

Propagation of two-particle correlations across the chaotic phase for interacting bosons

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We analyze the propagation of experimentally relevant two-particle correlations for one-dimensional interacting bosons, and give evidence that many-body chaos induces the emergence of an effective diffusive regime for the fully coherent correlation dynamics, characterized by an interaction dependent diffusion coefficient, which we estimate. This result supports very recent experimental observations, and paves the way toward an efficient description of the dynamical behavior of nonintegrable complex many-body systems. Furthermore, we show that the dynamical features within experimentally accessible time scales of a conveniently defined two-particle correlation transport distance provide a direct and unambiguous characterization of many-body quantum chaos in perfect agreement with its spectral identification.

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Introduction. Complex many-body dynamics ensue from an involved interplay between many-particle interference and interactions. The latter, in particular, are responsible for the appearance of chaotic regimes characterized by universal dictates [1–3], in which certain coarse-grained features of a many-body system become amenable to a description in terms of random matrix theory and statistical mechanics [4–8]. The specificity of the system in the chaotic phase, however, remains imprinted in certain observables [9–11], and the sensitivity to many-particle interference is in fact enhanced at the onset of such “universal” regime [12,13]. This raises a fundamental issue, how does the many-particle correlation dynamical response change upon the emergence of many-body quantum chaos?

This question bears practical implications, e.g., for the design of current quantum computing architectures [14–16], and its investigation finds an ideal experimental platform in systems of low dimensional ultracold bosons [17–34]. For one-dimensional bosons, beyond the dynamics of single-particle observables [20–24,35–39], studies on coherent relaxation of higher-order observables from nonequilibrium configurations have mostly focused on the propagation of two-particle correlation fronts, found to form a ballistic light cone—for any bosonic interaction strength—both theoretically [40–42] and experimentally [17].

Here, we show compelling evidence that the existence of many-body quantum chaos for one-dimensional bosons triggers a fundamental change in the coherent dynamics of two-particle correlations, giving rise to an effective diffusive regime governed by an interaction dependent diffusion

constant. This result is in significant agreement with recent experimental observations for hard-core bosons [34]. We find that the time development of correlations can be encoded into a suitable two-particle correlation transport distance, whose behavior—within experimentally reachable time scales—further permits an explicit identification of the chaotic phase in remarkable agreement with its spectral characterization [9–11,43–55]. Most interestingly, we show evidence that the equilibrium value of such correlation transport distance develops a nonanalyticity (as the thermodynamic limit is approached) as a function of the interaction strength at the onset of the chaotic regime.

We model an ensemble of N boson in an L -site one-dimensional optical lattice using the standard Bose-Hubbard hamiltonian (BHH) [56–58] with hard-wall boundary conditions,

$$H = -J \sum_j (b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j) + \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1), \quad (1)$$

and study the dynamics of the homogeneous Fock state with one boson per site, $|\psi_0\rangle = |1, 1, \dots, 1\rangle$, for varying relative tunneling strength $\gamma \equiv J/U > 0$. The time evolution is restricted to a maximum of 200 tunneling times, $\tau \equiv Jt/\hbar \leq 200$, in order to work within experimentally accessible time scales, as those probed in Refs. [27,30]. The chosen initial state ensures no significant net mass transport across the system in time but a γ -dependent development of many-particle correlations. We probe the dynamics of two-particle correlations via the experimentally accessible connected two-point density correlations

$$C_{j,k}(\tau) \equiv \langle \hat{n}_j(\tau) \hat{n}_k(\tau) \rangle_{\psi_0} - \langle \hat{n}_j(\tau) \rangle_{\psi_0} \langle \hat{n}_k(\tau) \rangle_{\psi_0} \quad (2)$$

between sites $j, k \in [1, L]$. One may conveniently define a two-particle correlation transport distance (CTD) as

$$\ell(\tau) \equiv \sum_{d=1}^{L-1} d \langle |C_{k,k+d}(\tau)| \rangle_k, \quad (3)$$

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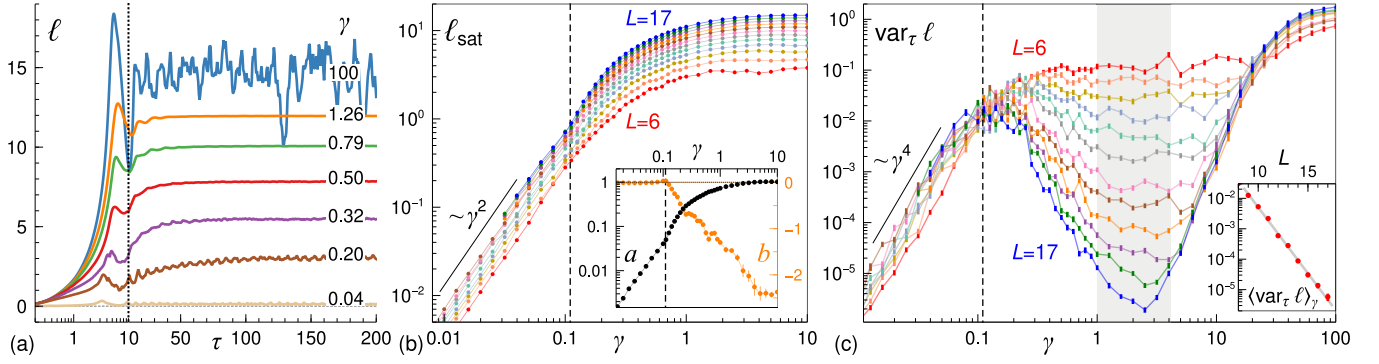


FIG. 1. Dynamical features of the two-particle correlation transport distance. (a) CTD [Eq. (3)] versus time for $L = 17$ and varying $\gamma = J/U$ as indicated. Note the log (linear) scale for $\tau \leq 10$, time step $\Delta\tau = 0.01$ ($\tau > 10$, $\Delta\tau = 0.5$). Panels (b) and (c) show, respectively, the time averaged value ℓ_{sat} and the CTD temporal variance in the interval $\tau \in [100, 200]$ versus γ for $L \in [6, 17]$. The vertical dashed line highlights the value $\gamma = 0.11$. The inset in (b) presents the parameters of the fit $\ell_{\text{sat}}(\gamma) = a(\gamma)L + b(\gamma)$. The inset in (c) displays $\text{var}_\tau \ell$ averaged over the shaded γ region for each L , with the solid line being the best fit $111.71e^{-L}$. When not visible, errors are contained within symbol size.

where angular brackets indicate an average over all site pairs $(k, k + d)$ for a given distance d , as was introduced and measured experimentally in Ref. [27] to discern thermal from many-body localized phases in the disordered BHH. The CTD encodes the magnitude of all two-point density correlations in the value of a “mean” correlation distance [59]. As a reference, in the noninteracting limit ($\gamma \rightarrow \infty$), the asymptotic temporal average of the CTD is $\overline{\ell(\tau)}_{\gamma \rightarrow \infty} = L + O(L^0)$ [59], becoming equal to the system size.

Saturation regime. The evolution of $|\psi_0\rangle$ for a given γ is performed numerically via an efficient expansion of the time-evolution operator using Chebyshev polynomials [60], which allows us to simulate accurately long time dynamics for systems up to $L = 17$ at unit density (with a Hilbert space size $\mathcal{D} \simeq 10^9$) [59].

An overview of the dynamical behavior of $\ell(\tau)$ for $L = 17$ is shown in Fig. 1(a). The CTD undergoes an initial growth in time and, due to the finiteness of the system, eventually reaches a γ -dependent saturation value which can be well identified for $\tau \in [100, 200]$. The most interesting feature of the signals in this time interval is the striking absence of temporal fluctuations for some γ values. The CTD saturation value, identified as the time average $\ell_{\text{sat}} \equiv \overline{\ell(\tau)}$ for $\tau \in [100, 200]$, is shown in Fig. 1(b) as a function of γ . The propagation of correlations is strongly suppressed for $\gamma < 0.1$, where $\ell_{\text{sat}} \ll 1$ and it exhibits a quadratic dependence on γ . As the relative tunneling strength further increases, ℓ_{sat} undergoes a pronounced surge, reaching a maximum followed by a subtle decay toward the noninteracting limit (see also Fig. S2 in the Supplemental Material). Our analysis shows that ℓ_{sat} exhibits a linear growth with L for all values of γ [59], $\ell_{\text{sat}} = aL + b$, albeit with markedly different slopes. In fact, around $\gamma = 0.11$ a change in the dependence of ℓ_{sat} on γ becomes visible for increasing L . This transition is distinctively witnessed by the fit parameters a and b [inset of Fig. 1(b)], indicating that a nonanalyticity develops in the vicinity of $\gamma = 0.11$ in the thermodynamic limit.

The CTD fluctuations after saturation can be characterized by the temporal variance $\text{var}_\tau \ell \equiv \ell^2(\tau) - \ell_{\text{sat}}^2$, which as observed in Fig. 1(c), features three different γ regimes. For low relative tunneling strength, fluctuations are weakly dependent

on L and strongly suppressed, growing as γ^4 up to a tipping point around $\gamma = 0.11$, marking the crossover to a new regime where $\text{var}_\tau \ell$ decreases dramatically with L and reaches a minimum around $\gamma \approx 3$. Note that $\text{var}_\tau \ell$ drops by nearly five orders of magnitude after a threefold increase in L . Indeed, here the fluctuations decrease exponentially with system size [inset of Fig. 1(c)]. This latter observation is one implication of the eigenstate thermalization hypothesis (ETH), by which temporal fluctuations around equilibrium values are reduced exponentially with the number of degrees of freedom [6,7] (i.e., the number L of modes in the BHH [11]). Fluctuations ultimately grow again with tunneling strength, and at $\gamma \simeq 20$ a third regime appears where $\text{var}_\tau \ell$ is slightly enhanced with system size.

It is remarkable that an intermediate γ regime of strongly suppressed temporal fluctuations in the CTD is so clearly visible for relatively small systems within experimentally accessible time scales of ≈ 100 tunneling times. Moreover, the relative fluctuations quantified by $(\text{var}_\tau \ell)/\ell_{\text{sat}}^2$, shown in the bottom panel of Fig. 2, exhibit a distinctive qualitative and quantitative behavior in this regime (which also emerges in the presence of periodic boundary conditions [59]).

The question now is to which extent this dynamical regime correlates with the system’s spectrally chaotic phase. The latter is best identified from the eigenvector structure [9–11,55], which is efficiently probed by the generalized fractal dimension

$$\tilde{D}_1 = - \sum_{|n\rangle} |\langle n|\psi\rangle|^2 \log |\langle n|\psi\rangle|^2 / \log \mathcal{D}, \quad (4)$$

for eigenstates $|\psi\rangle$ expanded in the on-site Fock basis $\{|n\rangle\}$. The top panel of Fig. 2 shows the energy resolved variance of $\tilde{D}_1$ for $L = 11$ calculated over sets with 1% of the eigenstates, as a function of γ and of the spectral percentage distance (sp) from the energy $E = 0$ of the initial state $|\psi_0\rangle$ taken as a reference value [i.e., the spectral percentage at energy E corresponds to $sp = 100 \times [\mathcal{N}(E) - \mathcal{N}(0)]/\mathcal{D}$, with $\mathcal{N}(E)$ the integrated density of states].

In this stationary picture, as demonstrated in Refs. [9–11,55], the chaotic phase is revealed by the region with suppressed $\text{var}(\tilde{D}_1)$, which correlates with the emergence

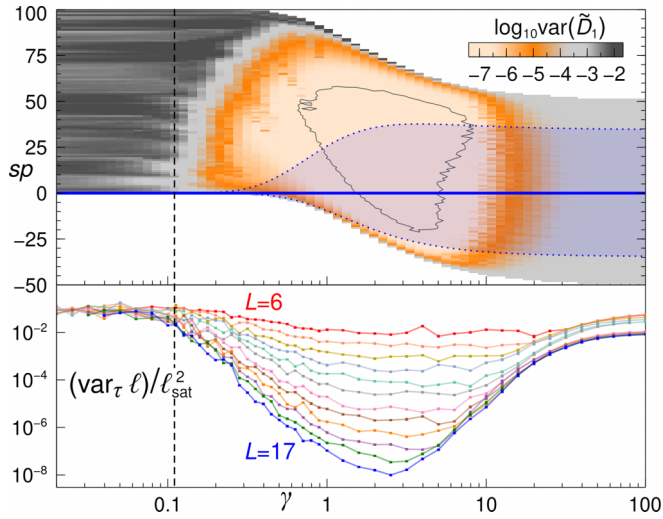


FIG. 2. Comparison of spectral (top) and dynamical (bottom) pictures of the chaotic phase. The top panel shows the variance of the eigenstate fractal dimension $\tilde{D}_1$ [Eq. (4)] for $L = 11$ (even parity subspace, $\mathcal{D} = 176484$) as a function of $\gamma \in [0.02, 100]$ calculated for sets with 1% of the eigenstates organized as the spectrum percentage (sp) measured from the initial state's energy (blue line, shaded area marks the corresponding energy width) (see main text). The contour line encircles the region where $\log_{10} \text{var}(\tilde{D}_1)$ lies within 5% of the RMT value. The bottom panel displays the CTD relative temporal variance versus γ for varying L .

of eigenstates that become extended ($\tilde{D}_1 \rightarrow 1$) in Fock space in the thermodynamic limit [59]. We note that the onset of the chaotic domain so identified does not change with increasing L [59]. To see how the initial state participates of the system eigenstates, the reference energy trajectory at $sp = 0$ is flanked by two bands corresponding to $\pm\sigma \equiv \pm(\langle H^2 \rangle_{\psi_0} - \langle H \rangle_{\psi_0}^2)^{1/2}$, which gives the energy width of the local density of states in Fock space at the point $|\psi_0\rangle$. The top panel of Fig. 2 thus shows how the spectrally chaotic phase develops as a function of γ from the perspective of $|\psi_0\rangle$, whose passage through the chaotic phase (accompanied by a marked broadening of the energy width) correlates unambiguously with the regime of suppressed temporal fluctuations of the CTD. Furthermore, $(\text{var}_\tau \ell) / \ell_{\text{sat}}^2$ is minimized in the γ range where the initial state participates maximally of eigenstates that exhibit the closest agreement with random matrix theory (RMT) (see contour line in the top panel of Fig. 2). These results also confirm that the dynamical emergence of ETH hinges on the existence of chaotic (extended in Fock space) eigenstates.

Presaturation dynamics. One should also ask if and how the emergence of the chaotic phase would manifest itself dynamically through the CTD before saturation. This requires challenging simulations for large enough systems, exhibiting a regime of steady growth for $\ell(\tau)$ sufficiently far away from the pronounced maxima that precede the onset of saturation [cf. Fig. 1(a)].

Let us first understand the CTD limiting behaviors. For short times, an exact calculation up to order τ^4 yields [59]

$$\ell(\tau) \underset{L \gg 1}{=} 4\tau^2 - \left(6 + \frac{1}{3\gamma^2}\right)\tau^4 + O(\tau^6). \quad (5)$$

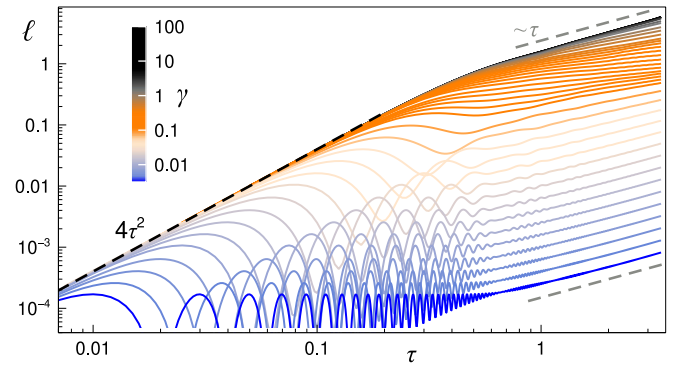


FIG. 3. Time evolution of the CTD for $L = 100$ and varying $\gamma \in [0.00316, 100]$ as indicated by the color scale. The black dashed line corresponds to the initial quadratic growth $4\tau^2$ [cf. Eq. (5)], while gray dashed lines highlight ballistic behavior.

Hence, the CTD exhibits an initial quadratic growth whose extension is maximal in the noninteracting case ($\gamma \rightarrow \infty$) and shrinks as $\sim \gamma$ for strong interactions ($\gamma \rightarrow 0^+$).

Asymptotically, for $L \gg \tau \rightarrow \infty$, the analysis in the absence of interactions reveals [59]

$$\ell(\tau) \underset{\gamma \rightarrow \infty}{=} \frac{16}{\pi^2} \tau + O\left(\frac{\log \tau}{\tau}\right), \quad (6)$$

and hence ballistic spreading. In the opposite limit of very strong interaction ($\gamma \ll 1$), making use of the fermionization approach of Ref. [41], we compute an analytical expression for the CTD for $L = \infty$ [59]. Most importantly, this expression provides the exact temporal asymptotic behavior in the very limit $\gamma \rightarrow 0^+$,

$$\ell(\tau) \underset{\gamma \rightarrow 0^+}{=} \frac{64\gamma^2}{\pi} \tau + O(\tau^{-1}). \quad (7)$$

Therefore, the spreading of two-point density correlations is also ballistic when approaching the $\gamma = 0$ integrable point. It is precisely in the dependence of the CTD's steady growth on γ where the emergence of the chaotic phase may be observable. (Recall that ℓ_{sat} depends linearly on L , and hence the CTD's increase as $L \gg \tau \rightarrow \infty$ is unbounded for all γ). In Fig. 3, we show the behavior of the CTD in time for $L = 100$ and varying γ , obtained using time evolving block decimation for matrix product states. The simulation parameters are carefully chosen to ensure convergence of the signals with a tolerance $\lesssim 0.5\%$ [59]. As can be observed, the initial growth is dictated by Eq. (5), where the extent of the quadratic regime wanes with the relative interaction strength. For dominating interactions ($\gamma \ll 1$), the departure from the τ^2 behavior is followed by an oscillatory regime governed by transitions to doublon and holon states, characterized by the single frequency $\omega = \gamma^{-1}$. As time progresses, the doublon-holon propagation and the participation of further states slowly builds up and eventually the CTD exhibits a sustained enhancement in time, even if with a very small amplitude. (This behavior can be analytically described in the limit $\gamma \rightarrow 0^+$ [59]). For weaker interactions, the oscillatory regime is progressively reduced and the quadratic time dependence transitions smoothly into a different behavior.

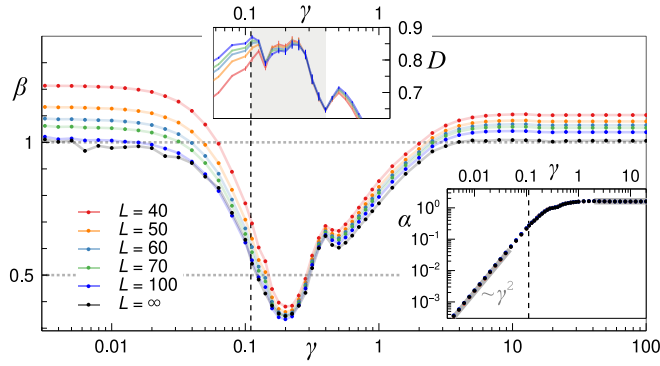


FIG. 4. Parameters β (main panel) and α (lower inset) of the CTD fit $\ell(\tau) = \alpha\tau^\beta$ in the interval $\tau \in [2.2, 3.3]$, and diffusion constant D (upper inset) as functions of γ for the indicated system sizes. Horizontal dotted lines highlight ballistic ($\beta = 1$) and diffusive ($\beta = 1/2$) behavior, while the vertical dashed line marks the value $\gamma = 0.11$. In the lower inset, all data for different L collapse, and the thick gray lines correspond to the coefficients of Eqs. (6) and (7). When not visible, errors are contained within symbol size.

The relevant steady growth of $\ell(\tau)$, after the uneventful τ^2 and oscillatory evolutions, is visible in Fig. 3 for $\tau > 1$ and for all γ , where the CTD seems to converge to a guiding power-law tendency. Note that, despite the limited simulation times, the predicted asymptotic ballistic behavior in the limits $\gamma \rightarrow \infty$ and $\gamma \rightarrow 0^+$ is unambiguously observable. For intermediate γ , however, the CTD's growth registers a noticeable slowdown.

In order to quantify the steady increase of $\ell(\tau)$, we show in Fig. 4 the parameters of the fit $\ell(\tau) = \alpha\tau^\beta$ for $\tau \in [2.2, 3.3]$ as functions of γ for $L \in [40, 100]$. In this time interval, the CTD shows a convergence toward a system-size independent form [59], allowing us to estimate the signal for $L = \infty$, and their corresponding fit parameters, included in Fig. 4. As revealed by the value of the exponent β , the growth of the CTD is indeed ballistic near the integrable limits $\gamma \rightarrow 0^+$ and $\gamma \rightarrow \infty$. As the relative tunneling strength departs from the region $\gamma \ll 1$, the value of β shows initially a timid decrease that eventually evolves into a marked drop reaching the value $1/2$ around $\gamma = 0.11$, the threshold identified in our previous analyses. Beyond this point, the exponent β seems to want to develop an oscillation around $1/2$, which is suddenly interrupted at $\gamma \approx 0.4$. For larger γ , the exponent approaches monotonically 1, and ballistic growth is fully restored for $\gamma \gtrsim 3$. It is worth emphasizing that the fit amplitude, α (lower inset to Fig. 4), is well converged with L , and its dependence on γ is consistent with the behavior observed for ℓ_{sat} (and its fit parameter a) [cf. Fig. 1(b)], as well as with Eq. (6) for $\gamma \rightarrow \infty$ and Eq. (7) for $\gamma \rightarrow 0^+$.

The results indicate that the onset of the chaotic phase correlates with the emergence of a diffusive behavior for the CTD. The spreading of two-particle correlations in the chaotic regime would then be governed by a diffusive process characterized by a single parameter, the diffusion constant $D(\gamma)$. Diffusion seems plausible in the range $0.1 \lesssim \gamma \lesssim 0.4$, that, however, does not correlate positively with the entire chaotic phase as identified previously. This (as well as the fluctuation of β around $1/2$) may be attributed to the short time scales

available: If the characteristic mean free time of the diffusive process exceeded the extent of the considered time interval then ballistic dynamics should be effectively realized. Accordingly, the observable diffusive regime is primarily limited by the accessible time scales, rather than by just the extension of the chaotic phase.

A comparison of the CTD against a simple diffusive model [59] permits a qualitative estimation of the diffusion constant, shown in the upper inset of Fig. 4. Upon entering the chaotic phase ($\gamma \approx 0.11$), the estimated D quickly collapses for all system sizes, and exhibits a slight decay upon approaching $\gamma \approx 0.4$, which is arguably the end of the observable diffusive regime. We note that the diffusion constant lingers around the value $D \approx 0.8Ja^2/\hbar$ (a being the lattice constant), in agreement with a very recent experimental observation for hard-core bosons on a ladder [34].

The identified diffusive γ range for two-particle correlations may be compared with the results for one-particle observables of Ref. [39], where diffusive mass transport of a single density defect was found for $\gamma \in [0.2, 0.5]$ on similar time scales, as well as with the pronounced slowdown in the expansion of a 1D bosonic cloud experimentally observed for $\gamma \approx 0.3$ in Ref. [21], conjectured to be due to the emergence of diffusive dynamics.

Note that diffusive spreading of correlations in the many-body chaotic phase lies beyond the reach of standard RMT ensembles. For the Gaussian orthogonal ensemble (GOE), one may evaluate $C_{j,k}^{\text{GOE}}(\tilde{\tau})$ analytically, with time $\tilde{\tau}$ in units of the inverse of the spectral width (results will be reported elsewhere). Here, correlations are independent of the distance $|j - k|$ and spread as $\sim \tilde{\tau}^2$ before saturation. Hence, the sparse Fock-space connectivity of typical many-body systems seems essential for the emergence of diffusive dynamics.

Conclusions. We have studied the propagation of experimentally relevant two-particle correlations for one-dimensional interacting bosons. Our results show evidence that in the chaotic regime certain aspects of the otherwise complex many-particle fully coherent dynamics are amenable to a simplified approach using a diffusive model specified by an interaction dependent diffusion coefficient. This observation goes in hand with very recent experimental results for low-dimensional hard-core bosons [34], and paves the way toward an efficient description of the dynamical behavior of nonintegrable complex many-body systems. Furthermore, the time development of two-point density correlations can be conveniently encoded in a two-particle correlation transport distance (CTD), whose features within experimentally accessible time scales provide a direct and unambiguous characterization of many-body quantum chaos. Besides demonstrating analytically the ballistic growth of the CTD in the vicinity of the system's integrable limits, we specifically show that (i) the onset of many-body chaos correlates with the emergence of a nonanalyticity in the equilibrium value of the CTD when approaching the thermodynamic limit, and (ii) the regime of suppressed CTD temporal fluctuations expose the chaotic phase in a one-to-one correspondence with its spectral identification.

Given that the system's dynamical response is fundamentally shaped by many-particle interference [61–70], controlled by particle distinguishability, most prominently at the onset of chaos [12,13], an analysis of the CTD and the diffusion

constant for bosonic mixtures could reveal how particle distinguishability may enter into effective macroscopic descriptions of many-body chaotic dynamics.

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