

Quantum dimer models with Rydberg gadgets

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The Rydberg blockade mechanism is an important ingredient in quantum simulators based on neutral atom arrays. It enables the emergence of a rich variety of quantum phases of matter, such as topological spin liquids. The typically isotropic nature of the blockade effect, however, restricts the range of natively accessible models and quantum states. In this work, we propose a method to systematically overcome this limitation, by developing gadgets, i.e., specific arrangements of atoms, that transform the underlying Rydberg blockade into more general constraints. We apply this technique to realize dimer models on square and triangular geometries. In these setups, we study the role of the quantum fluctuations induced by a coherent drive of the atoms and find signatures of $U(1)$ and $\mathbb{Z}_2$ quantum spin liquid states in the respective ground states. Finally, we show that these states can be dynamically prepared with high fidelity, paving the way for the quantum simulation of a broader class of constrained models and topological matter in experiments with Rydberg atom arrays.

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Introduction. Preparing and studying strongly correlated many-body quantum states is one of the central goals of near-term quantum devices [1]. Quantum spin liquids (QSLs) are a particularly interesting class of such states, that feature many exotic properties, including long-range entanglement, anyonic excitations, and topological ground state degeneracy [2–5]. In addition, QSLs offer intriguing possibilities for applications in quantum information processing, where they could form the basis of effective error-correcting schemes for robust quantum memories [6] or for topological quantum computation [7–9]. Recently, remarkable progress has been achieved in realizing some of these topologically ordered states with several quantum simulator platforms, using a variety of state preparation techniques. In digital approaches, the target states are prepared by applying circuits consisting of entangling gates to simple product states [10–12]. In contrast, in analog approaches, dynamical preparation protocols with a controllable, time-dependent Hamiltonian are employed, e.g., using the adiabatic principle [13–15]. The latter has the advantage that the prepared states can potentially be stabilized by a Hamiltonian and dynamical signatures of QSL phases could be probed.

Experimentally, the analog approach to realizing QSLs in synthetic quantum matter has been pioneered with Rydberg atom arrays [13]. These platforms enable the realization of highly programable spin models that have been explored for quantum simulation [13,16–20], quantum optimization [21], and quantum computation [22–25]. In these setups, atoms are

trapped in optical tweezers or lattices and internal states are employed to represent spin degrees of freedom [26]. Control is enabled by coherent laser drive and the geometric arrangement of the atoms [27,28]. A crucial ingredient is the so-called blockade effect [29–31], which prevents two atoms from being simultaneously excited to Rydberg states when their distance is smaller than the blockade radius. This mechanism, when combined with specific geometries of the array, can be used to introduce constraints on the allowed spin configurations, e.g., to mimic Gauss’s law of lattice gauge theories [32–35]. Moreover, it is a key ingredient for providing the geometric frustration that leads to the emergence of QSLs from many-body Rydberg Hamiltonians [36–41].

The Rydberg blockade is typically isotropic, strongly restricting the class of nontrivial constraints that can be implemented in Rydberg atom systems. To overcome this limitation various approaches have been proposed, ranging from choosing anisotropic Rydberg states [42] to making use of multilayer arrays [40]. Here, we employ a method that avoids these experimental challenges by generalizing the concept of *Rydberg gadgets* that have been developed to encode arbitrary classical constraints, e.g., for quantum optimization problems [43–47]. A gadget is a particular configuration of Rydberg atoms designed to provide a specific constraint solely as a result of the isotropic Rydberg blockade and the lattice geometry. While such constraints allow us to realize the frustration that can lead to the emergence of a QSL phase [47], understanding if the accessible quantum fluctuations actually stabilize such a phase in models constructed from Rydberg gadgets is an open question.

In this work, we propose simple gadgets that encode the constraint of dimer models and study the physics associated with the corresponding quantum fluctuations. Quantum dimer models [48–53] have recently sparked revived interest after

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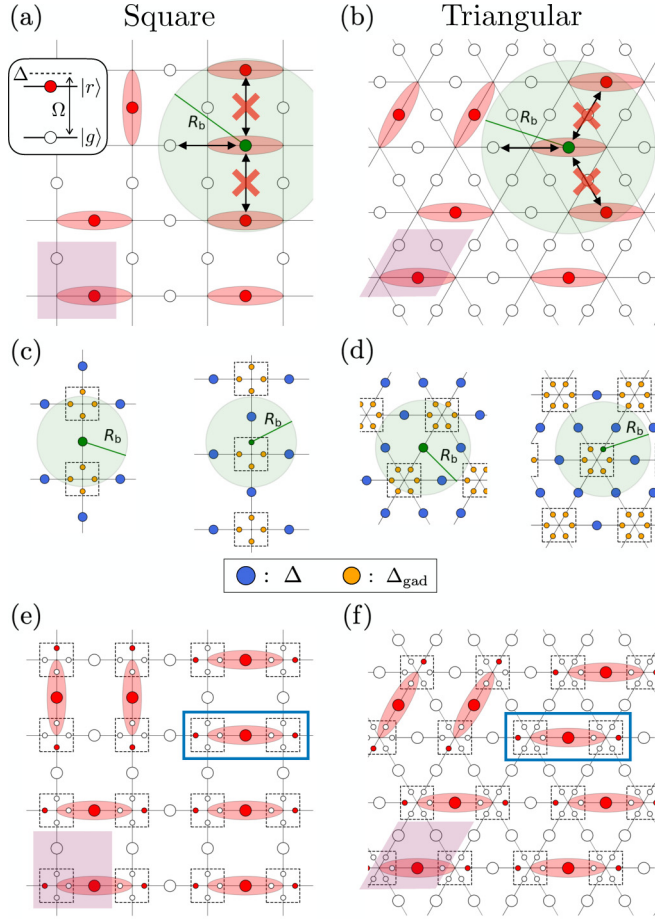


FIG. 1. Rydberg gadgets for the dimer constraint. Left and right panels correspond to the square and triangular lattices, respectively. [(a), (b)] Typical dimer fully packed configurations. Red (white) dots represent atoms in the Rydberg (ground) state [see the inset of (a)]. The red crosses indicate the undesired blockade for the selected blockade radius R_b . [(c), (d)] Illustration of the Rydberg gadgets. Gadgets are placed on the vertices of the lattice and consist of the orange atoms. The green disks indicate the blockade radius required to realize the dimer constraint. [(e), (f)] Rydberg configurations in the gadget model corresponding to the dimer coverings in (a), (d). Each dimer consists of one excited atom on the edge and two excited atoms on the gadgets at the extrema (blue rectangle). The purple-shaded square is the lattice unit cell.

the theoretical proposal [36] and corresponding experimental realization [13] with a system of Rydberg atoms positioned on the edges of a kagome lattice, realizing what we call “diluted dimer constraint” (see below). Here, we focus on geometries where this constraint is not natively provided by the Rydberg blockade [cf. Figs. 1(a) and 1(b)], namely the square and triangular lattices. We show that, in the Rydberg gadget model, i.e., the system in which gadgets are placed on the vertices of such lattices, the diluted dimer constraint is recovered in the limit of large density of Rydberg excitations. We address the effect of quantum fluctuations via analytical perturbation theory and numerical exact diagonalization (ED) of the Rydberg atom Hamiltonian. We analyze the ground state phase diagrams of these systems and demonstrate the onset of $U(1)$ and $\mathbb{Z}_2$ QSLs, respectively on the square and

triangular lattice. Moreover, by studying dynamical protocols to prepare them, we showcase the experimental relevance of our approach.

Diluted dimer models in Rydberg atom arrays. In a Rydberg atom array, neutral atoms are typically trapped into a configurable lattice geometry and approximated as two-level systems consisting of the ground state $|g\rangle$ and a highly excited Rydberg state $|r\rangle$. Each atom is driven by a coherent laser field with Rabi frequency Ω and detuning Δ . The Hamiltonian for this system is [26,54–56]

$$\hat{H}_{\text{Ryd}} = \Omega \sum_i \hat{\sigma}_i^x - \sum_i \Delta_i \hat{n}_i + \sum_{ij} U_{ij} \hat{n}_i \hat{n}_j, \quad (1)$$

where $\hat{\sigma}^x = |r\rangle\langle g| + |g\rangle\langle r|$, $\hat{n} = |r\rangle\langle r|$, and i labels the position $\mathbf{x}_i$ of each atom. The interaction is of the van der Waals form $U_{ij} \sim |\mathbf{x}_i - \mathbf{x}_j|^{-6}$, which is strongly repulsive at short distances and rapidly decays at large distances. The strong distance dependence of the interaction strength allows for a blockade approximation, where interactions within the so-called blockade radius R_b are assumed to be infinite and can be treated as the constraint $\hat{n}_i \hat{n}_j = 0$ for $|\mathbf{x}_i - \mathbf{x}_j| < R_b$, while interactions at larger distances are neglected.

In the blockade approximation, for certain atom arrangements and a properly chosen R_b , the Rydberg atom system can be exactly mapped into a dimer model subject to the diluted dimer constraint, where *at most* one dimer connects to each vertex of a lattice. The mapping consists of interpreting the atom positions as the midpoint of the edges of a lattice and identifying a Rydberg excitation with the presence of a dimer on the corresponding edge. This correspondence however only holds on certain geometries, such as the ruby [36] and kagome [15] lattices, which are equivalent to dimer models on the kagome and honeycomb lattices, respectively.

The effective Hamiltonian induced on the dimer degrees of freedom by Eq. (1) is given by $\hat{H} = \Omega \sum_i \hat{\sigma}_i^x - \Delta \sum_i \hat{n}_i$, where i labels the edges and $\hat{\sigma}_i^x$ adds or removes dimers if allowed by the diluted dimer constraint. We dub this model diluted dimer model (DDM) and observe that it is well defined regardless of the mapping into a Rydberg atom system. In the limit $\Delta/\Omega \rightarrow \infty$, its low-energy subspace is spanned by the fully packed dimer configurations with one dimer per vertex. At finite Δ/Ω , the off-diagonal term has two, competing effects on the DDM ground state: it favors coherence between maximal dimer coverings on the one hand and lowers the density of dimers on the other. Whereas the former typically stabilizes QSL phases, the latter could spoil it. Therefore, establishing the emergence of QSLs ground states in DDMs requires a nonperturbative treatment of quantum fluctuations, which we tackle below with ED methods.

In a generic lattice, the correspondence between Rydberg atom arrays in the blockade approximation and DDM does not hold. This is illustrated in Figs. 1(a) and 1(b) for the square and triangular lattices, respectively. For instance, on the square lattice [cf. Fig. 1(a)], the distance between atoms in the horizontal and vertical directions are both equal to the lattice spacing a . The diluted dimer constraint however would correspond to a blockade constraint that is active in one direction, but not in the other one, which is clearly incompatible with the isotropic blockade [see Fig. 1(a)]. In what follows,

we provide a solution to this issue by inserting gadgets on the lattice vertices [cf. Figs. 1(c) and 1(d)]. These gadgets are designed to transmit the blockade of an excited atom on an edge to all the atoms on adjacent edges, but to no atom on any other edge. We call the resulting model gadget model. Importantly, while this gadget model is designed to share similarities with the DDM on the same lattice, they differ in some key aspects. In particular, the diluted dimer constraint is enforced on the low-energy spectrum of the gadget model Hamiltonian only in the large Δ/Ω regime. Furthermore, the number of states in a unit cell is larger in the gadget model, resulting in an exponentially larger Hilbert space. Nevertheless, as we show below, the gadget model ground state satisfies the diluted dimer constraint in a wide parameter range and can even exhibit enhanced QSL features with respect to the DDM ground state.

Diluted dimer models with Rydberg gadgets. We now specify the Rydberg gadget models depicted in Figs. 1(c)–1(f). We consider the square and triangular lattices, with a Rydberg atom on each edge and a gadget on each vertex. The gadgets consist of 4 or 6 atoms on the square and triangular lattices, respectively. Edge and gadget atoms are represented by blue and orange dots, while the blockade radius is indicated by the green disk in Figs. 1(c) and 1(d). An edge atom blockades all the gadget atoms on adjacent vertices, except the two gadget atoms that are furthest away [cf. the left side of Figs. 1(c) and 1(d)]. A gadget atom blockades all atoms in the same gadget and all edge atoms adjacent to that vertex, excluding the farthest one [cf. the right side of Figs. 1(c) and 1(d)]. This atom arrangement yields the dimer constraint as follows. Whenever an atom in a gadget is excited to the Rydberg state, only atoms on the adjacent edges can be excited to the Rydberg state. This is akin to the constraint of having only one dimer on any edge connected to a vertex [cf. Figs. 1(e) and 1(f)]. Accordingly, we call a gadget *active* if it contains a Rydberg excitation. To favor active gadgets, the detuning for the gadget atoms, Δ_{gad} , is adjusted relative to the detuning of the edge atoms, Δ , i.e., $\Delta_{\text{gad}} = \alpha \Delta$. In the classical limit ($\Omega = 0$) gadgets are active when $\Delta > 0$ and $\alpha > 3/2$ and $\alpha > 1$ for square and triangular lattices, respectively [57]. In summary, as shown in Figs. 1(e) and 1(f), a single dimer in the gadget model is represented by an excited atom on an edge and it is always accompanied by two active gadgets at the neighboring vertices.

In this setup, the degenerate ground state subspace of the Rydberg Hamiltonian in the blockade approximation and for $\Omega = 0$ and $\Delta > 0$ is the same as in the associated DDM and it is composed of the fully packed dimer coverings. An important difference between Rydberg gadget models and DDMs is that the former allows for configurations in which there is more than one excited atom on edges connecting to the same vertex if the gadget on that vertex is inactive. However, as we will show below, when $\Delta \gtrsim 0$, the gadgets activate, thus penalizing these unwanted configurations at low energy [cf. Figs. 2(a) and 2(b)]. We will demonstrate that, although the gadget model is not guaranteed to reproduce the ground state properties of the DDM, it enables the observation of QSL features in the ground and dynamically prepared states.

Ground state phase diagram. We performed ED calculations for the ground state of DDM and gadget models on finite systems with periodic boundary conditions. We employ

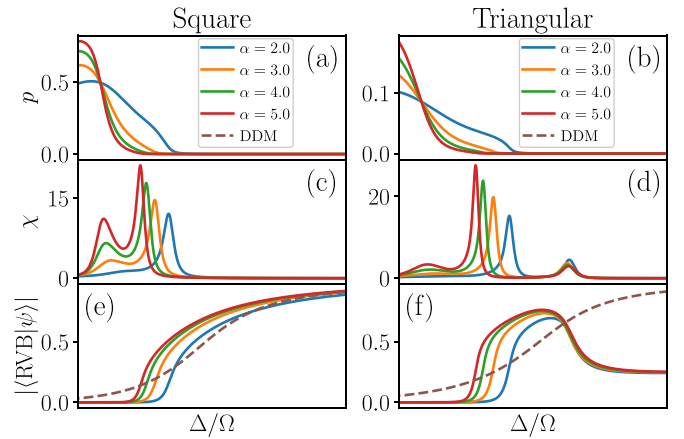


FIG. 2. Ground state phase diagram. Dimer-constraint violation probability [(a), (b)], fidelity susceptibility Eq. (3) [(c), (d)], and ground state RVB overlap [(e), (f)] as a function of Δ/Ω and for different values of $\alpha = \Delta_{\text{gad}}/\Delta$, on the square [(a), (c), (e)] and triangular [(b), (d), (f)] lattices. The dashed lines indicate the diluted dimer model. Results are obtained on 10 and 8 unit-cell clusters for the square and triangular lattices, respectively [57].

a unit cell of two and three edges for the square and triangular lattices, respectively [cf. Figs. 1(a) and 1(b)]. Since there is one gadget per vertex, the gadget models for these two lattices have 6 and 9 atoms per unit cell, resulting in a much larger Hilbert space. Therefore, we focus on periodic clusters of up to 8 and 10 unit cells for square and triangular lattices [57]. Our figure of merit for the characterization of a QSL is the overlap with the resonating valence bond (RVB) state [14,40,41], i.e., the equal-weight superposition of fully packed dimer coverings $\{|c\rangle\}$:

$$|\text{RVB}\rangle = \frac{1}{\sqrt{\mathcal{N}}} \sum_c |c\rangle, \quad (2)$$

where $\mathcal{N}$ is the total number of these configurations. These states are the simplest representatives of QSLs and have very different properties on bipartite and nonbipartite lattices, like the square and triangular lattices. In the former case they encode the physics of a compact $U(1)$ deconfined gauge theory and have power-law decaying correlations, while in the latter they are described by a deconfined $\mathbb{Z}_2$ gauge theory and their correlations are short range [2,48,58–62].

To identify the boundaries of the gadget model phase diagram, we also consider the ground state fidelity susceptibility [63,64]

$$\chi = -\frac{2}{\delta\lambda^2} \ln |\langle\psi_0(\lambda)|\psi_0(\lambda + \delta\lambda)\rangle|, \quad (3)$$

where $\lambda = \Delta/\Omega$, $|\psi_0(\lambda)\rangle$ is the ground state wave function, and $\delta\lambda$ is a small change in the parameter λ . Peaks in χ serve as a diagnostic tool for the location of phase transitions.

For the gadget model on the square lattices, χ exhibits two peaks, indicating two qualitative changes in the ground state [cf. Fig. 2(c)]. As we increase Δ/Ω , the system first undergoes a crossover from a trivially disordered regime to a “gadget-active” regime [65]. As shown in Fig. 2(a), this process is characterized by a sharp decrease in the ground

state probability p of violating the diluted dimer constraint, given, e.g., by configurations with two dimers connecting to one vertex with an inactive gadget. Subsequently, the system undergoes a transition to a phase characterized by a significant overlap with the RVB state, as shown in Fig. 2(e). Since $U(1)$ QSLs are expected to be unstable in 2D [66], we interpret this RVB phase as an emergence of a finite-size $U(1)$ QSL, which is not stable in the thermodynamic limit [15].

We note that the RVB overlaps for the DDM and gadget model on the square lattice converge to the same value in the classical limit $\Delta/\Omega \rightarrow \infty$, as can be seen from Fig. 2(e). This result is readily understood through perturbation theory, which yields the same leading-order effective Hamiltonian for the two models [57]

$$\hat{H}_{\square} = -t \sum_r \left(\left| \begin{array}{c} \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \end{array} \right| + \text{H.c.} \right), \quad (4)$$

where the thick lines correspond to dimers and the flipping strength t is proportional to Ω^4/Δ^3 and Ω^{12}/Δ^{11} for the diluted and gadget model, respectively.

In Figs. 2(d) and 2(f), we present the numerical results for the triangular lattice, where the fidelity susceptibility for the gadget model exhibits three peaks. Similarly to the square lattice, as Δ/Ω increases, the system first undergoes a crossover from the trivially disordered regime to a gadget-active regime [cf. Fig. 2(b)]. On the triangular lattice, however, a high-RVB overlap phase is observed only at intermediate values of Δ/Ω and a third peak in χ indicates a further transition to an ordered valence-bond solid (VBS) phase. We note that this is not the case for the DDM [dashed line in Fig. 2(f)], where the RVB overlap monotonically increases with Δ/Ω . In contrast to the square lattice, $\mathbb{Z}_2$ QSLs can be stable on the triangular lattice. Therefore, this high-RVB overlap phase could indicate the emergence of a $\mathbb{Z}_2$ QSL phase that survives in the thermodynamic limit. The fact that RVB overlaps for diluted dimer and gadget models do not match in the limit $\Delta/\Omega \rightarrow \infty$ can again be understood via perturbation theory. The effective Hamiltonians for the two models are [57]

$$\begin{aligned} \hat{H}_{\Delta, \text{diluted}} &= -t \sum_r \left(\left| \begin{array}{c} \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \end{array} \right| + \text{H.c.} \right), \\ \hat{H}_{\Delta, \text{gad}} &= -V \sum_r \left(\left| \begin{array}{c} \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \end{array} \right| + \left| \begin{array}{c} \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \end{array} \right| \right). \end{aligned} \quad (5)$$

The leading-order term for the diluted model is the dimer flipping term from fourth order, with strength $t \propto \Omega^4/\Delta^3$. For the gadget model, instead, the leading order is a nontrivial diagonal term with strength $V \propto \Omega^6/\Delta^5$ arising at sixth order. In this case, the dimer flipping term only occurs at twelfth order, with a strength proportional to Ω^{12}/Δ^{11} . This leads to the difference between the two RVB overlaps for $\Delta/\Omega \rightarrow \infty$.

We stress that the quantitative comparison of ground state RVB overlaps outlined in Figs. 2(e) and 2(f) between DDMs and gadget models is remarkable. In fact, for fixed Δ/Ω , RVB overlaps in the gadget model can be larger than in the DDM despite the much larger Hilbert space dimension of the former [67]. Moreover, the gadget system possesses the additional parameter $\alpha = \Delta_{\text{gad}}/\Delta$, which can be tuned to increase RVB fidelities further and thus enhance QSL features.

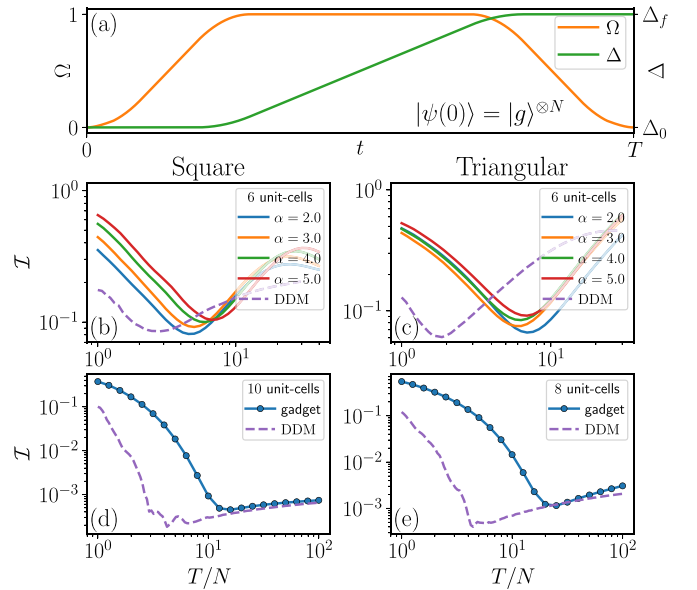


FIG. 3. RVB dynamical preparation infidelity. (a) Dynamical preparation protocol. [(b), (c)] Infidelities $\mathcal{I} \equiv 1 - |\langle \psi(T) | \text{RVB} \rangle|$ obtained from periodic clusters with 6 unit cells as a function of T/N with T the total preparation time and N the number of atoms. The dashed and solid lines are the infidelities obtained in the diluted dimer model and gadget model, for different values of $\alpha = \Delta_{\text{gad}}/\Delta$ in the latter. [(d), (e)] Same as [(b), (c)] but with $\alpha = 2$, for clusters with 10 and 8 unit cells for the square and triangular lattices, respectively.

Dynamical preparation. We now analyze dynamical schemes typically discussed for preparing QSLs in experiments. We focus on the preparation protocol illustrated in Fig. 3(a). The atoms are initialized in their ground state, corresponding to the many-body ground state of the Hamiltonian Eq. (1) when $\Omega = 0$ and $\Delta < 0$. The Rabi frequency is then gradually increased to its maximum value, which we set to 1 and fix our time units over the time interval $0 < t < T_1$. Following this, the detuning is linearly ramped up from an initial value $\Delta_0 < 0$ to a positive value Δ_f within the time span $T_1 < t < T_1 + T_2$. Finally, the Rabi frequency is gradually decreased to zero. Here, we set $T_2 = 0.5T$, $T_1 = T_3 = 0.25T$, $\Delta_0 = -5$, and $\Delta_f = 1.5$ [14].

When the annealing time T goes to infinity, the system evolves adiabatically and remains in the instantaneous ground state. The final state ends up in the ground state obtained in the classical limit $\Delta/\Omega \rightarrow \infty$, implying that the RVB overlap of the final state for $T \rightarrow \infty$ has to approach the RVB overlap of the ground state for $\Delta/\Omega \rightarrow \infty$ [cf. Figs. 2(e) and 2(f)]. Conversely, for small T , only a few atoms are excited throughout the dynamics, resulting in a significant number of defects and a suppressed RVB overlap. Our primary interest is on the semiadiabatic preparation in the intermediate regime of total preparation times T , where RVB fidelities are expected to be maximal [14]. In Figs. 3(b)–3(e), we plot the infidelity between the final state and the RVB state, i.e., $\mathcal{I} \equiv 1 - |\langle \psi(T) | \text{RVB} \rangle|$, for the square and triangular lattices. In both cases, we not only observe an enhanced RVB overlap at intermediate T with respect to the adiabatic case, but also an optimal overlap which is comparable in the DDMs and

gadget models, reaching $\mathcal{I} < 10^{-3}$ for the largest systems considered. In Figs. 3(b) and 3(c), we demonstrate how the extra control parameter α of the gadget model can be used to further minimize $\mathcal{I}$ [57]. Due to the large Hilbert space dimension, we did not perform such optimization in the larger periodic clusters in Figs. 3(d) and 3(e).

Conclusions. We proposed a method based on Rydberg gadgets to engineer the constraint of dimer models in neutral atom arrays in the regime of maximal density of Rydberg excitations. This method can be realized in experiments simply by placing the gadgets on the vertices of a lattice. We analyzed the effect of quantum fluctuations on the gadget model by numerically studying the ground state phase diagram on the square and triangular lattices and compared it to the simpler diluted dimer models, which are not naturally realized with an isotropic Rydberg blockade in these geometries. Making use of perturbation theory, we highlighted similarities and differences between DDMs and gadget models and showed that the ground state of the latter can exhibit enhanced QSL features with respect to their diluted dimer counterparts. Finally, we demonstrated that through nonadiabatic dynamical protocols, it is possible to achieve high-fidelity preparation of $U(1)$ and $\mathbb{Z}_2$ spin liquid states on the square and triangular lattice, respectively.

Our witness for topological order—the RVB overlap—is purely theoretical and cannot be directly probed in experiments. While nonlocal string operators have been devised to detect such QSLs in experiments [13,39], generalizing these operators in our gadget models can be challenging. However, alternative observables, such as the dimer density-density correlation function and the dimer structure factor [15,53], can

help distinguish topological RVB states from trivial ones. Another potential approach involves coherently transferring the population from the Rydberg state to a noninteracting hyperfine state [23], enabling state tomography on small systems.

In this work, we considered the blockade approximation of Rydberg Hamiltonians and ignored the long-range interaction tails, which is a crude approximation for real experiments. However, this approximation can be improved by properly tuning the blockade radius and utilizing different atomic species [57]. Although we only applied Rydberg gadgets to obtain the dimer constraint on square and triangular lattices, this method can be generalized to a larger class of models and used to engineer other kinds of constraints which, when combined with quantum fluctuations, could provide more exotic forms of topological order [47]. Specifically, we discuss the Rydberg gadgets for $\mathbb{Z}_2$ lattice gauge theory with dynamical matter in the Supplemental Material [57].

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