

Breaking local quantum speed limits with steering

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We show how quantum correlations allow us to break the local speed limits of physical processes using only local measurements and classical communication between two parties that share an entangled state. Inequalities that bound the minimal time of evolution of a quantum state by energy fluctuations can be violated in the presence of steering by conditioning on the measurement outcomes of a remote system. Our results open up pathways for studying how quantum correlations influence the dynamical properties of states and observables.

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Introduction. Quantum theory unveils fundamental constraints and principles that limit physical properties. The uncertainty principle, for instance, identifies limits on quantum fluctuations on noncommuting observables. Similarly, quantum speed limits (QSLs) identify bounds on the timescales of physical processes in terms of the available energy or its fluctuations. Apparent contradictions to such fundamental constraints often offer new insights into the nature of quantum systems. The Einstein-Podolsky-Rosen (EPR) paradox exemplifies one such contradiction [1], which can be understood as a noncompliance of a local complementarity principle when properties are conditioned on the measurement outcomes of a distant system.

Within the context of quantum information theory, we can attribute the EPR paradox to a strong form of quantum correlations, referred to as steering [2–5]. These correlations, in addition to their fundamental significance, play a pivotal role in applications of quantum science, including one-sided device-independent quantum cryptography [6] and subchannel discrimination [7]. Experimentally implementable criteria to detect the EPR paradox and to reveal the presence of steering have been derived from uncertainty principles, entropic uncertainty relations, and metrological complementarity [8–12].

In this Letter, we demonstrate that steering can lead to apparent violations of quantum speed limits. By conditioning the measurement results of a remote system, it becomes possible to achieve a quantum evolution with a characteristic

timescale that exceeds the quantum speed limit of the local system. We derive steering criteria based on conditional speed limits derived from Mandelstam and Tamm [13], as well as geometric approaches that leverage the quantum Fisher information as a metrological measure of quantum statistical speed. These results shed light on the EPR paradox by linking it to the evolution time of quantum mechanical systems and identifying ways to speed up physical processes in postselection using steering.

Quantum speed limits and the EPR paradox. The two main approaches towards deriving quantum speed limits are the *minimal time approach*, which relies on the effects of the statistical moments of the Hamiltonian that drives the system, and the *geometric approach*, which is based on the distinguishability of quantum states [14]. The first approach is based on an energy-time uncertainty relation by Mandelstam and Tamm [13], which is derived from Heisenberg's uncertainty principle and the Ehrenfest theorem. Considering

$$(\Delta\tau_{\hat{M}})_{\hat{\rho}} := \frac{(\Delta\hat{M})_{\hat{\rho}}}{\left|\frac{\partial}{\partial t}\langle\hat{M}\rangle_{\hat{\rho}}\right|}, \quad (1)$$

the minimal time for the mean value of the observable $\hat{M}$ to change by a standard deviation yields the following minimal time quantum speed limit:

$$(\Delta\tau_{\hat{M}})_{\hat{\rho}} \geq \frac{\hbar}{2(\Delta\hat{H})_{\hat{\rho}}}, \quad (2)$$

which corresponds to the energy-time uncertainty relation. Here, $(\Delta\hat{O})_{\hat{\rho}}^2 = \langle\hat{O}^2\rangle_{\hat{\rho}} - \langle\hat{O}\rangle_{\hat{\rho}}^2$ is the variance of the observable $\hat{O}$ for the state $\hat{\rho}$, and $\hat{H}$ is the Hamiltonian generating the unitary evolution.

The EPR paradox was first introduced as an apparent incompatibility with a local uncertainty principle. Depending on the choice of the measurement basis in a remote system, the conditional quantum state is given by either a position eigenstate or a momentum eigenstate. Based on Heisenberg's

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uncertainty principle, a quantitative criterion for observation of an EPR paradox and steering was first derived by Reid [15]. Steering excludes the possibility of explaining the joint measurement results in terms of a local hidden state (LHS) model.

A convenient description of the bipartite EPR steering scenario from Alice to Bob is given by assemblages $\mathcal{A}(a, \hat{X}) = p(a|\hat{X})\hat{\rho}_{a|\hat{X}}^B$, which map a result a for the measurement choice $\hat{X}$ by Alice to a non-normalized quantum state for Bob. Here, $p(a|\hat{X})$ is the probability for Alice to obtain the result a when measuring $\hat{X}$, and $\hat{\rho}_{a|\hat{X}}^B$ is the quantum state for Bob's subsystem conditioned on such an event. A local hidden state model exists if the measurement statistics can be modeled in terms of a set of local states $\hat{\sigma}_\lambda^B$, determined by the variable λ that is distributed with probability $p(\lambda)$ and also determines the conditional probability $p(a|\hat{X}, \lambda)$, such that $\mathcal{A}(a, \hat{X}) = \sum_\lambda p(\lambda)p(a|\hat{X}, \lambda)\hat{\sigma}_\lambda^B$.

The conditional fluctuations for the observable $\hat{O}$ are given by $(\Delta\hat{O})_{B|A}^2 := \min_{\hat{X}} \sum_a p(a|\hat{X})(\Delta\hat{O})_{\hat{\rho}_{a|\hat{X}}^B}^2$, i.e., by the average variance for each of the conditional states $\hat{\rho}_{a|\hat{X}}^B$ weighted by the probability of their occurrence. Alice's measurement setting is chosen such that the average fluctuations on Bob's side are minimized. Reid [15] showed that conditional variances do not necessarily satisfy the Heisenberg uncertainty bound that is defined by the commutator of two local observables on Bob's system and any violation of this bound constitutes an EPR paradox and therefore indicates the presence of steering. Hence, even though the average variances necessarily obey the uncertainty principle, it is possible, in the presence of steering to sort and average the measured variances in postselection using classical information about Alice's measurement choice and result. These conditional variances may break the uncertainty limit that is imposed by Heisenberg's uncertainty principle on Bob's local subsystem.

Breaking local quantum speed limit with steering. Let us first demonstrate that the EPR paradox can also manifest as a breach of a local quantum speed limit, assisted by steering and classical communication. To this end, we introduce the conditional characteristic time for variations of $\hat{M}$ as

$$(\Delta\tau_{\hat{M}})_{B|A} = \frac{(\Delta\hat{M})_{B|A}}{|\frac{\partial}{\partial t}\langle\hat{M}\rangle_{B|A}|}, \quad (3)$$

where $|\frac{\partial}{\partial t}\langle\hat{M}\rangle_{B|A}| = \max_{\hat{X}} \sum_a p(a|\hat{X})|\frac{\partial}{\partial t}\langle\hat{M}\rangle_{\hat{\rho}_{a|\hat{X}}^B}|$ are the conditional variations of the average value. The conditional timescale (3) is analogous to Eq. (1) but both the fluctuations and the mean value variations are processed separately for each conditional state in postselection.

Any assemblage that admits a LHS model must satisfy the local speed limit,

$$(\Delta\tau_{\hat{M}})_{B|A} \geq \frac{\hbar}{2(\Delta\hat{H})_{B|A}}, \quad (4)$$

for any local observable $\hat{M}$. The result is closely related to the Cavalcanti-Reid steering criterion [16] that improves over Reid's original result [15] by conditioning also on the result of the commutator [17]; see the Supplemental Material [18] for a proof. An interesting special case is found when $(\Delta\hat{M})_{B|A}$ is time independent. In the absence of steering, the time interval

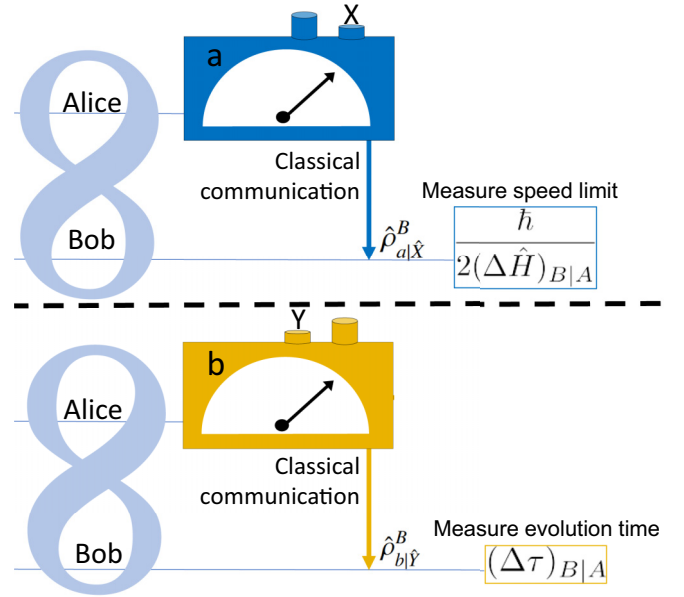


FIG. 1. Conditioned on Alice's measurement outcome and setting, Bob decides to measure either the QSL or the effective time of the evolution. The violation of the speed limit is a witness of quantum steering from Alice to Bob.

$\delta t = t_1 - t_0$ that is needed to induce the change $\delta\langle\hat{M}\rangle_B := |\langle\hat{M}\rangle_{\hat{\rho}^B(t_1)} - \langle\hat{M}\rangle_{\hat{\rho}^B(t_0)}|$ of the average value of $\hat{M}$ cannot be smaller than

$$\delta t \geq \frac{\hbar}{2(\Delta\hat{H})_{B|A}} \frac{\delta\langle\hat{M}\rangle_B}{(\Delta\hat{M})_{B|A}}. \quad (5)$$

This bound can be further sharpened if the change in the expectation value of $\hat{M}$ is also conditioned on Alice's measurement results. The proof and additional details are provided in the Supplemental Material [18]. For approaches that link steering to alternative definitions of the quantum speed limit via the Wigner-Yanase skew information and different measures of coherence, see Ref. [19]. In the following, we focus on our result, which employs the most widely used notions of the quantum speed limit.

As an example, consider a two-mode continuous-variable system that on Bob's side is subject to a translation generated by $\hat{U}(\delta t) = e^{-i\hat{p}\delta\langle\hat{x}\rangle_B/\hbar}$, where $\hat{p}$ is the momentum operator and an evolution time of δt leads to a displacement of $\delta\langle\hat{x}\rangle_B = p_0\delta t/m$. If Alice and Bob share a two-mode squeezed state, upper bounds on the conditional variances of the two quadratures obtained from quadrature measurements are given by $(\Delta\hat{p}_B)_{B|A}^2 \leq (\Delta p_0)^2 z/[z^2 \sin^2(\theta) + \cos^2(\theta)]$, $(\Delta\hat{x}_B)_{B|A}^2 \leq (\Delta x_0)^2 z/[z^2 \cos^2(\theta) + \sin^2(\theta)]$, where the squeezing parameter z ranges from 1, implying no squeezing, to 0, implying infinite squeezing. The parameter θ can be controlled via the transmissivity of a beam splitter that is used in the preparation of the state. Here, $(\Delta x_0)^2$ and $(\Delta p_0)^2$ are the variances of the quadratures of the state before applying the squeezing and they must satisfy the uncertainty principle $\Delta x_0 \Delta p_0 \geq \frac{\hbar}{2}$. We then consider the following protocol, depicted in Fig. 1: when Alice measures $\hat{p}$, Bob also measures $\hat{p}$; when Alice measures $\hat{x}$, Bob displaces its part of the state for a time δt

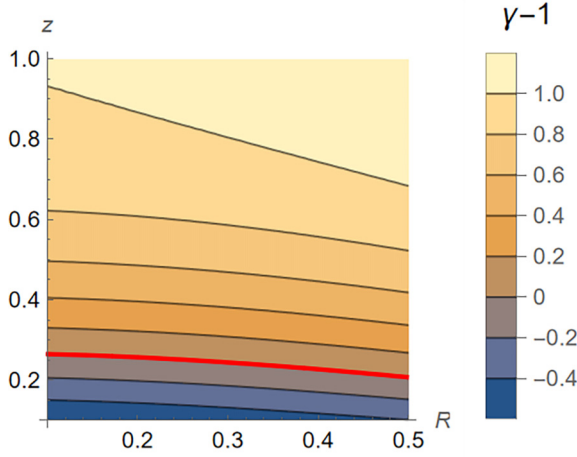


FIG. 2. Free particle violating local speed limit with steering. Density plot of the condition $\gamma - 1 > 0$ as a function of squeezing z and the ratio $R = \Delta p_0 / p_0$ between the uncertainty and the mean value of the momentum of the initial state, for $\theta = \pi/4$.

and then measures $\hat{x}$. By repeating the experiment many times and postselecting according to Alice's outcomes, Bob can infer the conditioned statistical moments of its quadratures. It is worth noting that, for Gaussian states, quadrature measurements are not always the optimal choice, as indicated in Ref. [20]. Nevertheless, they remain the most practical option for implementation in quantum optics and are well-suited for description within the framework of symplectic geometry. In this case, it is sufficient to apply Eq. (5) to obtain

$$\delta t \geq \frac{\hbar m}{2p_0(\Delta \hat{p})_{B|A}} \frac{\delta \langle \hat{x} \rangle_B}{(\Delta \hat{x})_{B|A}}. \quad (6)$$

It can be easily seen that, if $\theta = 0$, the time of the evolution is simply given by the ratio of the mean position with the velocity. Conversely, for $\theta > 0$ and $z < 1$, the state is entangled, and the bound is always violated.

As another case study, consider that of a free-propagating entangled particle with $\hat{H} = \hat{p}^2/2m$. Assuming that the initial average position of the particle is $\langle \hat{x}_B(0) \rangle = 0$ and the mean momentum (conserved through the evolution) is p_0 , we can derive a local bound for the evolution of an LHS model (for an explicit expression, see the Supplemental Material [18]). When the condition

$$\gamma := \frac{2(\Delta \tau_{\hat{M}})_{B|A}(\Delta \hat{H})_{B|A}}{\hbar} \geq 1 \quad (7)$$

is violated, we have detected steering. The expression is represented in Fig. 2 for the free particle with $\hat{M} = \hat{x}_B$. In addition, computing the evolution in time of the variance of $\hat{x}_B$, we can isolate δt and find the following bound for the physical time of the local evolution (see proof in the Supplemental Material [18]):

$$\delta t \geq \frac{(1 - \gamma)\hbar}{4R^2(\Delta \hat{H})_{B|A}^2}, \quad (8)$$

where any violation again implies steering, and $R = \Delta p_0 / p_0$. This criterion becomes increasingly harder to violate as time evolves due to the spreading of the wave function under free

evolution. The tightest bound is obtained at $\delta t = 0$ and, in this case, it is equivalent to the one obtained from Eq. (7).

Geometric speed limits. Geometric approaches to quantum speed limits measure the rate of change of a quantum state or an observable at any given instance of a trajectory between two points. By identifying optimal trajectories and determining the fastest possible approach from one point in state space to another, speed limits can be determined [14,21–24]. These approaches are based on a geometric notion of distance between states in Hilbert space, such as the Bures angular distance $D[\hat{\rho}, \hat{\sigma}] := \arccos(\text{Tr}[\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}])$ [25]. For infinitesimal system evolution $\hat{\rho}(t) \rightarrow \hat{\rho}(t + \epsilon)$, the rate of change in state space is given by $v(t) := \lim_{\epsilon \rightarrow 0} \frac{D[\hat{\rho}(t+\epsilon), \hat{\rho}(t)]}{\epsilon} = \frac{1}{2\hbar} \sqrt{F_Q[\hat{\rho}(t)]}$ [14,21]. Here, $F_Q[\hat{\rho}(t)]$ is the quantum Fisher information of the state $\hat{\rho}(t)$ with respect to changes of the time parameter t . This quantity further determines the ultimate precision limit for parameter estimation protocols that aim to extract the parameter t through measurements on $\hat{\rho}(t)$ [26].

In the scenario presented in Ref. [12], a steering-assisted phase-estimation protocol is employed, where Alice communicates the measurement setting $\hat{X}$ and outcome a to Bob. Bob can adapt the observable $\hat{X}$ based on the conditional state $\hat{\rho}_{a|\hat{X}}^B$ to maximize sensitivity. After multiple rounds of the protocol, Bob can achieve an average sensitivity characterized by the conditional quantum Fisher information $F_{B|A}[\hat{H}] := \max_{\hat{X}} \sum_a p(a|\hat{X}) F_Q[\hat{\rho}_{a|\hat{X}}^B, \hat{H}]$. In the case of a unitary evolution generated by $\hat{H}$ the quantity $F_Q[\hat{\rho}(t)] = F_Q[\hat{\rho}_{a|\hat{X}}^B, \hat{H}]$ becomes independent of t . It was shown in Ref. [12] that, for any assemblage $\mathcal{A}$ admitting a LHS model, the following bound holds: $F_{B|A}[\hat{H}] \leq 4(\Delta \hat{H})_{B|A}^2$.

Consequently, the conditional mean quadratic speed, defined as the conditional rate of change in the local state space of an assemblage, $\langle v^2 \rangle_{B|A} := \max_{\hat{X}} \sum_a p(a|\hat{X}) \frac{F_Q[\hat{\rho}_{a|\hat{X}}^B, \hat{H}]}{4\hbar^2}$, is, for LHS models, bounded by

$$\hbar^2 \langle v^2 \rangle_{B|A} \leq (\Delta \hat{H})_{B|A}^2. \quad (9)$$

This bound remains valid when integrated over time, allowing for a time average on both sides of the equation. For the unitary dynamics we are considering, this leads to the following final bound:

$$\delta t \geq \hbar \sqrt{\frac{\langle D(\delta t)^2 \rangle_{B|A}}{(\Delta \hat{H})_{B|A}^2}}, \quad (10)$$

where δt is the physical time Bob's conditioned system is left to evolve before being measured and $\langle D(\delta t)^2 \rangle_{B|A} = \max_{\hat{X}} \sum_a p(a|\hat{X}) D(\hat{\rho}_{a|\hat{X}}^B(\delta t), \hat{\rho}_{a|\hat{X}}^B(0))^2$ is the average quadratic distance between the initial and final state on Bob's side after conditioning on Alice's outcome. The bound above can be violated in the presence of steering and reduces to the standard QSL for arbitrary states and unitary evolution [27,28]. This type of bound was recently shown to set a fundamental bound on the minimum attainable phase estimation error through the quantum Cramér-Rao bound, relating the precision directly to the underlying dynamics of the system [29]. The proof of the above relation can be found in the Supplemental Material [18].

As an example, let us now consider a multipartite quantum state $\hat{\rho}$ with discrete degrees of freedom prepared in the form of a Greenberger-Horne-Zeilinger state with limited visibility p . In this case, Alice controls one qubit of the state and will measure it in either the $\hat{\sigma}_x$ or the $\hat{\sigma}_z$ basis. According to Alice's measurement setting, Bob will either let the system evolve for a time δt with the Hamiltonian $\hat{H}_B = \frac{\mu}{2} \sum_i \hat{\sigma}_z^{(i)}$, where μ is a coupling constant with the dimension of energy, or measure the energy variance of the state. By employing Eq. (10), we can derive a steering condition on the time of evolution, given by

$$\delta t \geq \frac{\hbar}{\mu} p \sqrt{\frac{N}{[2^{1-N}(1-p) + p][1-p + N(1-p)p]}}. \quad (11)$$

This is a bound on the physical time of evolution of the quantum state. It corresponds to a time-integrated formulation of the geometric criterion provided in Ref. [12]. This formulation opens up a path towards witnessing quantum steering based on the time evolution of a system that could be useful for some experimental platforms [30,31].

Conclusions. In this work, we have established a unified framework that links steering and quantum speed limits, opening up alternative applications for investigating quantum correlations. By introducing steering witnesses based on mechanical quantities, such as the time of flight of an entangled particle in rectilinear uniform motion, we have introduced alternative methods to certify quantum correlations in mechanical and thermodynamical scenarios. Our findings also suggest that steering can potentially provide a quantum advantage in mechanical tasks, such as speeding up certain physical processes in postselection.

A promising direction for further exploration would involve applying our framework to the ballistic scenario described in Ref. [32], to investigate the effect of steering on entangled projectiles and quantum backflow. Another application might be to study the effect of steering on the diffusive dynamics induced by continuous measurements [33]. Additionally, it would be intriguing to study the impact of nonclassical correlations in other scenarios where quantum speed limits are commonly employed, such as optimal quantum control systems [34,35], the entropy production rate of quantum engines [36], the maximal power of quantum battery arrays [37,38], the Landauer principle in finite time [39], or the minimum time of a quantum measurement [40].

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