

Engineering interactions by collective coupling of atom pairs to cavity photons for entanglement generation

Sankalp Sharma¹, Jan Chwedeńczuk², and Tomasz Wasak¹

¹*Institute of Physics, Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University in Toruń, Grudziądzka 5, 87-100 Toruń, Poland*

²*Faculty of Physics, University of Warsaw, ul. Pasteura 5, PL-02-093 Warszawa, Poland*



(Received 4 July 2024; revised 6 November 2024; accepted 22 January 2025; published 20 February 2025)

Engineering atom-atom interactions is essential both for controlling novel phases of matter and for efficient preparation of many-body entangled states, which are key resources in quantum communication, computation, and metrology. In this work, we propose a scheme to tailor these interactions by coupling driven atom pairs to optical cavity photons via a molecular state in the dispersive regime, resulting in an effective photon-field-dependent potential. As an illustrative example, by analyzing the quantum Fisher information, we show that such induced interactions can generate robust many-body entanglement in two-mode ultracold bosons in an optical cavity. By tuning the photon-induced interactions through the cavity drive, we identify conditions for preparing highly entangled states on timescales that mitigate decoherence due to photon loss. Our results show that entanglement formation rate scales strongly with both photon and atom number, dramatically reducing the timescale compared to bare atomic interactions. We also identify an optimal measurement for exploiting the metrological potential of the atomic state in an interferometric protocol with significant photon losses, saturating the quantum Cramer-Rao lower bound. Furthermore, we show that despite these losses the atomic state exhibits strong Bell correlations. Our results pave the way for engineering atom-atom interactions to study novel phases of light and matter in hybrid atom-photon systems, as well as for tailoring complex quantum states for new quantum technology protocols and fundamental tests of quantum mechanics.

DOI: [10.1103/PhysRevResearch.7.L012038](https://doi.org/10.1103/PhysRevResearch.7.L012038)

Introduction. Scalable many-body entangled states are essential resources for quantum technologies of the future, such as quantum communication, computation, and metrology. The generation of these states relies on precisely controlled interactions between the particles. They should enable fast creation of quantum correlations in order to mitigate the effects of decoherence, which is a fundamental obstacle even in most advanced equipments operating in well-isolated environments. In degenerate quantum gases, for instance, the formation of entanglement can be significantly accelerated by exploiting Feshbach resonances which increase the rate of two-body collisions [1–3]. This enhancement, however, comes at a cost—the magnetic field fluctuates, and, as the collisions intensify, the three-body losses come into play [4–6].

A promising approach for generating entangled states with tailored properties is based on cavity QED with ultracold atoms. This platform is particularly advantageous because it enables independent control over the light and matter components of the hybrid system. Such control allows for a strong coupling regime of the atom-photon interaction and the engineering of entangled states with specific desired properties [7]. Moreover, cavity QED systems can be probed in real time via

transmission spectra [8]. They are also a basis for the development of new nonequilibrium dissipation-controlled quantum dynamics protocols [9]. Finally, they enable quantum simulations of solid-state Hamiltonians and nonequilibrium effects in complex systems beyond the schemes known from condensed matter [10]. These unique possibilities critically depend on tunable-range photon-mediated atom-atom interactions [11–15]. The quantum dynamics of self-organization of atoms in an optical cavity was shown to create strong atom-photon entanglement that is, however, fragile to photon losses [16,17]. Recently, such photon-induced interactions in a cavity have been used experimentally to generate spin- and momentum-correlated atom pairs in a Bose gas [18]. Also, photon-atom scattering in a pumped ring cavity, causing the transverse self-organization of bosons, led to momentum multiparticle entangled Dicke-squeezed states [19].

Despite the versatility of the effects induced by the cavity, such as the generation of novel phases of matter [20–25], the photon-matter interaction has so far been limited to the dipole coupling of atoms, in which a single photon interacts with a single atom from a many-body system. However, a recent experiment reported the observation of universal pair-polaritons in a strongly interacting two-component Fermi gas [26]. Crucially, in this setup, the photons from a high-finesse optical cavity were directly coupled to pairs of atoms via a molecular state, which is in contrast to the regular scheme of dipole coupling where a single atom is excited by a single photon. This mechanism enabled a real-time weakly

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

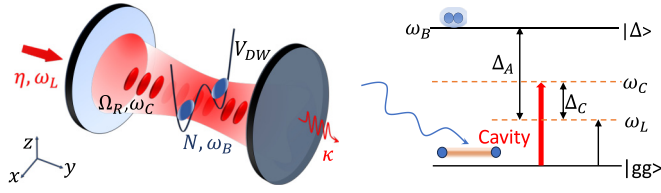


FIG. 1. Left: Illustration of the setup. N ultracold atoms with bosonic statistics are confined in a double-well potential V_{DW} inside an optical cavity. The laser pump η with frequency ω_L enters from one of the mirrors of the cavity with photon loss rate κ . The coupling between the atomic cloud and the cavity field is captured by the single-mode coupling strength Ω_R . Right: The diagram of the relevant energy levels of atomic pairs $|gg\rangle$ and the molecule $|\Delta\rangle$. By Δ_A and Δ_C we denote detunings from the molecular state with energy ω_B and cavity photon energy ω_C , respectively.

destructive probe of the pair correlation function of the atoms, and it holds promise for the development of novel methods to engineer atomic interaction potentials.

Here, we exploit this light-matter coupling to derive the form of the photon-induced atom-atom interactions under different system driving schemes. Based on this approach, we propose a method for the fast generation of many-body entangled states on a timescale controlled by the strength of the drive. The setup consists of ultracold bosons trapped in a double-well potential immersed in an optical cavity. Our results show that photon-induced atom-atom interactions lead to the generation of highly entangled atomic states which may trigger the development of new protocols and applications in quantum technologies [10,18,19,27,28].

Photon-atom pair interaction in a cavity. Our system of interest is a collection of N atoms of mass m held in a trap $V(x)$ in an optical cavity (see Fig. 1 with an example of a double-well potential). The atoms are in the ground state interacting through a contact potential of strength $g_0 = 4\pi a/m$, where a is the s -wave scattering length (we use $\hbar = 1$ units). We assume that the pairs of atoms have an optically accessible molecular state with energy ω_B [26].

The total Hamiltonian is $\hat{H} = \hat{H}_A + \hat{H}_\Delta + \hat{H}_C + \hat{H}_{CA} + \hat{H}_P$, where the terms describe ground-state atoms, molecules, cavity photons, light-matter interactions, and external pump, respectively [29]. The atoms are governed by

$$\hat{H}_A = \int d^3r \hat{\Psi}^\dagger \left[-\frac{\nabla^2}{2m} + V \right] \hat{\Psi} + \frac{g_0}{2} \int d^3r \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi}, \quad (1)$$

where $\hat{\Psi}(\mathbf{r})$ is the ground-state atom field operator for bosons [10,15], i.e., its commutator satisfies $[\hat{\Psi}(\mathbf{r}), \hat{\Psi}(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')$, with $\delta(\mathbf{r})$ the Dirac delta, and $V(\mathbf{r})$ is the trapping potential. For the molecular state of the pairs of atoms, we assume that they are tightly confined and effectively described by

$$\hat{H}_\Delta = \int d^3r \hat{\Psi}_\Delta^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2M} + V(\mathbf{r}) + \omega_B \right] \hat{\Psi}_\Delta(\mathbf{r}), \quad (2)$$

where $M = 2m$ is the molecule mass.

The high- Q optical cavity with the axis along the y direction is characterized by the resonant frequency ω_C and the mode of light is governed by the Hamiltonian $\hat{H}_C = \omega_C \hat{a}^\dagger \hat{a}$,

where $\hat{a}$ is the annihilation operator of the cavity photon with a position-dependent mode function $f(\mathbf{r})$ [29]. We model the atom-light interaction by the Hamiltonian in the rotating wave approximation,

$$\hat{H}_{CA} = -i\Omega_R \int d^3r f(\mathbf{r}) [\hat{\Psi}_\Delta^\dagger(\mathbf{r}) \hat{\Psi}^2(\mathbf{r}) \hat{a} - \text{H.c.}], \quad (3)$$

where Ω_R sets the single photon–single pair coupling strength, see also the Supplemental Material (SM) [29], while H.c. is the Hermitian conjugate. $\hat{H}_{CA}$ describes a process (and its reverse), see Fig. 1, where a single photon annihilates a pair of ground-state atoms, leading to the formation of a single molecule. Such a pair-molecule coupling mechanism [30], although without light quantization, was discussed in Ref. [31] in the context of composite Fermi-Bose superfluids and pairing by pair exchange through a molecular condensate. For bosons, it has been used to study the quantum dynamics of the photoassociation of molecules in an optical cavity from a single-mode Bose-Einstein condensate [32]. Recently, Ref. [26] exploited this coupling to observe universal pair-polariton formation (i.e., independent of the specific molecular transition used) in a strongly interacting two-component Fermi gas.

Below, we assume the cavity is pumped by a laser with a frequency ω_L , detuned from the cavity frequency by $\Delta_C = \omega_L - \omega_C$, through one of its mirrors. Additionally, in analogy with the regular single photon–single atom case of the dipole coupling [10], the direct transversal pumping of the molecular transition can be employed. Thus, the pumped single mode of radiation inside the cavity is described by $\hat{H}_P = -i\eta e^{-i\omega_L t} \hat{a}^\dagger - i\Omega_R \int d^3r \hat{\Psi}_\Delta^\dagger \hat{\Psi}^2 \eta_t + \text{H.c.}$, where η denotes the driving strength through the mirrors and $\eta_t(\mathbf{r})$ is the strength of the transversal pump mode.

If the atomic detuning $\Delta_A = \omega_L - \omega_B$ is much larger than the other characteristic frequency scales of the system, the population of the molecular state is small and follows the ground state adiabatically, and thus can be eliminated. Then, the effective Hamiltonian of the system is given by $\hat{H}$, where $\hat{H}_\Delta + \hat{H}_{CA}$ is replaced by the photon-induced interaction

$$\hat{H}_{AA}^C = U_0 \int d^3r [f^*(\mathbf{r}) \hat{a}^\dagger + \eta_t] [f(\mathbf{r}) \hat{a} + \eta_t] \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi}, \quad (4)$$

where $U_0 = \Omega_R^2/\Delta_A$ is the strength of the atom-atom interaction per photon induced by cavity photons [29]. Unlike conventional single photon–single atom coupling, where cavity photons mediate long-range atomic interaction after adiabatic elimination in the dispersive regime [10–14], the interaction $\hat{H}_{AA}^C$ retains the operator and dynamical character of cavity photons and local position dependence via the mode function. This enables the generation of highly entangled states and complex quantum dynamics offering exciting opportunities for quantum technology applications [33].

Cavity-assisted bosonic Josephson junction. To illustrate how the interaction from Eq. (4) influences the buildup of atomic correlations, we consider a bosonic Josephson junction (BJJ) with N atoms, without transverse driving, i.e., $\eta_t = 0$. We expand the field operator $\hat{\Psi}(\mathbf{r})$ into two modes, $\hat{b}_1$ and $\hat{b}_2$, with associated spatial mode functions localized around the minima of the double-well potential, see Fig. 1. In the rotating frame of the laser, we derive an effective atom-photon

Hamiltonian $\hat{H}_{\text{eff}} = -\tilde{\Delta}_c \hat{a}^\dagger \hat{a} - J \hat{J}_x + U \hat{J}_z^2 + 2W_0 \hat{a}^\dagger \hat{a} \hat{J}_z^2$, where the last term originates from Eq. (4) and the detuning is renormalized as $\tilde{\Delta}_c = \Delta_c - W_0 N^2/2$ [29]; by $\hat{J}_i$, where $i = x, y, z$, we denote the Schwinger-boson representation of the modes $\hat{b}_{1/2}$ [29,34]. Here, J is the tunneling amplitude and U is the on-site energy, while W_0 determines the strength of the photon-induced atomic interactions [29]. Importantly, the cavity modifies the atom-atom interaction in contrast to the reported cavity-assisted tunneling in the regular single photon–single atom coupling [35].

Generation of many-body entanglement in BJJ. We now demonstrate that the effective Hamiltonian $\hat{H}_{\text{eff}}$ generates strong entanglement between atoms. To illustrate this, we consider a scenario analogous to the one-axis twisting (OAT) protocol, which has been successfully employed to create many-body entangled states in two-mode atomic configurations [36–39]. Here, all atoms are initially placed in a single mode, and then the bare Josephson oscillation (governed by tunneling and in the absence of other terms) creates a separable state in which all atoms are in coherent spin state [34] with a symmetric superposition of the two modes, namely, $|\psi_0\rangle \propto (\hat{b}_1^\dagger + \hat{b}_2^\dagger)^N |\text{vac}\rangle$.

Next, the Josephson term is suppressed by raising the interwell barrier (and thus setting $J = 0$) and the atomic interactions correlate the particles. Without the cavity, this procedure is the standard OAT, because the Hamiltonian $\propto \hat{J}_z^2$ reduces atomic fluctuations, creating spin squeezing [27,40,41].

The standard OAT process is governed by two timescales. First, after $t \propto N^{-2/3}$, the Gaussian spin-squeezed states are created which yield sub-shot-noise sensitivity [29], i.e., $\Delta\theta < 1/\sqrt{N}$, of phase estimation in the Mach-Zehnder interferometer (MZI) [36,42]. Later, when $t \propto N^{-1/2}$ the system enters a non-Gaussian regime [43–45], where the many-body entangled states have to be characterized by the quantum Fisher information (QFI) [46], i.e.,

$$F_Q = 2 \sum_{i,j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle \Psi_i | \hat{J}_y | \Psi_j \rangle|^2, \quad (5)$$

which sets the Cramer-Rao lower bound (CRLB): $\Delta\theta \geq F_Q^{-1/2}$. Here, $\hat{J}_y$ is the generator of the MZI transformation [34,36], and the sum runs over the spectrum of the atomic density matrix, i.e., $\hat{\rho}_A = \sum_i \lambda_i |\Psi_i\rangle \langle \Psi_i|$. In the non-Gaussian regime, very strongly entangled states manifest through the Heisenberg scaling $F_Q \propto N^2$ [47].

We now demonstrate that even in the presence of photon losses, the light-induced atom-atom interaction from Eq. (4) can drastically accelerate the generation of strong entanglement compared to the above-mentioned rates characterizing the OAT procedure. Initially, atoms are in a state $|\psi_0\rangle$ and the photonic vacuum is dynamically populated by $\hat{H}_P$ with $\eta > 0$. In the Supplemental Material we discuss the other case, when photons are initially in a coherent state and pumping is absent [29]. This scenario is similar to the usual optical Feshbach resonance in free space with a coherent beam [1], which can lead to dynamical instabilities of atoms [48].

For the lossless case, an analytical expression for the atomic density matrix at time t , after tracing out the photonic

degrees of freedom, reads

$$\hat{\rho}_A(t) = \sum_{m,m'} C_m C_{m'} e^{-iU(m^2 - m'^2)t} u_{mm'}(t) |m\rangle \langle m'|, \quad (6)$$

where the time-dependent function captures the effect of photon-induced interactions:

$$u_{mm'}(t) = e^{-\frac{1}{2}(|\gamma_m|^2 + |\gamma_{m'}|^2)t} e^{\gamma_m \gamma_{m'}^*} e^{-i\eta t(\beta_m - \beta_{m'})} \times e^{-i\beta_m^2 \sin(\omega_m t)} e^{i\beta_{m'}^2 \sin(\omega_{m'} t)} \quad (7)$$

with $\omega_m = -\Delta_c + 2W_0 m^2$, $\beta_m = \eta/\omega_m$, and $\gamma_m = \beta_m(e^{-i\omega_m t} - 1)$, see SM for a detailed derivation [29]. This result is subsequently employed to quantify the entanglement strength between the atoms.

Although the calculation of the QFI for mixed states is typically challenging or even intractable, we found an analytical expression in the large- N limit under the following assumptions [29]. Namely, if the bare atom interactions can be neglected, i.e., $W_0 \bar{n}(t) \gg U$, where $\bar{n}(t)$ is the average number of photons, then in the short-time regime, defined by $\sqrt{N}W_0 t \ll 1$, where atomic coherences from the interaction remain minimal, the condition $\bar{n} \gg 1$ yields the following QFI:

$$F_Q \simeq \frac{N}{2} [1 + f_Q(t)] + \frac{N^2}{2} [1 - f_Q(t)], \quad (8)$$

with the dynamical crossover function reading

$$f_Q(t) = \exp \left[-2 \left(\frac{t}{\tau_Q} \right)^6 \right], \quad \tau_Q = \left(\frac{9}{4NW_0^2 \eta^4} \right)^{\frac{1}{6}}. \quad (9)$$

The value of the QFI grows rapidly from the shot-noise limit, i.e., $F_Q = N$, at $t = 0$ to half of Heisenberg limit $F_Q = N^2/2$ on a time given by τ_Q . Crucially, the timescale τ_Q can be shortened by injecting photons into the cavity. Additionally, the effective coupling strength that determines τ_Q is proportional to $\sqrt{N}W_0$, a result of the collective coupling between atoms and photons.

In Fig. 2(b), we display Husimi distribution [49–52], which is a positive quasi-probability distribution used to represent quantum states in phase space [52]. Specifically, for a two-mode atomic density matrix $\hat{\rho}_A$ (see SM [29]), the Husimi Q function is $Q(\theta, \phi) = \langle \theta, \phi | \hat{\rho}_A | \theta, \phi \rangle$, where $|\theta, \phi\rangle$ is a spin-coherent state expressed in terms of the bosonic creation operators as $|\theta, \phi\rangle = \frac{1}{\sqrt{N!}} (\cos \frac{\theta}{2} \hat{b}_1^\dagger + \sin \frac{\theta}{2} e^{i\phi} \hat{b}_2^\dagger)^N |0\rangle$ [53].

The rapid cavity dynamics result in a non-Gaussian state distributed around the equator of the Bloch sphere. Figure 2(a) shows the QFI from Eq. (5) (black) compared with the approximate formula Eq. (8) (green) and juxtaposed with the QFI for the OAT, i.e., $W_0 = 0$. Here, we take $\eta = 320U$ so that at $t = 0.02U^{-1}$, the average number of photons is $\bar{n}(t) \approx 40$, hence the pumping is fast enough to compete with the timescale τ_Q of the entanglement buildup. We take $N = 100$ atoms and the other parameters are set to unity, i.e., $W_0/U = \Delta_c/U = 1$. Although in a recent experiment [26] W_0 was less than U , the observation of polaritons in this setup is an indication that $W_0 \sim U$ should be achievable [29].

The approximation works exceptionally well recovering the timescale of entanglement buildup and the saturation level. In the SM [29], we demonstrate similar agreement for

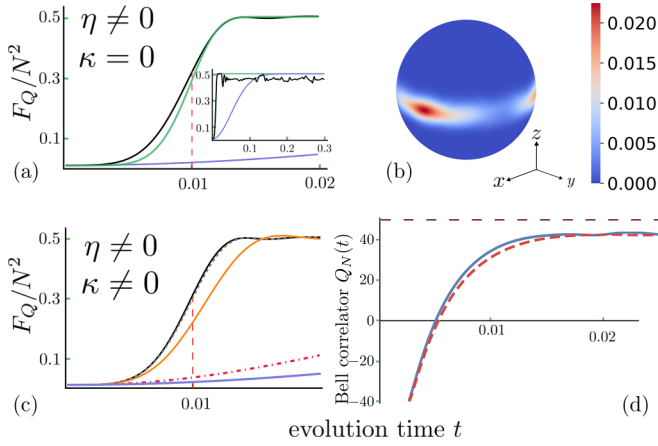


FIG. 2. (a), (c) QFI as a function of time (in units of U^{-1}), without (a) and with (c) photon losses. The black line shows F_Q for $\kappa = 0$ and the blue line for the OAT method. Inset shows long-time behavior. In (a), the solid green line is QFI from Eq. (8). In (c), QFI is for $\kappa\tau_Q = 0.1$ (dashed gray), 1 (orange), and 10 (dot-dashed red). The vertical dashed red line indicates $\tau_Q = 0.01U^{-1}$. (b) Husimi distribution at $t = 1/(4U\sqrt{N})$ shows a highly non-Gaussian state of atoms, $\kappa = 0$. Parameters for all panels: $U = W_0 = \Delta_c = 1$, $N = 100$ atoms, pump $\eta = 320$. (d) Bell correlator for $N = 50$ in the lossless case (blue solid line) and with losses $\kappa\tau_Q = 1$ (red dashed line). The black horizontal dashed line denotes the maximal possible value of $Q_N(t)$. All values of $Q_N(t) > 0$ indicate the presence of Bell correlations.

the coherent state case. The QFI shows a flat buildup, as the number of photons needs to accumulate to start driving the entanglement growth, manifesting in a large exponent in Eq. (9). Nevertheless, for a moderate number of photons, the improvement over the bare OAT procedure is impressive. The inset shows the long-time behavior. While the approximation, treating N as a continuous variable, cannot capture long-time oscillations (similar to the coherent-state approximation in the Jaynes-Cummings model [54]), it is sufficient to state that the photon coupling to atom pairs in the dispersive regime significantly enhances metrologically useful many-body entanglement.

Impact of photon losses. We now investigate the impact of photon losses on atomic entanglement. The QFI in Eq. (8) is given by the average number of photons, suggesting that on the timescale considered, F_Q is weakly sensitive to photonic decoherence.

We model the photon losses by the Lindblad master equation [55] governing the evolution of the system:

$$\partial_t \hat{\rho} = -i[\hat{H}_{\text{eff}}, \hat{\rho}] + \kappa(\hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\{\hat{\rho}, \hat{a}^\dagger\hat{a}\}), \quad (10)$$

where κ is the loss rate and $\{, \}$ stands for the anticommutator [53]. For the same initial state as in the lossless case, we now use $\kappa\tau_Q = 0.1, 1, 10$, so that losses span three orders of magnitude of the scale τ_Q that sets the entanglement buildup time. Figure 2(c) confirms that as long as $\kappa\tau_Q \lesssim 1$, F_Q remains roughly unaffected [29]. For the strong loss $\kappa\tau_Q \gg 1$, the initial growth slows down—the leakage of photons leaves the system in a bare OAT mode. Decoherence in the photonic subspace does not hinder the buildup of atomic entangle-

ment, as Heisenberg scaling of $F_Q \sim N^2$ is preserved even when $\kappa\tau_Q \sim 1$. However, it does influence the timescale over which entanglement is generated. Despite significant losses, the cavity-assisted OAT protocol remains effective in producing strong, genuine multiparticle entanglement. Moreover, after the entanglement is generated, the atomic state can be exploited for metrological purposes. Our calculations for $\kappa\tau_Q \sim 1$ show that F_Q remains large over a long timescale after the pump is switched off at $t \sim 2\tau_Q$, despite the rapid depletion of cavity photons [29].

Finally, we note that recent observations of pair polaritons in the strong coupling regime [26] for long observation times of the order of 0.5 s with frequent interrogations of the system with a probe beam indicate that, even in the dispersive regime considered in this work, where only virtual molecular transitions are involved, the effect of three-body losses can be neglected. Also, since the photons are far off-resonance from the molecular transition, the molecular population is significantly suppressed, minimizing the heating of the atomic gas due to spontaneous emission. We expect that photon loss from the cavity is the dominating source of noise, and we leave the impact of the heating for future research.

Optimal measurement. A physical measurement always yields classical Fisher information $F \leq F_Q$, where $F = \sum_m p_m(\theta) [\ln \partial_\theta p_m(\theta)]^2$, and the set of measurement operators $\{\hat{\Pi}_m\}$ defines the probabilities $p_m(\theta) = \text{Tr}[\hat{\rho}_A^\theta \hat{\Pi}_m]$ [33,44,46,56]. We find that the entanglement of $\hat{\rho}_A$ can be extracted by first rotating $\hat{\rho}_A$ by $\hat{V} = e^{i\vartheta \hat{J}_x}$ and then measuring population imbalance of the wells, see SM [29], i.e., the measurement is $\hat{\Pi}_m = \hat{V}|m\rangle\langle m|\hat{V}^\dagger$ with $\hat{J}_z|m\rangle = m|m\rangle$. Remarkably, selecting a single $\vartheta \approx \pi/2$ for all times, F is indistinguishable from F_Q in Fig. 2(a), even with losses $\kappa\tau_Q \sim 1$ [29]. The value of F can be extracted by the Hellinger distance method developed in Ref. [44].

Many-body Bell correlations. Finally, we demonstrate that, in addition to strong entanglement useful for metrology, the atoms exhibit many-body Bell correlations [57]. To this end, we use a Bell inequality developed in Refs. [58–60], namely, $Q_N(t) \leq 0$, where $Q_N(t) \equiv \ln[2^N \mathcal{E}_N(t)/(N!)^2]$, with $\mathcal{E}_N(t) \equiv |\text{Tr}[\hat{\rho}_A(t) \hat{J}_+^N]|^2$ and $\hat{J}_+ = \hat{J}_z + i\hat{J}_y$. Figure 2(d) shows that after the time $\sim \tau_Q/2$, $Q_N(t)$ becomes positive even in the presence of losses, signaling generation of Bell correlation. Moreover, for $t \gtrsim \tau_Q$, $Q_N(t) \sim N$, which indicates that $\sim N$ atoms become Bell correlated. Q_N can be experimentally extracted by measuring multiple quantum coherences technique [60–62]. Alternatively, since $\mathcal{E}_N$ probes a single element of $\hat{\rho}_A$ [58], it can be evaluated by quantum-state tomography [60].

Summary and conclusions. We have investigated a system's dynamics with interactions induced by coupling photons to pairs of atoms. Based on the example of the bosonic Josephson junction immersed into an optical cavity, in which photons are pumped through the cavity mirrors, we have shown how the strength of the atom-atom interaction, resulting from single photon–single pair coupling, can be controlled by injecting photons into the cavity. This allows us to fine-tune the time required for atoms to become entangled without having to rely on the potentially noisy magnetic fields that are typically used to control the strength of the atom-atom interaction. Our analytical results show that the strength of

entanglement, as measured by the quantum Fisher information, depends only on the intensity of the cavity light and not on the precise structure of the photonic state. The configuration is, therefore, robust to noise from photon losses. We analytically predicted the characteristic timescale for the generation of entanglement, which for realistic parameters can be orders of magnitude shorter than of bare atom-atom collisions. In turn, the short timescale implies that photon losses during the preparation of the atomic state can be overcome, since strong entanglement can be generated even with large losses. Additionally, we found the optimal measurement which saturates the quantum Cramer-Rao lower bound. Also, the atoms exhibit strong nonlocal many-body Bell correlations, which demonstrates that the photon-induced interaction can lead to a strongly entangled state that can serve for quantum technological purposes as well as for fundamental tests of quantum mechanics [58,63].

As a result, many-body entangled states of atoms, with their enormous potential for quantum applications, could be generated very quickly and reliably. In this way, one obstacle to the implementation of quantum solutions can be overcome. Our setup can be used in future quantum sensors or other devices where controllable and scalable many-body entanglement is required.

We have demonstrated that the photon-atom pair coupling adds an important ingredient to the techniques of engineering the interactions [10,64–66]. Due to the vanishing tunneling

rate, the dynamics in the protocol do not generate nonclassical light states, see SM [29]. However, since the photon-assisted atom-atom interaction depends in general on the quantum state of cavity photons, an intriguing direction for future research would be to explore the interplay of atomic and photonic nonclassicality, and how the nonclassical nature of photonic states could give rise to novel quantum phases of matter and light [67]. For instance, since the interaction is position dependent, it may lead to instabilities signaling novel types of self-ordering of atoms driven by correlations [10,48,68], such as supersolidity [69]. Finally, a promising research direction is to explore the potential for generating n -body photon-mediated interactions with $n > 2$ [70].

Acknowledgments. We are grateful to Jean-Philippe Brantut and Helmut Ritsch for their insightful comments on our work. This research is part of Project No. 2021/43/P/ST2/02911 cofunded by the National Science Centre and the European Union Framework Programme for Research and Innovation Horizon 2020 under the Marie Skłodowska-Curie Grant Agreement No. 945339. J.C. acknowledges the support of the National Science Centre, Poland, within the QuantERA II Programme that has received funding from the European Union's Horizon 2020 research and innovation programme under Grant Agreement No 101017733, Project No. 2021/03/Y/ST2/00195.

Data availability. The data presented in this article is available from [71].

-
- [1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Feshbach resonances in ultracold gases, *Rev. Mod. Phys.* **82**, 1225 (2010).
 - [2] S. Inouye, M. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, Observation of Feshbach resonances in a Bose-Einstein condensate, *Nature (London)* **392**, 151 (1998).
 - [3] E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Feshbach resonances in atomic Bose-Einstein condensates, *Phys. Rep.* **315**, 199 (1999).
 - [4] P. Massignan and H. T. C. Stoof, Efimov states near a Feshbach resonance, *Phys. Rev. A* **78**, 030701(R) (2008).
 - [5] D. Rätzel and R. Schützhold, Decay of quantum sensitivity due to three-body loss in Bose-Einstein condensates, *Phys. Rev. A* **103**, 063321 (2021).
 - [6] J. Stenger, S. Inouye, M. R. Andrews, H.-J. Miesner, D. Stamper-Kurn, and W. Ketterle, Strongly enhanced inelastic collisions in a Bose-Einstein condensate near Feshbach resonances, *Phys. Rev. Lett.* **82**, 2422 (1999).
 - [7] S. Haroche and J.-M. Raimond, *Exploring the Quantum: Atoms, Cavities, and Photons* (Oxford University Press, New York, 2006).
 - [8] I. B. Mekhov, C. Maschler, and H. Ritsch, Probing quantum phases of ultracold atoms in optical lattices by transmission spectra in cavity quantum electrodynamics, *Nat. Phys.* **3**, 319 (2007).
 - [9] N. Dogra, M. Landini, K. Kroeger, L. Hruby, T. Donner, and T. Esslinger, Dissipation-induced structural instability and chiral dynamics in a quantum gas, *Science* **366**, 1496 (2019).
 - [10] F. Mivehvar, F. Piazza, T. Donner, and H. Ritsch, Cavity QED with quantum gases: New paradigms in many-body physics, *Adv. Phys.* **70**, 1 (2021).
 - [11] P. Münstermann, T. Fischer, P. Maunz, P. W. H. Pinkse, and G. Rempe, Observation of cavity-mediated long-range light forces between strongly coupled atoms, *Phys. Rev. Lett.* **84**, 4068 (2000).
 - [12] V. D. Vaidya, Y. Guo, R. M. Kroeze, K. E. Ballantine, A. J. Kollár, J. Keeling, and B. L. Lev, Tunable-range, photon-mediated atomic interactions in multimode cavity QED, *Phys. Rev. X* **8**, 011002 (2018).
 - [13] M. A. Norcia, R. J. Lewis-Swan, J. R. Cline, B. Zhu, A. M. Rey, and J. K. Thompson, Cavity-mediated collective spin-exchange interactions in a strontium superradiant laser, *Science* **361**, 259 (2018).
 - [14] R. Mottl, F. Brennecke, K. Baumann, R. Landig, T. Donner, and T. Esslinger, Roton-type mode softening in a quantum gas with cavity-mediated long-range interactions, *Science* **336**, 1570 (2012).
 - [15] N. Masalaeva, H. Ritsch, and F. Mivehvar, Tuning photon-mediated interactions in a multimode cavity: From supersolid to insulating droplets hosting phononic excitations, *Phys. Rev. Lett.* **131**, 173401 (2023).
 - [16] C. Maschler, H. Ritsch, A. Vukics, and P. Domokos, Entanglement assisted fast reordering of atoms in an optical lattice within a cavity at $T = 0$, *Opt. Commun.* **273**, 446 (2007).
 - [17] A. Vukics, C. Maschler, and H. Ritsch, Microscopic physics of quantum self-organization of optical lattices in cavities, *New J. Phys.* **9**, 255 (2007).

- [18] F. Finger, R. Rosa-Medina, N. Reiter, P. Christodoulou, T. Donner, and T. Esslinger, Spin- and momentum-correlated atom pairs mediated by photon exchange and seeded by vacuum fluctuations, *Phys. Rev. Lett.* **132**, 093402 (2024).
- [19] I. Krešić, G. R. M. Robb, G.-L. Oppo, and T. Ackemann, Generating multiparticle entangled states by self-organization of driven ultracold atoms, *Phys. Rev. Lett.* **131**, 163602 (2023).
- [20] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Dicke quantum phase transition with a superfluid gas in an optical cavity, *Nature (London)* **464**, 1301 (2010).
- [21] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Cold atoms in cavity-generated dynamical optical potentials, *Rev. Mod. Phys.* **85**, 553 (2013).
- [22] J. Klinder, H. Keßler, M. R. Bakhtiari, M. Thorwart, and A. Hemmerich, Observation of a superradiant Mott insulator in the Dicke-Hubbard model, *Phys. Rev. Lett.* **115**, 230403 (2015).
- [23] R. Landig, L. Hruby, N. Dogra, M. Landini, R. Mottl, T. Donner, and T. Esslinger, Quantum phases from competing short-and long-range interactions in an optical lattice, *Nature (London)* **532**, 476 (2016).
- [24] J. Léonard, A. Morales, P. Zupancic, T. Esslinger, and T. Donner, Supersolid formation in a quantum gas breaking a continuous translational symmetry, *Nature (London)* **543**, 87 (2017).
- [25] J. Léonard, A. Morales, P. Zupancic, T. Donner, and T. Esslinger, Monitoring and manipulating Higgs and goldstone modes in a supersolid quantum gas, *Science* **358**, 1415 (2017).
- [26] H. Konishi, K. Roux, V. Helson, and J.-P. Brantut, Universal pair polaritons in a strongly interacting Fermi gas, *Nature (London)* **596**, 509 (2021).
- [27] K. Gietka, P. Szańkowski, T. Wasak, and J. Chwedeńczuk, Quantum-enhanced interferometry and the structure of twisted states, *Phys. Rev. A* **92**, 043622 (2015).
- [28] A. Niezgoda, J. Chwedeńczuk, T. Wasak, and F. Piazza, Cooperatively enhanced precision of hybrid light-matter sensors, *Phys. Rev. A* **104**, 023315 (2021).
- [29] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevResearch.7.L012038> for the detailed step-by-step derivation and discussion.
- [30] M. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Resonance superfluidity in a quantum degenerate Fermi gas, *Phys. Rev. Lett.* **87**, 120406 (2001).
- [31] E. Timmermans, K. Furuya, P. W. Milonni, and A. K. Kerman, Prospect of creating a composite Fermi-Bose superfluid, *Phys. Lett. A* **285**, 228 (2001).
- [32] C. P. Search, J. M. Campuzano, and M. Zivkovic, Quantum dynamics of cavity-assisted photoassociation of Bose-Einstein-condensed atoms, *Phys. Rev. A* **80**, 043619 (2009).
- [33] C. L. Degen, F. Reinhard, and P. Cappellaro, Quantum sensing, *Rev. Mod. Phys.* **89**, 035002 (2017).
- [34] C. Gross, Spin squeezing, entanglement and quantum metrology with Bose-Einstein condensates, *J. Phys. B: At. Mol. Opt. Phys.* **45**, 103001 (2012).
- [35] G. Szirmai, G. Mazzarella, and L. Salasnich, Tunneling dynamics of bosonic Josephson junctions assisted by a cavity field, *Phys. Rev. A* **91**, 023601 (2015).
- [36] J. Esteve, C. Gross, A. Weller, S. Giovanazzi, and M. Oberthaler, Squeezing and entanglement in a Bose-Einstein condensate, *Nature (London)* **455**, 1216 (2008).
- [37] C. Gross, T. Zibold, E. Nicklas, J. Esteve, and M. K. Oberthaler, Nonlinear atom interferometer surpasses classical precision limit, *Nature (London)* **464**, 1165 (2010).
- [38] M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and P. Treutlein, Atom-chip-based generation of entanglement for quantum metrology, *Nature (London)* **464**, 1170 (2010).
- [39] T. Berrada, S. van Frank, R. Bücker, T. Schumm, J.-F. Schaff, and J. Schmiedmayer, Integrated Mach-Zehnder interferometer for Bose-Einstein condensates, *Nat. Commun.* **4**, 2077 (2013).
- [40] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Squeezed atomic states and projection noise in spectroscopy, *Phys. Rev. A* **50**, 67 (1994).
- [41] A. Sørensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Many-particle entanglement with Bose-Einstein condensates, *Nature (London)* **409**, 63 (2001).
- [42] L. Pezzé and A. Smerzi, Phase sensitivity of a Mach-Zehnder interferometer, *Phys. Rev. A* **73**, 011801(R) (2006).
- [43] L. Pezzé and A. Smerzi, Entanglement, nonlinear dynamics, and the Heisenberg limit, *Phys. Rev. Lett.* **102**, 100401 (2009).
- [44] H. Strobel, W. Muessel, D. Linnemann, T. Zibold, D. B. Hume, L. Pezzé, A. Smerzi, and M. K. Oberthaler, Fisher information and entanglement of non-Gaussian spin states, *Science* **345**, 424 (2014).
- [45] L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Quantum metrology with nonclassical states of atomic ensembles, *Rev. Mod. Phys.* **90**, 035005 (2018).
- [46] S. L. Braunstein and C. M. Caves, Statistical distance and the geometry of quantum states, *Phys. Rev. Lett.* **72**, 3439 (1994).
- [47] P. Hyllus, W. Laskowski, R. Krischek, C. Schwemmer, W. Wieczorek, H. Weinfurter, L. Pezzé, and A. Smerzi, Fisher information and multiparticle entanglement, *Phys. Rev. A* **85**, 022321 (2012).
- [48] T. Wasak, V. V. Konotop, and M. Trippenbach, Atom laser based on four-wave mixing with Bose-Einstein condensates in nonlinear lattices, *Phys. Rev. A* **88**, 063626 (2013).
- [49] M. Jääskeläinen and P. Meystre, Coherence dynamics of two-mode condensates in asymmetric potentials, *Phys. Rev. A* **73**, 013602 (2006).
- [50] W. P. Schleich, *Quantum Optics in Phase Space* (John Wiley & Sons, New York, 2011).
- [51] B. Juliá-Díaz, T. Zibold, M. K. Oberthaler, M. Melé-Messeguer, J. Martorell, and A. Polls, Dynamic generation of spin-squeezed states in bosonic Josephson junctions, *Phys. Rev. A* **86**, 023615 (2012).
- [52] Y. L. Loh and M. Kim, Visualizing spin states using the spin coherent state representation, *Am. J. Phys.* **83**, 30 (2015).
- [53] J. R. Johansson, P. D. Nation, and F. Nori, QuTiP: An open-source Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **183**, 1760 (2012).
- [54] C. Gerry and P. Knight, *Introductory Quantum Optics* (Cambridge University Press, New York, 2004).
- [55] D. Manzano, A short introduction to the Lindblad master equation, *AIP Adv.* **10**, 025106 (2020).
- [56] P. Hyllus, L. Pezzé, and A. Smerzi, Entanglement and sensitivity in precision measurements with states of a fluctuating number of particles, *Phys. Rev. Lett.* **105**, 120501 (2010).
- [57] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, *Rev. Mod. Phys.* **86**, 419 (2014).

- [58] A. Niezgoda and J. Chwedeńczuk, Many-body nonlocality as a resource for quantum-enhanced metrology, *Phys. Rev. Lett.* **126**, 210506 (2021).
- [59] J. Chwedeńczuk, Many-body Bell inequalities for bosonic qubits, *SciPost Physics Core* **5**, 025 (2022).
- [60] M. Płodzień, T. Wasak, E. Witkowska, M. Lewenstein, and J. Chwedeńczuk, Generation of scalable many-body bell correlations in spin chains with short-range two-body interactions, *Phys. Rev. Res.* **6**, 023050 (2024).
- [61] M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet, *Nat. Phys.* **13**, 781 (2017).
- [62] M. Gärttner, P. Hauke, and A. M. Rey, Relating out-of-time-order correlations to entanglement via multiple-quantum coherences, *Phys. Rev. Lett.* **120**, 040402 (2018).
- [63] R. Schmied, J.-D. Bancal, B. Allard, M. Fadel, V. Scarani, P. Treutlein, and N. Sangouard, Bell correlations in a Bose-Einstein condensate, *Science* **352**, 441 (2016).
- [64] S. Zeytinoğlu, A. İmamoğlu, and S. Huber, Engineering matter interactions using squeezed vacuum, *Phys. Rev. X* **7**, 021041 (2017).
- [65] E. J. Davis, A. Periwal, E. S. Cooper, G. Bentsen, S. J. Evered, K. Van Kirk, and M. H. Schleier-Smith, Protecting spin coherence in a tunable Heisenberg model, *Phys. Rev. Lett.* **125**, 060402 (2020).
- [66] A. Periwal, E. S. Cooper, P. Kunkel, J. F. Wienand, E. J. Davis, and M. Schleier-Smith, Programmable interactions and emergent geometry in an array of atom clouds, *Nature (London)* **600**, 630 (2021).
- [67] O. Dmytruk and M. Schirò, Controlling topological phases of matter with quantum light, *Commun. Phys.* **5**, 271 (2022).
- [68] R. Yamazaki, S. Taie, S. Sugawa, and Y. Takahashi, Submicron spatial modulation of an interatomic interaction in a Bose-Einstein condensate, *Phys. Rev. Lett.* **105**, 050405 (2010).
- [69] A. Recati and S. Stringari, Supersolidity in ultracold dipolar gases, *Nat. Rev. Phys.* **5**, 735 (2023).
- [70] C. Luo, H. Zhang, C. Maruko, E. A. Bohr, A. Chu, A. M. Rey, and J. K. Thompson, Realization of three and four-body interactions between momentum states in a cavity through optical dressing, *arXiv:2410.12132*.
- [71] S. Sharma, J. Chwedeńczuk, and T. Wasak, Datasets for “Engineering interactions by collective coupling of atom pairs to cavity photons for entanglement generation”, RepOD 2025, <https://doi.org/10.18150/D0SEQ6>.