

Dopant pairing in a disordered magnetic spin ladder

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I demonstrate a pairing mechanism of dopants in a magnetic lattice, emerging from the underlying *high-temperature disorder* of the spins. The effect is demonstrated in a mixed-dimensional model, where dopants travel along a two-leg ladder, while the spins feature Ising interactions along the rungs and the legs of the ladder. The dynamics following the sudden immersion of two nearest neighbor dopants in an infinite temperature spin lattice shows that thermal spin disorder frustrates the relative motion of the dopants and enforces them to copropagate. The predictions are shown to be realistically testable in quantum simulation experiments.

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Understanding pairing of dopants in magnetic lattices is an immense challenge in modern quantum many-body theory. The main goal is to gain a deeper understanding of strongly correlated systems, arising, e.g., in the notoriously difficult high-temperature superconductors [1–3]. While solid state experiments continue to pursue such inquiries after decades of research [4], a highly interesting and promising pathway has appeared in quantum simulation experiments with ultracold atoms in optical lattices [5–12]. Single-site resolution capabilities [13,14] makes it possible to measure low-temperature spatial correlations [8–11], as well as track them over time by repeated realizations of the same initial state. In particular, single hole dynamics consistent with magnetic polaron formation [15] in quantum antiferromagnets has been measured [6]. Moreover, in such experiments one can tinker with the simulated models, otherwise fixed in the solid state. This has enabled the observation of dopant pairing in a mixed-dimensional magnetic lattice [12], also seen to be a plausible explanation [16] for high-temperature superconductivity in a specific nickelate compound [17]. More broadly speaking, the appearance of pairing is conventionally tied to the distortion of some underlying order, be it the underlying antiferromagnetic order in the above example [18,19], or how, in conventional superconductors, electrons distort the underlying crystal lattice, inducing an attraction between them [20–22]. In materials supporting high-temperature superconductivity, there may be an important counterexample to this generic scenario, in which preformed Cooper pairs [23,24] appearing above the superconducting phase transition might explain [25,26] the pseudogap phase [27].

In this Letter, I demonstrate a pairing mechanism that *does not* rely on being in nor in the vicinity of a phase transition to an ordered phase. In particular, the discovered mechanism

remains at *infinite* temperatures and is a direct consequence of the thermally disordered spin lattice. I analyze the effect in a simplistic setup of a mixed-dimensional model [Fig. 1(a)], where two holes are placed in two legs of a spin ladder. The spin degree of freedom is assumed to be at infinite temperature and completely disordered. Conventional wisdom, therefore, suggests that the holes should not be able to pair. Nevertheless, when the holes are initialized on the same rung in the infinite temperature spin lattice, the strong thermal disorder of the system frustrates the relative motion of the dopants. Indeed, tracking the hole-hole correlator in time [Fig. 1(b)] shows a significant correlation along the diagonal $x_1 = x_2$, where the holes are on the same rung of the ladder. One may equivalently observe that the interhole distance is always smaller than the traversed distance of their center of mass [Fig. 1(c)]. This results in a strictly *positive* correlation between the holes' positions, signaling an effective attraction between them [Fig. 1(d)]. Finally, we observe that the probability of finding the holes on the same rung is easily increased by a factor of 4, as compared to what one expects from their marginal distributions. See the relative hole-hole correlator in Fig. 1(e). This phenomenon arises due to an effective induced interaction that has a disordered character, encouraging the holes to copropagate, and is closely tied to the recent discovery of thermally induced localization in such systems [28,29]. In the extreme limit of $J_{\parallel} = 0$, the holes even dislodge from their original positions and spread *subdiffusively as a pair*. I emphasize that the discovered effect would appear in a wide range of systems, though the robustness of the pairing in more generic setups is presently unclear.

Model. I consider spins in a two-leg ladder with Ising interactions and hopping *along* the ladder,

$$\hat{H} = \sum_{\mathbf{i}, \mathbf{j}} \frac{J_{\mathbf{i}-\mathbf{j}}}{2} \hat{S}_{\mathbf{i}}^{(z)} \hat{S}_{\mathbf{j}}^{(z)} - t \sum_{\langle \mathbf{i}, \mathbf{j} \rangle_{\parallel}, \sigma} [\tilde{c}_{\sigma \mathbf{i}}^{\dagger} \tilde{c}_{\sigma \mathbf{j}} + \tilde{c}_{\sigma \mathbf{j}}^{\dagger} \tilde{c}_{\sigma \mathbf{i}}], \quad (1)$$

i.e., a mixed-dimensional t - J_z model [30]. I focus on the case of nearest-neighbor couplings along, $J_{\parallel} = J_{\pm \mathbf{e}_x}$, and across, $J_{\perp} = J_{\pm \mathbf{e}_y}$, the ladder, with $\hat{S}_{\mathbf{i}}^{(z)}$ the z projection of the spin operator $\hat{\mathbf{S}}_{\mathbf{i}}$. The hopping, $\hat{H}_t$, is only allowed along the ladder

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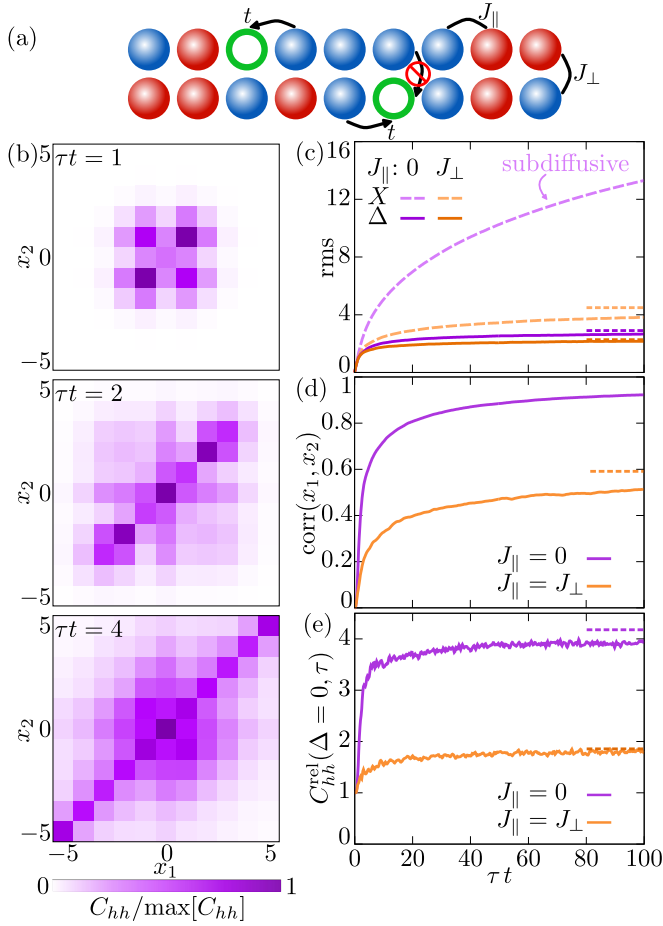


FIG. 1. Pairing dynamics. (a) Setup: holes move in a magnetic (red: $\uparrow$; blue: $\downarrow$) two-leg ladder with Ising interactions $J_{\perp}, J_{\parallel}$ via nearest-neighbor hopping t along the ladder. (b) Hole-hole correlator $C_{hh}(x_1, x_2; \tau) = \langle \hat{n}_h(x_1) \hat{n}_h(x_2) \rangle_{\tau}$ for $J_{\parallel} = 0$ at indicated times normalized to its maximum after the holes are initialized on the same rung, $(l, x_1) = (1, 0), (l, x_2) = (2, 0)$. (c) Dynamics of the root-mean-square dynamics of the center-of-mass $X = (x_1 + x_2)/2$ and relative $\Delta = (x_1 - x_2)/2$ coordinates, (d) correlation coefficient $\text{corr}(x_1, x_2) = \langle x_1 x_2 \rangle_{\tau} / [\sigma(x_1) \sigma(x_2)]$, and (e) relative correlation function for $\Delta = 0$. Steady state values are shown as short dashed lines. In all, $J_{\perp} = 5t$.

and only to vacant sites, enforced by the constrained operators $\tilde{c}_{\sigma i}^{\dagger} = \hat{c}_{\sigma i}^{\dagger} [1 - \hat{n}_i]$, where $\hat{n}_i = \sum_{\sigma} \hat{n}_{\sigma i} = \sum_{\sigma} \hat{c}_{\sigma i}^{\dagger} \hat{c}_{\sigma i}$ is the local density operator. The effective spin degree of freedom generally describes an internal degree of freedom with the Ising-type coupling in $\hat{H}_J$, the first term in Eq. (1), and since there are no exchange processes of the dopants, both fermions and bosons can realize the model. To have an efficient description of the hole and spin degrees of freedom, I perform a Holstein-Primakoff transformation on top of the *ferromagnetic* ground state $|\text{FM}\rangle = |\cdots \uparrow \uparrow \cdots\rangle$, with all spins in the $|\uparrow\rangle$ state. The spin Hamiltonian

$$\hat{H}_J = \sum_{i,j} \frac{J_{i-j}}{2} \left[\frac{1}{2} - \hat{s}_i^{\dagger} \hat{s}_i \right] \left[\frac{1}{2} - \hat{s}_j^{\dagger} \hat{s}_j \right] [1 - \hat{h}_i^{\dagger} \hat{h}_i] [1 - \hat{h}_j^{\dagger} \hat{h}_j] \quad (2)$$

thereby describes the interaction of bosonic spin flip excitations $\hat{s}_i^{\dagger}$, corresponding to the creation of a spin $\downarrow$ on site i , and

holes created by the operator $\hat{h}_i^{\dagger}$, which inherit the statistics of the underlying particles, be it fermionic *or* bosonic [19]. Equivalently, the hopping Hamiltonian

$$\begin{aligned} \hat{H}_t = t \sum_{(i,j)} & [\hat{h}_j^{\dagger} F(\hat{h}_i, \hat{s}_i) F(\hat{h}_j, \hat{s}_j) \hat{h}_i \\ & + \hat{h}_j^{\dagger} \hat{s}_i^{\dagger} F(\hat{h}_i, \hat{s}_i) F(\hat{h}_j, \hat{s}_j) \hat{s}_j \hat{h}_i] + \text{H.c.} \end{aligned} \quad (3)$$

describes two distinct ways in which holes may hop: (1) a hole hopping from site i to j in the absence of a spin flip on site j , and (2) hopping in the presence of a spin flip, whereby the hole and spin flip swap places. The constraint $F(\hat{h}, \hat{s}) = \sqrt{1 - \hat{s}^{\dagger} \hat{s} - \hat{h}^{\dagger} \hat{h}}$ ensures at most one spin per site. The initial state at time $\tau = 0$ of two holes on the same rung of the ladder in a spin background σ is then $|\Psi_{\sigma}(\tau = 0)\rangle = \prod_{l=1}^2 \hat{h}_{(l,0)}^{\dagger} \prod_{j \in S_{\sigma}^l} \hat{s}_{(l,j)}^{\dagger} |\text{FM}\rangle$, in which S_{σ}^l is the subset of spins in leg l , which are initially in the $|\downarrow\rangle$ state. Here, I write $i = (l, j)$, with leg index $l = 1, 2$ and position j along the leg. Since spins only exchange places with the holes along the ladder, the state at any later stage is

$$\begin{aligned} |\Psi_{\sigma}(\tau)\rangle = & \sum_{x_1, x_2} a_{\sigma}(x_1, x_2; \tau) \prod_{l=1}^2 \hat{h}_{(l, x_l)}^{\dagger} \\ & \times \prod_{\substack{j \in S_{\sigma}^l \\ j \in [\text{sgn}(x_l), x_l]}} \hat{s}_{(l, j - \text{sgn}(x_l))}^{\dagger} \prod_{\substack{j \in S_{\sigma}^l \\ j \notin [\text{sgn}(x_l), x_l]}} \hat{s}_{(l, j)}^{\dagger} |\text{FM}\rangle. \end{aligned} \quad (4)$$

Here, $\text{sgn}(x)$ is the sign function. Despite the rather formal description, the physics is simple and clear: if the hole in leg l moves $|x_l|$ times to the right (left), it surpasses $|x_l|$ spins that all move one step to the left (right). The corresponding probability amplitudes $a_{\sigma}(x_1, x_2; \tau)$ of finding the holes at positions x_1, x_2 at time τ obey

$$\begin{aligned} i\partial_{\tau} a_{\sigma}(x_1, x_2; \tau) = & V_{\sigma}(x_1, x_2) a_{\sigma}(x_1, x_2; \tau) \\ & + t \sum_{\substack{\delta_1, \delta_2 \\ |\delta_1| + |\delta_2| = 1}} a_{\sigma}(x_1 + \delta_1, x_2 + \delta_2; \tau), \end{aligned} \quad (5)$$

derived from the Schrödinger equation, $i\partial_{\tau} |\Psi_{\sigma}(\tau)\rangle = \hat{H} |\Psi_{\sigma}(\tau)\rangle$, with the initial condition that holes are on the same rung: $a_{\sigma}(x_1 = 0, x_2 = 0; \tau = 0) = 1$. The sum extends over configurations reached in a single hop. The induced interaction $V_{\sigma}(x_1, x_2)$ emerges, because the motion of the holes alter the spin bonds in the ladder [Fig. 2(a)]. For a random spin background, the induced interaction is also random [Fig. 2(b)], characterized in the relative and center-of-mass coordinates $\Delta = (x_1 - x_2)/2$ and $X = (x_1 + x_2)/2$. I indeed show [31] that if all spin backgrounds are equally likely—as in an infinite temperature spin state—then $V_{\sigma}(x_1, x_2)$ has zero mean and a variance $\text{Var}[V_{\sigma}(x_1, x_2)] \propto J_{\perp}^2 |\Delta|$, linear in the relative distance $2|\Delta|$ between the holes, describing a classical random walk of V in $|\Delta|$ [Fig. 2(b)]. I then focus on the evolution in specific spin backgrounds by resolving the dynamics of the density matrix in terms of these states. Starting from the initial density matrix

$$\hat{\rho}(0) = \sum_{\sigma_1, \sigma_2} \hat{c}_{(2,0), \sigma_2} \hat{c}_{(1,0), \sigma_1} \hat{\rho}_J \hat{c}_{(1,0), \sigma_1}^{\dagger} \hat{c}_{(2,0), \sigma_2}^{\dagger}, \quad (6)$$

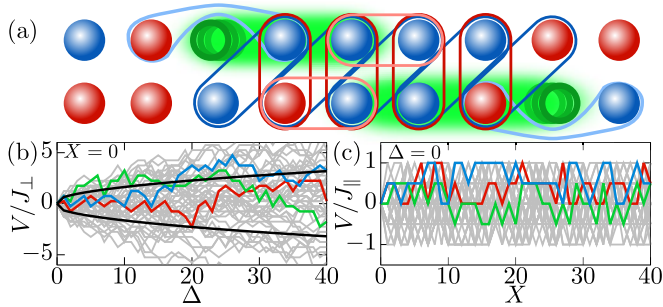


FIG. 2. Pairing mechanism. (a) As the holes move (green shaded region), spin bonds in the initial configuration are broken across (dark blue) and within (light blue) the ladder, and new bonds are established (dark and light red, respectively), giving a random induced interaction between the holes. (b) Induced interaction for vanishing center of mass, $X = 0$, as a function of the relative coordinate Δ for 50 realizations of the spin background (grey and colored lines), performing a random walk with standard deviation $\sim |J_\perp| \sqrt{|\Delta|}$ (black lines). (c) Induced interaction at $\Delta = 0$, as a function of X .

where $\hat{\rho}_J = e^{-\beta \hat{H}_J} / Z_J$ is the Gibbs state of the spins at inverse temperature $\beta = 1/(k_B T)$ and $\sigma_l = \uparrow, \downarrow$ designates the spin configuration at the origin of each leg, l , the ensuing dynamics $\hat{\rho}(\tau) = e^{-i\hat{H}\tau} \hat{\rho}(0) e^{+i\hat{H}\tau}$ —provided that the system is assumed to be closed—is the Boltzmann-weighted sum $\hat{\rho}(\tau) = Z_J^{-1} \sum_{\sigma} e^{-\beta E_J(\sigma)} |\Psi_{\sigma}(\tau)\rangle \langle \Psi_{\sigma}(\tau)|$ [28,29], where $E_J(\sigma)$ is the magnetic energy of the spin realization σ before the spins σ_1, σ_2 are removed. At infinite temperatures all pure state evolutions $|\Psi_{\sigma}(\tau)\rangle$ are sampled equally, $e^{-\beta E_J(\sigma)} / Z_J = 2^{-N}$, for system size N , and the ensuing dynamics is *exactly equal* for antiferromagnetic ($J_\perp, J_\parallel > 0$) and ferromagnetic couplings ($J_\perp, J_\parallel < 0$). The calculation of the hole dynamics then follows the recipe (1) generate a sample S of $N_S = 400$ random spin configurations σ and calculate $V_{\sigma}(x_1, x_2)$, (2) solve the equations of motion in Eq. (5) using exact diagonalization for $N = 201 \times 2$, and (3) compute the hole-hole correlator

$$C_{hh}(x_1, x_2; \tau) = \langle \hat{n}_h(x_1) \hat{n}_h(x_2) \rangle_{\tau} = \sum_{\sigma \in S} \frac{|a_{\sigma}(x_1, x_2; \tau)|^2}{N_S}, \quad (7)$$

as a function of time [Fig. 1(b)]. This describes the probability of simultaneously finding holes at positions $(1, x_1), (2, x_2)$, with $\hat{n}_h(x_l) = \hat{h}_{(l, x_l)}^\dagger \hat{h}_{(l, x_l)}$ the hole density operator. Figure 1(b) manifestly breaks the individual inversion symmetries $x_1 \rightarrow -x_1$ or $x_2 \rightarrow -x_2$ of uncorrelated holes, and shows a clear tendency for them to be on the same rung ($x_1 = x_2 \Leftrightarrow \Delta = 0$). Their motion is further analyzed by the root-mean-square value of the center-of-mass and relative coordinates [Fig. 1(c)],

$$A_{\text{rms}}(\tau) = \left[\sum_{X, \Delta} A^2 C_{hh}(X + \Delta, X - \Delta; \tau) \right]^{1/2}, \quad (8)$$

for $A = X, \Delta$. Since $X_{\text{rms}} > \Delta_{\text{rms}}$, the holes are closer together than they have propagated into the lattice. This is further

characterized by the correlation coefficient

$$\text{corr}(x_1, x_2) = \frac{\langle x_1 x_2 \rangle_{\tau}}{\sigma(x_1) \sigma(x_2)} = \frac{X_{\text{rms}}^2 - \Delta_{\text{rms}}^2}{X_{\text{rms}}^2 + \Delta_{\text{rms}}^2}, \quad (9)$$

taking values in $[-1, +1]$. Positive values, as in Fig. 1(d), correspond to $X_{\text{rms}} > \Delta_{\text{rms}}$, and signify *attraction*. Likewise, negative values correspond to effective repulsion, while $\text{corr}(x_1, x_2) = 0$ corresponds to no linear correlations. To further substantiate the findings, I analyze the relative hole-hole correlator

$$C_{hh}^{\text{rel}}(\Delta, \tau) = \frac{\sum_X C_{hh}(X + \Delta, X - \Delta; \tau)}{C_{hh}^{\text{disc}}(\Delta, \tau)}, \quad (10)$$

giving the probability of finding the holes at distance $2|\Delta|$ *relative* to what is expected from their marginals, i.e., the disconnected correlator $C_{hh}^{\text{disc}}(\Delta, \tau) = \sum_X \langle \hat{n}_h(X + \Delta) \rangle \langle \hat{n}_h(X - \Delta) \rangle$. This is shown for $\Delta = 0$ in Fig. 1(e), demonstrating a dynamically increasing probability of finding the holes on the same rung.

For vanishing intraleg spin coupling, $J_\parallel = 0$, the holes will continue to spread into the system. Indeed, by analyzing the center-of-mass dynamics [black-filled colored lines in Fig. 3(a)], I find that it transitions from an initial ballistic expansion with velocity t to *subdiffusive* behavior, $X_{\text{rms}} \sim (\tau \cdot t)^\nu$. This persistent spreading comes from a vanishing potential for holes on the same rung, $V(x, x) = 0$. In fact, in the limit of $J_\perp \gg t$, I gain simplified quantitative insights by assuming the holes move at most one site away from each other. The resulting effective model allows me to push the numerics to larger systems and longer times, verifying the subdiffusive behavior, and is seen to perfectly match the full dynamics for large $J_\perp/t$ on all timescales [gray-filled colored lines in Fig. 3(a)]. The subdiffusive exponent [bottom of Fig. 3(a)] slowly decreases for increasing $J_\perp/t$, reaching a plateau at $\nu \simeq 0.32$ for $J_\perp/t \gg 1$. Eventually, the system settles into a steady state, such that $X_{\text{rms}} \rightarrow l_X, \Delta_{\text{rms}} \rightarrow l_\Delta$ [Fig. 3(b)]. The associated asymptote of the correlation coefficient [Fig. 3(c)] shows attractive correlations over the entire investigated range. Moreover, the relative hole-hole correlator in Figs. 3(d) and 3(e) show that the probability of finding holes in close proximity can be up to an order of magnitude larger than expected from their marginals. For $J_\perp = J_\parallel \geq 3t$, there are minor finite-size corrections, while smaller spin couplings results in more spread out holes and significant corrections [31]. Indeed, for $J_\parallel = 0$, the subdiffusive behavior leads to a system-size dependent steady state, and I find $l_X \sim \sqrt{N}$. In turn, the correlation coefficient [Fig. 3(b)] behaves as $1 - \text{corr}(x_1, x_2) \propto l_\Delta^2/N$. The relative hole-hole correlator is, however, found to be well-converged for any $J_\perp > 3t$ [31]. For $J_\perp \leq 3t$ and both values of $J_\parallel$, there are substantial finite-size effects that suppress the pairing signals, because it foremost restricts the center-of-mass motion. In this manner, pairing is apparent across the entire investigated regime and persists for $J_\perp, J_\parallel < t$.

The origin of this dynamical pairing is *exactly* the same as for the thermal localization phenomenon discovered recently for the motion of single holes [28,29]. Indeed, because the induced interaction $V_{\sigma}(x_1, x_2)$ inevitably fluctuates to large values in the relative distance $2|\Delta| = |x_1 - x_2|$ [Fig. 2(b)],

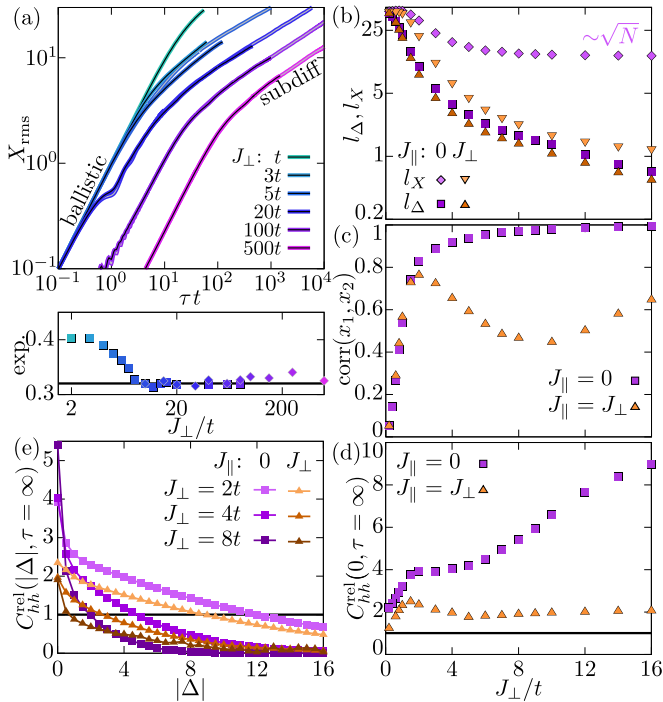


FIG. 3. Pair subdiffusion and steady state. (a) For $J_{\parallel} = 0$, the hole pair spreads subdiffusively, $X_{\text{rms}} \sim \tau^{\nu}$ at long times [top] with an exponent ν decreasing with $J_{\perp}/t$ [bottom]. Black- and gray-filled lines [top] give the dynamics for the full [Eq. (1)] and effective models, with corresponding black- and gray-outlined points for the fitted exponents [bottom]. (b) Steady state rms distances l_X and l_{Δ} of the center-of-mass, X , and relative, Δ , coordinates of the holes as a function of $J_{\perp}/t$ for specified $J_{\parallel}$. For $J_{\parallel} = 0$, l_X saturates at $\sim \sqrt{N}$. (c) Correlation coefficient $\text{corr}(x_1, x_2) = \langle x_1 x_2 \rangle / [\sigma(x_1) \sigma(x_2)]$ and (d) relative hole-hole correlator [Eq. (10)] for holes on the same rung ($\Delta = 0$) vs $J_{\perp}/t$. (e) Relative hole-hole correlator vs $|\Delta|$ for indicated values of $J_{\perp}/t$ and $J_{\parallel}$.

the holes, at the very least, back-reflect once they meet their classical turning points, corresponding to emergent Anderson localization [32] for *strong disorder* [33]. For nonzero $J_{\parallel}$, the holes remain localized, having a finite l_X , because even when they are on the same rung they will encounter a fluctuating potential taking on values in the interval $[-|J_{\parallel}|, +|J_{\parallel}|]$ [Fig. 2(c)], leading to Anderson localization for *weak disorder* [33].

Rydberg-dressed atoms. Quantum simulators based on Rydberg-dressed atoms in optical lattices natively realize spin-specific density interactions [34]

$$\hat{H}_J = \sum_{i \neq j} \frac{J_{i-j}}{2} \hat{n}_{i\uparrow} \hat{n}_{j\uparrow} \quad (11)$$

of the internal atomic state $|\uparrow\rangle$, by optically dressing it with a higher-lying Rydberg state. Here, J_{i-j} takes on a soft core shape [31,35]. Such systems were recently demonstrated to enter the itinerant regime [36]. Finally, a Feshbach resonance used to enhance the onsite interaction deep into the Mott-insulating phase ensures at most one spin per site, negligible superexchange, and thereby realizes the desired itinerant spin model governed by $\hat{H}_J$ in Eq. (11) and the constrained hopping $\hat{H}_t$ in Eq. (1). Precise control of the temperature in

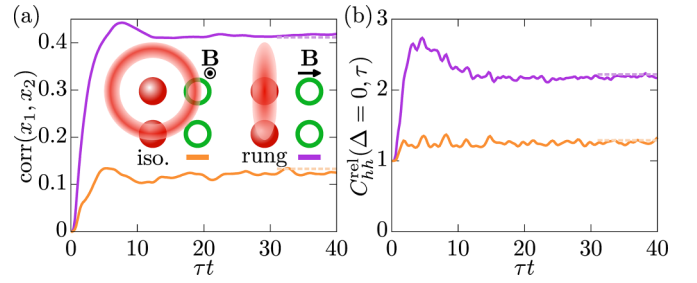


FIG. 4. Rydberg dressing. (a) Dynamics of the correlation coefficient for isotropic (orange) and strong rung (purple) couplings. (b) Relative correlator for holes on the same rung, $\Delta = 0$. Dashed lines: steady state values. Nearest neighbor rung interaction: $J_{e_y} = 8t$. Insets: $30P_{3/2}$ orbital orientation dictated by the magnetic fields, **B**. System size: $N = 21 \times 2$.

such systems is difficult, but the hole dynamics may still be investigated in the following manner [28]. (1) initialize holes at $x_1 = x_2 = 0$ by applying a strong repulsive light field to those sites [6], (2) rotate all spins from the fully polarized spin sample in the non-interacting $|\downarrow\rangle$ state into the equal superposition state $|\Psi_0\rangle = \prod_{i \neq (0,0),(0,1)} \frac{1}{\sqrt{2}} [\hat{c}_{i\uparrow}^\dagger + \hat{c}_{i\downarrow}^\dagger] |0\rangle$, and (3) turn off the focused light fields, freeing the holes to move along the ladder [12].

This initially seems distinct from the infinite temperature scenario above. The hole-hole correlator dynamics, $C_{hh}(x_1, x_2; \tau) = 2^{-N} \sum_{\sigma} |a_{\sigma}(x_1, x_2; \tau)|^2$, is however unchanged, whereby the holes move as at infinite temperature. The polarized Ising form of the interaction is crucial for this exact equivalence, and one should note that *spin* correlators would presumably differ from their infinite temperature counterpart. The computation now follows the same recipe as before. The resulting correlation coefficient dynamics [Eq. (9)] and relative correlation function [Eq. (10)] are given in Figs. 4(a) and 4(b), respectively. This is averaged over $N_S = 1000$ realizations and given in two instances [inset of Fig. 4(a)]: (1) isotropic couplings akin to the $J_{\parallel} = J_{\perp}$ case, and (2) anisotropic couplings where the $30P_{3/2}$ orbital of the Rydberg state is rotated to lie along the rungs using an external magnetic field [34], suppressing coupling in other directions similar to low values of $J_{\parallel}$ [31]. Crucially, we observe that the appearing disorder-induced attraction between the holes can be probed for experimentally feasible system sizes and timescales. Indeed, stroboscopic dressing [36] makes 20 tunneling events feasible, while longer timescales are currently inhibited by the intrinsic lifetime of the Rydberg state [37].

Conclusions and outlook. I have demonstrated that thermal spin disorder can frustrate the relative motion of dopants and lead to dynamical pairing. The nature of the effect is very general: it comes down to a randomly fluctuating field emerging as the dopants separate. It is closely related to the thermally induced localization phenomenon recently discovered for single holes [28,29]. By carefully analyzing its origin and behavior at infinite temperature, it is clear that the effect persists for all values of the spin coupling versus the hopping. For diminishing temperatures, ferromagnetic interactions should lead to a crossover to delocalized and uncorrelated holes. For antiferromagnetic interactions, a crossover from

fluctuation induced pairing to *confinement* of the holes emerges at low temperatures [19]. As demonstrated, my predictions are feasibly testable using available techniques with ultracold atoms in optical lattices. Alternative platforms include Rydberg arrays [38–42] and polar molecules [43]. The generality of the discovered effect may be tested by adding trans-leg hopping, $t_{\perp}$, or spin superexchange $\propto \alpha J$. I expect pairing to be robust as long as the hopping along the ladder dynamically dominates the perturbations, $t \gg t_{\perp}, \alpha J$. Such computations go beyond the presently described methodology, but

may be amenable to density matrix renormalization group (DMRG) approaches [44]. This will be addressed in future work.

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Data availability. The data that support the findings of this article are openly available [45].

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