

Formation of Rydberg crystals induced by quantum melting in one dimension

Zeki Zeybek,^{1,2,*} Peter Schmelcher^{1,2} and Rick Mukherjee^{2,†}

¹The Hamburg Centre for Ultrafast Imaging, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

²Zentrum für Optische Quantentechnologien, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany



(Received 29 May 2024; accepted 5 November 2024; published 13 January 2025)

Quantum fluctuations in frustrated systems can lead to the emergence of complex many-body phases. However, the role of quantum fluctuations in frustration-free lattices is less explored and could provide an interesting avenue for exploring new physics, and perhaps is easier to realize compared to frustrated lattice systems. Using Rydberg atoms with tunable interactions as a platform, we leverage strong van der Waals interactions and obtain a constrained model in one dimension with nonlocal fluctuations given by dipolar interactions alongside local fluctuations. The combined effect of such processes leads to intrinsically quantum-ordered Rydberg crystals through the order-by-disorder mechanism. Finite-size analyses indicate that combined fluctuations drive the transition from disordered to ordered phases, contrary to the expected direction. We provide a theoretical description to understand the physics of order-by-disorder in one-dimensional systems, which are typically seen only in higher dimensions.

DOI: [10.1103/PhysRevResearch.7.L012009](https://doi.org/10.1103/PhysRevResearch.7.L012009)

Introduction. Quantum fluctuations give rise to diverse phenomena ranging from spin fluctuation-induced superfluidity of helium-3 [1,2] to superconductivity in metals [3]. In the case of frustrated systems, quantum fluctuations lift the macroscopic classical ground-state degeneracy [4–8] and stabilize exotic many-body phases, such as the quantum spin liquids [6,7], quantum spin nematics [9–11], and long-range ordered states through order-by-disorder [12–18], as well as lead to complex phase transitions [19–21]. However, it can be hard to realize these phases in frustrated materials particularly when they involve nontrivial geometries such as kagome [19,20,22], honeycomb [15,23,24], triangle [25,26], and pyrochlore [27,28] lattices. Further issues arise from environmental noise, system disorder, and limited control of competing interactions [18,29].

Platforms based on Rydberg atoms have proven to be highly controllable quantum simulators [30–35] as their large dipole moments provide tunable strong interactions with different characters such as van der Waals (vdW) and dipolar interactions [36–39]. The quantum spin-liquid phase was observed recently involving a fluctuation-driven Rydberg setup with only vdW interactions [40,41]. However, fluctuation-driven phenomena were not explored using both vdW and dipolar interactions, and there is a growing interest in studying the combined effects of these processes [32,42–46]. Especially, a question arises on whether the flexibility of tuning

both vdW and dipolar interactions can be leveraged to observe such phenomena in simple frustration-free setups such as a one-dimensional (1D) lattice.

In this Letter, we address this question by studying a 1D Rydberg system in the limit of strong vdW interactions, where a constrained model is derived. We report the order-by-disorder mechanism in a 1D frustration-free lattice, which has not been shown before, making use of a new constrained model beyond the PXP paradigm [47–49]. We reveal that the long-range order is established through the cooperative action of local fluctuations caused by laser coupling and nonlocal fluctuations due to dipolar interactions. The ordered phase is a Rydberg crystal without any classical analog since it does not arise by minimizing the vdW energy, differing from earlier results on Rydberg crystals [50–52]. We present an intuitive physical picture of the underlying physics and motivate a variational ansatz that describes both the disordered and ordered regimes. The nature of the quantum phase transition (QPT) is examined by finite-size-scaling analyses. Our results highlight a novel use of the Rydberg platform, enabling the simulation of fluctuation-driven phenomena and the exploration of new physics with different constrained models.

Theory. We consider a 1D system of trapped highly excited neutral atoms that are modeled as two-level systems with a pair of Rydberg states coupled by a microwave laser with the Rabi frequency $\Omega_{\mu\nu}$ and detuning $\Delta_{\mu\nu}$ as shown in Fig. 1(a). These distinct highly excited Rydberg states represent the hard-core bosonic degrees of freedom given by $\{|\circ\rangle, |\bullet\rangle\}$, where $|\circ\rangle$ ($|\bullet\rangle$) denotes the absence (presence) of a boson [Fig. 1(a)]. The state $|\circ\rangle$ corresponds to occupying the lower-lying Rydberg state, unlike the ground state used in conventional Rydberg simulators. Similarly, $|\bullet\rangle$ indicates occupation of the higher-lying Rydberg state.

This encoding leads to expressing the vdW and dipolar interacting Rydberg atoms by an extended Bose-Hubbard model

*Contact author: zzeybek@physnet.uni-hamburg.de

†Contact author: rmukherj@physnet.uni-hamburg.de

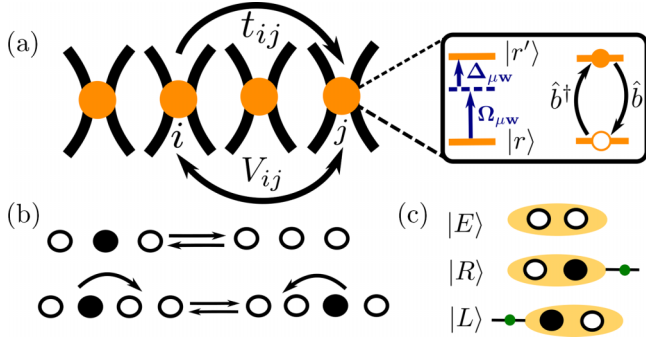


FIG. 1. (a) Schematic of a 1D system of vdW (V_{ij}) and dipolar (t_{ij}) interacting Rydberg atoms. Coupling of levels $|r\rangle \leftrightarrow |r'\rangle$ by a microwave laser with Rabi frequency $\Omega_{\mu w}$ and detuning $\Delta_{\mu w}$ corresponds to the creation (annihilation) of a boson with $\hat{b}^\dagger$ ($\hat{b}$). (b) Constrained local and nonlocal fluctuations are generated by single- and two-site processes. (c) Dimer degrees of freedom are given by combining two sites into one, which makes up a single dimer site. Auxiliary fermions are at links connecting dimer sites to ensure the occupation constraint.

(EBHM) [42,44] as given in the following,

$$\hat{H}_{\text{Ryd}} = \sum_{i < j} t_{ij} (\hat{b}_i^\dagger \hat{b}_j + \text{H.c.}) + \sum_{i < j} V_{ij} \hat{n}_i \hat{n}_j - \Delta_{\mu w} \sum_i \hat{n}_i + \Omega_{\mu w} \sum_i (\hat{b}_i^\dagger + \hat{b}_i), \quad (1)$$

where $\hat{b}_i^\dagger$ ($\hat{b}_i$) is the bosonic creation (annihilation) operator at site i with $(\hat{b}_i^\dagger)^2 = 0$, and $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ is the number operator. vdW interactions give rise to long-range density interaction with strength $V_{ij} > 0$ with $1/|i - j|^6$ scaling. The detuning $\Delta_{\mu w}$ term corresponds to the chemical potential which controls the density of excitations $|r'\rangle$ (bosons). Differing from other EBHMs, Eq. (1) includes both local and nonlocal quantum fluctuations that are provided by the Rabi term and the dipolar interactions involving single-site and two-site processes, respectively. The Rabi term $\Omega_{\mu w}$ introduces particle fluctuations at a site with creation/annihilation processes. The dipolar interactions encode particle exchange involving two sites with strength $t_{ij} > 0$ with $1/|i - j|^3$ scaling. Such single- and two-site processes lead to the local and nonlocal melting of crystalline phases into a disordered and a Luttinger liquid phase [42,53], respectively.

In the classical limit ($\Omega_{\mu w}, t \rightarrow 0$) of Eq. (1) with only nearest-neighbor interactions and $\Delta_{\mu w} = 0$, all the configurations without nearest-neighbor boson occupation are degenerate with energy $E = 0$ and the degeneracy scales exponentially as $e^{L \ln \alpha}$ with the system size L and $\alpha = (\sqrt{5} + 1)/2$ [54,55]. This can form the required degenerate classical ground-state manifold in the limit of strong nearest-neighbor interactions ($V_{i,i+1} \rightarrow \infty$) where the Hilbert space is constrained such that configurations with boson occupation in nearest-neighbor sites such as $|\cdots \bullet \bullet \cdots\rangle$ are excluded. Therefore, the degenerate manifold is provided by the occupation constraint, differing from the models with frustration. The resulting model in this projected subspace is obtained perturbatively [56]. With vanishing longer-range

terms ($t_{i,i+k}, V_{i,i+k} = 0, k > 1$) and detuning ($\Delta_{\mu w} = 0$), the Hamiltonian up to first order is expressed as

$$\hat{H} = \Omega_{\mu w} \sum_i \hat{P}_{i-1} (\hat{b}_i^\dagger + \hat{b}_i) \hat{P}_{i+1} + t \sum_i \hat{P}_{i-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}) \hat{P}_{i+2}, \quad (2)$$

where $\hat{P}_i = |\circ\rangle_i \langle \circ|$ is the projector operator to the absence of a boson. In the first term, dressing the local fluctuations with the projectors leads to the well-studied PXP Hamiltonian [47–49,57–59]. It involves single-site processes [Fig. 1(b)] where creation/annihilation at a given site is allowed if empty sites surround it. Configurations with different particle numbers are coupled; therefore, $U(1)$ symmetry is violated. In the second term, dressing the nonlocal fluctuations [Fig. 1(b)] with the projectors gives rise to constrained hoppings [60,61] where empty sites must surround the two sites involved in the particle exchange. Such processes preserve $U(1)$ symmetry since the particle number is conserved. The interplay between local and nonlocal fluctuations dictates the ground-state properties of the model. Differing from previous models, this aspect will play a crucial part in stabilizing long-range order in 1D, which has been previously reported to exist only in higher-dimensional constrained/frustrated models [62,63].

We provide a theoretical description of the model to capture the ground-state properties, where we split the lattice into interacting dimers consisting of two sites [56]. With this partitioning, a single dimer consists of two sites and encodes the dimer degrees of freedom with the allowed configurations given by $\{|\circ\circ\rangle, |\circ\bullet\rangle, |\bullet\circ\rangle\}$. We attach a fermion to the left (right) link of the configuration $|\bullet\circ\rangle$ ($|\circ\bullet\rangle$), which we denote as left (right)-dimer $|L\rangle$ ($|R\rangle$), and the configuration $|\circ\circ\rangle$ corresponds to the empty dimer $|E\rangle$ [Fig. 1(c)]. Fermionic links naturally implement the occupation constraint where the configurations of the form $|\cdots RL \cdots\rangle$ are excluded due to the Pauli exclusion principle. Motivated by this formulation, we propose an ansatz to describe the ground state in the following form,

$$|\Psi\rangle = \frac{1}{\mathcal{N}} \sum_{\Lambda} u^{N_E^{\Lambda}} v^{N_L^{\Lambda}} w^{N_R^{\Lambda}} |\Lambda\rangle, \quad (3)$$

where $N_{E,L,R}^{\Lambda}$ denotes the number of empty, left, and right dimers in the many-body configuration $|\Lambda\rangle$ of dimer states. The variational parameters are u , v , and w , and $\mathcal{N}$ is the normalization constant. The chosen ansatz is well suited to capture the ground-state properties as it addresses various aspects of the system: (i) Configurations $|\Lambda\rangle$ automatically satisfy the occupation constraint. (ii) The coherent superposition form of the ansatz can resonate a subset of classical configurations $|\Lambda\rangle$ which may be favored by the quantum fluctuations. (iii) The superposition can include configurations with different numbers of left/right dimers. Therefore, the ansatz can interpolate between $U(1)$ -preserving/ $U(1)$ -violating regions accounting for the effects of both local and nonlocal fluctuations. (iv) The weights of the configurations $|\Lambda\rangle$ in the superposition are determined according to the number of dimer types therein. Therefore, the correlations

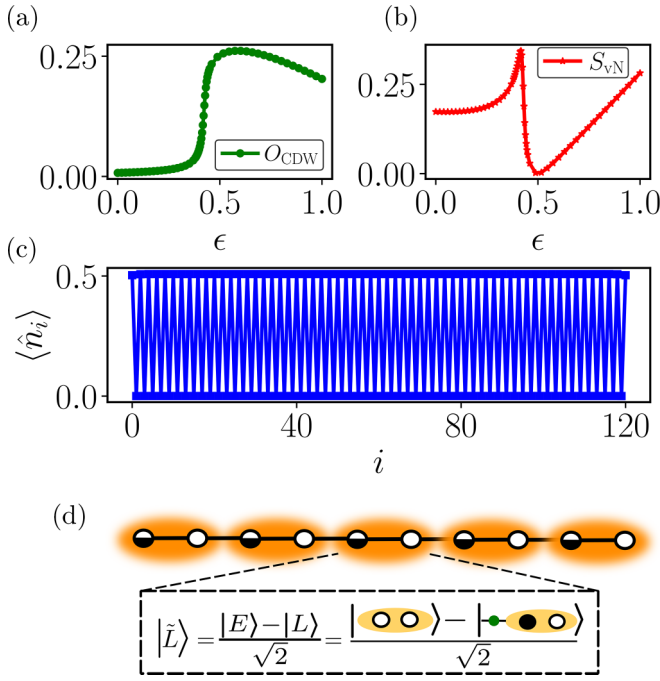


FIG. 2. (a) CDW order parameter O_{CDW} and the (b) bipartite von Neumann entanglement entropy S_{vN} as a function of $\epsilon = t/\Omega_{\mu w}$ is depicted for a system size of $L = 121$. (c) The expectation value of the density operator $\hat{n}_i$ for $L = 121$ with $\epsilon = 0.5$ is shown. (d) The schematic of the ground state with CDW order is displayed. The unit cell is given by the equal superposition of $|E\rangle$ and $|L\rangle$ as $|\tilde{L}\rangle = (|E\rangle - |L\rangle)/\sqrt{2}$.

emanating from the fluctuations of the dimer densities in a given $|\Lambda\rangle$ can be captured.

Results. We demonstrate that even in a simple 1D lattice, the combination of local- and nonlocal quantum fluctuations helps stabilize long-range order, even though they do not promote order individually. The nature of the QPT is examined by performing finite-size-scaling analyses. Numerical results are obtained by employing density-matrix-renormalization-group [64–68] simulations.

In the limit $\epsilon = t/\Omega_{\mu w} \rightarrow 0$, local quantum fluctuations become dominant and lift the degeneracy by resonating classical configurations. In this mechanism, the classical configurations from the degenerate manifold are selected such that their superposition state can have low-energy fluctuations when acted upon by the $\Omega_{\mu w}$ term. For example, the configuration $|\bullet \circ \cdots \bullet \circ\rangle$ has high-energy fluctuations in the sense that applying $\Omega_{\mu w}$ terms at neighboring sites causes coupling to constraint-violating states (high-energy states). Therefore, the classical configurations with a high density of low-energy mode are favored when acted upon by the $\Omega_{\mu w}$ term. In one-dimension, this stabilizes a disordered phase exhibiting homogeneous density without long-range order. This can be seen in Fig. 2(a) with vanishing $O_{\text{CDW}} = (1/L) \sum_i (-1)^i \langle \hat{n}_i \rangle$ which measures the staggered density. To gain more insight into the superposition, we compute the overlap of the ansatz $|\Psi\rangle$ in Eq. (3) with the exact ground state $|\Psi_{\text{ED}}\rangle$ and obtain $|\langle \Psi | \Psi_{\text{ED}} \rangle| \sim 0.99$ with the variational parameters $u = 0.92$, $v = -0.58$, and $w = -0.58$ [56]. Equal weight given to the left-dimer/right-dimer state demonstrates no

preference for selecting either side. We find that the variationally optimized ansatz $|\Psi\rangle$ can be composed by $\bigotimes_i |\mathcal{D}\rangle_i$, with $|\mathcal{D}\rangle_i = [(|E\rangle_i/\sqrt{2}) - (|L\rangle_i + |R\rangle_i)/2]$ and i denoting a single dimer site [56]. This implies that each dimer site has an equal probability of being in $|L\rangle$ or $|R\rangle$. In light of this, the role of the geometry becomes apparent when the state selection mechanism from the degenerate manifold is considered. Due to the low connectivity in one dimension, both the left- and right-dimer dominant $|\Lambda\rangle$ configurations have low-energy fluctuations due to the $\Omega_{\mu w}$ term; thereby, the disordered phase with equal favoring of $|L\rangle$ and $|R\rangle$ is promoted. Compared to 2D lattices where order-by-disorder has been extensively studied, configurations that equally favor $|L\rangle$ and $|R\rangle$ are not seen as in our 1D case. For example, the square lattice limits the accessible $|\Lambda\rangle$ configurations from the degenerate manifold due to the occupation constraint and increased connectivity. The system cannot facilitate having a superposition of $|L\rangle$ and $|R\rangle$ in the form $|\mathcal{D}\rangle$ in each dimer site since this leads to configurations with high-energy fluctuations. Considering the square lattice as stacked 1D chains, the $\Omega_{\mu w}$ term gives rise to configurations of the form $|\tilde{L}\rangle$ or $|\tilde{R}\rangle$ in the neighboring rows where the occupation constraint is violated. Consequently, the geometric structure of the square lattice restricts the classical ground-state manifold more severely, resulting in fewer available states compared to the one-dimensional case. This leads to spontaneous symmetry breaking with Neel (checkerboard) order [62] with a square-lattice configuration with alternating rows of $|\tilde{R}\rangle = (|E\rangle - |R\rangle)/\sqrt{2}$ and $|\tilde{L}\rangle = (|E\rangle - |L\rangle)/\sqrt{2}$.

In the regime $\epsilon \sim 0.5$ –1, both fluctuation terms are comparable in magnitude. This results in a crystalline phase with long-range order. In Fig. 2(a), finite values of O_{CDW} imply that the unit cells double and the translational symmetry is spontaneously broken. This is also corroborated by the behavior of the bipartite von Neumann entanglement entropy $S_{\text{vN}} \equiv -\text{Tr}(\rho_r \ln \rho_r)$ of the ground state as a function of ϵ , where ρ_r is the reduced density matrix of half of the chain. As seen in Fig. 2(b), S_{vN} makes a peak near the quantum critical point (QCP) and drops in the ordered phase. This vanishing of S_{vN} around $\epsilon \sim 0.5$ [Fig. 2(b)] implies that the ordered phase becomes an exact product state. As $\epsilon \rightarrow 1$, the ordered state becomes entangled yet still stays close to a product state form. CDW character is also reflected in the alternating density oscillations which imply that the bosons occupy every alternating site on the chain as shown in Fig. 2(c). Differing from conventional CDW phases, the unit cell is given by $|\tilde{L}\rangle$ or $|\tilde{R}\rangle$ instead of $|L\rangle$ or $|R\rangle$ as given in Fig. 2(d). We reveal the ordering mechanism by introducing both local and non-local fluctuations to the ordered system. This can be inferred from the coupling of the unit cell $|\tilde{L}\rangle = (|E\rangle - |L\rangle)/\sqrt{2}$ to $[(\Omega_{\mu w} - t)|R\rangle - \Omega_{\mu w}(|E\rangle - |L\rangle)]/\sqrt{2}$. Specifically, the individual components $|E\rangle$ and $|L\rangle$ couple to $|R\rangle$ via local fluctuations and nonlocal fluctuations (Sec.2 in Ref. [56]), respectively. Therefore, the system can be further constrained to prefer left dimers by tuning t to lower the coupling to $|R\rangle$. This also explains why having $t < 0$ in Eq. (2) does not help restore the long-range order but rather promotes the disordered state (Sec. 2 in Ref. [56]). Therefore, having nonlocal fluctuations mimics the increased connectivity of two dimensions by lowering the coupling of $|\tilde{L}\rangle$ ($|\tilde{R}\rangle$) to $|R\rangle$ ($|L\rangle$). This is also

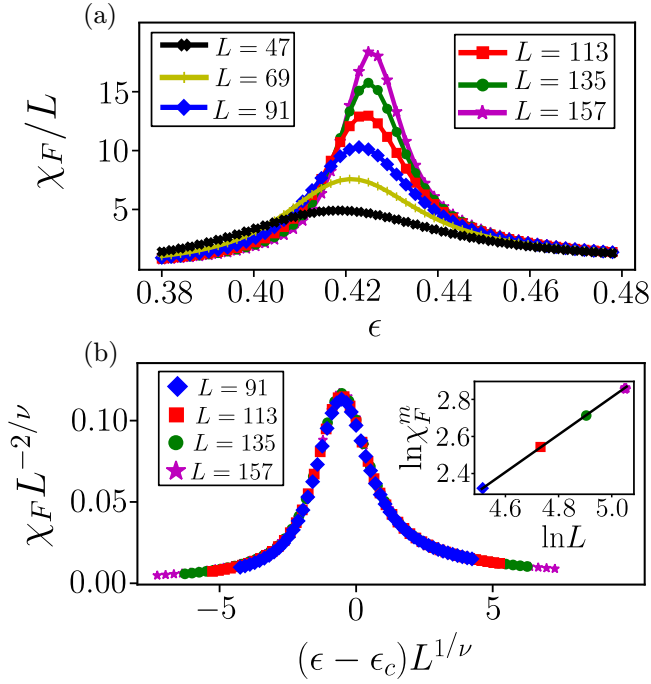


FIG. 3. (a) The fidelity susceptibility per site χ_F/L exhibits a peak around the critical point. The peak becomes more pronounced as the system size L gets larger. (b) Rescaled χ_F displays a data collapse for the critical point $\epsilon_c = 0.429$ and the critical exponent $\nu = 1.011$ for different system sizes. Fidelity susceptibility at pseudocritical points χ_F^m as a function of system size in logarithmic scale is shown in the inset.

corroborated by the ansatz overlap $|\langle \Psi | \Psi_{ED} \rangle| \sim 0.99$ with variational parameters $u = 0.92$, $v = -0.92$, and $w = 0.01$ (Sec. 2 in Ref. [56]), where vanishing w indicates that $|\Psi\rangle$ is favored. Therefore, the ansatz leads to a product state of the form $\bigotimes_i |\tilde{L}\rangle_i$. Restoring order in one dimension requires both local and nonlocal fluctuations, a key aspect that sets our setup apart from typical constrained models in conventional Rydberg simulators.

In the unconstrained case with $V > \Delta_{\mu w} > \Omega_{\mu w}$ and $t = 0$, the transition from a disordered to an ordered phase occurs via the Ising transition [69]. Although fluctuations work against ordering, we show that the same transition takes place in our constrained model by performing finite-size analysis of the fidelity susceptibility $\chi_F = \lim_{d\epsilon \rightarrow 0} 2[1 - \langle \phi(\epsilon) | \phi(\epsilon + d\epsilon) \rangle] / (d\epsilon)^2$ [69–71], where $|\phi(\epsilon)\rangle$ is the ground state of $\hat{H}(\epsilon)$ in Eq. (2). χ_F probes the QPT by detecting the abrupt change in the ground-state character as ϵ varies. This can be seen in Fig. 3(a) where χ_F/L makes a peak around the QCP with the peak becoming more pronounced as the system size L increases. This divergent behavior is captured in the inset of Fig. 3(b) by analyzing the power-law behavior of the fidelity susceptibility given by $\chi_F^m(\epsilon_m, L) \propto L^{2/\nu}$ [70,72] at pseudocritical points ϵ_m , where ν is the correlation critical exponent and is extracted as $\nu = 1.011$ by the slope of the fitted line. To locate the QCP, the finite-size-scaling law for χ_F in the form $\chi_F(\epsilon, L) = L^{2/\nu} \mathcal{F}_F[L^{1/\nu}(\epsilon - \epsilon_c)]$ is analyzed, where ϵ_c is the QCP and $\mathcal{F}_F$ is an unknown scaling function [72,73]. Figure 3(b) displays the behavior of the scaling function $\mathcal{F}_F$

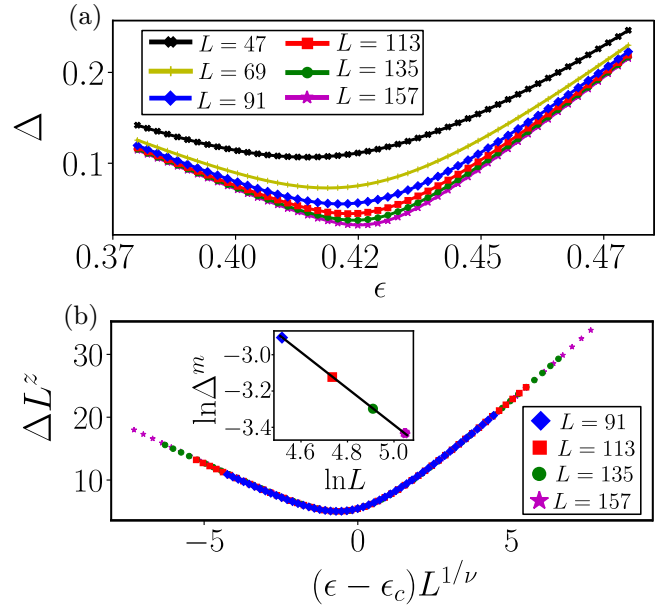


FIG. 4. (a) Δ forms a dip in the vicinity of the critical point. As the system size L gets larger, Δ approaches 0, thus making the dip sharper. (b) Rescaled Δ exhibits a data collapse for the critical point $\epsilon_c = 0.429$, the correlation critical exponent $\nu = 1.011$, and the dynamic critical exponent $z = 1.001$ for various system sizes. The inset depicts the neutral gap at pseudocritical points Δ^m as a function of the system size in logarithmic scale.

for various system sizes by using rescaled variables $\chi_F L^{-2/\nu}$ and $(\epsilon - \epsilon_c) L^{1/\nu}$. A good data collapse is achieved by substituting the previously obtained exponent $\nu = 1.011$ and tuning the $\epsilon_c = 0.429$ as shown in Fig. 3(b). A similar analysis of the neutral gap $\Delta = E_1 - E_0$ for the scaling forms $\Delta^m(\epsilon_m, L) \propto L^{-z}$ and $\Delta(\epsilon, L) = L^{-z} \mathcal{F}_\Delta[L^{1/\nu}(\epsilon - \epsilon_c)]$ is performed [74]. Here, E_0 and E_1 are the ground-state and first-excited-state energy of $\hat{H}$ in Eq. (2). The dynamical critical exponent is determined as $z = 1.001$ as shown in Figs. 4(a) and 4(b). We conclude that the order-by-disorder transition belongs to the $(1+1)$ D Ising universality class with $\nu = 1$ and $z = 1$. The key difference from earlier results is that combining local and nonlocal fluctuations drives the transition in the disordered-to-ordered direction where the long-range order is restored. In the constrained model, the vdW interaction is approximated using only nearest-neighbor terms. Therefore, the effects of the vdW tail should be minimized such that longer-range density-density interaction terms cannot lift the degeneracy and suppress the quantum fluctuations tuned by $\Omega_{\mu w}$ and t . To achieve this, large interatomic distances and high principle quantum numbers should be considered as elaborated in Sec. 3 of Ref. [56].

Conclusion and outlook. While quantum fluctuations give rise to a wide range of phenomena in frustrated systems, we study their role in frustration-free systems using strongly interacting Rydberg atoms. A constrained model is derived which differs from previous models [47,57] by putting local and nonlocal fluctuations on an equal footing. The order-by-disorder mechanism in one dimension requires both fluctuations and is well captured using the variational ansatz within our dimer description of the model. For future works, it

would be interesting to see the physics arising from combining local and nonlocal fluctuations in higher-dimensional lattices with occupation constraints.

Acknowledgments. This work is funded by the Cluster of Excellence “CUI: Advanced Imaging of Matter” of the

Deutsche Forschungsgemeinschaft (DFG), EXC 2056, Project ID No. 390715994. This work is funded by the German Federal Ministry of Education and Research within the funding program “Quantum Technologies—From Basic Research to Market” under Contract No. 13N16138.

- [1] P. W. Anderson and W. F. Brinkman, Anisotropic superfluidity in ^3He : A possible interpretation of its stability as a spin-fluctuation effect, *Phys. Rev. Lett.* **30**, 1108 (1973).
- [2] W. F. Brinkman, J. W. Serene, and P. W. Anderson, Spin-fluctuation stabilization of anisotropic superfluid states, *Phys. Rev. A* **10**, 2386 (1974).
- [3] D. Fay and J. Appel, Coexistence of p -state superconductivity and itinerant ferromagnetism, *Phys. Rev. B* **22**, 3173 (1980).
- [4] E. F. Shender and P. C. W. Holdsworth, Order by disorder and topology in frustrated magnetic systems, in *Fluctuations and Order: The New Synthesis*, Institute for Nonlinear Science, edited by M. Millonas (Springer US, New York, 1996), pp. 259–279.
- [5] A. G. Green, G. Conduit, and F. Krüger, Quantum order-by-disorder in strongly correlated metals, *Annu. Rev. Condens. Matter Phys.* **9**, 59 (2018).
- [6] Y. Zhou, K. Kanoda, and T.-K. Ng, Quantum spin liquid states, *Rev. Mod. Phys.* **89**, 025003 (2017).
- [7] L. Balents, Spin liquids in frustrated magnets, *Nature (London)* **464**, 199 (2010).
- [8] C. M. F. Lacroix and P. Mendels, *Introduction to Frustrated Magnetism*, Springer Series in Solid-State Sciences (Springer, Berlin, 2011), Vol. 164.
- [9] S. Jiang, J. Romhányi, S. R. White, M. E. Zhitomirsky, and A. L. Chernyshev, Where is the quantum spin nematic? *Phys. Rev. Lett.* **130**, 116701 (2023).
- [10] Y. Kohama, H. Ishikawa, A. Matsuo, K. Kindo, N. Shannon, and Z. Hiroi, Possible observation of quantum spin-nematic phase in a frustrated magnet, *Proc. Natl. Acad. Sci. USA* **116**, 10686 (2019).
- [11] A. Mulder, R. Ganesh, L. Capriotti, and A. Paramekanti, Spiral order by disorder and lattice nematic order in a frustrated Heisenberg antiferromagnet on the honeycomb lattice, *Phys. Rev. B* **81**, 214419 (2010).
- [12] J. Villain, R. Bidaux, J.-P. Carton, and R. Conte, Order as an effect of disorder, *J. Phys. Fr.* **41**, 1263 (1980).
- [13] C. L. Henley, Ordering due to disorder in a frustrated vector antiferromagnet, *Phys. Rev. Lett.* **62**, 2056 (1989).
- [14] C. L. Henley, Ordering by disorder: Ground-state selection in fcc vector antiferromagnets, *J. Appl. Phys.* **61**, 3962 (1987).
- [15] S. Jiang, S. R. White, and A. L. Chernyshev, Quantum phases in the honeycomb-lattice J_1 – J_3 ferro-antiferromagnetic model, *Phys. Rev. B* **108**, L180406 (2023).
- [16] S. Sachdev, Kagomé- and triangular-lattice Heisenberg antiferromagnets: Ordering from quantum fluctuations and quantum-disordered ground states with unconfined bosonic spinons, *Phys. Rev. B* **45**, 12377 (1992).
- [17] O. A. Starykh, Unusual ordered phases of highly frustrated magnets: a review, *Rep. Prog. Phys.* **78**, 052502 (2015).
- [18] J. G. Rau and M. J. Gingras, Frustrated quantum rare-earth pyrochlores, *Annu. Rev. Condens. Matter Phys.* **10**, 357 (2019).
- [19] V. Grison, P. Viot, B. Bernu, and L. Messio, Emergent Potts order in the kagome J_1 – J_3 Heisenberg model, *Phys. Rev. B* **102**, 214424 (2020).
- [20] H. Li and T. Li, Competing emergent Potts orders and possible nematic spin liquids in the kagome J_1 – J_3 Heisenberg model, *Phys. Rev. B* **106**, 035112 (2022).
- [21] S. Khatua, M. J. P. Gingras, and J. G. Rau, Pseudo-goldstone modes and dynamical gap generation from order by thermal disorder, *Phys. Rev. Lett.* **130**, 266702 (2023).
- [22] J. Pitts, F. L. Buessen, R. Moessner, S. Trebst, and K. Shtengel, Order by disorder in classical kagome antiferromagnets with chiral interactions, *Phys. Rev. Res.* **4**, 043019 (2022).
- [23] M. Gohlke, L. E. Chern, H.-Y. Kee, and Y. B. Kim, Emergence of nematic paramagnet via quantum order-by-disorder and pseudo-goldstone modes in Kitaev magnets, *Phys. Rev. Res.* **2**, 043023 (2020).
- [24] E. Sela, H.-C. Jiang, M. H. Gerlach, and S. Trebst, Order-by-disorder and spin-orbital liquids in a distorted Heisenberg-Kitaev model, *Phys. Rev. B* **90**, 035113 (2014).
- [25] J. Willsher, H.-K. Jin, and J. Knolle, Magnetic excitations, phase diagram, and order-by-disorder in the extended triangular-lattice Hubbard model, *Phys. Rev. B* **107**, 064425 (2023).
- [26] D. Yamamoto, C. Suzuki, G. Marmorini, S. Okazaki, and N. Furukawa, Quantum and thermal phase transitions of the triangular $\text{SU}(3)$ Heisenberg model under magnetic fields, *Phys. Rev. Lett.* **125**, 057204 (2020).
- [27] K. A. Ross, Y. Qiu, J. R. D. Copley, H. A. Dabkowska, and B. D. Gaulin, Order by disorder spin wave gap in the XY pyrochlore magnet $\text{Er}_2\text{Ti}_2\text{O}_7$, *Phys. Rev. Lett.* **112**, 057201 (2014).
- [28] A. W. C. Wong, Z. Hao, and M. J. P. Gingras, Ground state phase diagram of generic xy pyrochlore magnets with quantum fluctuations, *Phys. Rev. B* **88**, 144402 (2013).
- [29] M. J. P. Gingras and P. A. McClarty, Quantum spin ice: a search for gapless quantum spin liquids in pyrochlore magnets, *Rep. Prog. Phys.* **77**, 056501 (2014).
- [30] V. Lienhard, S. de Léséleuc, D. Barredo, T. Lahaye, A. Browaeys, M. Schuler, L.-P. Henry, and A. M. Läuchli, Observing the space- and time-dependent growth of correlations in dynamically tuned synthetic Ising models with antiferromagnetic interactions, *Phys. Rev. X* **8**, 021070 (2018).
- [31] E. Guardado-Sanchez, P. T. Brown, D. Mitra, T. Devakul, D. A. Huse, P. Schauß, and W. S. Bakr, Probing the quench dynamics of antiferromagnetic correlations in a 2D quantum Ising spin system, *Phys. Rev. X* **8**, 021069 (2018).
- [32] P. Scholl, H. J. Williams, G. Bornet, F. Wallner, D. Barredo, L. Henriot, A. Signoles, C. Hainaut, T. Franz, S. Geier, A. Tebben, A. Salzinger, G. Zürn, T. Lahaye, M. Weidemüller, and A. Browaeys, Microwave engineering of programmable xxz Hamiltonians in arrays of Rydberg atoms, *PRX Quantum* **3**, 020303 (2022).

- [33] G. Abumwis, M. T. Eiles, and A. Eisfeld, Extended coherently delocalized states in a frozen Rydberg gas, *Phys. Rev. Lett.* **124**, 193401 (2020).
- [34] S. Ebadi, T. T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, W. W. Ho, S. Choi, S. Sachdev, M. Greiner, V. Vuletić, and M. D. Lukin, Quantum phases of matter on a 256-atom programmable quantum simulator, *Nature (London)* **595**, 227 (2021).
- [35] P. Scholl, M. Schuler, H. J. Williams, A. A. Eberharter, D. Barredo, K.-N. Schymik, V. Lienhard, L.-P. Henry, T. C. Lang, T. Lahaye, A. M. Läuchli, and A. Browaeys, Quantum simulation of 2D antiferromagnets with hundreds of Rydberg atoms, *Nature (London)* **595**, 233 (2021).
- [36] H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, A Rydberg quantum simulator, *Nat. Phys.* **6**, 382 (2010).
- [37] A. Browaeys, D. Barredo, and T. Lahaye, Experimental investigations of dipole-dipole interactions between a few Rydberg atoms, *J. Phys. B: At., Mol. Opt. Phys.* **49**, 152001 (2016).
- [38] A. Browaeys and T. Lahaye, Many-body physics with individually controlled Rydberg atoms, *Nat. Phys.* **16**, 132 (2020).
- [39] S. Whitlock, A. W. Glaetzle, and P. Hannaford, Simulating quantum spin models using Rydberg-excited atomic ensembles in magnetic microtrap arrays, *J. Phys. B: At., Mol. Opt. Phys.* **50**, 074001 (2017).
- [40] R. Verresen, M. D. Lukin, and A. Vishwanath, Prediction of toric code topological order from Rydberg blockade, *Phys. Rev. X* **11**, 031005 (2021).
- [41] G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletić, and M. D. Lukin, Probing topological spin liquids on a programmable quantum simulator, *Science* **374**, 1242 (2021).
- [42] Z. Zeybek, R. Mukherjee, and P. Schmelcher, Quantum phases from competing Van der Waals and dipole-dipole interactions of Rydberg atoms, *Phys. Rev. Lett.* **131**, 203003 (2023).
- [43] Z. Zeybek, P. Schmelcher, and R. Mukherjee, Bond-order density wave phases in dimerized extended Bose-Hubbard models, *Phys. Rev. B* **110**, 075111 (2024).
- [44] S. de Léséleuc, V. Lienhard, P. Scholl, D. Barredo, S. Weber, N. Lang, H. P. Büchler, T. Lahaye, and A. Browaeys, Observation of a symmetry-protected topological phase of interacting bosons with Rydberg atoms, *Science* **365**, 775 (2019).
- [45] A. Signoles, T. Franz, R. Ferracini Alves, M. Gärttner, S. Whitlock, G. Zürn, and M. Weidemüller, Glassy dynamics in a disordered Heisenberg quantum spin system, *Phys. Rev. X* **11**, 011011 (2021).
- [46] L. Homeier, S. Hollerith, S. Geier, N.-C. Chiu, A. Browaeys, and L. Pollet, Supersolidity in Rydberg tweezer arrays, *arXiv:2407.12752*.
- [47] P. Fendley, K. Sengupta, and S. Sachdev, Competing density-wave orders in a one-dimensional hard-boson model, *Phys. Rev. B* **69**, 075106 (2004).
- [48] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations, *Phys. Rev. B* **98**, 155134 (2018).
- [49] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Weak ergodicity breaking from quantum many-body scars, *Nat. Phys.* **14**, 745 (2018).
- [50] H. Weimer and H. P. Büchler, Two-stage melting in systems of strongly interacting Rydberg atoms, *Phys. Rev. Lett.* **105**, 230403 (2010).
- [51] E. Sela, M. Punk, and M. Garst, Dislocation-mediated melting of one-dimensional Rydberg crystals, *Phys. Rev. B* **84**, 085434 (2011).
- [52] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, Probing many-body dynamics on a 51-atom quantum simulator, *Nature (London)* **551**, 579 (2017).
- [53] Z. Lan, W. Li, and I. Lesanovsky, Quantum melting of two-component Rydberg crystals, *Phys. Rev. A* **94**, 051603(R) (2016).
- [54] C. Domb, On the theory of cooperative phenomena in crystals, *Adv. Phys.* **9**, 149 (1960).
- [55] R. J. Baxter, Exactly solved models in statistical mechanics, in *Integrable Systems in Statistical Mechanics* (World Scientific, Singapore, 1985).
- [56] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevResearch.7.L012009> for the derivation of the constrained model, details on the dimer description and the variational ansatz with additional numerical results, details on the experimental parameters, and which includes Refs. [75–81].
- [57] V. Khemani, C. R. Laumann, and A. Chandran, Signatures of integrability in the dynamics of Rydberg-blockaded chains, *Phys. Rev. B* **99**, 161101(R) (2019).
- [58] F. M. Surace, P. P. Mazza, G. Giudici, A. Lerose, A. Gambassi, and M. Dalmonte, Lattice gauge theories and string dynamics in Rydberg atom quantum simulators, *Phys. Rev. X* **10**, 021041 (2020).
- [59] D. Bluvstein, A. Omran, H. Levine, A. Keesling, G. Semeghini, S. Ebadi, T. T. Wang, A. A. Michailidis, N. Maskara, W. W. Ho, S. Choi, M. Serbyn, M. Greiner, V. Vuletić, and M. D. Lukin, Controlling quantum many-body dynamics in driven Rydberg atom arrays, *Science* **371**, 1355 (2021).
- [60] F. C. Alcaraz and R. Z. Bariev, An exactly solvable constrained xxz chain, *arXiv:cond-mat/9904042*.
- [61] R. Verresen, A. Vishwanath, and F. Pollmann, Stable Luttinger liquids and emergent $U(1)$ symmetry in constrained quantum chains, *arXiv:1903.09179*.
- [62] M. Yue, Z. Wang, B. Mukherjee, and Z. Cai, Order by disorder in frustration-free systems: Quantum Monte Carlo study of a two-dimensional PXP model, *Phys. Rev. B* **103**, L201113 (2021).
- [63] M. Sarkar, M. Pal, A. Sen, and K. Sengupta, Quantum order-by-disorder induced phase transition in Rydberg ladders with staggered detuning, *SciPost Phys.* **14**, 004 (2023).
- [64] J. Hauschild and F. Pollmann, Efficient numerical simulations with Tensor Networks: Tensor Network Python (TeNPy), *SciPost Phys. Lect. Notes* **5** (2018).
- [65] S. R. White, Density matrix formulation for quantum renormalization groups, *Phys. Rev. Lett.* **69**, 2863 (1992).
- [66] S. R. White, Density-matrix algorithms for quantum renormalization groups, *Phys. Rev. B* **48**, 10345 (1993).
- [67] U. Schollwöck, The density-matrix renormalization group, *Rev. Mod. Phys.* **77**, 259 (2005).

- [68] U. Schollwöck, The density-matrix renormalization group in the age of matrix product states, *Ann. Phys.* **326**, 96 (2011).
- [69] X.-J. Yu, S. Yang, J.-B. Xu, and L. Xu, Fidelity susceptibility as a diagnostic of the commensurate-incommensurate transition: A revisit of the programmable Rydberg chain, *Phys. Rev. B* **106**, 165124 (2022).
- [70] W.-L. You, Y.-W. Li, and S.-J. Gu, Fidelity, dynamic structure factor, and susceptibility in critical phenomena, *Phys. Rev. E* **76**, 022101 (2007).
- [71] S. Chen, L. Wang, Y. Hao, and Y. Wang, Intrinsic relation between ground-state fidelity and the characterization of a quantum phase transition, *Phys. Rev. A* **77**, 032111 (2008).
- [72] A. F. Albuquerque, F. Alet, C. Sire, and S. Capponi, Quantum critical scaling of fidelity susceptibility, *Phys. Rev. B* **81**, 064418 (2010).
- [73] S.-J. Gu, H.-M. Kwok, W.-Q. Ning, and H.-Q. Lin, Fidelity susceptibility, scaling, and universality in quantum critical phenomena, *Phys. Rev. B* **77**, 245109 (2008).
- [74] R. Samajdar, S. Choi, H. Pichler, M. D. Lukin, and S. Sachdev, Numerical study of the chiral z_3 quantum phase transition in one spatial dimension, *Phys. Rev. A* **98**, 023614 (2018).
- [75] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University, Cambridge, England, 2011).
- [76] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1998).
- [77] S. Bravyi, D. P. DiVincenzo, and D. Loss, Schrieffer–Wolff transformation for quantum many-body systems, *Ann. Phys.* **326**, 2793 (2011).
- [78] Y. Cheng and C. Li, Gauge theory description of Rydberg atom arrays with a tunable blockade radius, *Phys. Rev. B* **107**, 094302 (2023).
- [79] Y. Cheng, C. Li, and H. Zhai, Variational approach to quantum spin liquid in a Rydberg atom simulator, *New J. Phys.* **25**, 033010 (2023).
- [80] L. Pan and H. Zhai, Composite spin approach to the blockade effect in Rydberg atom arrays, *Phys. Rev. Res.* **4**, L032037 (2022).
- [81] N. Šibalić, J. Pritchard, C. Adams, and K. Weatherill, ARC: An open-source library for calculating properties of alkali Rydberg atoms, *Comput. Phys. Commun.* **220**, 319 (2017).