

Mapping the topology-localization phase diagram with quasiperiodic disorder using a programmable superconducting simulator

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We explore a topology-localization phase diagram by simulating a one-dimensional Su-Schrieffer-Heeger model with quasiperiodic disorder using a programmable superconducting simulator. We experimentally map out and identify various trivial and topological phases with extended, critical, and localized bulk states. We find that with increasing disorder strength, some extended states can be first replaced by localized states and then by critical states before the system finally becomes fully localized. The critical states exhibit typical features such as multifractality and self-similarity, which lead to surprisingly rich phases with different types of mobility edges and scaling behaviors on the phase boundaries. Our results shed light on the investigation of the topological and localization phenomena in condensed-matter physics.

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Introduction. The interplay between topology and disorder has stimulated considerable theoretical [1–8] and experimental [9–13] interests recently. While strong disorder can drive a trivial material into an Anderson localized state [14], moderate disorder in many cases leads to a nontrivial state with nonzero topological number and zero-energy edge modes, which is referred to as a topological Anderson insulator (TAI) [1,2]. Also, depending on the specific types of disorder, the resulting bulk properties may differ considerably. With random disorder in low dimensions, the TAIs are gapless and contain only localized bulk states without mobility edges [4,5,9]. With quasiperiodic disorder that is deterministic, the critically localized states appear, with unique properties like critical wave function, multifractality, self-similarity, and slow dynamics [13,15–35], distinct from those of the extended or fully (Anderson) localized states.

The critical states in quasiperiodic systems have attracted much attention and are often investigated in the

framework of the generalized Aubry-André-Harper (gAAH) model [13,15–29], with a few works concerning the topological phenomena [13,20,24]. They have also been a hot topic for the quantum Ising systems [30–34]. On the other hand, there are many studies of the topological states [8,36–42] based on the Su-Schrieffer-Heeger (SSH) model [43–45]. In particular, topological phase diagrams with quasiperiodic disorder have been theoretically studied recently [8,39,40], but it is not clear whether the critical states may exist and fundamentally change the properties of the phase diagram.

Quantum simulators provide a valuable tool for the study of various problems ranging from condensed-matter physics and quantum chemistry to high-energy physics and cosmology [46,47], even in the present NISQ era [48]. Many problems are often hard to study in real materials. For instance, topological [36,49] and localization [50] states and quantum scars [51] have been simulated using superconducting quantum processors. The TAIs have been studied and demonstrated in the ultracold atom [9] and photonic [10,11] systems by tracing along particular parameter paths across the phase boundaries. Nevertheless, the experimental study and mapping of an overall topological phase diagram remain largely unexplored, especially when associated with different localization properties of the bulk states.

In this work, we present an experiment of mapping the topology-localization phase diagram with quasiperiodic disorder using a programmable superconducting quantum simulator [52] developed by flip-chip technology [53]. The simulator has 63 tunable qubits and 105 tunable couplers

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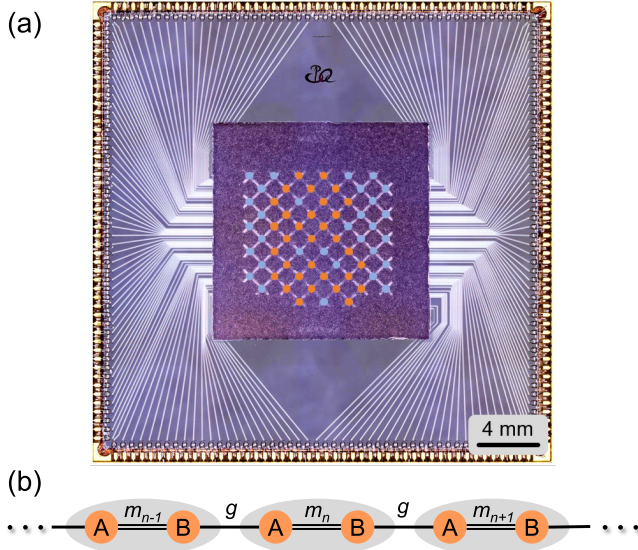


FIG. 1. (a) Photograph of the programmable superconducting simulator. The circuits on the top half-transparent sapphire substrate are, on the back side, seen as a grid image and color circles are added at the nodes to indicate qubits, with those in orange forming the 32-qubit chain used in this work. (b) Schematic of the gSSH model. The intercell tunneling (single lines) is homogeneous, while the intracell tunneling (double lines) is modulated with quasiperiodic disorder.

arranged in a two-dimensional (2D) square lattice with extremely small next-nearest-neighbor (diagonal) coupling on the order of $2\pi \times 30$ kHz, allowing engineering of Hamiltonians conveniently and accurately by controls over qubits and couplers via programming. A chain of 32 qubits is used in this work as shown in Fig. 1(a). By simulating the one-dimensional SSH model with quasiperiodic disorder, we experimentally map out a complete topology-localization phase diagram and identify various trivial and topological phases having extended, critical, and localized bulk states. With increasing disorder strength, some extended states can be first replaced by localized states and then by critical states, and finally, the system becomes fully localized. We find that the critical states have typical features such as multifractality and self-similarity. Their presence changes the structure of the phase diagram as compared to the previous works [8,39,40], leads to different types of mobility edges, and affects the scaling behaviors and critical exponents on the phase boundaries.

Model. The SSH model was originally proposed to describe spinless fermions in dimerized chains [see Fig. 1(b)] [43–45]. It is the simplest model widely used in the studies of topological states [36] and TAI states with random [9] and quasiperiodic [8,37–39] disorders. For a dimerized chain with L lattice sites or $N_c = L/2$ unit cells, the model with quasiperiodic disorder reads

$$\hat{H}_{\text{gSSH}}/\hbar = \sum_{n=1}^{N_c} m_n \hat{a}_n^\dagger \hat{b}_n + g \sum_{n=1}^{N_c-1} \hat{a}_{n+1}^\dagger \hat{b}_n + \text{H.c.}, \quad (1)$$

where $\hat{a}_n$ ($\hat{b}_n$) is the fermionic annihilation operator for the A (B) site in the n th unit cell. This generalized SSH (gSSH)

model can be mapped by the Jordan-Wigner transformation to a spin- $\frac{1}{2}$ system that can be conveniently simulated by a superconducting qubit chain [36,54]. We consider a simple case where the intercell tunneling g is homogeneous and quasiperiodic disorder is introduced in the intracell tunneling term:

$$m_n = m + W \cos(2n\pi\beta + \delta). \quad (2)$$

Here W quantifies the disorder strength, $\beta = (\sqrt{5} - 1)/2$ is an irrational number [17], and δ is an arbitrary phase. δ can be used to generate different disorder realizations for finite-size systems, which are averaged to recover the thermodynamic results. The disorder in this simple form is favorable for the observations of TAI and critical states and the determination of phase boundaries [52]. The topological and localization properties of the 1D disordered chiral system can be characterized by the real-space winding number N_w and the spectrally averaged inverse participation ratio $\overline{\text{IPR}}$, namely, $N_w = 0$ and 1 for topological trivial and nontrivial states, and $\overline{\text{IPR}} = 0$ and $\overline{\text{IPR}} > 0$ for extended and localized states, respectively [8].

Experiment. We use 32 qubits from the simulator forming a 1D system for the simulation of the gSSH model (see Fig. 1). The active qubits are biased to a common frequency of $\omega_{\text{ref}} = 2\pi \times 5.495$ GHz. The nearest-neighbor couplings are tuned to yield an inter-cell tunneling strength of $g = 2\pi \times 1.5$ MHz and the ratio m/g (W/g) is varied in the range $[0, 2]$ ($[0, 4]$). In the experiment, we first align the frequencies of 32 active qubits, then parametrize the coupling strength of each nearest-neighbor qubit pair, and finally compensate for the coupler-induced dispersive shifts. By a fast control procedure, we are able to run the experiment with a switching rate of 18 target Hamiltonians per hour. The statistical fidelity, i.e., $F(p, q) = \sum_x \sqrt{p_x q_x}$, between the measured and ideal population distribution [54] in the dynamical evolution averaged over different target Hamiltonians is over 91% within the maximum evolution time.

We quantitatively extract the topological and localization properties, i.e., N_w and $\overline{\text{IPR}}$, from the time-averaged expectation values of the chiral displacement operator and the survival probability in quantum-walk experiments:

$$C_t = \frac{2}{t} \int_0^t \langle \psi(t') | \hat{\Gamma} \hat{X} | \psi(t') \rangle dt', \quad (3)$$

$$S_t = \frac{1}{t} \int_0^t |\langle l_0 | \psi(t') \rangle|^2 dt'. \quad (4)$$

Here $\hat{\Gamma} \equiv \sum_n (\hat{a}_n^\dagger \hat{a}_n - \hat{b}_n^\dagger \hat{b}_n)$ and $\hat{X} \equiv \sum_n n (\hat{a}_n^\dagger \hat{a}_n + \hat{b}_n^\dagger \hat{b}_n)$. The quantum-walk dynamics is governed by $|\psi(t)\rangle = \exp(-i\hat{H}_{\text{gSSH}}t/\hbar)|l_0\rangle$, where $|l_0\rangle$ is the initial Fock state with only the l_0 th site excited. N_w can be obtained from C_t [55], while $\overline{\text{IPR}} = \lim_{t \rightarrow \infty} \overline{S}_t$, with $\overline{S}_t = \frac{1}{L} \sum_{l_0} S_t(l_0)$ [52]. In the experiment, we construct eight disorder realizations and four initial single-excitation states for a given target Hamiltonian. The dynamical properties of C_t and S_t for the 8×4 quantum-walk instances with given parameters W/g and m/g are measured to obtain the instance-averaged $\overline{C}_t$ and $\overline{S}_t$. Although the evolutions of C_t and S_t depend on the specific disorder realization and initial state, $\overline{C}_t$ and $\overline{S}_t$, after taking the average over the 32 instances, converge to N_w and $\overline{\text{IPR}}$, respectively.

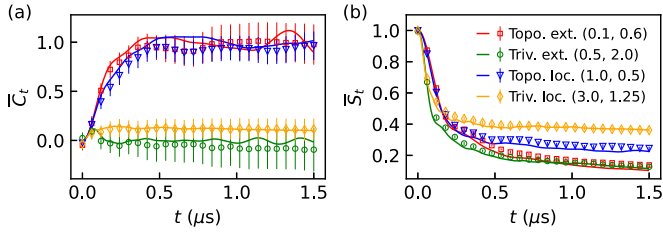


FIG. 2. Dynamical properties of (a) averaged chiral displacement and (b) survival probability in different topological and localization phases with parameters $(W/g, m/g)$ indicated in the legend. Here symbols and lines are the experimental and numerical results, respectively. For the experimental results, the error bars represent the standard error of the mean arising from the sampling error in the projective measurement with 1024 repetitions.

Figure 2 shows the measured $\bar{C}_t$ and $\bar{S}_t$ for four $(W/g, m/g)$ parameter sets, corresponding to different topological and localization properties in the W - m plane described below. Quantum walks with these parameters show that the edge excitations in the topological phase remain largely at the edge in time evolution no matter whether the bulk states are localized or extended, while in the trivial case the excitations will remain at the edge for localized states and spread to all qubits for extended states [52]. In Fig. 2(a), we find that the quantity $\bar{C}_t$ almost converges to the corresponding values in the long-time limit, say $N_w = 1$ (0) for topological (trivial) phases, while it is less straightforward for the results concerning the localization property as shown in Fig. 2(b). Ideally, $\bar{S}_t$ should vanish for systems of infinite size at infinite time in the extended state. In the present case, it remains finite due to the finite size and time in the experiment. However, we observe that $\bar{S}_t$ decreases with time at rates clearly higher for extended states than those for localized states. The values of $\bar{S}_t$ near the end of evolution still appear separate enough to discriminate the localized states from the extended ones, which can be verified by numerical simulations in the corresponding cases with infinite size and time.

Phase diagram. To map out the phase diagram, we sweep the model parameters in the W - m plane and the experimental results are presented in Figs. 3(a) and 3(c). Here we use the measured values at the maximum evolution time of $\tau = 1.5 \mu\text{s}$, denoted by $\bar{C}_\tau$ and $\bar{S}_\tau$, as the experimental quantities for evaluating topological and localization properties. We choose 114 points unevenly distributed in the W - m plane and obtain the real-space winding number N_w ($\bar{C}_\tau$) and the spectrally averaged inverse participation ratio $\overline{\text{IPR}}$ ($\bar{S}_\tau$) in the long-time limit. Numerical results calculated [56] using $L = 32$ and an open boundary condition without free fitting parameters are shown in Figs. 3(b) and 3(d), demonstrating good agreements between experiment and theory.

The experimental results in Figs. 3(a) and 3(c) reveal remarkably rich phases for the gSSH model. First of all, there exist two distinct regions in Fig. 3(a), namely, topologically trivial and nontrivial, as separated by dashed lines for N_w ($\bar{C}_\tau$) values close to 0 and 1, respectively. Second, as can be seen from increasing $\overline{\text{IPR}}$ ($\bar{S}_\tau$) in Fig. 3(c), the bulk states change monotonically from extended to localized ones for $m/g \sim 2$ as the disorder parameter W increases. However, for m/g

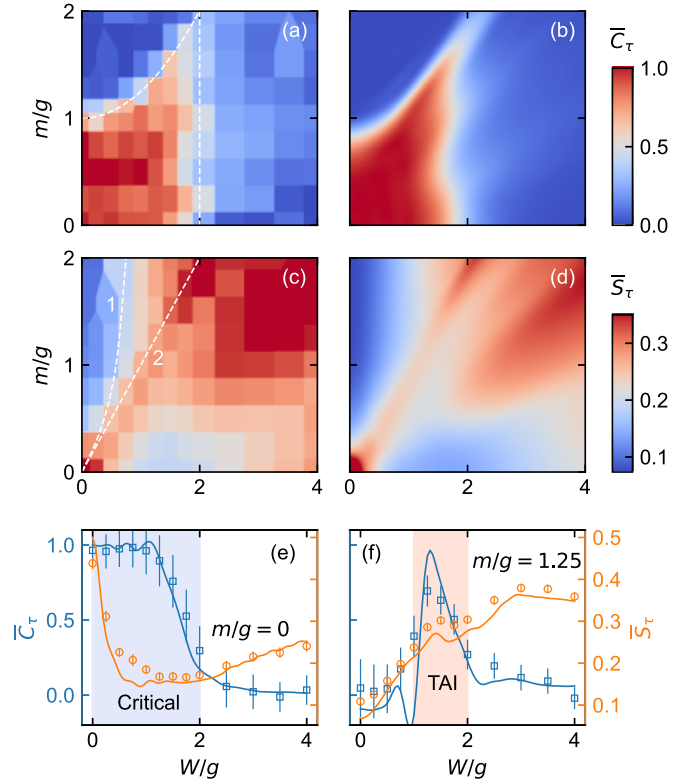


FIG. 3. Topology-localization phase diagram seen from (a) experimental and (b) numerical averaged chiral displacement $\bar{C}_\tau$ and (c) experimental and (d) numerical averaged survival probability $\bar{S}_\tau$. Each plaquette in panels (a) and (c) represents one data point on the parameter plane. White dashed lines indicate calculated phase boundaries in Fig. 4(d) considering extended qubit numbers. Two representative cross sections of the parameter space are shown in panels (e) and (f) with symbols and lines being the experimental and numerical results, and shaded areas indicating pure critical states and TAI regimes. The colorbar for panels (c) and (d) is set for better contrast with the data near origin out of range as can be seen in panel (e).

around 1, we find that $\overline{\text{IPR}}$ ($\bar{S}_\tau$) first increases for W/g up across the dashed straight line (phase boundary 2) and then decreases before finally increasing to a more localized state. We also observe a clear opposite trend near $m/g = 0$. Namely, the bulk states change from more localized to less localized ones with increasing W , and further increasing W leads to a more localized state again (note that at $m/g = W/g = 0$ the system is completely localized due to the disconnectivity of the chain). Figures 3(e) and 3(f) show N_w ($\bar{C}_\tau$) and $\overline{\text{IPR}}$ ($\bar{S}_\tau$) versus W for $m/g = 0$ and 1.25, respectively, in which the pure critical states and TAI regimes are indicated.

To further understand the experimental phase diagram in Fig. 3, we turn to the numerical results calculated using Eqs. (1) and (2) with larger $L = 1220$ and a periodic boundary condition for more general circumstances closer to the thermodynamic limit. Figure 4(a) shows the result of $-\ln \overline{\text{IPR}} / \ln L$ as a function of W/g , which corresponds to the fractal dimension D_f when L tends to infinity [23,57]. For $m/g = 0$, the system is seen to be critically localized for $0 < W/g < 2$ in terms of $D_f \sim 0.75$. Surprisingly, a small

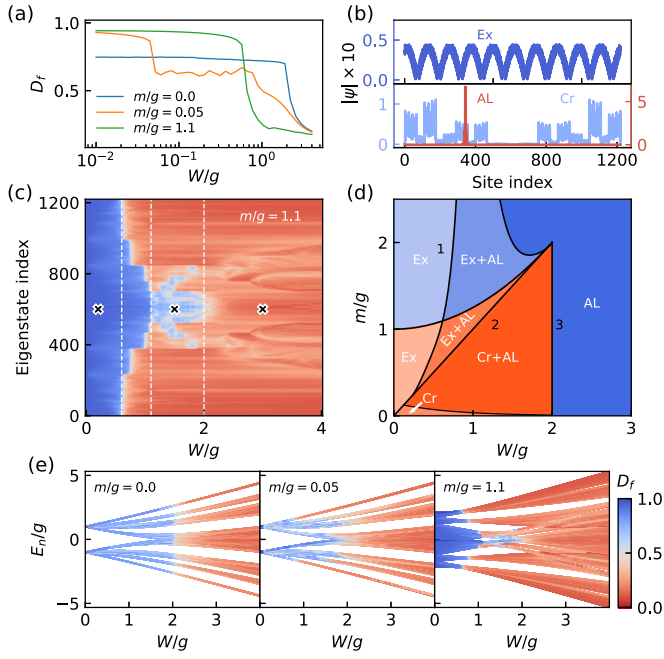


FIG. 4. (a) Fractal dimension D_f versus W/g . (b) Wave functions for $m/g = 1.1$ and three different W/g marked by crosses from left to right in panel (c), corresponding to the extended, critical, and localized states, respectively. (c) Eigenstate D_f vs W/g for $m/g = 1.1$, sharing the same colorbar in panel (e). Three vertical lines from left to right indicate the points on phase boundaries 1, 2, and 3 of bulk-state transitions in panel (d), respectively. (d) Phase diagram. Orange (blue) color indicates the topologically nontrivial (trivial) phase, with increasing darkness for more localized states. In panels (b) and (d), Ex, Cr, and AL denote extended, critical, and Anderson localized states, respectively. (e) Eigenstate D_f versus W/g for three m/g values. All results are calculated using Eqs. (1) and (2) with $N_c = 610$ and a periodic boundary condition.

increase of m/g from 0 to 0.05 can lead to an abrupt change of the bulk-state property. The system in this case changes from extended ($D_f \sim 1$) to critically localized ($D_f \sim 0.6$) and finally to fully localized ($D_f \sim 0$) states as W increases. For further increased $m/g = 1.1$, the extended state persists to larger W/g before finally turning to the fully localized state.

The critical states have unique properties compared to the extended and fully (Anderson) localized states. They have distinct critical wave functions as shown in Fig. 4(b) and are characterized by multifractality as seen from D_f and self-similarity [17,35] as described in Ref. [52]. Figure 4(c) shows the eigenstate D_f against W/g for $m/g = 1.1$, with the values of D_f for critical states lying between those for the extended and Anderson localized states. We see that the whole region is occupied by the extended states for small W/g . As W/g increases, some extended states are replaced by the Anderson localized states, and further by the critical states, resulting in the absence of the extended states, which correspond to the points on phase boundaries 1 and 2 in Fig. 3(c), respectively. Following this there exists an increase of the critical states against Anderson localized states in a narrow range of W/g . Finally, the Anderson localized states expand in number and become the only bulk states for W/g above 2. A complete

phase diagram is shown in Fig. 4(d) [52]. It is worth noting that the pure critical phase is discovered in the present experiment partly due to the finite-size effect that enlarges the parameter ranges in the phase diagram [compare Figs. 3(c) and 4(d)].

Discussion. The simulated gSSH model has some interesting relations with the gAAH and Ising models, either in limiting cases [52] or under exact mapping [58]. For example, with $m \gg g, W$, the gSSH chain can be reduced to two sub-AAH chains, with the average on-site potentials of m and $-m$, respectively, and the intra- and interchain coupling of $g/2$. The interchain coupling can be neglected due to large m . The disorder W/g drives the extended-to-localized transition inside each AAH chain, with the critical value of $W/g = 1$, which corresponds to phase boundary 1 in Figs. 3(c) and 4(d) for large m . In the case of $g \gg m, W$, relating to the phase diagram near origin, the gSSH chain can be reduced to two off-diagonal AAH chains with the on-site potentials homogeneously shifted to g and $-g$, such that the interchain tunneling terms, with the tunneling strength proportional to m and W , can be safely neglected. The extended-to-critical transition occurs as W/m increases, with the critical line of $m = W$ [16], corresponding to phase boundary 2 near the origin in Figs. 3(c) and 4(d).

The Ising model with quasiperiodic disorder exhibits some magnetic phase boundaries similar in form to those in Fig. 4(d) [31]. Following the discussion there, we consider a 2D tight-binding model on a square lattice, with each row described by the SSH model [52]. The intracell quasiperiodic disorder is introduced by adding complex couplings over diagonal sites between adjacent rows within the same unit cell. For $m = 0$, the square lattice decouples to two honeycomb lattices, with a flux of 2β penetrating each hexagon and the tunneling strengths of the three bonds connected to the same site of $W/2$, $W/2$, and g , respectively. As discussed in Ref. [31], the system possesses extra rotational symmetry when $W/g = 2$, leading to a critical-to-localized phase transition. The gSSH model with $m = 0$ exhibits an energy-independent critical-to-localized transition at $W/g = 2$, as shown in the left panel of Fig. 4(e). However, this simple picture is only valid for $m = 0$, while increasing m introduces energy-dependent critical-to-localized transition. This yields generalized mobility edges which separate critical and localized states in the regime of moderate disorder, as shown in the middle panel of Fig. 4(e). As m further increases, the regime of critical states shrinks to the vicinity of the zero-energy gap, while mobility edges between extended and localized states emerge, which are shown in the right panel of Fig. 4(e). In this case, the zero-energy gap is almost closed at the disorder-free limit and reopened in the moderate disorder regime, which indicates the existence of the TAI [8].

The boundaries separating topologically trivial (blue) and nontrivial (orange) phases in Figs. 3(a) and 4(d) can be obtained analytically by considering the inverse localization length of zero-energy modes with the open boundary condition [52]. They are identified as $m = g + W^2/4g$ and $W = 2g$ for $m > W$ and $m < W$, respectively. By analyzing the scaling behaviors of the inverse localization length and energy gap at half filling [4,31], we find their critical exponents ν

and z to be $\nu = z = 1$ for $m > W$, and $\nu = 1$ but a nonuniversal z varying around 1.5 for $m < W$. These results show that the topological transitions across the two boundaries above and below the diagonal line $m = W$ belong to two different classes. Notably, in the regions above and below the diagonal line, the critical states are absent and present, which implies that the nonuniversal critical exponents may result from the transition between the critical and localized states.

Finally, we mention that both the energy spectra and the wave functions of the critical states exhibit perfect self-similarity in the gSSH model under periodic and open boundary conditions [52]. Under the open boundary condition, some eigenstates will appear lying between the subbands of the fractal spectrum and become localized at the edges. These in-gap localized states are not protected by the chiral

symmetry, thus different from the topological zero-energy edge states.

Conclusion. We have experimentally mapped out a topology-localization phase diagram by simulating the gSSH model and demonstrated the potential of NISQ devices for solving fundamental problems in condensed-matter physics. Topologically trivial and nontrivial phases with (mixed) extended, critical, and localized bulk states have been found, offering a rich phase diagram that is valuable for further studies.

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