

## Flow through porous media at the percolation transition

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We study low-Reynolds-number fluid flow through a two-dimensional porous medium modeled as a Lorentz gas. Using extensive finite-element simulations we fully resolve the flow fields for packing fractions approaching the percolation threshold. Near the percolation transition, we find a power-law scaling of the flow rate versus the packing fraction with an exponent of  $\approx 5/2$ , which was predicted earlier by mapping the macroscopic flow to a discrete flow network [B. I. Halperin *et al.*, *Phys. Rev. Lett.* **54**, 2391 (1985)]. Importantly, we observe a rounding of the scaling behavior at small system sizes, which can be rationalized via a finite-size scaling ansatz. Finally, we show that the distribution of the kinetic energy exhibits a power-law scaling over several decades at small energies, originating from collections of self-similar, viscous eddies in the dead-end channels. Our results lay the foundation for unraveling critical behavior of complex fluids omnipresent in biological and geophysical systems.

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Transport processes through complex media play fundamental roles in diverse areas, ranging from ancient irrigation systems [1] to modern technological applications, such as oil recovery from tight reservoirs [2], water filtration [3], processing of polymeric materials [4], and electrolyte transport in batteries [5], to geophysics [6,7], where pollutants contaminate rocky materials, to biology. The latter involves the circulation of blood through microvascular networks [8], microbial motion through wet soil and dense tissues [9–19], and molecular transport in the interior of crowded cells [20] and unicellular organisms [21–23], to name a few. In these heterogeneous environments, fluid flow encounters highly disordered porous structures with many dead-end pockets and corners, yielding numerous regions of stagnant flow alongside a few pores of rapid fluid streams [24–30]. These intricate flow fields can generate hydrodynamic dispersion of individual constituents [31–34] and are strongly affected by complex fluid properties [4,35]. Thus, the interplay of hydrodynamic flows and densely packed, disordered environments represents an exciting topic at the interface of fluid mechanics and statistical physics with several fundamental questions for various fields remaining still open, even in the Stokes regime.

In the realm of low-Reynolds-number flows, the behavior of viscous fluids traversing porous media is conventionally

captured by Darcy's law. This phenomenological law establishes a relationship, valid at length scales vastly exceeding the pore structure, between the average fluid velocity  $\langle \mathbf{u} \rangle$ , the pressure gradient  $\nabla p$  imposed across the porous medium, and the scalar permeability  $k$  alongside the fluid viscosity  $\mu$ :  $\mu \langle \mathbf{u} \rangle = -k \nabla p$ . While anisotropic features of the porous medium can be incorporated in a permeability tensor  $\mathbf{K}$  [36], replacing  $k$ , such a continuum description tends to overlook the intricate details of the local pore structure. Recent experimental investigations, particularly within dense environments, have illuminated the critical role of strong spatial variations in fluid flow [25,26,37], potentially exerting a profound influence on the macroscopic flow rate.

Important insights into the underlying physics could come from percolation theory [38], which deals with system properties that emerge as the density of obstacles increases to a critical packing fraction  $\phi_c$ , where a sample-spanning cluster first appears. At this percolation threshold, observables are predicted to exhibit power-law scaling behaviors, akin to classical phase transitions. Importantly, a scaling theory predicts a power-law behavior of the flow rate of Stokes flow through a porous medium, modeled as a Lorentz gas, near the critical packing fraction:  $Q \sim (\phi_c - \phi)^\alpha$  with  $\alpha \approx 5/2$  in two dimensions (2D) and  $\alpha \approx 4.4$  in three dimensions (3D) [39]. This scaling prediction relies on mapping the complex flow through the Lorentz gas onto a flow network by accounting for its geometrical features. In contrast to phenomenological laws, this would provide fundamental physical insights from first principles, but has not yet been confirmed, neither numerically nor experimentally.

Here, we compute the flow field of a viscous fluid passing through a 2D porous medium, described by the Lorentz gas, for a wide range of packing fractions using extensive

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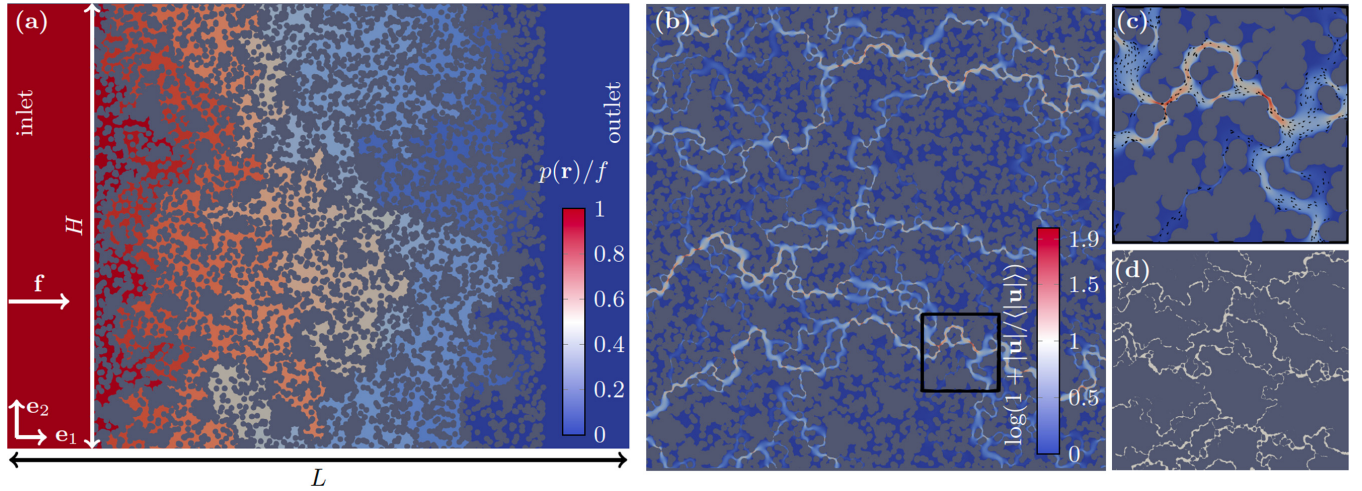


FIG. 1. (a) Model setup. A porous medium, modeled by overlapping disks in a domain of size  $H \times H$ , is placed at the center of a channel of length  $L$  and width  $H$ . Fluid is pushed at a force  $\mathbf{f}$  from the channel inlet to the outlet. The background color indicates the pressure field  $p(\mathbf{r})/f$ . Here, the width and length are  $H = 150a$  and  $L = 1.4H$ , respectively, where  $a$  denotes the disk radius. The packing fraction of overlapping disks is  $\phi \approx 0.92\phi_c$ . Disconnected void space is shown in gray. [(b), (c)] Velocity fields. (b) Rescaled magnitude of the velocity  $|\mathbf{u}|/\langle|\mathbf{u}|\rangle$  shown in logarithmic scale (base 10), where  $\langle|\mathbf{u}|\rangle$  is the velocity magnitude averaged over the connected part of the sample. (c) Zoom of the black frame in (b). Black spikes indicate the flow direction. (d) Backbone of the velocity field, where regions of velocities  $\geq \langle|\mathbf{u}|\rangle$  are indicated in light gray.

finite-element simulations. Our results reveal a power-law behavior of the flow rate near the percolation threshold with exponent  $\approx 5/2$ , thereby confirming the prediction by Halperin *et al.* [39]. Importantly, these findings demonstrate that for dense porous media the macroscopic flow rate is effectively determined by flow through a discrete random network of the form of the “nodes-links-blobs” model describing the geometric structure of the Lorentz gas close to percolation. For small systems the power-law behavior disappears and the data can be collapsed via a finite-size scaling ansatz with theoretically predicted exponents as input. Further, we find a power-law scaling of the distribution of the kinetic energy close to percolation that can be rationalized by collections of viscous, self-similar eddies present in the dead-end pores of the stagnant zones.

**Model.** We consider incompressible, viscous flow through a porous medium in 2D. The quasisteady, spatially dependent fluid velocity  $\mathbf{u}(\mathbf{r})$  and pressure fields  $p(\mathbf{r})$  are described by the Stokes and continuity equations,

$$\mu \nabla^2 \mathbf{u} - \nabla p + \mathbf{f} = \mathbf{0} \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0, \quad (1)$$

where  $\mu$  denotes the fluid viscosity and  $\mathbf{f}$  external forces. The porous medium is modeled using the Lorentz gas (or “Swiss-cheese model”) [39], composed of  $M$  randomly distributed, overlapping disks of radius  $a$  in a square domain of width  $H$ . We further add empty patches to the channel inlet and outlet so that the resulting domain is a rectangular channel of length  $L = 1.4H$  and width  $H$  [Fig. 1(a)]. Fluid can move through the free space between the disks, denoted by  $\Omega$ . We impose no-slip boundary conditions (BCs)  $\mathbf{u} = \mathbf{0}$  at  $\partial\Omega$  and periodic BCs along  $\mathbf{e}_2$ . Further, we apply a constant force (density) at the inlet of the channel  $\mathbf{f} = f\delta_{\text{in}}(\mathbf{r})\mathbf{e}_1$ , where  $f > 0$  and  $\delta_{\text{in}}$  is the Kronecker delta identifying the inlet of the channel. This leads to a macroscopic pressure drop along the channel ( $\mathbf{e}_1$ ),

$\Delta p = p_{\text{inlet}} - p_{\text{outlet}}$ , hence mimicking pressure-driven flow. By using  $a$  as relevant length scale,  $fa/\mu$  as characteristic velocity, and  $f$  as pressure scale, we identify one nondimensional parameter:  $H/a$ . We solve Eqs. (1) numerically using a finite-element scheme based on Taylor-Hood elements and properly adapt the mesh close to tight spaces and sharp corners [40].

The porosity of the Lorentz gas is characterized by the packing fraction of the disks:  $\phi = 1 - \exp(-M\pi a^2/H^2)$  [38]. In percolation theory, an important quantity represents the probability that two points at a given distance  $r$  belong to the same cluster, which is encoded in the pair-connectedness function  $p_2(r) \sim \exp(-r/\xi)$  [41] with correlation length  $\xi$  as relevant length scale. As the percolation threshold (i.e., the density of obstacles  $\phi_c$  at which a sample-spanning cluster first appears) is approached, it is predicted to obey  $\xi \sim (\phi_c - \phi)^{-\nu}$  with critical packing fraction  $\phi_c \approx 0.67637$  [42,43] and universal exponent  $\nu = 4/3$  in 2D [38,41]. In our work, we consider systems of width  $H = [12.5, 150]a$  and use packing fractions of  $0.05\phi_c$  to  $0.98\phi_c$ . Quantities such as the flow rate and velocity distributions are obtained by averaging over at least 30 different statistically independent geometries [40].

**Power-law scaling of the flow rate.** Figure 1 displays the spatially varying pressure field  $p(\mathbf{r})$  in a highly heterogeneous, porous environment. We observe large regions of constant pressure, whose magnitude decreases from the channel inlet to its outlet [Fig. 1(a)]. These regions result from very few open pores that enable fluid flow from one area of the pore space to another and can extend across the entire channel width. Consequently, this pressure field then leads to the formation of very few streams of high velocities through the open channels of the porous medium [Figs. 1(b) and 1(c)], forming the flow backbone [Fig. 1(d)], and large stagnant zones of vanishing flow in the areas of constant pressure.

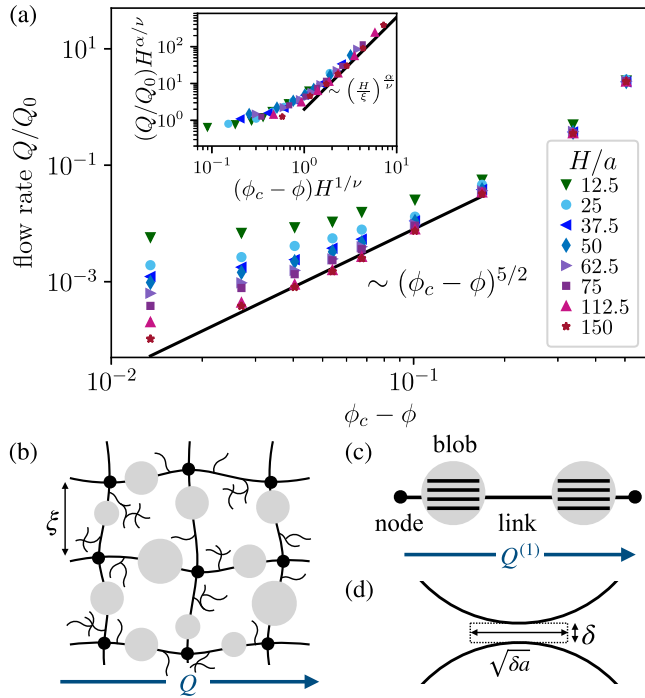


FIG. 2. (a) Flow rate  $Q$  as a function of the packing fraction  $\phi_c - \phi$  for different system sizes  $H/a$ . The solid line indicates the scaling prediction. The flow rate is rescaled by  $Q_0 = f/(\mu H)$ . Inset: Finite-size scaling of  $Q$  following Eq. (6). Here,  $\xi \sim (\phi_c - \phi)^{-\nu}$  denotes the correlation length. [(b), (c)] Sketch of a flow network composed of nodes connected through links and blobs. The tangling ends correspond to dead-end channels of the porous medium and constitute the dominant part of the void space. The flow rate between two nodes is denoted by  $Q^{(1)}$ . (d) Sketch of a characteristic channel of the porous medium having a width  $\delta$  and length  $\sqrt{\delta a}$ .

This highly heterogeneous flow field  $\mathbf{u} = (u_1, u_2)$  can be quantified in terms of the macroscopic flow rate via  $Q = \int_0^H u_1(x, y) dy$ . In practice, we also average over the length of the porous domain  $x \in [0, H]$ . Strikingly, we observe a power-law scaling of the flow rate as the percolation threshold  $\phi_c$  is approached,  $Q \sim (\phi_c - \phi)^\alpha$ . Fitting the data for  $H/a = 150$  and  $\phi_c - \phi \lesssim 0.02$  yields an exponent of  $\alpha \simeq 2.44 \approx 5/2$  [Fig. 2(a)].

Our findings agree with the scaling exponent previously predicted by Halperin *et al.* [39]. In their approach, the void space of the porous medium is mapped to a discrete random network (the “nodes-links-blobs” model [44]) composed of *nodes* which are separated at a distance  $\sim \xi$  and connected through sequences of *links* (composed of connected channels) and *blobs* (a group of connected channels in parallel); see Figs. 2(b) and 2(c). As dead-end channels do not contribute to transporting fluid across the medium, we do not consider them here. To obtain the flow rate–pressure drop ( $Q^{(1)} - \Delta p$ ) relation between two nodes, we ignore the resistance due to the blobs and compute the flow through a single link. Therefore, we first write the pressure drop as the sum of pressure drops  $\Delta p_i$  of  $N$  individual channels:  $\Delta p = \sum_{i=1}^N \Delta p_i$ . Taking into account the geometry of a narrow channel of width  $\delta_i$  composed of two opposed disks of radius  $a$ , one identifies its

length as  $\sqrt{a\delta_i}$  [Fig. 2(d)], leading to

$$\Delta p_i = 12Q^{(1)}\mu\delta_i^{-5/2}a^{1/2}. \quad (2)$$

To evaluate the resulting relation,

$$\Delta p \sim Q^{(1)} \sum_{i=1}^N \delta_i^{-5/2}, \quad (3)$$

a few observations are of importance. First, the bonds  $\delta_i$  of a Lorentz gas obey a distribution  $\varphi(\delta)$ , which is finite as  $\delta \rightarrow 0^+$  [38,45]. Next, the number of channels  $N$  increases with increasing  $\phi$  and scales as  $N \sim (\phi_c - \phi)^{-1}$  for two points separated at a distance  $\sim \xi$  [44]. These aspects allow us to replace Eq. (3) by an integral

$$\Delta p \sim Q^{(1)} N \int_{\delta_0}^{\infty} \varphi(\delta) \delta^{-5/2} d\delta, \quad (4)$$

where  $\delta_0$  denotes the minimum bond width of the string. As the width of the channels  $\delta$  becomes smaller for increasing  $\phi$ , we can assume  $\delta_0 \approx 1/[\varphi(0)N]$  (for large  $N$ ) and consequently find that the dominating contribution stems from the weakest bond [39], leading to  $Q^{(1)} \sim N^{-5/2} \sim (\phi_c - \phi)^{5/2}$ . Finally, the macroscopic flow rate  $Q$  is related to  $Q^{(1)}$  in 2D by considering a network of  $L/\xi$  links connected in series and  $L/\xi$  of the series of links in parallel:

$$Q \sim (\phi_c - \phi)^{5/2}. \quad (5)$$

Most importantly, our excellent agreement with numerical simulations suggests that the scaling behavior of the flow rate  $Q$  is fully determined by a flow network, which requires only knowledge of flow through a single narrow channel and geometric features of the Lorentz model. Thus it is quite remarkable that despite the complexity of our system (see Fig. 1), our numerical measurements of the flow rate are indeed explained by the scaling theory [39], where, for instance, the contribution of blobs is ignored, and analytical predictions for channel flow suffice to describe the macroscopic flow rate in dense media.

*Finite-size scaling.* We finally stress that this critical behavior in principle holds for infinitely large systems ( $H \rightarrow \infty$ ) and is expected to be recovered for finite system sizes that are larger than the correlation length of the void space,  $H \gtrsim \xi$ . Our results show that deviations appear for smaller system sizes,  $H/a \lesssim 100$ , where the curves of the flow rate bend and the power-law behavior becomes rounded. To interpret these results we perform a finite-size scaling ansatz. In particular, we note that the scaling prediction for the flow rate can be expressed by  $Q \sim \xi^{-\alpha/\nu}$ . For small systems,  $H \ll \xi$ ,  $H$  assumes the role of  $\xi$ , modifying the prediction to  $Q \sim H^{-\alpha/\nu}$  for  $\phi \rightarrow \phi_c$ . Thus, we use the following ansatz [38,46],

$$Q/Q_0 = H^{-\alpha/\nu} f(H^{1/\nu}(\phi_c - \phi)), \quad (6)$$

with scaling function  $f(x) = \text{constant}$  for  $H \ll \xi$  and  $f(x) \sim x^\alpha$  for  $H \gg \xi$ . This ansatz indeed nicely collapses our data covering sizes of  $12.5 \leq H/a \leq 150$  at high packing fractions [Fig. 2(a), inset]. Our results thus suggest that  $\xi$  is the relevant length scale of our system, and the expected scaling with exponent  $\approx 5/2$  is only smeared out by finite-size effects.

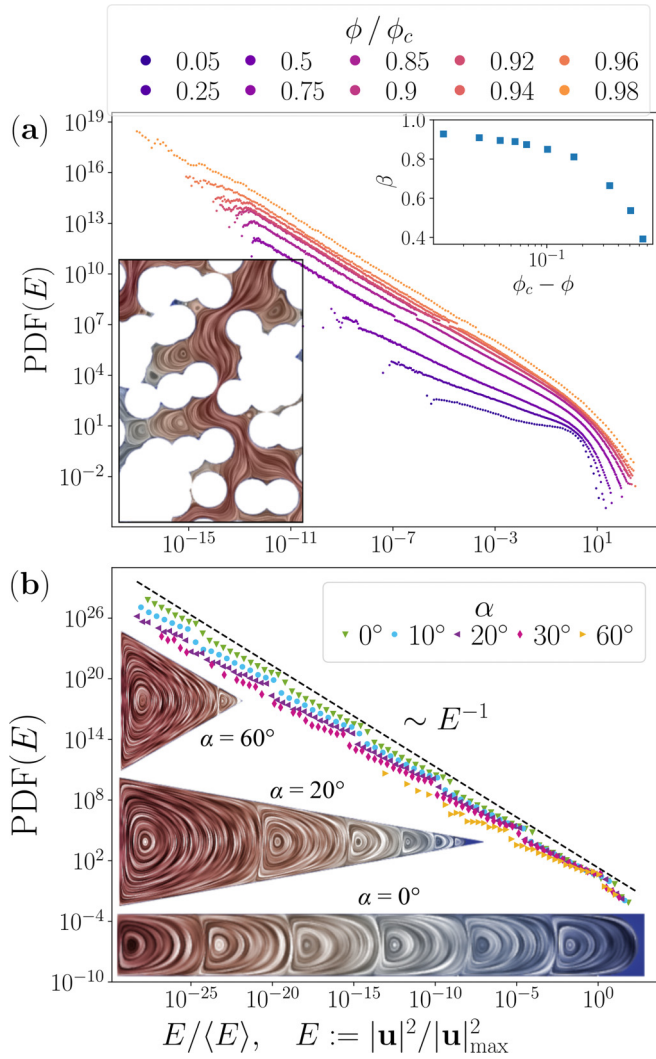


FIG. 3. (a) Probability density function (PDF) of the rescaled kinetic energy,  $E = |\mathbf{u}|^2 / |\mathbf{u}|_{\max}^2$ , where  $|\mathbf{u}|_{\max}$  denotes the maximal velocity. Top right: Exponent  $\beta$  of the power-law behavior for small kinetic energies. Bottom left: Example of a stagnant zone with eddies in the dead-end pores induced by the tangent velocity of the backbone flow (dark red). (b) PDF( $E$ ) for corner flows in different channel geometries. Inset: Collection of viscous eddies emerging in different channel geometries. Here,  $\alpha$  denotes the angle between the two long sides of the channels, where  $\alpha = 0^\circ$  corresponds to two parallel plates. The height of the channel is kept constant for different  $\alpha$ . The channels are rescaled for visualization purposes. Magnitudes of the velocities are shown in a logarithmic scale, with red corresponding to  $O(1)$  and dark blue to  $O(10^{-15})$ .

**Kinetic energy.** We further analyze the distribution of the kinetic energy, denoted as  $E = |\mathbf{u}|^2 / |\mathbf{u}|_{\max}^2$  [Fig. 3(a)]. As the packing fraction increases, we observe a noteworthy power-law scaling behavior,  $\text{PDF}(E) \sim E^{-\beta}$ , spanning several decades in the range of  $\sim 10^{-9}$ – $10^{-1}E$ , with the exponent  $\beta$  approaching  $\approx 0.9$ . This power-law scaling emerges at small kinetic energies, which can be primarily found in the dead-end pores of the medium (i.e., the tangling ends of the random resistor network). Within these regions, the flow consists of collections of viscous eddies of varying sizes

[Fig. 3(a), bottom left]. To rationalize whether the eddies contribute to the power-law scaling of  $\text{PDF}(E)$ , we investigate Stokes flow across different channel geometries. These reveal collections of eddies [Fig. 3(b)] with a significant reduction in velocities (and consequently energies) spanning orders of magnitude, consistent with predictions for viscous corner flows in terms of asymptotic, self-similar solutions [47]. The resulting PDFs exhibit a power-law scaling with a geometry-dependent exponent of  $\beta \sim 1$  for straight channels ( $\alpha = 0^\circ$ ) that decreases for triangular shapes ( $\beta \sim 0.8$  for  $\alpha = 60^\circ$ ). Notably, these exponents closely resemble those observed in the porous medium, implying that the  $\text{PDF}(E)$  at low  $E$  is predominantly influenced by a collection of such viscous eddies within dead-end channels. Thus, we anticipate that the power-law behavior emerges due to the self-similar nature of the individual viscous eddies and the extended power law represents a collection of these.

**Conclusions.** Our findings on Stokes flow through porous media demonstrate a power-law scaling of the macroscopic flow rate with an exponent  $\approx 5/2$  at the percolation threshold, thus confirming analytical predictions by Halperin *et al.* [39]. We observe that in very dense environments, the underlying flow field becomes highly heterogeneous, characterized by a few fast streams transporting fluid across the porous medium. This feature allows mapping the behavior of the macroscopic flow to a discrete flow network, whose structure relies on geometric features of the Lorentz gas [39]. Moreover, our study has provided a firm foundation for understanding Stokes flow through dense porous media via a finite-size scaling analysis.

Looking ahead, our findings lay the foundation to study the physics of complex fluids in disordered media and unraveling power-law behaviors for, e.g., non-Newtonian [4,35] or active fluids [9–12,17,48]. In particular, scaling relations for their critical behavior may be derived for stationary, non-Newtonian flows using analytical predictions for their flow rate through narrow channels as input [49,50], in a regime where memory effects can be ignored. We anticipate that testing these predictions through numerical and experimental means will enhance our understanding of non-Newtonian flows through disordered media, potentially utilizing substances like xanthan gum solution [51], for experimental validation. Furthermore, it will be interesting to study the impact of inertial effects, where, in addition to the flow rate, the distribution of the kinetic energy may follow a different scaling behavior due to the emergence of inertial vortices in narrow corners.

While our focus has been on Stokes flow through the Lorentz gas, we believe our results can be extended to other porous media models, especially those close to percolation, and are anticipated to be universal for media obeying similar pore size distributions. However, even for models featuring different channel shapes [25] and pore size distributions, the void space may similarly be mapped onto the “nodes-links-blobs” network.

We further note that transport of finite-sized particles through porous media can significantly change the properties of the velocity field [30] and so the flow rate can be used as measure for the dynamic properties of natural porous media, such as sediments. As a side note, interesting physics also

emerge for diffusive particles transported in porous media flows. The vortex structure in dead-end channels [47] has been argued to lead to power-law behaviors of their exit times [29], which dictate the overall transport across the porous medium, thus emphasizing the role of velocity regions that are slow yet comparable to Brownian noise.

Finally, several (semi)empirical expressions for the permeability, introduced via Darcy's law, have been derived to quantify and predict flow through porous media (e.g., the Katz-Thompson or the Kozeny-Carman relations) [25,52–54]. These typically depend on the conductivity of the fluid phase and microscopic parameters (e.g., the surface area of the porous structure or the grain radius [54]) and thus serve as a measure of the geometric features of the porous medium. Yet, the scaling exponent of the flow rate (or equivalently,

permeability) close to percolation has remained under debate, as previous work pointed out that the flow rate is not related to the conductivity due to the different nature of the Laplace and Stokes equations [38]. Our study now resolves this aspect by providing a numerical confirmation of the scaling prediction of Halperin *et al.* [39] and offers valuable guidance for future experiments, particularly through a comprehensive finite-size analysis.

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