

Dynamical control of topological unidirectional guided resonances via external magnetic field

Bao Jie Zheng¹, Wei Jie Shi¹, Hui Yuan Dong², Yong Tao Li², Jia Qi Li¹, Zheng-Gao Dong¹, and Jin Wang^{1,*}

¹Key Laboratory of Quantum Materials and Devices of Ministry of Education, School of Physics, Southeast University, Nanjing 211189, China

²School of Science, Nanjing University of Posts and Telecommunications, Nanjing 210003, China



(Received 11 October 2024; accepted 8 January 2025; published 23 January 2025)

Asymmetric radiation in upward and downward directions supported by leaky modes can be generally exhibited in the reciprocal but geometrical up-down mirror-symmetry-breaking structures, i.e., photonic crystal slabs. Here, we theoretically propose and numerically demonstrate a scheme to break the reciprocity, and to generate and manipulate the unidirectional guided resonances (UGRs) in a one-dimensional magneto-optical (MO) grating, which radiate solely toward one side with no emission in the opposite direction. Under an external magnetic field, it is found that genuine UGRs (or quasi-UGRs) in topological nature, depicted by a phase winding number of far-field radiation, can be created by hybridizing the different orthogonal bands in such a nonreciprocal grating. We further investigate the evolution, annihilation, and band transition for residential position of UGRs in three-dimensional parameter space, and we also notice that the emission direction may be switched from one side to the other due to the tuning of the asymmetric coupling. Finally, the excitation of UGRs is examined in a lossless system related to directional coupling of external waves, and it can also be utilized to realize perfect absorption in a lossy MO grating structure under single-sided illumination. The general principles presented here may lead to an efficient and active approach for the dynamic manipulation of topological radiation asymmetry, and they may serve as new inspiration to control flexibly the resonant and topological properties of light-matter interactions.

DOI: [10.1103/PhysRevResearch.7.013091](https://doi.org/10.1103/PhysRevResearch.7.013091)

I. INTRODUCTION

Highly asymmetric radiation (or unidirectional light emission) is of vital importance for the development of key technologies such as large-scale photonic integration [1] and photonic chips [2], and it has been used extensively in various optoelectronic devices, including high-performance grating couplers [3–5], energy-efficient laser emitters [6–8], and optical antennas [9–11]. The conventional approach to forbid light radiation in undesired channels relies on the use of reflective materials or structures, i.e., multiple layers and grooves [12,13], and blazing gratings [14,15]. However, the introduction of such a “mirror” usually adds complexity to the fabrication process due to its intrinsic bulky, dispersive, and dissipative features, thereby imposing restrictions on the practical applications for compact and on-chip photonic devices.

Recently, a new class of specific eigenstates, termed unidirectional guided resonances (UGRs), were predicted theoretically [16–27] and verified experimentally [28,29] in photonic crystal slabs (PCs). Such an eigenstate shows unique

characteristics that emit light towards just one side of the PCs, whereas no mirror needs to be placed on the other side. Interestingly, UGRs are closely related to distinctive topological properties, e.g., phase singularities [23,25,26] or polarization singularities [20,28,30–32] in momentum space, such as bound states in the continuum (BICs) or circularly polarized states (C points) [32–36]. For the purpose of constructing the UGRs, one may initially break the in-plane C_2 symmetry as well as the up-down mirror symmetry (σ -symmetry) of the PCs, then generate and manipulate two C points spawned from the void [20] or bound states in the continuum (BICs) [37–42], and eventually merge the paired C points together to form the UGRs. For instance, previous relevant work has been demonstrated, but it was mainly restricted to reshaping the real geometrical structure, such as changing the tilt angle [28], dislocating the double-layered structure [20], and adding specific geometry material [17]. Very recently, Yin *et al.* [23] and Lee *et al.* [27] reported a different mechanism to realize UGRs through tuning the interband interaction, but retaining the in-plane C_2 symmetry of the structure. To some extent, this proposal may contribute to simplifying the practical fabrication process of the UGRs. Nevertheless, most of the current unidirectional radiation strategies are based on static element surfaces, hindering the dynamic switching and optimization of radiation [17,18,23–28]. Therefore, it remains a challenge to achieve a practical and controllable platform for the goal of high-performance unidirectional guided radiation.

In this work, we propose an alternative mechanism to actively generate and dynamically manipulate topological

*Contact author: jinwang@seu.edu.cn

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UGRs in a nonreciprocal one-dimensional (1D) magneto-optical (MO) grating. Due to the addition of the external magnetic field, a series of symmetries in the proposed grating structure are broken, such as in-plane C_2 symmetry, up-down σ symmetry, and reciprocity. Accordingly, the original two orthogonal energy bands with different transverse parity are perturbed, become interacting with each other, and eventually give rise to the possibility of the formation of UGRs near the band crossing through delicately tuning the external magnetic field. Distinct from most of the previous studies focusing on the complete evolution process of C points, we instead give a full picture of the trajectories of multiple UGRs in three-dimensional (3D) parameter space through overlapping different orthogonal bands, and we display several intriguing phenomena involved in the UGRs, containing the generation and annihilation of the UGRs, the band transition for their residential position, as well as the flipping of the radiation direction supported by UGRs. Meanwhile, we confirm the topological nature of the UGRs by examining the topological index—the phase winding number of far-field radiation—and we notice that the overall topological indices produced by the UGRs remain constant in various cases. In addition, the UGRs can be well excited in the lossless grating, which is strongly dependent on the incident direction of external waves. And once intrinsic material loss is considered in the proposed MO grating, the UGRs can also be used to attain close to full absorption in the structure without an additional reflective mirror, except for a light beam incident from a single side.

II. MODEL AND METHODS

Let us consider a 1D MO grating with lattice constant a as depicted in the inset of Fig. 1(a). The unit cell consists of one gyrotropic bar with width w , thickness h , and the axis of the bars is in the y direction. The gyrotropic material is homogeneously magnetized along the y direction in the Voigt configuration, which is perpendicular to the wave vector of light radiation. The optical response of such gyrotropic material can be expressed by an asymmetric permittivity tensor $\tilde{\epsilon}$:

$$\tilde{\epsilon} = \begin{pmatrix} \epsilon_r & 0 & i\delta \\ 0 & \epsilon_r & 0 \\ -i\delta & 0 & \epsilon_r \end{pmatrix}, \quad (1)$$

with δ in the off-diagonal elements describing the magnetization-induced gyration of the material, which is proportional to the external magnetic field. Without loss of generality, we take realistic material parameters for the typical gyrotropic materials, i.e., Bi-substituted yttrium iron garnet (Bi:YIG) in the optical or infrared frequency. For example, we take the diagonal element $\epsilon_r = 4$, the maximum absolute value $|\delta| \approx 0.5$ for the off-diagonal element of the permittivity tensor, but we neglect the intrinsic dispersive and dissipative features for theoretical prediction. Similar experimental data in the permittivity tensor elements for Bi:YIG have been recorded in the presence of a feasible magnetic field $B \approx 1$ T [43,44]. The background medium is set to be vacuum. To investigate the UGRs induced by the gyrotropic effect, we focus only on the leaky modes supported by this MO grating structure in transverse magnetic

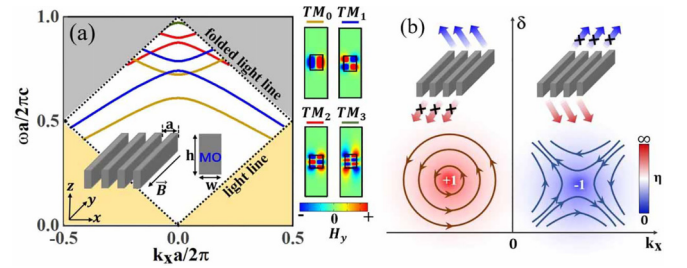


FIG. 1. (a) Calculated dispersion diagram of TM-polarized leaky modes above the light cone for a 1D MO grating with no applied external magnetic field, corresponding to the zero gyration ($\delta = 0$) in the permittivity tensor $\tilde{\epsilon}$ of MO material. The colored dispersion curves represent band TM_0 (yellow line), TM_1 (blue line), TM_2 (red line), and TM_3 (green line), respectively. The associated magnetic field (H_y) distributions for these four different bands are also shown on the right side of (a). The inset of (a) illustrates the schematic of MO grating, consisting of an array of infinitely long MO bars with 1D periodicity in the x direction. The grating period, bar thickness, and width are denoted, respectively, by a , h , and w . Here, the parameters $h = 1.45a$ and $w = 0.58a$ are used in the calculated band structure of (a). (b) Schematics of a nonreciprocal MO grating supporting a unidirectional guided resonance (UGR), when an external magnetic field is applied along the $-y$ direction ($\delta \neq 0$). The color maps display the asymmetric ratio between upward and downward radiative loss, $\eta = \gamma_t/\gamma_b$, approaching infinity (zero) at the genuine UGRs (DUGRs). A typical phase vector field of far-field radiation (red/blue) shows the topological indices $S = +1$ (-1) associated with the phase winding around the UGRs (DUGRs).

(TM) polarization, i.e., with a magnetic field polarized along the bars in the y direction. Throughout this work, the $e^{-i\omega t}$ time-dependent convention for the harmonic field is used, and all numerical calculations on the eigenmodes, eigenstates, as well as radiation polarization vectors are performed with a finite-element solver (COMSOL MULTIPHYSICS).

We start by analyzing the typical TM-polarized eigenmodes [shown in Fig. 1(a)], supported by the MO grating in the absence of an external magnetic field ($\delta = 0$). For the case of a fixed $k_y = 0$, the normalized eigenfrequency $\omega a/2\pi c$ has been calculated in Fig. 1(a) as a function of the x component of the normalized wave vector $k_x a/2\pi$. The dispersion curves of the four leaky modes TM_0 , TM_1 , TM_2 , and TM_3 are displayed inside the reduced Brillouin zone for the grating of $h = 1.45a$ and $w = 0.58a$. It is found that several energy bands, i.e., TM_0 and TM_1 (or TM_1 and TM_2), can perfectly cross with each other when the external field is absent, due to the complete orthogonality of their respective even and odd transverse magnetic field profiles. Notice that here we only focus on the zero-order diffracted waves supported in the white region of Fig. 1(a) due to the fact that truly guided modes are not leaky below the light line of the background (yellow region), and high-order spectral response is less practical above the folded light line (gray region).

As the external magnetic field along the $-y$ direction increases gradually from zero ($\delta \neq 0$), the mirror symmetry σ_z -symmetry as well as the in-plane C_2^z symmetry is broken in the MO grating structure, the radiation becomes asymmetric in the upward and downward directions, and the UGRs may

emerge near the band crossing (discussed below) in the 2D parameter space (k_x, δ), with an asymmetric ratio of $\eta = \gamma_t/\gamma_b$ approaching an extreme value as shown in Fig. 1(b). Here, γ_t and γ_b denote decay rates in the upward (+z) and downward (-z) directions, respectively. Moreover, once the Bloch wave vector k_x changes from negative to positive, the emission direction from UGRs flips from upward to downward accordingly, due to the reserved inversion symmetry (P-symmetry) for our proposed MO grating structure even in the presence of external magnetic field.

The presence of external magnetic field may give rise to the perturbation and hybridization of the original orthogonal eigenstates in the grating, thereby providing an opportunity for us to manipulate the distribution of far-field radiation. According to coupled mode theory [18,23,27,45], two leaky modes, i.e., TM_m and TM_n with their respective complex eigenfrequencies $\Omega_m = \omega_m - i\gamma_m$ and $\Omega_n = \omega_n - i\gamma_n$, are assumed in a resonator system, the Hamiltonian can be expressed as follows:

$$H = \begin{pmatrix} \omega_m & \alpha \\ \alpha & \omega_n \end{pmatrix} - i \begin{pmatrix} \gamma_m & \beta \\ \beta & \gamma_n \end{pmatrix}, \quad (2)$$

where α and β represent the near-field and far-field coupling between these two modes, respectively. The eigenvalues of H can be written as

$$\Omega_{+,-} = \frac{\Omega_m + \Omega_n}{2} \pm \sqrt{\frac{(\Omega_m - \Omega_n)^2}{4} + (\alpha - i\beta)^2}, \quad (3)$$

and the corresponding hybrid eigenstates can be expressed by

$$\Psi_{+,-}(\mathbf{k}_{\parallel}) = a_{+,-}\Psi_m(\mathbf{k}_{\parallel}) + b_{+,-}\Psi_n(\mathbf{k}_{\parallel}), \quad (4)$$

where $[a, b]_{+,-}^T$ denotes the eigenvector of H , $\Psi_{m,n}$ represents the unperturbed eigenstate of the $\text{TM}_{m,n}$ modes, and $\mathbf{k}_{\parallel} = (k_x, k_y)$ represents the in-plane wave vector.

The far-field polarization vector of hybrid eigenstates projected onto the x - y plane can then be obtained as

$$\mathbf{c}_{+,-}^l(\mathbf{k}_{\parallel}) = a_{+,-}\mathbf{c}_m^l(\mathbf{k}_{\parallel}) + b_{+,-}\mathbf{c}_n^l(\mathbf{k}_{\parallel}), \quad (5)$$

where $\mathbf{c}_j^l$ ($j = m, n$) denotes the radiation amplitudes of Ψ_j , with superscripts $l = t(b)$ referring to the light emission in the upward (+z) or downward (-z) direction, respectively. Notably, the eigenstates $\Psi_{+,-}$ as well as their radiation properties $\mathbf{c}_{+,-}^l$ can be tuned by the interband coupling.

In the numerical simulations, the far-field polarization vector $\mathbf{c}_{+,-}^l$ can be readily computed by the associated eigenstates via [20,36]

$$\mathbf{c}_{+,-}^l = [c_{x;+,-}^l, c_{y;+,-}^l] = \frac{\int_{\Omega} \mathbf{E}_{+,-}^l(\mathbf{r}) \mathbf{e}^{-i\mathbf{k}_{\parallel} \cdot \mathbf{r}} d\mathbf{r}}{\int_{\Omega} d\mathbf{r}}, \quad (6)$$

where Ω refers to a unit cell in the x - y plane. To demonstrate the topological characteristics, we consider the phase vector field $[\text{Re}(c_{x(y);+,-}^l/c_0), \text{Im}(c_{x(y);+,-}^l/c_0)]$ (c_0 is a normalized constant), and we define a gauge-invariant topological index (phase winding number) as [23]

$$S_{x(y);+,-}^l = \frac{1}{2\pi} \oint_C d \arg \left[\frac{c_{x(y);+,-}^l}{c_0} \right], \quad (7)$$

through an integral of phase on a closed clockwise loop C around the UGRs. So once the upward (downward) genuine

UGRs [termed UUGRs (DUGRs), solely radiating upward (downward)] are formed on Ψ_{+} , the asymmetric ratio between decay rates in the upward and downward directions should tend to be infinity or zero [$\eta = \gamma_t(\gamma_b) \rightarrow \infty(0)$], or the vanishing of radiation modulus with $c_{x,+}^{b(u)} = c_{y,+}^{b(u)} = 0$, which is also equivalent to a singular value of its phase. Such a singularity naturally causes a vortex in the polarization vector field centered on the UGRs, as it does for Ψ_{-} .

For this reason, we can unveil the topological properties of UGRs from the phase singularity of a far-field polarization vector by virtue of the phase winding number to connect UGRs with topological phenomena: as one encircles the UGR center in the parameter space, the polarization vector has to come back to itself after the closed loop, implying that the overall angle change of the polarization vector must be an integer multiple of 2π , and then the associated phase winding number S in Eq. (7) must be an integer. Note that the connection between the winding number and UGRs is general and not restricted to the model used in this work. For example, a recent study by Yin *et al.* [23] showed that UGRs may carry an integer phase winding number in a freestanding Si_3N_4 photonic slab patterned with square-lattice air holes.

In view of the TM-polarized light radiation (E_x, H_y, E_z) for a typical example of our case, we illustrate schematically in Fig. 1(b) the phase singularities upon the x -component of a radiating electric field E_x , and we find the phase changes 2π (-2π) upon $c_{x,+}^{b(u)}$ for Ψ_{+} when one encircles the phase vector field clockwise around the UUGRs (DUGRs), implying that the UGRs carry opposite topological indices: $S = +1$ for UUGRs and $S = -1$ for DUGRs.

In the following, we investigate and analyze the critical influence of an external magnetic field on the dispersion relations, Q factors, asymmetric radiation ratio η , as well as the complex phase vector field c_x near the multiple band crossing, such as orthogonal band crossing between TM_0 and TM_1 , or TM_1 and TM_2 . Then we aim to construct the topological UGRs, and eventually give a summary on the dynamic control of UGRs via external magnetic field.

III. RESULTS

A. Generation and manipulation of topological UGRs

1. Near the band crossing of TM_0 and TM_1

In Fig. 2, we first consider a particular case of two orthogonal eigenmodes having opposite symmetry, i.e., TM_0 and TM_1 , in the proposed MO grating. In the absence of external magnetic field ($\delta = 0$), the two eigenmodes can directly cross with each other [Fig. 2(b), top panel] due to the complete orthogonality in the transverse magnetic field profiles: TM_0 is even and TM_1 is odd with respect to z . Meanwhile, a symmetric-protected BIC appears at the Γ point ($k_x = 0, k_y = 0$) for TM_0 with a divergent Q factor—numerically larger than 10^7 [Fig. 2(b), middle panel]. Due to the existence of up-down mirror symmetry (σ_z) in the structure, the light radiation remains identical in the upward or downward directions for these two resonances with the asymmetric ratio $\eta = 0$ [Fig. 2(b), bottom panel].

When the gyration in MO materials is present by applying external magnetic field, i.e., $\delta = 0.532$, the energy bands of

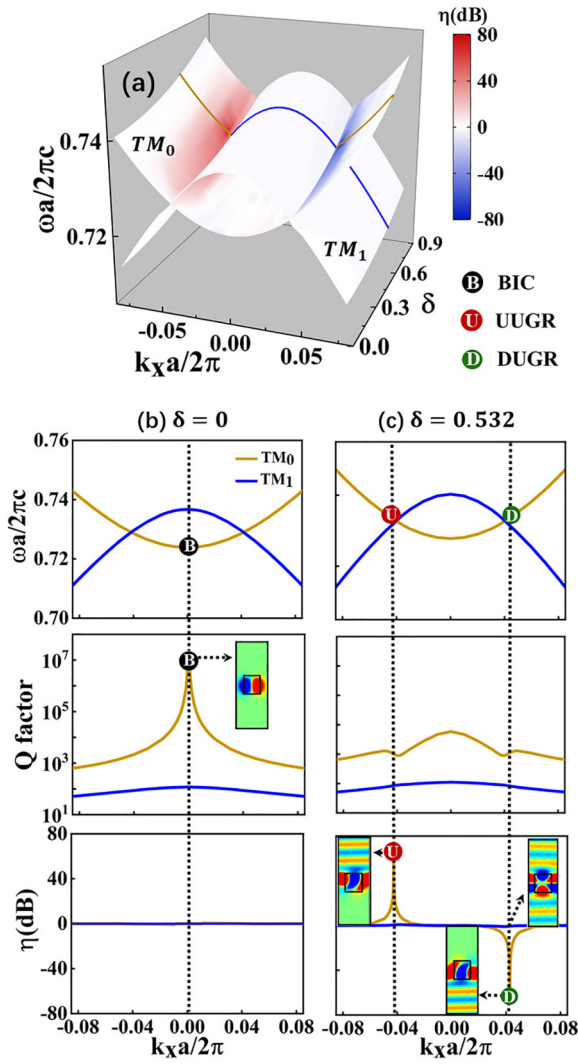


FIG. 2. (a) Coupling scenario between TM_0 and TM_1 modes in the parameter space (k_x, δ) with $k_y = 0$. The color map shows the asymmetric ratio η for two perturbed eigenstates $\Psi_{\pm}$, and the band crossing between two specific dispersion curves in (a) denotes the emergence of genuine UGRs under a given gyration of MO material $\delta = 0.532$. (b),(c) Two different cases of dispersion curves (upper panel), radiative Q factors (middle panel), and asymmetric radiation ratio η (lower panel) in 1D-momentum space for original eigenstates $\Psi_{0,1}$ with $\delta = 0$ (b), and perturbed eigenstates $\Psi_{+,-}$ with $\delta = 0.532$ (c). The black dot in (b) and red (green) dots in (c), respectively, mark the locations of symmetry-protected BIC on the original eigenstate Ψ_0 , or genuine UGRs (DUGRs) on the perturbed eigenstate Ψ_+ . For comparison, profiles of $\Psi_{+,-}$ at normalized wave vectors $k_x a/2\pi = \pm 0.0427$ are, respectively, plotted in the inset of the bottom panel of (c), exhibiting the presence or absence of genuine UGRs. Other geometrical parameters and settings in (a)–(c) are the same as those in Fig. 1(a).

TM_0 and TM_1 shift up slightly [Fig. 2(c), top panel], but the Q factor for TM_0 varies promptly, particularly at the Γ point, only with finite values, and the original BIC vanishes [Fig. 2(c), middle panel]. More importantly, the radiation upon TM_0 starts to exhibit significant asymmetry in upward and downward directions near the band crossing between TM_0

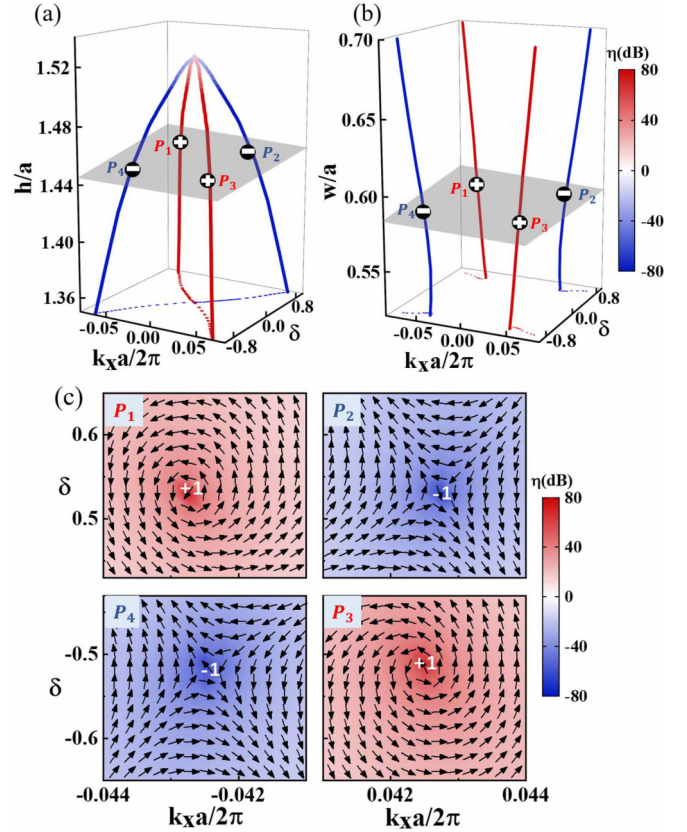


FIG. 3. Evolution of UGRs in parameter space, induced by TM_0 - TM_1 coupling. (a),(b) The trajectories of UGRs for eigenstate Ψ_+ in the 3D parameter space (k_x, δ, h) with fixed $w = 0.58a$ (a), or parameter space (k_x, δ, w) with fixed $h = 1.45a$ (b). The color maps in different trajectories indicate the asymmetric radiation ratio η . The gray plane is pierced by the lines at $h = 1.45a$ (a) or $w = 0.58a$ (b), with four intersection points $P_1, P_2, P_3,$ and P_4 representing four UGRs in the subspace. (c) The associated phase vector field and asymmetric ratio around points $P_1, P_2, P_3,$ and P_4 in the 2D parameter space (k_x, δ) . The topological indices $S = +1, -1$ are also labeled at the UGRs, respectively. Other geometrical parameters and settings in (a)–(c) are the same as those in Fig. 2.

and TM_1 . It is indicated [Fig. 2(c), bottom panel] that a sharp peak of asymmetric ratio $\eta > 70$ dB appears at $k_x a/2\pi = -0.0427$ upon the band TM_0 , thereby creating a genuine UGR. Similarly, an ideal UGR is also formed at $k_x a/2\pi = +0.0427$ for TM_0 , with a distinct dip of $\eta < -70$ dB. In contrast, the response, i.e., the Q factor and radiation distribution, associated with the band TM_1 is tiny. To clarify the influence on the TM_0 of external magnetic field, we also plot in Fig. 2(a) the asymmetric ratio η as well as the band structure in 2D parameter space (k_x, δ) . Notably, an introduction of magnetic field may drastically change the radiation distribution on TM_0 , and the optimal parameters for the genuine UGRs can be found by scanning a series of δ as well as grating thickness h upon band TM_0 (see more discussions in Appendix A).

We next investigate in Fig. 3(a) the dynamic evolution of the aforementioned UGRs in a 3D parameter space (k_x, δ, h) with fixed $k_y = 0$ and $w = 0.58a$. Four UGR points can be found in a 2D subspace (k_x, δ) for a wide range of the grating thickness h . The topological index S may change

the sign ($+S \rightarrow -S$) upon flipping the direction of external magnetic field ($+\delta \rightarrow -\delta$). As h increases gradually shown in Fig. 3(a), topological indices tend to move toward each other, coalesce, and finally annihilate at the Γ point for $h = 1.52a$, retaining a conserved quantity for the total S upon Ψ^+ , $S^+ = (-1) + 1 + 1 + (-1) = 0$, in the 2D subspace (k_x, δ) . For h exceeding $1.52a$ in this case, the bands TM_0 and TM_1 start to separate out, and no intersection points exist between them. As a consequence, no UGRs appear in the subspace, and the radiation goes back to being nearly identical for both energy bands of TM_0 and TM_1 (see details in Appendix B). Also, a second trajectory diagram of the UGRs in another 3D parameter space (k_x, δ, w) with fixed $h = 1.45a$ is plotted in Fig. 3(b), indicating that the stability and robustness of the UGRs existed in a broader MO grating structure. We further check the topological nature of UGRs in Fig. 3(c) for several specific examples, corresponding to four intersection points P_1, P_2, P_3 , and P_4 in the gray plane of Fig. 3(a) or 3(b). Clearly, the phase singularity of the polarization vector occurs at the extreme point of η with topological indices $S = +1, -1$, resulting in the formation of topological UGRs in the 2D subspace (k_x, δ) . Due to the inversion symmetry of the structure, the distribution of the asymmetrical ratio as well as the phase vector field appears to be identical around points P_1 and P_3 , or P_2 and P_4 .

So far, we have restricted ourselves to investigating the evolution process of the UGRs for the case of $k_y = 0$. It should be emphasized that the creation of UGRs can also be explained from the generation and coalescence of a pair of C points in the whole 2D $k_{||}$ momentum space (k_x, k_y) . See discussions in Appendix C.

2. Near the band degeneracy of TM_1 and TM_2

Let us turn to a second case of two eigenmodes (TM_1 and TM_2) still having opposite symmetry, and analyze the formation of UGRs near the band crossing of these two orthogonal bands, due to the presence of external magnetic field. In comparison to case I of TM_0 and TM_1 in Fig. 2, it is shown in Figs. 4(c)–4(e) that the real bands of TM_1 and TM_2 may experience crossing to anticrossing at a critical magnetic field corresponding to $\delta = 0.094$, whereas the imaginary parts ($\propto 1/Q$) of these two bands change conversely from anticrossing to crossing. This can be understood from the theoretical analysis in Eq. (3) of Sec. II, which states that a degenerate point with the same eigenvalues and eigenstates, termed the exceptional point (EP) [46–52], may be created once the condition $(\Omega_1 - \Omega_2)^2/4 + (\alpha - i\beta)^2 = 0$ is fulfilled for two leaky modes with their respective eigenfrequencies Ω_1 and Ω_2 . However, it should be noted that there is no direct connection between EP and UGR, and the condition on their formation is totally different, though both of them are involved in the interband coupling. In the bottom panel of Fig. 4(e), it is seen that highly asymmetric radiation ($\eta \approx 50$ dB) stemming from the emergence of quasi-UUGRs on band TM_2 appears under a specific configuration ($\delta = 0.098$), which is close to but actually different from the EP. For the purpose of clearly demonstrating the influence of external magnetic field on the band as well as the radiation distribution, we also illustrate in Figs. 4(a) and 4(b) the band structure, radiative Q factor,

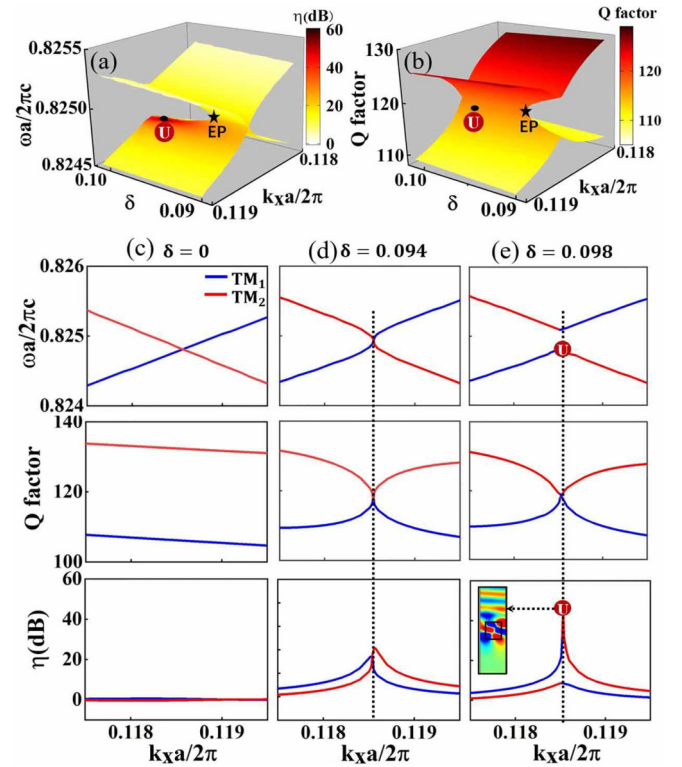


FIG. 4. (a) Real band and (b) radiative Q factors for perturbed eigenstates evolved from the coupling effect between TM_1 and TM_2 in the parameter space (k_x, δ) with $k_y = 0$. The color maps represent the asymmetric ratio (a) and the Q factor (b), respectively. Note that the occurring UGR and EP are also marked in (a),(b). (c)–(e) Three different cases on dispersion curves (upper panel), radiative Q factors (middle panel), and asymmetric radiation ratio η (lower panel) in 1D momentum space for original eigenstates $\Psi_{1,2}$ with $\delta = 0$ (c), and perturbed eigenstates $\Psi_{+,-}$ with $\delta = 0.094$ (d) and $\delta = 0.098$ (e). The dotted lines in (d),(e) depict the locations at which EP or UUGR appears in 1D momentum space. The associated profile of magnetic field H_y at the UUGR is shown in the inset of the bottom panel of (e). Other geometrical parameters and settings in (a)–(e) are the same as those in Fig. 1(a).

and the asymmetric ratio η in 2D parameter space (k_x, δ) with zero k_y . A single EP is found to be at the degenerate point $(0.1184, 0.094)$ (marked by a black star), whereas the UGR with the maximum η is located at a different location $(0.1185, 0.098)$ (denoted as a black dot), of which the real bands of TM_1 and TM_2 are anticrossing.

Subsequently, we aim to track in Fig. 5(a) the evolution of the UGRs that emerged from the interaction between the bands TM_1 and TM_2 in the 3D parameter space (k_x, δ, h) . Compared to case I of the UGRs related to the band crossing between TM_0 and TM_1 [Fig. 3(a)], a more sophisticated and intriguing phenomenon is indicated in Fig. 5(a) for case II, including the band transition for the residential position of UGRs, flipping of the radiation direction supported by UGRs, as well as the variation of topological properties involved in UGRs. To clarify the evolution process of UGRs, we cut the trajectory line of UGRs by using a set of surfaces [denoted in Fig. 5(a)], divide the whole space into several different regions, and then make a summary on the UGRs in Table I:

TABLE I. Evolution process of the UGRs that emerged from the interaction between the bands TM_1 and TM_2 .

Region	Thickness of grating h	Occurring band	Radiating direction	Topological index S
I	$h < 1.27a$	Not considered		
II	$1.27a < h < 1.39a$	TM_1	downward	-1
III	$1.39a < h < 1.735a$	TM_2	upward	+1
IV	$1.735a < h < 1.82a$	TM_2	downward	+1
V	$1.82a < h < 2.2a$	TM_1	upward	-1
VI	$h > 2.2a$	No UGRs		

(i) Region I ($h < 1.27a$): The intersection point of bands TM_1 and TM_2 shifts above the folded light line and enters

into the high-order diffraction parts [gray region in Fig. 1(a)]. So this case is not considered here.

(ii) Region II ($1.27a < h < 1.39a$): The quasi-DUGR appears on the band TM_1 , but it prefers to radiate downward with the minimum asymmetric ratio η . The topological index remains unchanged ($S = -1$) in this region.

(iii) Region III ($1.39a < h < 1.735a$): The UGRs transfer from band TM_1 to TM_2 , while the emission direction flips from downward to upward, and also the sign of the topological index changes from $S = -1$ to $+1$.

(iv) Region IV ($1.735a < h < 1.82a$): The emission direction flips again from upward to downward, while other information on the UGRs remains unchanged compared to that in region III, such as the occurring band and the topological index.

(v) Region V ($1.82a < h < 2.2a$): All situations are changed once more. For example, the UGRs occur again at TM_1 , the emission radiates upward, and the topological index is restored to $S = -1$. As h is increased, the intersection of the two bands moves towards the Γ point, and they finally merge at $h = 2.2a$.

(vi) Region VI ($h > 2.2a$): The two bands are separated from each other and no longer intersect for $h > 2.2a$. Therefore, no UGRs exist in this region.

We also examine the dispersion curve, Q factor, asymmetric ratio η (see details in Appendix D), as well as the phase vector field c_x [shown in Fig. 5(b)] by taking the typical examples at points Q_1 , Q_2 , Q_3 , and Q_4 in regions II, III, IV, and V, respectively. It is revealed that the UGRs prefer to reside at a band with a higher Q factor, due to its unique unidirectional radiation characteristics. A previous relevant study demonstrated that the UGRs may transit from one band to another, through the Fermi arc connected by paired EPs or a high-symmetry point, i.e., the Γ point. We find here that the conversion related to the UGRs, i.e., the occurring band, the radiating direction, or the phase singularity upon the UGRs, may happen on the high-symmetry line (k_x axis) in momentum space under appropriate configurations, when the external magnetic field is turned off ($\delta = 0$). Moreover, during the evolution process of the UGRs, it should be noticed that the overall topological indices S are always conserved ($S = 0$) in the (k_x, δ) plane at arbitrary h , which is in agreement with previous reports in the context of phase winding [23] or a polarization topological charge picture [20]. Additionally, the thickness of grating h or nonzero gyration δ in MO material may cause the variation of EPs accordingly, but no direct interplay with the UGRs is observed (not shown here).

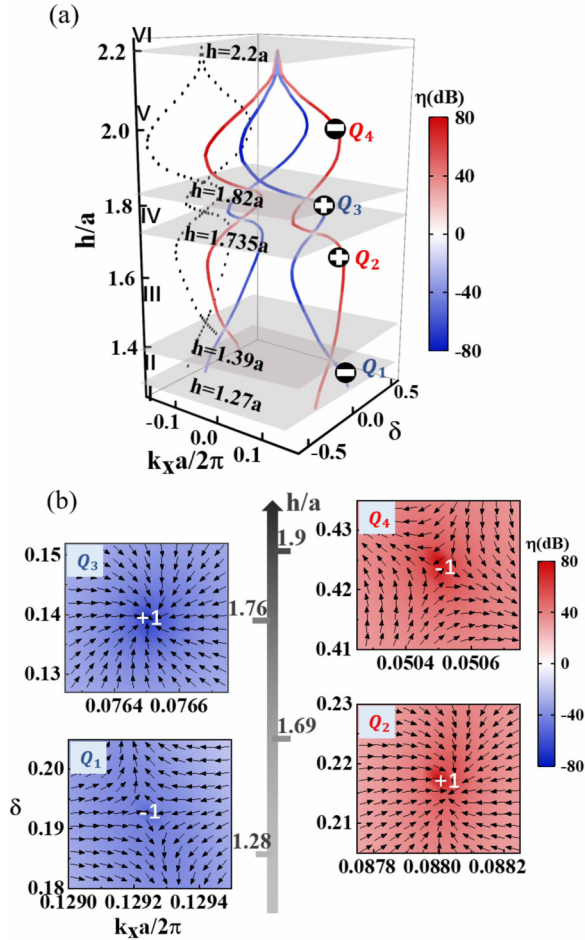


FIG. 5. (a) Evolution of UGRs in 3D parameter space (k_x, δ, h) , induced by TM_1 - TM_2 coupling. A series of gray planes cuts the trajectory line of UGRs and divides the whole space into several different regions I-VI, illustrating a vivid picture of the evolution process of UGRs. The color maps in different trajectories indicate the asymmetric radiation ratio. (b) The associated phase vector field and asymmetric ratio at four particular points Q_1 , Q_2 , Q_3 , and Q_4 in regions II, III, IV, and V, corresponding to the respective thickness of MO bars $h = 1.28a, 1.69a, 1.76a$, and $1.9a$. The topological indices $S = +1, -1$ are also labeled at the UGRs, respectively. Other geometrical parameters and settings in (a), (b) are the same as those in Fig. 4.

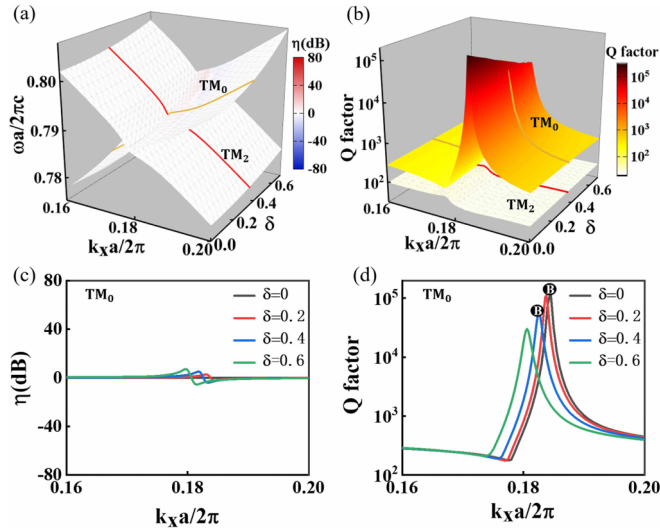


FIG. 6. (a) Real band and (b) Q factors for perturbed eigenstates evolved from the coupling effect between TM_0 and TM_2 in the parameter space (k_x, δ) with $k_y = 0$. The color maps represent the asymmetric ratio (a) and Q factor (b), respectively. (c) Asymmetric ratio and (d) Q factors for a series of gyrations of MO materials $\delta = 0, 0.2, 0.4$, and 0.6 , representing the absence of UGRs with small η and the emergence of quasi-BICs with highly radiative Q factors near the band crossing of TM_0 and TM_2 . The black dots in (d) mark the locations of BIC in 1D momentum space. Other geometrical parameters and settings in (a)–(d) are the same as those in Fig. 1(a).

B. Absence of UGRs and emergence of quasi-BICs

We now proceed to investigate the third case of nonorthogonal band coupling between TM_0 and TM_2 . Though both of the guided resonances TM_0 and TM_2 have even (or evenlike) transverse magnetic field profiles, their real bands are still likely to cross each other under some appropriate configurations, as shown in Fig. 6(a). As the magnetic field is applied externally, no remarkable difference (the maximum asymmetric ratio $|\eta| \approx 5$ dB) is seen in Fig. 6(c) between the waves radiating in the upward and downward directions, which implies that the UGR is absent in case III. However, it is indicated in Figs. 6(b) and 6(d) that the quasi-BICs upon band TM_0 may be formed with high radiative Q factor ($Q \approx 10^5$) near the band crossing of TM_0 and TM_2 , whether or not the magnetic field is present. Therefore, in contrast to that in case I (orthogonal band hybridization of TM_0 and TM_1 in Sec. III A 1) or case II (orthogonal interband coupling of TM_1 and TM_2 in Sec. III A 2), the nonorthogonal band overlapping between TM_0 and TM_2 in case III does not lead to the formation of the UGRs, but it results in the emergence of quasi-BICs.

C. Formation mechanisms of UGRs and BICs

The comparative analysis of the above three typical examples (Secs. III A and III B) and the specific description of the coupled mode theory (Sec. II) may suggest that the interband-coupling-induced UGRs or BICs should be strongly dependent on the interference patterns of radiating waves due to these two waveguide modes with different spatial parities:

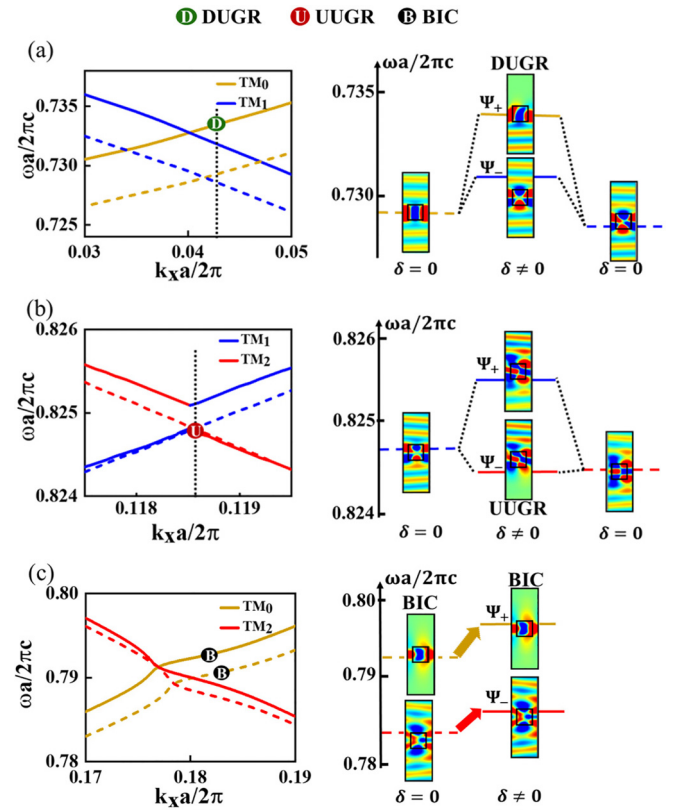


FIG. 7. A comparison of the formation mechanisms of UGRs and BICs for three different cases, due to the interband coupling effect of (a) TM_0 - TM_1 , (b) TM_1 - TM_2 , and (c) TM_0 - TM_2 . The left panel of (a)–(c): dispersion diagrams on the coupling scenario between their original bands ($\delta = 0$) with dotted lines and perturbed bands [(a) $\delta = 0.532$, (b) $\delta = 0.098$, and (c) $\delta = 0.4$] with solid lines. The right panel of (a)–(c): the associated schematic diagrams on the formation of (a) DUGRs, (b) UUGRs, and (c) BICs. DUGRs (UUGRs) can be created due to the destructive interference between radiating waves emitted from two orthogonal eigenmodes TM_0 and TM_1 (TM_1 and TM_2) in the upward (downward) directions. Conversely, BICs arise when radiating waves from modes TM_0 and TM_2 with identical transverse parities undergo destructive interference in both upward and downward directions simultaneously.

constructive or destructive interaction at two opposite radiating channels.

To illustrate this point, we then aim to elucidate the formation mechanism of UGRs or BICs in Fig. 7 by looking into the tuning of energy bands in 1D momentum space for the proposed MO grating structure as well as unique coupled eigenstates that originated from the presence of external magnetic field. As an example in Fig. 7(a) (also case I in Sec. III A 1), we consider the case of the orthogonal band coupling between TM_0 and TM_1 , possessing evenlike and oddlike transverse magnetic field distributions at $\delta = 0$, respectively. Initially, these two modes cross each other near the Γ point ($k_x = k_y = 0$), and the emission supported by the resonance remains identical in the upward and downward directions due to the existence of up-down mirror symmetry (σ_z symmetry) for this reciprocal grating with no external magnetic field ($\delta = 0$). Further, the presence of external magnetic field ($\delta \neq 0$)

may cause weak modulation in both near-field and far-field coupling of the two modes. Nevertheless, these two dispersion curves can still cross as straight lines, just shifting up slightly, and the band gap remains closed, as shown in the left panel of Fig. 7(a). However, due to the breaking of σ_z symmetry as well as the violation of reciprocity in the proposed MO grating ($\delta \neq 0$), the phase angle of radiating waves from their respective modes may be changed, which is strongly dependent on the emission direction. For instance, at a given wave vector of $k_x a/2\pi = +0.0427$ with $\delta = 0.532$, the phase difference $\Delta\phi_u$ in upward directions between radiating waves supported from the perturbed eigenstates of TM_0 and TM_1 is π , while that value $\Delta\phi_b$ in downward directions becomes 0. So the hybrid eigenstate Ψ_+ becomes DUGRs that radiate the power only toward the bottom of the MO grating structure, due to the destructive (constructive) interference between radiating waves in the upward (downward) direction. The associated magnetic field profile (H_z) upon Ψ_+ is confirmed in the right panel of Fig. 7(a). In contrast, it is seen that the emission remains identical on another coupled eigenstate Ψ_- with nearly the same phase difference in two opposite channels. As a consequence, the emergence of UGRs here comes from the hybridization of two guided resonances, of which their respective phase angles can be readily tuned by the applied magnetic field.

For another example shown in Fig. 7(b) (also case II in Sec. III A 2), a similar situation happens with the formation of UGRs near the band degeneracy of TM_1 and TM_2 away from the Γ point in 1D momentum space. On this occasion, at a specific wave vector of $k_x a/2\pi = +0.1185$ with $\delta = 0.098$, the coupled eigenstate Ψ_- turns into UUGRs that radiate in the upward direction [Fig. 7(b), right panel], because the radiating waves from oddlike TM_1 mode and evenlike TM_2 mode interfere destructively and cancel each other out in the downward direction. Unlike the above case in Fig. 7(a), the band gap may open up [depicted in Fig. 7(b)] due to the significant enhancement of the coupling effect between two bands by applying the magnetic field.

Finally, Fig. 7(c) illustrates the physical picture of forming BIC (quasi-BICs). In comparison with that of UGRs, two eigenmodes with the same spatial parities, such as TM_0 and TM_2 , may interact with each other, and the resultant coupled eigenstates bring out the quasi-BICs near the band crossing when radiating waves from these modes interfere destructively at both the upward and downward channels simultaneously. Under these circumstances, it is noticed that the presence or absence of external magnetic field has less of an impact on the emergence of quasi-BICs, giving rise to just a bit of movement at the occurring locations of BICs in 1D parameter space.

D. Excitation of UGRs related to directional coupling

As a validation of the prediction of UGRs based on mode analysis, we can directly excite and observe the eigenstate supported from the UGRs in this nonreciprocal MO grating structure under the illumination of a light beam. Figure 8(a) shows the overall transmittance spectrum T through the proposed MO grating within the reduced Brillouin zone when the gyration of MO material is tuned with $\delta = 0.532$. The responses in several specific regions are not included, such

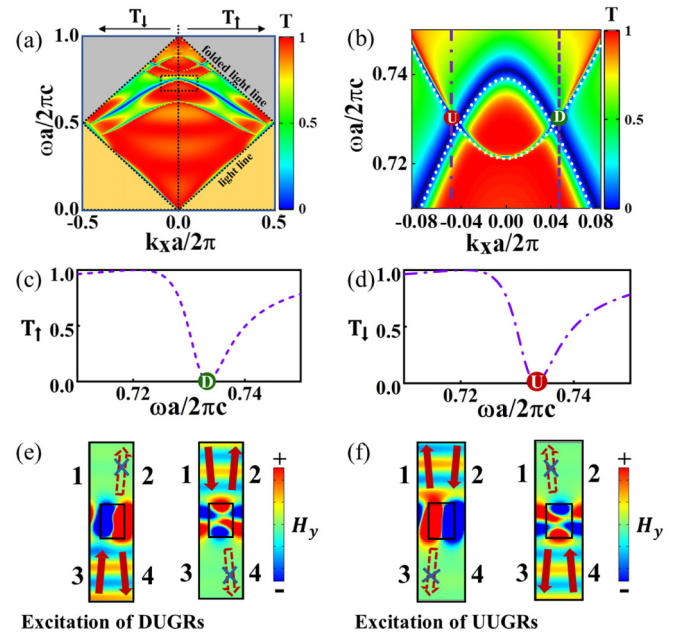


FIG. 8. Excitation of UGRs related to directional coupling. (a) The transmittance spectra in the reduced Brillouin zone for 1D MO grating with $\delta = 0.532$, with $T_\downarrow$ and $T_\uparrow$ being the transmittance for downward ($k_x < 0$) and upward ($k_x > 0$) rays either illuminated from port 1 (4) or 3 (2), respectively. (b) The same as (a), but at a zoom-in view on the transmittance around the vicinity of TM_0 - TM_1 band crossing. For comparison, the dispersion curves of TM_0 and TM_1 are also plotted with white dotted lines. The transmittance spectra of (c) $T_\uparrow$ and (d) $T_\downarrow$ for $k_x a/2\pi = +0.0427$ or -0.0427 , respectively, shown in (b) by the dashed line and the dashed-dotted line. The transmittance dips associated with the excitation of DUGRs (UUGRs) are marked in (b)–(d) with the green (red) dots. (e),(f) Profiles of magnetic field H_y for a lossless grating structure, when the light beam is incident from port 1 (3) (e) or port 4 (2) (f). Other geometrical parameters and settings in (a)–(d) are the same as those in Fig. 1(a).

as that below the light line (yellow region) and high-order diffraction parts above the folded light line (gray region). Other geometrical parameters and settings are the same as those in Fig. 1(a). Due to the reserved P-symmetry in the structure, the transmittance for light beams incident from port 1 (3) should be identical to that from port 4 (2) [note that the orientations of four ports 1, 2, 3, and 4 are labeled around the MO grating structure in Fig. 8(e) or 8(f)]. So herein we denote $T_\downarrow$ and $T_\uparrow$ as the transmittance for downward ($k_x < 0$) and upward ($k_x > 0$) rays either illuminated from port 1 (4) or 3 (2), respectively. It is found that the transmittance dips shown as dark blue colors in Fig. 8(a) are in excellent agreement with those radiative modes in Fig. 1(a). To see this more clearly, we zoom in and get a closeup view of the transmittance near the Γ point over a certain frequency window [shown in Fig. 8(b)] whose minimum values are mapped well onto its associated purely real band curves of TM_0 and TM_1 (indicated by white dotted lines). For comparison, the transmittances $T_\uparrow$ and $T_\downarrow$ are given, respectively, in Figs. 8(c) and 8(d) at a normalized wave vector of $k_x a/2\pi = +0.0427$ or -0.0427 . Notice that these two transmittance spectra remain almost the same, although the σ_z symmetry in the structure is broken. At

the normalized frequency $\omega a/2\pi c = 0.7335$ with zero transmittance (or unit reflectance), the DUGRs manifest as the excitation of the coupled eigenstate Ψ_+ , which can be verified from the spatial field distributed in the MO grating structure. Due to the fact that the DUGRs radiate waves solely toward the bottom of the grating structure, the DUGRs (predicted in Sec. III A 1) can be well excited, but only for the case of upward rays from the channels below. Apparently, the coupled eigenstate Ψ_+ associated with the DUGRs is observed in the grating structure when the wave travels from port 3 at an incident angle $\theta = +3.34^\circ$ corresponding to $k_x a/2\pi = +0.0427$ [Fig. 8(e), left panel]. In contrast, for the case of downward incidence when the light beam is incident from port 1 at the same angle, the eigenstate Ψ_+ fails to be excited, and another hybrid eigenstate Ψ_- is seen instead that can radiate waves in both the upward and downward directions [Fig. 8(e), right panel]. As might be expected, the UUGRs at the same frequency of $\omega a/2\pi c = 0.7335$ with $k_x a/2\pi = -0.0427$ can also be excited accordingly through the rays with an incident angle $\theta = -3.34^\circ$, but illuminating from port 2 instead of port 4 shown in Fig. 8(f).

E. Perfect absorption under single-sided illumination

Until now, we have considered a nonreciprocal grating structure composed of lossless MO material by applying static magnetic field, where the emergence of UGRs provides us with an opportunity to achieve highly directional coupling with external incident waves: the excited resonance can decay radiatively into only one direction, whereas the radiation coupling to the opposite emission channel is greatly suppressed. Further, once the intrinsic loss is incorporated into the MO material [i.e., the diagonal element in Eq. (1) is replaced by the complex permittivity $\epsilon_{\text{MO}} = \epsilon_r + i\epsilon'$ with finite loss ϵ'], a nonradiative decay channel is then added to the structure. By combining these different channels and matching the decay lifetimes between the radiative and nonradiative processes, we may expect to achieve perfect absorption for light incident from a single side with strong coupling to the structure.

To verify this prediction, we finally investigate in Fig. 9 the directional absorption stemming from the excitation of UGRs in the grating structure in the presence of intrinsic material loss as well as external magnetic field. Followed by the preceding section, a similar example is given in Figs. 9(a) and 9(b) at the occurring location of DUGRs with the normalized wave vector $k_x a/2\pi = +0.0427$ and normalized frequency $\omega a/2\pi c = 0.7335$ [indicated as the green dot in Fig. 8(b)], except that the loss in MO material is taken into account. In Figs. 9(a) and 9(b), we make a comparison between the absorptance $A_\uparrow$ and $A_\downarrow$ for a lossy MO grating in the 2D parameter space (ϵ', δ) . Here, $A_\uparrow$ ($A_\downarrow$) represents the upward (downward) absorptance when the wave is incident from port 3 (port 1), respectively. Notably, herein lies the remarkable distinction in absorption, which is strongly dependent on the incident direction. More importantly, appropriate tuning of loss in MO material contributes to achieving nearly perfect absorption in one of the incident directions. For instance, a sharp peak in absorptance $A_\uparrow$ occurs at (0.006, 0.532) [marked with a green dot in Fig. 9(a)] where the DUGRs are supported, whereas absorptance $A_\downarrow$ varies smoothly and remains

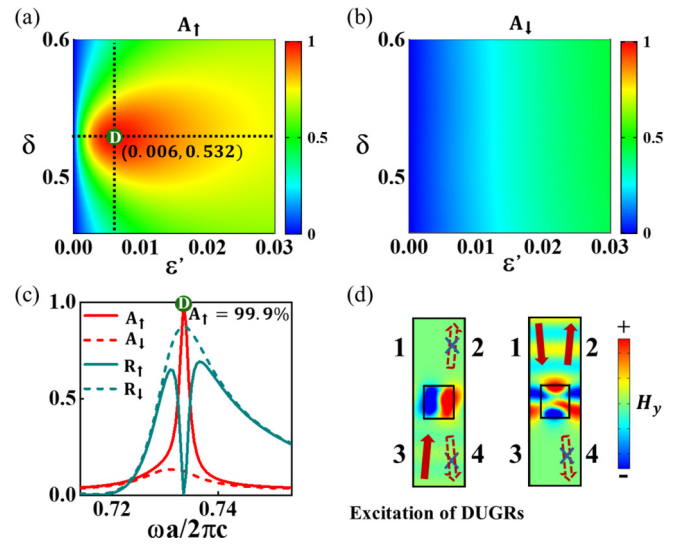


FIG. 9. Perfect absorption under single-sided illumination. (a) The absorptance $A_\uparrow$ and (b) $A_\downarrow$ vs the loss ϵ' and gyration δ in MO material at the normalized wave vector $k_x a/2\pi = +0.0427$ and normalized frequency $\omega a/2\pi c = 0.7335$. Near-perfect absorption in $A_\uparrow$ occurs at (0.006, 0.532) [marked with a green dot in (a)], where DUGRs are supported. (c) The absorptance spectra ($A_\uparrow$ and $A_\downarrow$) as well as the reflectance spectra ($R_\uparrow$ and $R_\downarrow$) around the DUGRs. (d) The same as Fig. 8(e), except that the finite loss $\epsilon' = 0.006$ in MO material is considered. Here, almost complete absorption is seen for upward-illuminated structure, due to the excitation of DUGRs.

relatively low intensity in the same vicinity of 2D parameter space (ϵ', δ) . For clarity, we further plot in Fig. 9(c) the absorptance spectra ($A_\uparrow$ and $A_\downarrow$) as well as the reflectance spectra ($R_\uparrow$ and $R_\downarrow$) around the DUGRs. [Note that the transmittance spectra $T_\uparrow$ and $T_\downarrow$ remain identical even if the material loss is considered, and meanwhile the magnitude of $T_\uparrow$ or $T_\downarrow$ in the case of a small loss rate ϵ' is close to that in the lossless case shown in Fig. 8(c) (not shown here)]. It is clearly seen that the absorptance could be as high as $A_\uparrow = 99.9\%$ and also the reflectance $R_\uparrow$ drops to zero at a given normalized frequency $\omega a/2\pi c = 0.7335$, implying that near-perfect absorption can be attained when the wave is incident from port 3. By contrast, the weak absorption $A_\downarrow$ and strong reflection $R_\downarrow$ are observed at the same point, under the illumination of a light beam from port 1. Eventually, we confirm in Fig. 9(d) the obtained results associated with the occurrence of DUGRs [marked with a green dot in Fig. 9(a) or 9(c)] by illustrating the field distribution in the vicinity of MO grating structure. Due to the excitation of DUGRs for an upward-illuminated structure, almost complete absorption is seen with the generation of eigenstate Ψ_+ , accompanied by both zero transmission and zero reflection [Fig. 9(d), left panel]. On the other hand, the downward-illuminated structure acts as a reflector with the excitation of Ψ_- [Fig. 9(d), right panel]. In this regard, the situation is analogous to that for the lossless case [Fig. 8(d), right panel].

IV. DISCUSSION AND CONCLUSION

In conclusion, we show that the periodic MO grating structure in violation of reciprocity can enable intriguing

single-sided radiation through the creation of topological UGRs. With the addition of external static magnetic field associated with the gyration of MO material, multiple UGRs are generated by coupling two energy bands with different spatial parities, such as TM_0/TM_1 or TM_1/TM_2 . The dynamic evolution process of topological UGRs is investigated in 3D parameter space, providing a vivid picture for actively manipulating the optical radiation as well as the phase singularity of a far-field polarization vector. The underlying mechanism of the formation of UGRs can be explained by the interference patterns of radiating waves, which are strongly dependent on the emission channels. Finally, the single-sided radiation characteristic is verified by the excitation of UGRs in the grating structure. For example, the hybrid eigenstate associated with UGRs is observed in the lossless MO grating, which is closely related to directional coupling of external waves. And near perfect absorption under single-sided illumination could also be achieved in a lossy system by delicately tuning the intrinsic absorption loss in MO material.

In contrast with previous studies on the achievement of topological UGRs, herein we break the requisite symmetries in geometry, i.e., the up-down σ_z symmetry, by incorporating MO material into the grating structure. In general, the response of MO material could be gyroelectric [i.e., bismuth iron garnet (BIG) in the optical or infrared frequency [31,43,44], or magnetized plasma–indium antimonide (InSb) operating at THz frequencies [53,54]] or gyromagnetic [i.e., yttrium-iron-garnet (YIG) at microwave frequencies [55–57]]. And MO devices can possess the unique nonreciprocal effect with broad magnetic tunability, universally applied in nonreciprocal transmission [58,59], polarization control [60], magnetic field sensing [61], and other fields [62]. Therefore, the design principle of nonreciprocal structure with MO effects may provide an alternative strategy to generate and control dynamically the UGRs in a more flexible manner, relaxing the limitation on the specific options in geometrical settings for structure. Additionally, the MO material in the proposed grating is magnetized in the Voigt configuration, where light propagates in the plane perpendicular to the y axis, as shown in the inset of Fig. 1(a). In this geometry, TM and TE modes (i.e., H or E fields polarized along the y direction, respectively) are completely decoupled. So we mainly focus on the asymmetric radiation supported from TM modes, whereas TE modes still possess reciprocity, and the corresponding emissions remain identical in two opposite directions.

In addition, we emphasize that the UGRs demonstrated in this work are a direct consequence of the coupling effect between two crossing bands. So they may be termed interband-coupling-induced UGRs [23], which are quite different from the accidental UGRs supported on the isolated band via tuning parameters. In this regard, such interband-coupling-induced UGRs may seem to be analogous to the Friedrich-Wintgen (FW) BICs [63] (also corresponding to the case in Sec. III B). However, there is no direct correlation between them. First, the symmetry requirement for two crossing modes is different. Upon forming the interband-coupling UGRs, two bands possess different vertical symmetries (i.e., an evenlike TM_0 mode and an oddlike TM_1 mode), while for FW BICs, the same vertical symmetries for two modes should

be preserved (i.e., both of evenlike TM_0 and TM_2 modes). And then, it is not necessary to generate interband-coupling UGRs that evolved from a FW-BIC. For instance, TM_0 and TM_1 modes are completely orthogonal without applied magnetic field in the case of Sec. III A 1, and no interaction occurs between them, implying that no FW BIC is formed at the initial stage. [Despite the fact that there exists a BIC at the Γ point of band TM_0 shown in Fig. 2(b), it belongs to a class of symmetry-protected BIC, originating substantially from the in-plane C_2 symmetry of structure.]

We finally remark that the design principle based on nonreciprocal MO grating structure provides an efficient and active approach to exploit the guided resonance in topological nature (i.e., phase singularity or polarization singularity for a far-field radiation field), and to dynamically control the optical radiation, such as highly directional radiation or perfect absorption from a single side. It may greatly benefit applications such as surface-emitting lasers, high-performance grating couplers, and optical antennas, and it may also optimize the design and improve the efficiency of photodetectors and solar cells. In a follow-up work, the nonreciprocal MO grating structure may be extended to other geometries (i.e., the Faraday configuration in which light propagates along the magnetization direction) or high-dimensional space, exploring a richer picture on the UGRs for manipulating light-matter interactions.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grants No. 12174052 and No. 22473060).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: THE RELATIONSHIP BETWEEN THE ASYMMETRIC RADIATION RATIO η , THE GYRATION δ IN MO MATERIAL, AND THE GRATING THICKNESS h

In Sec. III A 1 of the main text, we illustrate the first case of TM_0 - TM_1 coupling, where a genuine DUGR arises for a particular gyration $\delta = 0.532$ in MO material. In Fig. 10(a) of this Appendix, we then check into the evolution of such a genuine DUGR by investigating the radiation ratio η curves as δ varies. It is evident that a quasi-DUGR occurs as δ increases from zero, and it transforms progressively into a genuine DUGR with the minimum η numerically less than -70 dB at $k_x a/2\pi = +0.0427$ when $\delta = 0.532$. However, this DUGR reverts to a quasi-DUGR with a further increase of δ . Furthermore, we discuss in Fig. 10(b) the impact of grating thickness h on the minimum radiation ratio $\eta_{\min}$ for DUGRs. Notably, $\eta_{\min}$ experiences a notable decrease around a specific grating thickness of $h_{\text{down}} \equiv h/a = 1.45$. Similarly, we also depict in Fig. 10(c) how a quasi-UUGR emerges

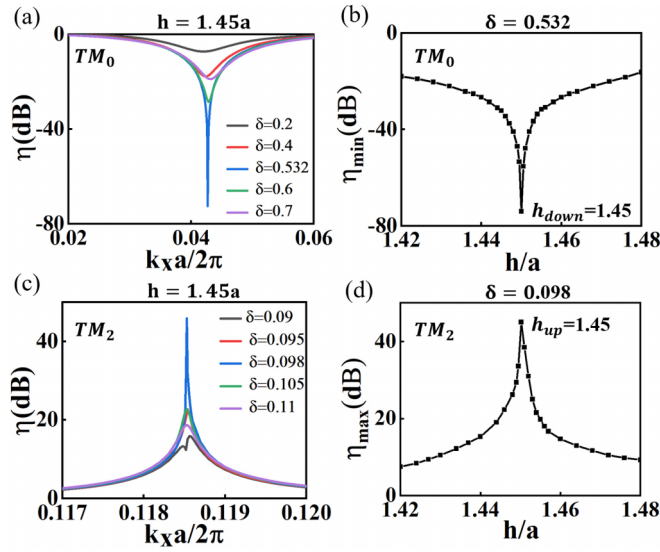


FIG. 10. (a), (c) The evolution of asymmetric ratio η curves for perturbed eigenstates Ψ_+ near TM_0 - TM_1 coupling (a) or TM_1 - TM_2 coupling (c), as a function of k_x under the variation of the gyration δ in MO material. The grating thickness is fixed with $h = 1.45a$. (b) Simulated $\eta_{\min}$ for the DUGRs in (a), or (d) $\eta_{\max}$ for the UUGRs in (c) as a function of h . The gyration in MO materials is fixed with $\delta = 0.532$ in (b) and $\delta = 0.098$ in (d), respectively. Other geometrical parameters and settings in (a)–(d) are the same as those in Fig. 1(a).

for the second case of TM_1 - TM_2 coupling in Sec. III A 2, with a variation of δ . In this case, the maximum η beyond 40 dB is achieved at $k_x a/2\pi = +0.098$ when $\delta = 0.098$. And there is a significant increase of η at the same grating thickness of $h_{\text{up}} \equiv h/a = 1.45$ as that in the first case, shown in Fig. 10(d). Consequently, it is available for the generation and manipulation of multiple UGRs in a given MO grating structure, through tuning the single parameter of gyration δ in MO material via external magnetic field.

APPENDIX B: THE ANNIHILATION OF UGRS ARISING FROM AVOIDED BAND CROSSING

As a supplementary figure to Fig. 3(a) in the main text, we give two more examples to clarify the evolution process of UGRs near the band crossing between TM_0 and TM_1 . As the grating thickness increases to $h = 1.518a$, bands TM_0 and TM_1 still cross each other, and two intersection points tend to collide at the Γ point as shown in the upper panel of Fig. 11(a). Meanwhile, there remains the unique characteristics of highly asymmetrical emission in upward and downward directions due to the presence of DUGRs and UUGRs, i.e., $\eta = -49$ dB at $k_x a/2\pi = +0.0087$ or $\eta = 49$ dB at $k_x a/2\pi = -0.0087$ as indicated in the lower panel of Fig. 11(a). Further, a bit more of an increase of grating thickness, i.e., $h = 1.54a$, can cause a visible gap between bands TM_0 and TM_1 at the Γ point. Correspondingly, the UGRs vanish and the emission goes back to being almost identical ($\eta \rightarrow 0$) in the opposite directions over the entire 1D momentum space, shown clearly in Fig. 11(b). Therefore, it is confirmed here that the generation, evolution, and annihilation of UGRs can be readily controlled

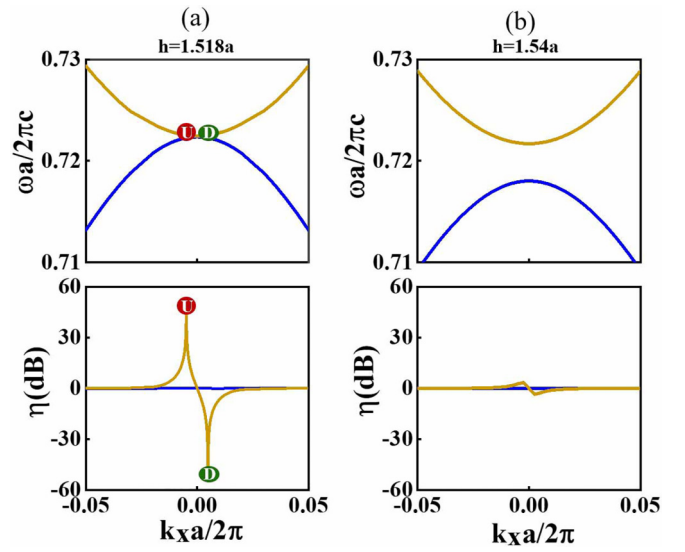


FIG. 11. Two specific cases on dispersion curves (upper panel) and asymmetric radiation ratio η (lower panel) in 1D momentum space for perturbed eigenstates $\Psi_{+,-}$ near TM_0 - TM_1 coupling with a grating thickness of (a) $h = 1.518a$ and (b) $h = 1.54a$. The gyration in MO material is fixed with $\delta = 0.065$.

by tuning the interband coupling effect in such a nonreciprocal MO grating structure.

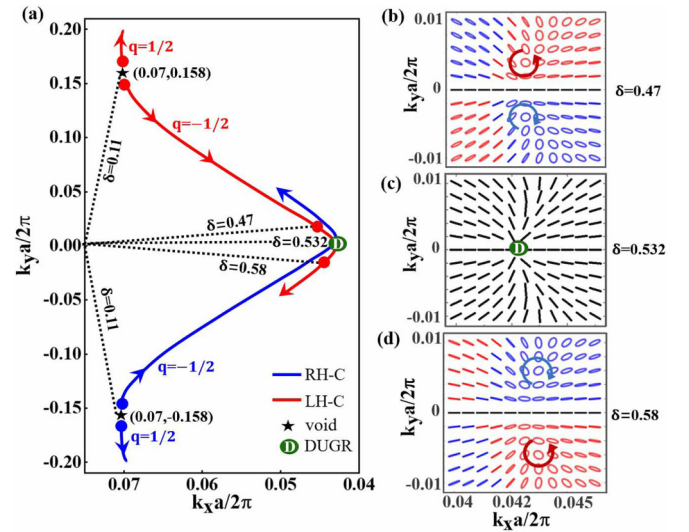


FIG. 12. (a) The generation, evolution, and coalescence of a pair of C points in the right half in 2D $\mathbf{k}_{\parallel}$ momentum space (k_x, k_y), as δ increases for upward radiation. The left-handed (right-handed) circular polarization states with a topological charge of $q = \pm 1/2$ are marked by the red (blue) dot, respectively. A UGR with a green D marker lies in the k_x axis. (b)–(d) Far-field polarization patterns of perturbed eigenstates in upward radiation for $\delta = 0.47$ (b), $\delta = 0.532$ (c), and $\delta = 0.58$ (d). The red (blue) ellipse denotes the left-handed (right-handed) elliptic polarizations with different major axes orientation. The black line represents the linear polarization.

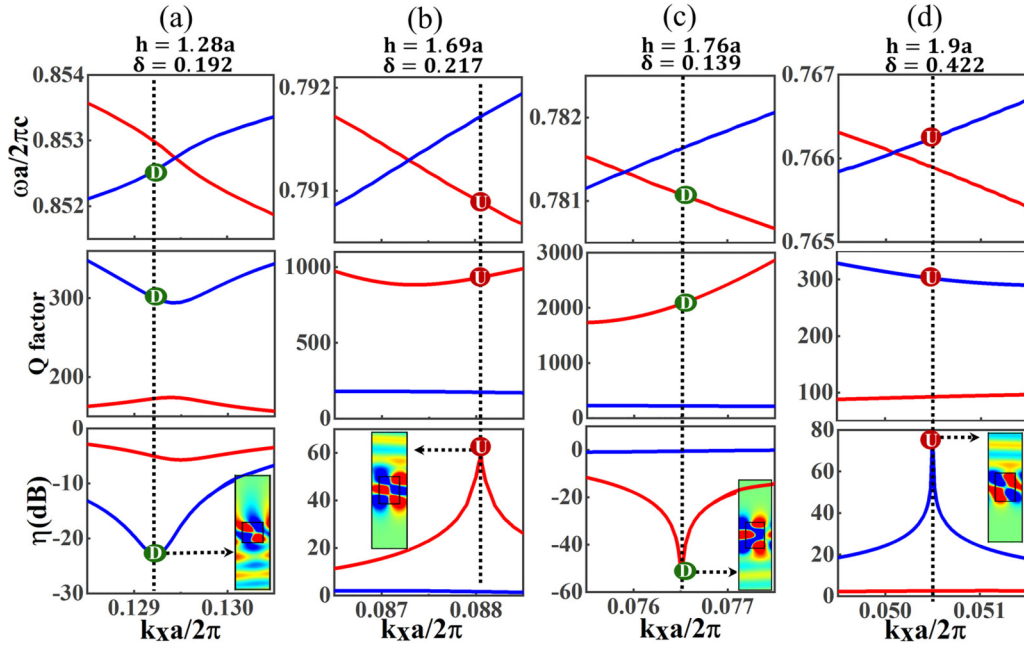


FIG. 13. (a)–(d) Band structures (upper panel), Q factor (middle panel), and asymmetric radiation ratio (lower panel) for points Q_1 , Q_2 , Q_3 , and Q_4 in Fig. 5(b) of the main text. The green (red) dots mark the locations of UGRs. Insets of the bottom panel in (a)–(d): The associated profiles of magnetic field H_y at the DUGR (UGR) for perturbed eigenstates Ψ_+ or Ψ_- .

APPENDIX C: REALIZATION OF UGRS BY MERGING A PAIR OF C POINTS

In previous work, the generation of UGRs on energy bands could usually be explained from the viewpoint of polarization topological charges—the singularity for a polarization vector $\mathbf{c}(\mathbf{k}_{\parallel}) = c_x(\mathbf{k}_{\parallel}) + ic_y(\mathbf{k}_{\parallel})$ of far-field radiation in 2D $\mathbf{k}_{\parallel}$ momentum space (k_x, k_y) , defined as [20,35,37]

$$q = \frac{1}{2\pi} \oint_L d\mathbf{k}_{\parallel} \cdot \nabla_{\mathbf{k}_{\parallel}} \phi(\mathbf{k}_{\parallel}). \quad (\text{C1})$$

Here, $\phi(\mathbf{k}_{\parallel})$ is the azimuthal angle of the polarization major axis, and L is a closed path in the $\mathbf{k}_{\parallel}$ space that travels around the singularity point in a counterclockwise direction. Note that nonzero topological charge q arises when the polarization states are ill-defined in the momentum space, i.e., q is a half-integer for C points (circular polarization points) or q is an integer for a V point (the vortex center of polarization fields).

In this Appendix, we demonstrate in Fig. 12(a) the formation of DUGRs by utilizing a topological charge picture on the generation, evolution, and coalescence of C points in 2D momentum space (k_x, k_y) . Initially, two pairs of left-handed (right-handed) C points carrying half-integer charges $q = \pm 1/2$ for perturbed eigenstates Ψ_+ in upward radiation are spawned from two mirror-positioned voids (marked by stars) when $\delta = \delta_v = 0.11$. As δ increases, the C points begin to evolve from $\mathbf{k}_{\parallel}a/2\pi = (0.07, \pm 0.158)$, respectively. However, the evolution process is quite different in positive (negative) charges for C points: the positive charges $q =$

$1/2$ depart towards the far end of the $\pm k_y$ axis, whereas the negative charges $q = -1/2$ move towards each other. At $\delta = \delta_u = 0.532$, a pair of C points with opposite handedness and identical charges $q = -1/2$ meet at the k_x axis, $\mathbf{k}_{\parallel}a/2\pi = (0.0427, 0)$, becoming a V point with an integer charge $(-1/2) + (-1/2) \rightarrow -1$. Then, the upward radiation is fully suppressed, creating a solely downward radiating DUGR (green D marker). Eventually, a further increase of δ ($> \delta_u$) causes the splitting of the integer charge as $-1 \rightarrow (-1/2) + (-1/2)$, resulting in the disappearance of DUGR and again the generation of a pair of C points. For the purpose of giving insight into the emergence of DUGRs, we also calculate the far-field polarization states in upward radiation near the merging point of DUGRs, and we plot them in Figs. 12(b)–12(d) for $\delta = 0.47, 0.532$, and 0.58 , respectively. It is seen that the chiral dichroism in the optical response can be generally supported for left- and right-handed circularly polarized light in upward radiation, except for the accidental appearance of a unidirectional polarization vortex center when $\delta = 0.532$, shown in Fig. 12(c). Therefore, we give here an explicit example to display and validate the realization of DUGRs by merging a pair of C points with identical half-integer charges via magnetic modulation.

APPENDIX D: SUPPLEMENTARY FIGURE

Figure 13 is a supplementary figure to Fig. 5(b) in the main text.

[1] R. Nagarajan, C. H. Joyner, R. P. Schneider, J. S. Bostak, T. Butrie, A. G. Dentai, V. G. Dominic, P. W. Evans, M. Kato, M.

Kauffman *et al.*, Large-scale photonic integrated circuits, *IEEE J. Sel. Top. Quantum Electron.* **11**, 50 (2005).

- [2] W. Luo, L. Cao, Y. Shi, L. Wan, H. Zhang, S. Li, G. Chen, Y. Li, S. Li, Y. Wang *et al.*, Recent progress in quantum photonic chips for quantum communication and internet, *Light Sci. Appl.* **12**, 175 (2023).
- [3] D. Vermeulen, S. Selvaraja, P. Verheyen, G. Lepage, W. Bogaerts, P. Absil, D. Van Thourhout, and G. Roelkens, High-efficiency fiber-to-chip grating couplers realized using an advanced CMOS-compatible silicon-on-insulator platform, *Opt. Express* **18**, 18278 (2010).
- [4] A. Mekis, S. Gloeckner, G. Masini, A. Narasimha, T. Pinguet, S. Sahni, and P. D. Dobbelaere, A grating-coupler-enabled CMOS photonics platform, *IEEE J. Sel. Top. Quantum Electron.* **17**, 597 (2011).
- [5] H. Subbaraman, X. Xu, A. Hosseini, X. Zhang, Y. Zhang, D. Kwong, and R. T. Chen, Recent advances in silicon-based passive and active optical interconnects, *Opt. Express* **23**, 2487 (2015).
- [6] M. Yoshida, M. De Zoysa, K. Ishizaki, Y. Tanaka, M. Kawasaki, R. Hatsuda, B. Song, J. Gelleta, and S. Noda, Double-lattice photonic-crystal resonators enabling high-brightness semiconductor lasers with symmetric narrow-divergence beams, *Nat. Mater.* **18**, 121 (2019).
- [7] R. Sakata, K. Ishizaki, M. De Zoysa, S. Fukuhara, T. Inoue, Y. Tanaka, K. Iwata, R. Hatsuda, M. Yoshida, J. Gelleta *et al.*, Dually modulated photonic crystals enabling high-power high-beam-quality two-dimensional beam scanning lasers, *Nat. Commun.* **11**, 3487 (2020).
- [8] R. Morita, T. Inoue, M. De Zoysa, K. Ishizaki, and S. Noda, Photonic-crystal lasers with two-dimensionally arranged gain and loss sections for high-peak-power short-pulse operation, *Nat. Photon.* **15**, 311 (2021).
- [9] T. H. Taminiau, F. D. Stefani, F. B. Segerink, and N. F. Van Hulst, Optical antennas direct single-molecule emission, *Nat. Photon.* **2**, 234 (2008).
- [10] T. Kosako, Y. Kadoya, and H. F. Hofmann, Directional control of light by a nano-optical Yagi-Uda antenna, *Nat. Photon.* **4**, 312 (2010).
- [11] C. Belacel, B. Habert, F. Bigourdan, F. Marquier, J.-P. Hugonin, S. Michaelisde Vasconcellos, X. Lafosse, L. Coolen, C. Schwob, C. Javaux *et al.*, Controlling spontaneous emission with plasmonic optical patch antennas, *Nano Lett.* **13**, 1516 (2013).
- [12] M. Fan, M. A. Popović, and F. X. Kärtner, High directivity vertical fiber-to-chip coupler with anisotropically radiating grating teeth, in *Conference on Lasers and Electro-Optics* (Optica Publishing Group, 2007), p. CTuDD3.
- [13] J. Song, H. Heo, S. Lee, and S. Kim, Mirror-less unidirectional radiation in an asymmetric single resonator, *J. Light. Technol.* **40**, 5163 (2022).
- [14] W. Streifer, R. D. Burnham, and D. R. Scifres, Analysis of grating-coupled radiation in GaAs:GaAlAs lasers and waveguides - II: Blazing effects, *IEEE J. Quantum Electron.* **12**, 494 (1976).
- [15] J. M. Miller, N. de Beaucoudrey, P. Chavel, J. Turunen, and E. Cambril, Design and fabrication of binary slanted surface-relief gratings for a planar optical interconnection, *Appl. Opt.* **36**, 5717 (1997).
- [16] K. X. Wang, Z. Yu, S. Sandhu, and S. Fan, Fundamental bounds on decay rates in asymmetric single-mode optical resonators, *Opt. Lett.* **38**, 100 (2013).
- [17] H. Zhou, B. Zhen, C. W. Hsu, O. D. Miller, S. G. Johnson, J. D. Joannopoulos, and M. Soljačić, Perfect single-sided radiation and absorption without mirrors, *Optica* **3**, 1079 (2016).
- [18] S.-G. Lee, S.-H. Kim, and C.-S. Kee, Bound states in the continuum (BIC) accompanied by avoided crossings in leaky-mode photonic lattices, *Nanophotonics* **9**, 4373 (2020).
- [19] S. Mukherjee, J. Gomis-Bresco, D. Artigas, and L. Torner, Unidirectional guided resonances in anisotropic waveguides, *Opt. Lett.* **46**, 2545 (2021).
- [20] Y. Zeng, G. Hu, K. Liu, Z. Tang, and C.-W. Qiu, Dynamics of topological polarization singularity in momentum space, *Phys. Rev. Lett.* **127**, 176101 (2021).
- [21] Z. Zhang, F. Wang, H. Wang, Y. Hu, X. Yin, W. Hu, and C. Peng, All-pass phase shifting enabled by symmetric topological unidirectional guided resonances, *Opt. Lett.* **47**, 2875 (2022).
- [22] C. Zhao, G. Hu, Y. Chen, Q. Zhang, Y. Zhang, and C.-W. Qiu, Unidirectional bound states in the continuum in Weyl semimetal nanostructures, *Photon. Res.* **10**, 1828 (2022).
- [23] X. Yin, T. Inoue, C. Peng, and S. Noda, Topological unidirectional guided resonances emerged from interband coupling, *Phys. Rev. Lett.* **130**, 056401 (2023).
- [24] M. Gong, J. Liu, L. Ge, H. Xiang, and D. Han, Multipolar perspective on unidirectional guided resonances, *Phys. Rev. A* **108**, 013522 (2023).
- [25] Z.-P. Zhuang, H.-L. Zeng, X.-D. Chen, X.-T. He, and J.-W. Dong, Topological nature of radiation asymmetry in bilayer metagratings, *Phys. Rev. Lett.* **132**, 113801 (2024).
- [26] X. Ni, Y. Liu, B. Lou, M. Zhang, E. L. Hu, S. Fan, E. Mazur, and H. Tang, Three-dimensional reconfigurable optical singularities in bilayer photonic crystals, *Phys. Rev. Lett.* **132**, 073804 (2024).
- [27] S.-G. Lee, S.-H. Kim, and W.-J. Lee, Transition from quasi-unidirectional to unidirectional guided resonances in leaky-mode photonic lattices, *Laser Photon. Rev.* **18**, 2301339 (2024).
- [28] X. Yin, J. Jin, M. Soljačić, C. Peng, and B. Zhen, Observation of topologically enabled unidirectional guided resonances, *Nature (London)* **580**, 467 (2020).
- [29] Z. Zhang, X. Yin, Z. Chen, F. Wang, W. Hu, and C. Peng, Observation of intensity flattened phase shifting enabled by unidirectional guided resonance, *Nanophotonics* **10**, 4467 (2021).
- [30] Y. Zhang, A. Chen, W. Liu, C. W. Hsu, B. Wang, F. Guan, X. Liu, L. Shi, L. Lu, and J. Zi, Observation of polarization vortices in momentum space, *Phys. Rev. Lett.* **120**, 186103 (2018).
- [31] C. Zhao, S. Dong, Q. Zhang, Y. Zeng, G. Hu, and Y. Zhang, Magnetic modulation of topological polarization singularities in momentum space, *Opt. Lett.* **47**, 2754 (2022).
- [32] H. Luo, L. Liu, Z. Xi, Y. Lu, and P. Wang, Dynamics of diverse polarization singularities in momentum space with far-field interference, *Phys. Rev. A* **107**, 013504 (2023).
- [33] W. Liu, B. Wang, Y. Zhang, J. Wang, M. Zhao, F. Guan, X. Liu, L. Shi, and J. Zi, Circularly polarized states spawning from bound states in the continuum, *Phys. Rev. Lett.* **123**, 116104 (2019).
- [34] X. Zhang, Y. Liu, J. Han, Y. Kivshar, and Q. Song, Chiral emission from resonant metasurfaces, *Science* **377**, 1215 (2022).
- [35] T. Yoda and M. Notomi, Generation and annihilation of topologically protected bound states in the continuum and circularly

- polarized states by symmetry breaking, *Phys. Rev. Lett.* **125**, 053902 (2020).
- [36] W. Ye, Y. Gao, and J. Liu, Singular points of polarizations in the momentum space of photonic crystal slabs, *Phys. Rev. Lett.* **124**, 153904 (2020).
- [37] B. Zhen, C. W. Hsu, L. Lu, A. D. Stone, and M. Soljačić, Topological nature of optical bound states in the continuum, *Phys. Rev. Lett.* **113**, 257401 (2014).
- [38] W. Głowadzka, M. Wasiak, and T. Czyszanowski, True- and quasi-bound states in the continuum in one-dimensional gratings with broken up-down mirror symmetry, *Nanophotonics* **10**, 3979 (2021).
- [39] P. Hu, J. Wang, Q. Jiang, J. Wang, L. Shi, D. Han, Z. Q. Zhang, C. T. Chan, and J. Zi, Global phase diagram of bound states in the continuum, *Optica* **9**, 1353 (2022).
- [40] K. Koshelev, A. Bogdanov, and Y. Kivshar, Engineering with bound states in the continuum, *Opt. Photon. News* **31**, 38 (2020).
- [41] X. Du, L. Xiong, X. Zhao, S. Chen, J. Shi, and G. Li, Dual-band bound states in the continuum based on hybridization of surface lattice resonances, *Nanophotonics* **11**, 4843 (2022).
- [42] H. M. Doeleman, F. Monticone, W. den Hollander, A. Alú, and A. F. Koenderink, Experimental observation of a polarization vortex at an optical bound state in the continuum, *Nat. Photon.* **12**, 397 (2018).
- [43] E. Jesenska, T. Yoshida, K. Shinozaki, T. Ishibashi, L. Beran, M. Zahradnik, R. Antos, M. Kučera, and M. Veis, Optical and magneto-optical properties of Bi substituted yttrium iron garnets prepared by metal organic decomposition, *Opt. Mater. Express* **6**, 1986 (2016).
- [44] D. Yan and H. Wang, Magneto-optical properties and gyration vectors of iron garnet films containing different bismuth ions, *Appl. Phys. A* **129**, 510 (2023).
- [45] W. Suh, Z. Wang, and S. Fan, Temporal coupled-mode theory and the presence of non-orthogonal modes in lossless multi-mode cavities, *IEEE J. Quantum Electron.* **40**, 1511 (2004).
- [46] C. Dembowski, H.-D. Gräf, H. L. Harney, A. Heine, W. D. Heiss, H. Rehfeld, and A. Richter, Experimental observation of the topological structure of exceptional points, *Phys. Rev. Lett.* **86**, 787 (2001).
- [47] M. V. Berry, Physics of nonhermitian degeneracies, *Czech. J. Phys.* **54**, 1039 (2004).
- [48] W. D. Heiss, The physics of exceptional points, *J. Phys. A* **45**, 444016 (2012).
- [49] B. Zhen, C. W. Hsu, Y. Igarashi, L. Lu, I. Kaminer, A. Pick, S.-L. Chua, J. D. Joannopoulos, and M. Soljačić, Spawning rings of exceptional points out of Dirac cones, *Nature (London)* **525**, 354 (2015).
- [50] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching, *Nature (London)* **537**, 76 (2016).
- [51] H. Zhou, C. Peng, Y. Yoon, C. W. Hsu, K. A. Nelson, L. Fu, J. D. Joannopoulos, M. Soljačić, and B. Zhen, Observation of bulk Fermi arc and polarization half charge from paired exceptional points, *Science* **359**, 1009 (2018).
- [52] M.-A. Miri and A. Alú, Exceptional points in optics and photonics, *Science* **363**, eaar7709 (2019).
- [53] Q. Y. Shi, H. Y. Dong, K. H. Fung, Z.-G. Dong, and J. Wang, Optical non-reciprocity induced by asymmetrical dispersion of Tamm plasmon polaritons in terahertz magnetoplasmonic crystals, *Opt. Express* **26**, 33613 (2018).
- [54] Q. Mu, F. Fan, S. Chen, S. Xu, C. Xiong, X. Zhang, X. Wang, and S. Chang, Tunable magneto-optical polarization device for terahertz waves based on InSb and its plasmonic structure, *Photon. Res.* **7**, 325 (2019).
- [55] W. Wang, W. T. Yau, Y. Cui, J. Wang, and K. H. Fung, Maxwell's demon-like nonreciprocity by non-Hermitian gyrotropic metasurfaces, *Phys. Rev. Res.* **3**, L022006 (2021).
- [56] J. Wang, Y. J. Liu, H. Y. Dong, Z.-G. Dong, and K. H. Fung, Nonreciprocal transparency in asymmetric gyrotropic trimers, *Phys. Rev. Res.* **4**, 013147 (2022).
- [57] J. Wang, K. H. Fung, H. Y. Dong, and Z.-G. Dong, Multi-frequency and wide-angle nonreciprocity arising from non-uniform magnetic field, *Appl. Phys. Lett.* **123**, 171704 (2023).
- [58] W. Yan, Y. Yang, S. Liu, Y. Zhang, S. Xia, T. Kang, W. Yang, J. Qin, L. Deng, and L. Bi, Waveguide-integrated high-performance magneto-optical isolators and circulators on silicon nitride platforms, *Optica* **7**, 1555 (2020).
- [59] V. S. Asadchy, C. Guo, B. Zhao, and S. Fan, Sub-wavelength passive optical isolators using photonic structures based on Weyl semimetals, *Adv. Opt. Mater.* **8**, 2000100 (2020).
- [60] T. Arikawa, X. Wang, A. A. Belyanin, and J. Kono, Giant tunable Faraday effect in a semiconductor magneto-plasma for broadband terahertz polarization optics, *Opt. Express* **20**, 19484 (2012).
- [61] S. Chen, F. Fan, X. He, M. Chen, and S. Chang, Multifunctional magneto-metasurface for terahertz one-way transmission and magnetic field sensing, *Appl. Opt.* **54**, 9177 (2015).
- [62] F. Fan, S. Chen, W. Lin, Y.-P. Miao, S.-J. Chang, B. Liu, X.-H. Wang, and L. Lin, Magnetically tunable terahertz magnetoplasmons in ferrofluid-filled photonic crystals, *Appl. Phys. Lett.* **103**, 161115 (2013).
- [63] H. Friedrich and D. Wintgen, Interfering resonances and bound states in the continuum, *Phys. Rev. A* **32**, 3231 (1985).