X_i}^\pm \equiv (\sigma_{X_i}^x \pm i\sigma_{X_i}^y)/2$ and the coupling constant $c_{A_k B_l}$ is nonzero only if the two qubits A_k and B_l are of equal frequency $\omega_{A_k} = \omega_{B_l}$. The latter commutation relation should lead to constraints on H_{AB}^I in Eq. (8) and H_{XI} . However, we neglect H_{XI} since we find it does not contribute to the heat transfer [$\Delta\mathcal{J}_A = 0$ in Eq. (5)]. Deriving the instantaneous heat transfer equations for the model defined by Eqs. (6) and (8), we obtain a no-go theorem:

Theorem 1. In the presence of only 2B intersystem interactions, instantaneous AHT is impossible if all the qubit triplets $X_m X_k (\neq m) Y_l$, where $(X, Y) \in \{(A, B), (B, A)\}$, have no initial quantum coherence [47].

The proof of Theorem 1 can be found in the SM [37]. We formally decompose the initial state ρ into three components, $\rho = \otimes_{X,k} \rho_{X_k} + \mathcal{D} + \chi$, where ρ_{X_k} is the Gibbs state of the qubit X_k as defined in Eq. (3), and $\mathcal{D}$ (diagonal) and χ (with diagonal entries all zero) encode the classical correlation and quantum coherence among the qubits, respectively. Theorem 1 implies that, if intersystem interactions are limited to the 2B type, the initial classical correlation $\mathcal{D}$ alone ($\chi = 0$) cannot induce AHT. Furthermore, not any initial quantum coherence can induce AHT. For example, an initial state with the coherence component $\chi = (\otimes_{X,k} \sigma_{X_k}^+ + \text{H.c.})$ does not exhibit AHT. Lastly, Theorem 1 imposes constraints on the entropy flux terms in Eq. (5). If there is no initial coherence in all the triplets, we have $\Delta\mathcal{T}_A < 0$ (any of subsystems of A gaining energy) and $\Delta\mathcal{T}_A > -\Delta S - \Delta\mathcal{I}_{AB}$ (B losing energy), which gives a lower bound for the MI, $\Delta\mathcal{I}_{AB} > -\Delta S$.

In general, the quantum coherence among a triplet $X_m X_k Y_l$ can be decomposed into doublet components and triplet collective components nonexistent in any doublet [48,49]. In correspondence, we find that the intersystem coherence in doublets $X_m Y_l$ and $X_k Y_l$ only contributes to the heat transfer rate dQ/dt , while the intrasystem coherence in doublet $X_m X_k$ as well as the triplet collective coherence contribute to the instantaneous heat transfer convexity d^2Q/dt^2 (see the SM [37]). The AHT induced by intersystem doublet coherence has been realized in nuclear spin systems [22]. When this intersystem doublet coherence vanishes ($dQ/dt = 0$), the intrasystem doublet coherence can also induce AHT ($d^2Q/dt^2 > 0$), which might be interesting for experimental verification.

(II) *3B intersystem interactions.* We further investigate the qubit systems in which the intersystem interactions are of the 3B form $H_I = H_{AAB}^I + H_{ABB}^I$ and the intrasystem interactions are of 2B form. The HTC leads to three

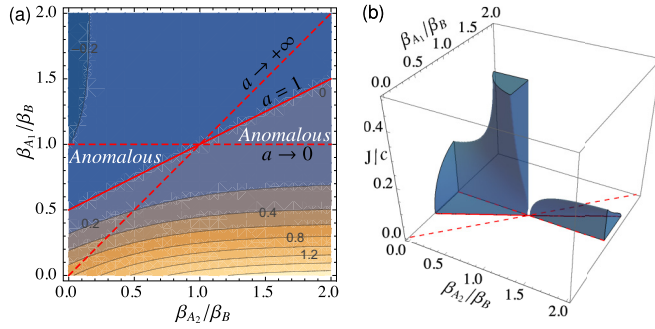


FIG. 2. (a) Contour plot for the heat transfer convexity at the initial time [Eq. (10)]. The AHT region lies between the red solid line ($a = 1$) and the red dashed line ($a \rightarrow 0$). (b) AHT region (blue) when the 3B interaction (9) is perturbed by a 2B interaction with strength J . Panel (a) corresponds to $J/c = 0$.

independent commutation relations $[H_{A0} + H_{B0}, H_{AAB}^I] = [H_{A0} + H_{B0}, H_{ABB}^I] = [H_{AI} + H_{BI}, H_I] = 0$, which impose constraints on the interaction parameters.

The first commutator leads to three types of interactions depending on the frequencies of the qubits involved. For any three qubits $A_{k,l}$ and B_m , the interaction denoted by H_{A_k,A_l,B_m}^I takes the forms as follows: For $\omega_u = \omega_v + \omega_w$ or $\omega_v = \omega_w$, where $(u, v, w) \in \{(A_k, A_l, B_m), (A_l, A_k, B_m), (B_m, A_k, A_l)\}$, we obtain $H_{A_k,A_l,B_m}^I = c_{uvw} \sigma_u^+ \sigma_v^- \sigma_w^- + \text{H.c.}$ or $H_{A_k,A_l,B_m}^I = c'_{uvw} \sigma_u^z \sigma_v^- \sigma_w^+ + \text{H.c.}$, respectively, where c_{uvw} and c'_{uvw} are arbitrary complex constants. For unconstrained frequencies, we obtain $H_{A_k,A_l,B_m}^I = c''_{klm} \sigma_k^z \sigma_l^z \sigma_{B_m}^z$, where c''_{klm} is an arbitrary real constant. The second commutator gives H_{AAB}^I of a similar form as H_{ABB}^I . The third commutator introduces further constraints to interaction parameters. We note that, unlike the 2B interaction case, the intersystem interaction terms $\sigma_{A_k}^z \sigma_{A_l}^z \sigma_{B_m}^z$ as well as the intrasystem interactions H_{XI} cannot be omitted since they can contribute to heat transfer. We note that, exploiting 3B intersystem interactions, all three types of AHT can be realized. In particular, AHT may occur even if the initial state is the product state ρ_0 in Eq. (4).

AHT in three-qubit systems. We study a three-qubit system $\{A_{1,2}, B\}$ with a combination of 2B and 3B intersystem interactions. The derivation of the Hamiltonians and designing details of the AHT mechanism can be found in the SM [37].

(I) In Fig. 1(b), we show an example of temperature-inhomogeneity-dominant AHT. We take $H_{AI} = 0$ and the intersystem 3B interaction takes the simple expression

$$H_{A_1A_2B}^I = c \sigma_{A_1}^+ \sigma_{A_2}^- \sigma_B^- + \text{H.c.}, \quad (9)$$

where c is the coupling strength. We choose the initial state as the product state ρ_0 in Eq. (4). Therefore, the MI variation $\Delta\mathcal{I}_{AB} \geq 0$, and the intrasystem interaction contribution $\Delta\mathcal{I}_A = 0$. According to Eq. (5), when $-\Delta\mathcal{T}_A > \Delta\mathcal{S} + \Delta\mathcal{I}_{AB}$, AHT occurs.

For this model the initial heat transfer rate vanishes: $dQ/dt = 0$. Phase transition between AHT and normal heat transfer is shown in Fig. 2(a), which we can obtain by calculating the initial heat transfer convexity,

$$\frac{d^2 Q}{dt^2} = \frac{2|c|^2 \omega_B [e^{(\beta_{A_2} \omega_{A_2} + \beta_B \omega_B)} - e^{\beta_{A_1} \omega_{A_1}}]}{(1 + e^{\beta_{A_1} \omega_{A_1}})(1 + e^{\beta_{A_2} \omega_{A_2}})(1 + e^{\beta_B \omega_B})}, \quad (10)$$

and further parametrizing the qubit frequencies as $\omega_{A_1} = (1 + a)\omega_B$ and $\omega_{A_2} = a\omega_B$ with $a \in (0, +\infty)$ (see the SM [37]). The phase boundary is defined by $d^2 Q/dt^2 = 0$, which gives $(1 + a)\beta_{A_1}/\beta_B = 1 + a\beta_{A_2}/\beta_B$ [the red lines in Fig. 2(a)]. As a varies from 0 to ∞ , the slope of the phase boundary varies from 0 to 1 correspondingly. In the top-right region where $\beta_{A_2} > \beta_{A_1} > \beta_B$, for a fixed β_{A_1} , $\Delta\mathcal{T}_A$ becomes negative enough, such that $-\Delta\mathcal{T}_A > \Delta\mathcal{S} + \Delta\mathcal{I}_{AB} (> 0)$, to realize AHT when β_{A_2} is large enough to cross the phase boundary.

As the 3B interaction (9) is perturbed by weak 2B interactions, which breaks the HTC, AHT is robust. For example, we take the 2B perturbation $H_b^I = J \sum_{u,v} \sum_a \sigma_u^a \sigma_v^a$ with a uniform coupling constant J . It can be proven that H_b^I gives no correction to the initial heat transfer rate, while it gives a correction of the order of J^2 to the convexity (see the SM [37]). As shown in Fig. 2(b), when $J/c = 0$, we recover the phase diagram of the unperturbed system [Fig. 2(a)]. As J/c increases, the region of AHT shrinks but remains finite.

(II) In Fig. 1(c), we show an example of intrasystem-interaction-dominated AHT. We take the intersystem interaction $H_I = 0.005\omega_0[(2i\sigma_{A_1}^- \sigma_{A_2}^+ + \text{H.c.})\sigma_B^y + (\sigma_{A_2}^z - \sigma_{A_1}^z)\sigma_B^x]$ being a combination of 2B and 3B types, and the intrasystem interaction $H_{AI} = 0.5\omega_0(\sigma_{A_1}^- \sigma_{A_2}^+ + \text{H.c.})$. We choose the product state ρ_0 in Eq. (4) as the initial state with the qubits $A_{1,2}$ being of the same temperature $\beta_A = \beta_{A_{1,2}}$. Hence $\Delta\mathcal{I}_{AB} \geq 0$ and $\Delta\mathcal{T}_A = 0$ during the evolution. We observe that $-\Delta\mathcal{T}_A > \Delta\mathcal{S} + \Delta\mathcal{I}_{AB}$ can induce AHT.

(III) In Fig. 1(d), we show an example of intrasystem-correlation-dominated AHT. The intrasystem interaction is set to be zero: $H_{AI} = 0$. We prepare the initial state $\rho = (\rho_{A_1} \otimes \rho_{A_2} + \chi_A) \otimes \rho_B$, where the coherence term $\chi_A = 0.24(\sigma_{A_1}^+ \otimes \sigma_{A_2}^- + \text{H.c.})$ encodes quantum correlation within A and $A_{1,2}$ share the same temperature $\beta_A = \beta_{A_{1,2}}$. The joint system evolves under the 2B interaction $H_I = 0.005\omega_0(\sigma_{A_1}^- \sigma_B^+ + \sigma_{A_2}^- \sigma_B^+ + \text{H.c.})$. Obviously, we have $\Delta\mathcal{T}_A = \Delta\mathcal{I}_A = 0$ and $\Delta\mathcal{I}_{A:B} \geq 0$ during the evolution. Hence AHT can only be realized by consuming the correlation between the qubits A_1 and A_2 , such that $-\Delta\mathcal{T}_A > \Delta\mathcal{S} + \Delta\mathcal{I}_{A:B}$, which is consistent with Theorem 1. This mechanism has been discussed in Ref. [46].

Discussion. We have shown that both intrasystem temperature inhomogeneity and intrasystem interactions are able to induce AHT in quantum systems, aside from initial inter- and intrasystem correlation. For qubit systems in which the interactions are limited to the 2B type, we have proven that AHT occurs only if the initial states contain quantum coherence. Realizing AHT via the other mechanisms requires 3B interactions, which are usually weak and overwhelmed by 2B interactions in nature. However, 3B and multibody interactions have been simulated in the laboratory with various quantum techniques from trapped ions to superconducting qubits [50–54]. It would be interesting to simulate the AHT phenomenon dominated by the mechanisms beyond the initial-state correlation.

A quantum system exhibiting AHT is a potential platform for engineering heat pumps, quantum absorption refrigerators, and quantum autonomous thermal machines [55–59]. A qubit heat pump driven entirely by quantum correlation as fuel has been proposed theoretically [45], and quantum

refrigerators utilizing three thermal baths have been demonstrated [60,61]. Moreover, thermodynamic computing using autonomous quantum thermal machines, where different operation regimes are tuned by temperature difference, has been proposed [62]. Our AHT classification offers an atlas for searching the heat engine cycles and optimizing the efficiency performance, in particular, by exploring the intrasystem degrees of freedom.

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