

Floquet engineering of polaritonic amplification in dispersive photonic time crystals

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In this study, we investigate the dynamics of dispersive photonic time crystals (PTCs) and their potential applications for controlling light-matter interaction. Using the Lorentz-Drude as dispersive material model, we analyze the effects of periodic modulation of dispersion parameters through both theoretical and numerical analysis, revealing the emergence of hybrid bandgaps from the interaction of polaritonic branches with distinctive characteristics. Our study demonstrates that dispersive PTCs offer novel excitation channels and amplification possibilities, that require lower modulation frequencies compared to nondispersive systems thus alleviating experimental challenges for the realization of PTCs in the optical regime. These findings pave the way for advancements in polaritonic lasing and resonant Raman scattering.

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I. INTRODUCTION

The study of time-varying photonic media has a history spanning several decades [1–4]. Nevertheless, recent years have witnessed a resurgence of interest in this field revealing numerous new physical phenomena [5] such as the creation of squeezed and entangled photon states [6–9], spatiotemporal beam steering [10,11], and the breaking of Lorentz reciprocity [12,13]. One of the most notable outcomes of engineering temporal dynamics is the concept of photonic time crystals (PTCs) [4,14], where the permittivity of the host material undergoes a periodic modulation with time, analogous to spatial photonic crystals which are periodically patterned in real space [15,16]. Thanks to the non-Hermitian properties of PTCs, these systems exhibit unique light-matter interactions, leading to intriguing phenomena such as generation and amplification of radiation fields from free electrons and dipoles [17,18].

Abrupt and strong changes in material's optical properties over time result in reflections and refractions at temporal interfaces [19,20], akin to spatial boundaries. However, unlike spatial boundaries, temporal ones require dynamical modulation of optical properties, creating an open system and allowing time-reflected and refracted waves to have different energies than the incident beam. PTCs exploit this to create and amplify electromagnetic radiation in their momentum bandgap region, where time-reflected and refracted waves constructively interfere. [17,21] Notably, the wave propagation and amplification in photonic time crystals can

be solely understood in terms of Floquet engineering and the physics of parametric scattering of photons from time harmonic excitations of the matter. Fundamentally, this amplification resembles the well known parametric amplification [1,3,22,23] but differs in two distinct features. First, the presence of strong modulation of material parameters, such as strongly modulating permittivity around the epsilon near zero point [24,25], breaks the applicability limits of the conventional perturbative nonlinear treatment [24,26]. Second, it is assumed that the entire medium undergoes simultaneous modulation while the probe beam propagates inside the medium, relaxing the typical phase-matching conditions [27].

Amplification within the bandgap of PTCs results from energy transfer from the medium's modulation to the probe. This modulation also enables interactions among various harmonics of the parametrically scattered fields enriching the spectral content of the probe beam. In a nondispersive medium, where only a single frequency mode is supported for each momentum, effective amplification demands the modulation frequency be twice that of the probe beam. Consequently, time reflection and exponential growth in momentum bandgaps have been observed experimentally only in the microwave domain where sub-cycle modulation is readily achievable [28,29]. Recent advances in optical materials modulated on a femtosecond timescale [24,30–32] offer a potential for PTCs in the optical domain [33,34]. These materials, typically strongly dispersive and lossy, may introduce new constraints into PTC realization [35]. The dispersive nature of materials creates hybrid states of light and matter, known as polaritons, which are quasiparticles with well-defined energy and momentum. Building on the nondispersive foundations of recent PTC studies [2,17,18,21,36], we include material dispersion in our analysis to investigate the interaction between matter and the radiation field under optical modulation.

Recent studies have explored plane wave propagation in dispersive time-varying media, for example, the effects from a single temporal-interface [37], utilizing a two-level model

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with periodic modulation of resonance frequency [38,39], and the widening of the momentum bandgap near material or structural resonance [40]. A broader investigation of polaritonic interactions and their effect on PTC amplification is necessary. This paper analyzes the overall PTC dispersion, referred to as the optical band diagram, in the presence of Lorentz-Drude-type material dispersion. We specifically explore the modulation of the coupling between different polaritonic branches. Our findings reveal that when the parameters of dispersive materials are modulated, additional bandgaps emerge that require lower modulation speeds. These hybrid bandgaps, accessible from the modes of the unmodulated medium, enable excitation channels for coherently interfering time-reflected and refracted waves, establishing exponentially growing fields. This relaxes the fast modulation requirement of nondispersive materials, suggesting more accessible experimental pathways for observing such effects with much slower modulation frequencies.

As amplification in PTCs stems from nonperturbative nonlinear interactions, the amplification of waves within the polaritonic PTCs is intricately connected to the nonlinear interaction among polaritons [41]. Thus, by examining polaritonic PTCs we may gain valuable insights into the physics of these strong, nonperturbative interactions among polaritons that pave the way forward for enhanced resonant Raman scattering and polaritonic lasing [42,43].

II. POLARITONIC BANDS WITH MODULATION

When the optical properties of a medium are periodically modulated with a frequency Ω , its properties remain time-translation invariant under a temporal shift of $T = 1/\Omega$, i.e., $\Psi(t + T) = \Psi(t)$ where Ψ is a physical quantity of the system. This temporal periodicity causes the ω - k dispersion of the supported modes to also become periodic in frequency with a period of Ω . Therefore, the dispersion of the system is completely characterized by the frequency range of 0 to Ω , known as the 1st Brillouin zone. Owing to this periodicity, the modes of the medium can be described using the Floquet theory, by expanding the relevant quantities in the form $\Psi(t) = e^{i\omega_k t} \sum_{n=-\infty}^{\infty} \Psi_n e^{in\Omega t}$, with $\omega_k + 2\pi n/T = \omega_k$, where n is an integer.

In the case of a stationary, nondispersive optical medium, the dispersion relation of propagating photons is given by the expression $\omega = \pm kc/n$, where c is the speed of light in a vacuum, k is the wave number, and n is the refractive index. This relation defines a single line for positive ω and k shown in Fig. 1 by black dashed lines. The effect of periodic modulation of the dielectric function on the dispersion relation for such nondispersive materials can be visualized as dispersion lines of the unmodulated medium being translated along the frequency axis by integer multiples of the modulation frequency Ω . In this case, the dispersion for negative frequencies becomes relevant as they get shifted to the positive frequency domain and crosses with other dispersion lines. The translation of the dispersion lines causes the branches in different Brillouin zones to cross each other leading to complex frequency eigenvalues, ω_k , with degenerate real parts as observed in Fig. 1 at $|\Omega/2|$. Following the analogy of the frequency band gaps in spatial photonic crystals, these regions

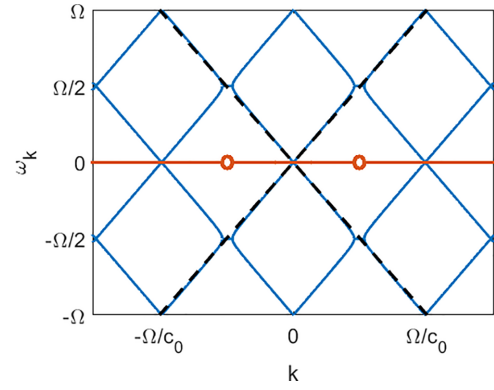


FIG. 1. Band diagram of a nondispersive photonic time crystal with electric permittivity $\epsilon = 1 + 0.3 \sin(\Omega t)$. The dashed black line shows the dispersion relation of the unmodulated medium with $\epsilon = 1$. The blue lines indicate the real parts of eigenvalues while the red indicates the imaginary parts.

with imaginary eigenfrequencies have been called momentum bandgaps with the characteristic signature of exponentially growing photon fields [17,18]. The modes, as they are linear, cross each other at half of the modulation frequency $\Omega/2$ resulting in a bandgap at that frequency.

In the case of a dispersive medium, the energy no longer remains purely in the fields. The electromagnetic waves hybridize with material excitations and form polaritons. The response of the materials that possess bound charges can be described classically with Lorentz-type frequency-dependent dielectric function:

$$\epsilon(\omega) = \epsilon_{\infty} + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \quad (1)$$

where ω_0 , γ , and ϵ_{∞} are, respectively, the resonant frequency, damping, and high-frequency permittivity of the material, and $\omega_p^2 = 4\pi e^2 N/m^*$ is the plasma frequency of the material defined by free carrier concentration N and effective mass m^* . This form of dielectric function follows from the harmonically oscillating polarization density of N noninteracting bound charges excited by electromagnetic waves of the same frequency whose equation of motion is given by

$$\frac{d^2 P}{dt^2} + \gamma \frac{dP}{dt} + \omega_0^2 P = \frac{Ne^2}{m^*} E. \quad (2)$$

The dispersion relation for such material follows from Maxwell's equations and is given by $k^2 = \omega^2 \epsilon(\omega)/c^2$. The solution of this quadratic equation leads to two dispersion branches above and below characteristic frequency $\omega_0/\sqrt{\epsilon_{\infty}}$, which are conventionally called upper and lower polaritonic branches. These two branches arise from the coupling of bound charges and photonic dispersion lines. Due to the coupling, the branches bend outward and form an avoided crossing with an energy difference $\Delta E = \hbar\omega_p/\sqrt{\epsilon_{\infty}}$ known as the Rabi splitting. An example of such dispersion curves is shown in Fig. 2(a) calculated for normalized oscillator parameters $\omega_p = 1.6\Omega$, $\omega_0 = 0.6\Omega$, and $\epsilon_{\infty} = 1$.

We now incorporate temporal changes in material properties using dispersion parameters. Experimentally the most accessible modulation is through optical pumping of

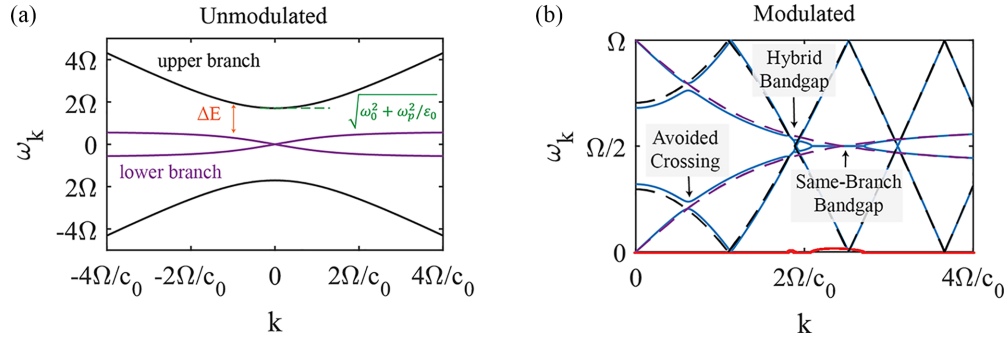


FIG. 2. (a) The dispersion diagram of a Lorentz-type material with $\omega_p = 1.6\Omega$, $\omega_0 = 0.6\Omega$, and $\varepsilon_\infty = 1$. (b) PTC band diagram of the dispersive material presented in panel (a). The plasma frequency is modulated with $\omega_p(t) = 1.6\Omega + 0.8\Omega \sin(\Omega t)$. The blue lines show the real and the red lines show the imaginary part of the frequency eigenvalues. The translated dispersion branches of the unmodulated medium is presented with dashed lines with the same colors as panel (a), i.e., purple for lower branch and black for upper branch. Crossing of branches causes the real part of the frequency eigenvalues to split and give rise to the bandgaps. The same-branch bandgap is highlighted for the amplification region where the two lower branch crosses each other. Similarly, the crossing of lower and upper branches is labeled as Hybrid Bandgap. For clarity the y axis is scaled to show only the first Brillouin zone (from 0 to Ω) to better highlight the features as the bands are the same in each Brillouin zone.

semiconductors and changing conduction band electron density or effective mass. These two parameters directly affect the plasma frequency, therefore in our model, we focus on modulating the plasma frequency. Although our model includes dispersive effects the response of dispersion parameters to external stimuli is still assumed to be instantaneous and follow a harmonic time profile:

$$f(t) = \omega_p^2(t) = \omega_{p0}^2[1 + \Delta^2 \cos(\Omega t)]. \quad (3)$$

This assumption is commonly made in studies of time-varying media, although experimental methods for achieving it are still under investigation. Due to the periodicity, the fields inside the medium can be written as Floquet expansion and then coupled to each other through Maxwell's equation. The following eigenvalue equation for the field array inside the medium can be obtained:

$$\tilde{A}(k)\Psi = \omega_k \Psi, \quad (4)$$

where $\tilde{A}(k)$ is a generalized coefficient matrix, Ψ is an array of the field eigenvectors, and ω_k is the corresponding frequency eigenvalue for a given momentum k . Detailed derivation of this equation is given in the Supplemental Material [46]. Analytically, ω_k eigenvalues are obtained for every k value resulting in the band diagram for the dispersive photonic time crystal along with the corresponding field eigenfunctions. Introducing modulation to a dispersive medium, like in the nondispersive case, causes the dispersion branches to be translated with the modulation frequency. The overlapping polaritonic branches interact and give rise to bandgaps or avoided crossings as shown in Fig. 2(b). The dashed lines show the translated dispersion lines of the stationary medium and the blue lines show the dispersion line of the modulated medium. The dispersion parameters and frequency eigenvalue of the Floquet modes are expressed relative to the modulation frequency Ω .

The interaction between polaritonic branches of the same type causes the emergence of bandgaps with characteristics similar to those of nondispersive PTC bandgaps. These bandgaps, arising from two interacting lower branches, is

highlighted in Fig. 2(b) as same branch bandgaps. These bandgaps occur in the middle of the Brillouin zone at frequency $\Omega/2$, with a flat dispersion line. The modulation needs to be twice the probe frequency as this is the energy required to create a counter propagating wave of the incident wave. Through multiple temporal reflections they coherently add up and cause amplification. One distinction of these bandgaps from the nondispersive case is the additional excitation channels made available by the upper branch with matching momentum. The temporal dynamics of this system is further investigated in the Supplemental Material [46] using dipole and plane wave excitations.

The unique property of dispersive PTCs is the ability of polaritonic branches of different types to interact, which is otherwise not observed in nondispersive medium. The result of this interaction is shown in Fig. 2(b) as a hybrid bandgap which stems from the interaction of the upper and lower polaritonic branches. In this case, the amplification mechanism differs significantly from that in nondispersive or same-branch bandgap cases. Rather than generating counter-propagating waves, the modulation induces frequency sidebands $\pm\Omega$ in the incident wave. When these sidebands overlap with a different dispersion branch, the resonant energy transfer between the branches occurs, leading to amplification. If the overlapping branches exhibit similar dispersion—i.e., their slopes and group velocities match—the energy exchange and phase matching is sustained for a larger range of momentum values. This extended interaction results in greater amplification with higher gain.

To illustrate this effect, we consider the dispersion parameters of hexagonal boron nitride (h-BN), a well-known phononic optical material (see Table I) [44]. Recently, h-BN has been explored for THz amplification based on phononic nonlinearities [45]. The band diagram for the stationary medium is shown in Fig. 3(b). In this analysis, we use the same modulation strength as in Fig. 2(b), but we choose the modulation frequency so that the group velocities of the upper and lower branches match where they overlap [see Figs. 3(b) and 3(c)]. With the modulation, the branches are

TABLE I. Dispersion parameters of hBN in tangential direction. Modulation frequency $\Omega = 26$ THz.

ω_p	ω_0	γ	ϵ_∞
58 THz	41 THz	0.2 THz	4.9
1920 cm^{-1}	1360 cm^{-1}	7 cm^{-1}	
$\sqrt{\epsilon_\infty} \Omega$	1.7Ω	0.034Ω	4.9

translated by the modulation frequency. Additionally, because the plasma frequency is periodically modulated, the shapes of the branches also oscillate. The dashed lines in Fig. 3(b) indicate how the branches evolve at the peak and valley of the modulation cycle. Since the slopes of the upper and lower branches are similar in this case, the overlap region increases in width both horizontally and vertically. Consequently, the interaction region between the translated and modulated branches expands and the bandgap extends over a wider momentum range [see Fig. 3(a)] with higher imaginary values. When the modulation frequency is detuned from the range where the slopes of the branches (and therefore the group velocities) are different, the overlap region shrinks. This is apparent from Fig. 2(b) where the hybrid bandgap is much smaller (also see Supplemental Material [46] for band structure of hBN when modulation is detuned). Therefore, the amplification is maximum if the modulation is chosen to couple the branches where they have similar shapes.

In fact, this region, where the maximum coupling between the branches occurs, is also the range where the energy separation between the bands is minimum and where the Rabi splitting happens between the photonic and the material modes. So the modulation creates an interaction between polaritons which are split due to strong coupling. For a phononic medium, the minimum separation of the lower and upper branches is equal to $\omega_p/\sqrt{\epsilon_\infty}$ which is proportional to the difference between the transverse phonon (TO) and longitudinal phonon (LO) frequencies such that $\Delta\omega = \sqrt{\omega_{LO}^2 - \omega_{TO}^2}$ [36]. In Fig. 3 the selected modulation frequency is equal to this value.

A crucial point to emphasize for technological relevance is that unlike the nondispersive case, it is not required to create counter propagating waves for amplification. Instead, it is sufficient for the modulation to be equivalent to the frequency difference between the upper and lower branches. Consequently, the nondispersive PTC requirement for the modulation speed to be twice the probe frequency is lifted. Additionally, the amplification happens directionally in the incident waves' initial propagation direction. This is apparent in Fig. 3 as the bandgap is located on a slanted line rather than a on flat line.

To further study the time dynamics of the hybrid band gap we look into the excitation of the modes via a probe wave using FDTD simulations. The details of the calculation are provided in the Supplemental Material [46]. The modulation frequency is chosen to be 13 THz in this case, which is the half of the minimum separation between the upper and lower branches so the interaction occurs through the harmonic of the modulation.

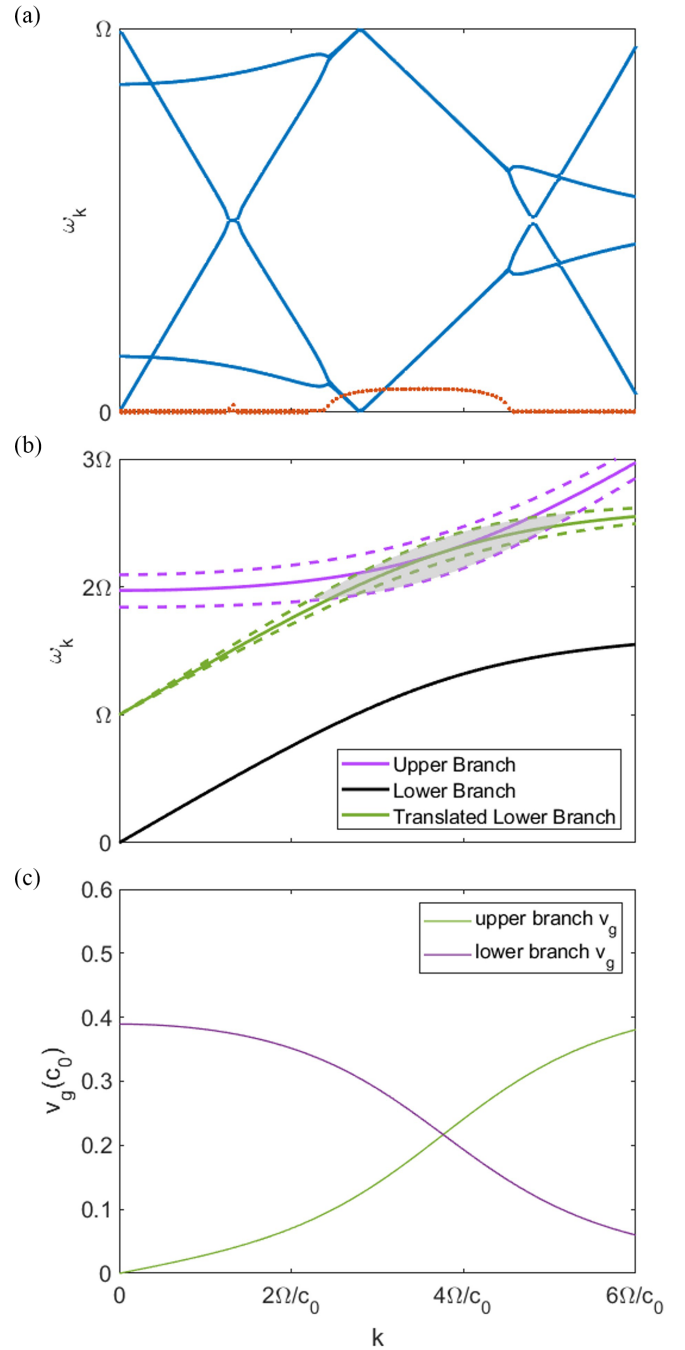


FIG. 3. (a) PTC band diagram of the hBN-like dispersive material presented in Table I. The plasma frequency is modulated with $\omega_p(t) = 0.61\Omega + 0.3\Omega \sin(\Omega t)$, where $\Omega = 13\text{THz}$. The blue lines show the real part whereas the red lines show the imaginary part of the frequency eigenvalues. (b) This figure highlights the interaction of polaritons with modulation. Translated lower branch crosses with the upper branch. The dashed lines show how the modulation changes throughout the modulation cycle, at the maximum and minimum values of plasma frequency. Gray region highlights the extends of the crossing. (c) Group velocity of the upper and lower branches for the case of unmodulated hBN dispersion.

Before modulation, the probe wave can have momentum and frequency residing only on one of the dispersion branches of the nonmodulated medium. Once the modulation starts, the waves's momentum remains constant while its frequency

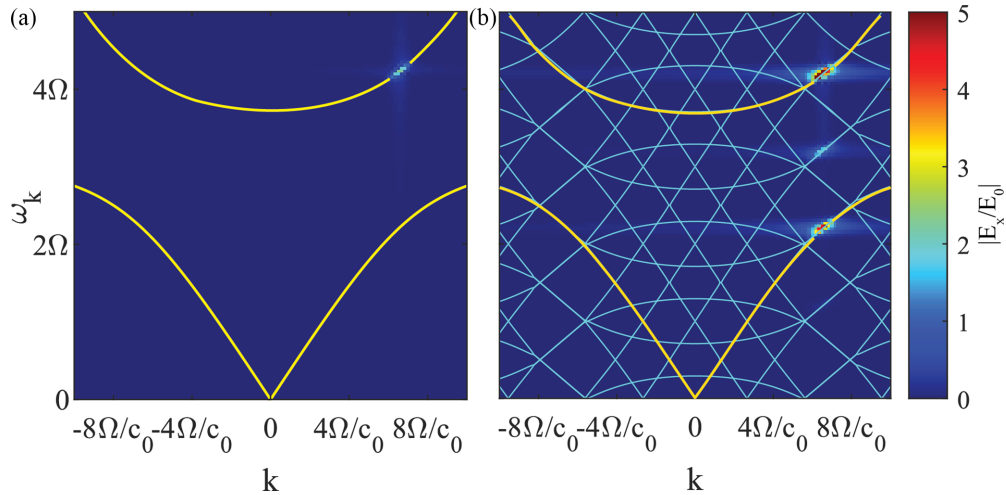


FIG. 4. The ω - k diagram for the plane wave excitation of the hybrid bandgap. (a) Frequency-momentum relation inside the medium with hBN-like dispersion without modulation. The yellow lines show the dispersion relation of the unmodulated medium. (b) The frequency-momentum relation of the field inside the dispersive PTC for the incident wave given in part a. The light blue lines show the band diagram of the PTC overlaid with the yellow lines showing the dispersion lines of the unmodulated medium.

couples to the Floquet modes. When the probe wave momentum is within the bandgap, the wave couples into the momentum gap mode and starts growing. For a nondispersive medium, the bandgap always occurs at $k = \Omega/(2n_{\text{avg}}c)$ [17,18]. In such a medium, for a wave to have this momentum, its frequency should always be $\Omega/2$. Hence, exciting the bandgap requires very fast and strong modulation of the optical properties, making it challenging to possibly observe photonic time crystals in the optical regime.

This is not the scenario for the hybrid bandgap. To couple into the bandgap mode, we excite from the upper branch at $\omega = 4.23\Omega$ and $k = 6.7\Omega/c$ as shown in Fig. 4(a). These values are chosen so that the excitation beam overlaps with the bandgap of the modulated medium. After the modulation is introduced, the bandgap occurs on the branch where the wave was propagating before the modulation, therefore all the energy directly couples to the bandgap mode resulting in amplification. Notably, the bandgap in this case is not a flat band as in the same-branch case but is instead slanted. Therefore, the resulting amplified beam propagates with nonzero group velocity. Figure 4(b) shows the ω - k relationship of the waves inside the medium during the modulation where one can observe the growth in the field amplitude and the excitation of the lower branches Ω away in frequency from the probe beam. Supplementary Video 1 [46] shows the temporal evolution of this excitation. The amplification is also evident from Fig. 4 as the amplitude of the beam has increased when the modulation is introduced.

We emphasize the relaxation of modulation frequency criteria by noting that amplification occurs for a modulation speed that is more than 8 times lower than that of a nondispersive medium ($\Omega/2$). The interaction between the polaritonic branches allows for PTC bandgaplike amplification to happen at significantly slower modulation. In the example shown a modulation at 13 THz gives rise to directional amplification of a wave at 55 THz. This feature is a clear path forward for observing PTC bandgaps in the infrared (IR) regime and

possibly in the optics regime with appropriate selection of material and modulation properties.

When modulated with a frequency proportional to the frequency separation of the LO and TO phonons, phononic materials can be used for polaritonic amplification. Although creating strong optically induced changes in these materials are challenging, it has been shown that it is possible to drive nonlinear response in phononic materials through Born effective charge and dielectric screening of the electric field by electrons. This nonlinearity has been used to observe parametric amplification of optical phonons [23,45].

Apart from employing the phononic material dispersion, polaritonic amplification could be realized by constructing the desired dispersion lines with structured and patterned surfaces. For example, bulk and surface plasmons hybridize in thin nanoscale films at the epsilon near zero region giving rise to Berreman and ENZ modes. The energy spacing between these modes can be quite small allowing coupling with relatively low frequency modulation.

III. CONCLUSIONS

In summary, the interaction of polaritonic modes in dispersive photonic time crystals (PTCs) has the potential for amplification across a broad electromagnetic spectrum. This enhancement can impact the scope and applications of polaritonic lasing, resonant Raman scattering, and polaritonic nonlinearities.

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