

Type-II heavy Fermi liquids and the magnetic memory of $4Hb$ -TaS₂Elio J. König 

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The interplay of quantum spin liquids with itinerant conduction electrons is of crucial interest for understanding layered structures composed of frustrated magnet and metal monolayers. Using parton-mean-field theory, we demonstrate here that a type-II heavy Fermi liquid, which is characterized by a vortex lattice in the slave boson condensate, can occur in the vicinity of the quantum phase transition separating fractionalized and heavy Fermi liquid phases. The magnetic flux threading each such vortex is about $v_f/137c$ times smaller than the magnetic flux threading vortices in type-II superconductors, where v_f is the speed of magnetic excitations and c the speed of light. This makes a magnetic observation of this effect challenging. We propose scanning tunneling spectroscopy instead and investigate its signatures. If a type-II heavy Fermi liquid is cooled into a type-II superconductor, vortices in the slave boson condensate and in the superconducting condensate mutually attract. We argue that the type-II heavy Fermi liquid thereby provides a compelling explanation for the magnetic memory observed recently [Persky *et al.*, *Nature (London)* **607**, 692 (2022)] in thermal cycles of $4Hb$ -TaS₂.

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Introduction. Unconventional superconductors emerging in quantum materials with strongly entangled spin degrees of freedom are fascinating phases of matter of prime theoretical and experimental interest. While classic examples are heavy fermion superconductors [1] and cuprates [2], this interest was recently boosted by the advent of van der Waals materials [3]. In particular, the past decade witnessed tremendous experimental progress in the ability to synthesize, stack, align, twist, and gate a variety of two-dimensional materials with superconducting, magnetic, semimetallic, and semiconducting properties.

This Letter is motivated by experiments on allotropes of the transition metal dichalcogenide TaS₂, which are all layered and exfoliable. 2H-TaS₂ is a metal turning into an Ising superconductor below $T_c = 0.7$ K [4,5]. In contrast, 1T-TaS₂ forms a $\sqrt{13} \times \sqrt{13}$ charge density wave at about 200 K with a typical “star-of-David” deformation of the Ta atoms. Below this temperature, the electronic degrees of freedom are commonly described by a single-orbital Hubbard model on the emergent triangular superlattice, where the distance between the centers of the unit cell ensures relatively flat bands in comparison with the interaction scale. This leads to Mott localization [6–9] (cf. [10–17] for related materials). Fascinatingly, 1T-TaS₂ seems to avoid magnetic order down to lowest accessible temperatures [7,18,19] and given thermodynamic evidence for low-lying (fermionic) excitations [7,9] it was proposed [20,21] that 1T-TaS₂ realizes a (gapless) quantum spin liquid (QSL) [22,23]. Finally, $4Hb$ -TaS₂, composed of

alternating 1T-TaS₂ and 1H-TaS₂ layers, is an unconventional superconductor with enhanced $T_c = 2.7$ K (as compared to 2H-TaS₂) [24–27] in which the μ SR signal has a drastic

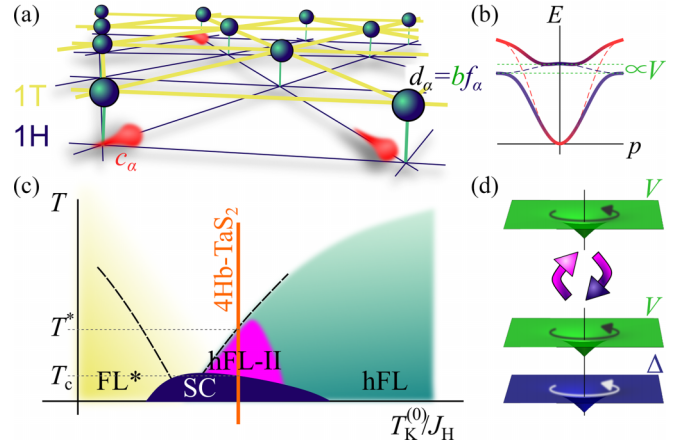


FIG. 1. (a) Triangular superlattice of a 1T-1H bilayer of TaS₂, i.e., the building block of the layered $4Hb$ bulk. Strong Coulomb repulsion amongst d electrons in the 1T layer is treated using slave bosons b . (b) Once $\langle b \rangle \propto V$ condenses, heavy f fermions hybridize with light c electrons leading to a heavy Fermi liquid (hFL). (c) Schematic phase diagram (temperature T versus the ratio of bare Kondo temperature $T_K^{(0)}$ and spin-exchange J_H between adjacent 1T sites). The quantum critical point between U(1) fractionalized Fermi liquid (FL*, b uncondensed) and heavy Fermi liquid (hFL, b condensed) is buried under a superconducting (SC) dome close to a type-II hFL phase (hFL-II) displaying vortices in V upon application of external magnetic fields. (d) Mechanism for magnetic memory: below T_c , vortices in V and in the SC order parameter Δ attract. Upon heating to temperatures $T \in [T_c, T^*]$, SC order disappears. Part of the vortices in V remains pinned even when the external magnetic field is turned off and nucleates SC vortices in subsequent cooling cycles.

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TABLE I. Comparison of theories of the magnetic memory in $4Hb$ -TaS₂, as outlined in the main text.

Theory	TRSB above T_c	Carrier of magnetic memory	Topological order	Generalizable to mixed valence
Chiral spin liquid [28]	Yes	Chiral order parameter χ_Δ	Yes: U(1) [29]	No
Spinon-vortex interconversion [30]	Yes	Flux in $\mathbf{a}$, trapped spinons	No	Yes
Vison-vortex interconversion [31]	No	Vison	Yes: $\mathbb{Z}_2$	No
Type-II heavy Fermi liquid (<i>this work</i>)	No	Vortex in hybridization V	No	Yes

change at T_c , consistent with time-reversal symmetry breaking (TRSB) concomitant with the onset of superconductivity.

A major puzzle relates to the magnetic memory effect [28] of $4Hb$ -TaS₂. In this experiment, first, the sample is field-cooled below T_c ; then the resulting vortex configuration, which is rather irregular, is mapped out using the scanning SQUID technique. Subsequently, the sample is heated to a temperature $T_{\text{cycle}} > T_c$, before being zero field cooled (ZFC) below T_c , again. The magnetic memory consists in the fact that a fraction of vortices (up to 12%) survives the cycle, as long as $T_{\text{cycle}} < T_* \simeq 3.6$ K. At the same time, the measured magnetic field in the normal state is virtually absent, which is consistent with previously mentioned μ SR data [24]. Thermodynamic evidence for spontaneous TRSB above T_c is not reported in the literature. Therefore, a vestigial [32] TRSB normal phase [33,34], similar to the one recently reported [35] in strongly hole-doped Ba_{1-x}K_xFe₂As₂, is likely not responsible for the magnetic memory. An alternative scenario based on point-like impurities alone (e.g., residual local magnetic moments) seems to be implausible, as point defects are energetically unable to nucleate entire vortex lines threading the 3D bulk crystals.

Theories of $4Hb$ -TaS₂. It is a topic of present debate whether $4Hb$ -TaS₂ should be viewed as a Kondo lattice system. While Kondo physics was clearly demonstrated in tunneling spectroscopy of 1T-TaS₂ monolayers on 1H-TaS₂ [10], first principle theoretical calculations [36] highlight the importance of charge transfer from 1T to 1H layers, as spectroscopically observed in 1T monolayers on $4Hb$ [37]. This leads to a “mixed-valence picture” at intermediate charge transfer. While ARPES data suggests that the dispersion of the individual constituent layers of $4Hb$ -TaS₂ is retained [24], very recent STM experiments unveil both charge transfer and Kondo physics in distinct spatial areas on the surface of $4Hb$ -TaS₂ [38]. Generally, both mixed valence systems and Kondo lattice materials can display (type-I) heavy Fermi liquid behavior, i.e., a Fermi liquid with substantial mass renormalization, which is obtained from coherent hybridization of local strongly interacting fermions and conduction electrons; Fig. 1(b).

We briefly review existing theories of the magnetic memory effect; see Table I for comparison. Kalisky and coauthors of the experimental discovery paper [28] suggest that the magnetic memory originates from a chiral spin liquid (CSL), which is a gapped QSL with spontaneous TRSB with a nonzero pseudoscalar order parameter $\chi_\Delta = \vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$ of spins $\vec{S}_{1,2,3}$ on the corners of the triangles of the superlattice. This scenario is at odds with the experimental report of TRSB at low $T \leq T_c$, only [24]. To reconcile the contradiction it was argued that the measurable magnetization generated by the CSL is small.

The details of vortex nucleation in the thermal cycling process within an extended CSL scenario were elaborated subsequently by Lin [30], who proposes the condensation of the Kondo hybridization V , in addition to a nonzero χ_Δ . At this point, the internal gauge theory is Higgsed, i.e., confining, and thus topological order is lost. Assuming that V is homogeneous, flux lines in the physical U(1) field $\mathbf{A}$ imply flux lines in the emergent U(1) field $\mathbf{a}$ [39,40], which can trap spinons above T_c . These spinons subsequently nucleate Abrikosov vortices upon successive cooling below T_c .

Chen [31] proposes an alternative to the CSL scenario, i.e., that the magnetic memory originates from a vison-vortex interconversion due to a postulated underlying $\mathbb{Z}_2$ topological order of local moments in the normal state. Expanding on earlier ideas by Senthil and Fisher [41], vortices in the physical superconducting order parameter Δ bind fluxes in the emergent $\mathbb{Z}_2$ gauge field (“visons”). When heated above T_c , the emergent fluxes remain and nucleate spontaneous formation of vortices in Δ in the next cooling cycle. This scenario does not assume TRSB above T_c , yet the sign of the vortices reappearing during ZFC cycles is only weakly imposed by fields in the environment.

Type-II heavy Fermi liquids. Here we consider an alternative scenario for the magnetic memory effect, based on a phase we call “type-II heavy Fermi liquids (hFLs).” In full analogy to type-II superconductors, it relies on vortices in the complex mean-field order parameter V of hFLs, Fig. 1(d), which denotes the Kondo hybridization in Kondo lattice systems or the condensate wave function of slave bosons in theories of mixed-valence systems [40], respectively. We explicitly demonstrate that, below T_c and in a magnetic field, vortices in the superconducting order parameter Δ and vortices in V mutually attract, thereby stabilizing a vortex lattice of superimposed Δ and V vortices. Upon heating, Δ vanishes, yet vortices in V persist as long as the temperature is smaller than the Kondo temperature T_K (identified with T_*). Some V vortices may remain pinned once the magnetic field is turned off. Importantly, the physical magnetic field they generate is small and possibly below the threshold of experimental sensitivity [42]. During subsequent cooling cycles and by means of the aforementioned attraction mechanism these vortices in V nucleate vortices in Δ . We first discuss this effect phenomenologically, demonstrate its energetic stability, and provide microscopic, justifying calculations. We then discuss the magnetic and tunneling spectroscopic signatures of this phase.

Main assumptions of the theory. Given that we study magnetic field-induced vortices, the first and main assumption of our theory is the tacit supposition that heavy fermion physics is at play and that orbital field effects dominate over Zeeman effects (“Pauli limit”), not only for Δ but also for V .

The latter assumption is motivated by recent susceptibility measurements [7] uncovering a minute magnetic moment per star of David in 1T-TaS₂ and by early theoretical proposals [6] advocating a negligible g factor in the same material. It is however most strongly motivated by the experimental fact that the fields applied in the magnetic memory experiments [28] are below 1 mT, while the splitting of the Kondo peak in STM is not observed in 4Hb-TaS₂ up to 9 T [38] and occurs at about 10 T in 1T-TaS₂ on 1H-TaS₂ bilayers [10]. The second assumption to stabilize type-II hFLs is the condition of large effective penetration depth λ_K as compared to the Kondo coherence length ξ_K . This is very much akin to the standard condition in type-II superconductors. As we explicitly demonstrate, close to the quantum phase transition (QPT) in Fig. 1(c), this assumption is met by the presence of sufficiently strong ring-exchange terms which occur close to Mott delocalization. These terms, sometimes argued to stabilize quantum spin liquid ground states [43], may additionally help to energetically stabilize the vortex lattice in the Kondo hybridization V by translating the applied external magnetic field to a net flux of the emergent gauge field [44,45].

Overview. Abrikosov's [46] explanation of the Ryabinin-Shubnikov superconducting phase [47] with imperfect Meißner effect involves vortex lattices in the complex, charge $2e$ order parameter $\Delta \sim \langle c_\uparrow c_\downarrow \rangle$, where c_α is the annihilation operator of a conduction fermion of spin α . The condition to stabilize the flux phase in type-II superconductors is a large London penetration depth λ as compared to the coherence length ξ (which sets the scale of the vortex core diameter).

Kondo lattice systems are commonly described using a fractionalized representation of spins [48,49] $\hat{\mathbf{S}} = f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$, where f are fermionic spinon operators of emergent gauge charge e_* . The appearance of e_* and an associated emergent electromagnetic vector potential $\mathbf{a}$ (in addition to the standard electromagnetism with vector potential $\mathbf{A}$ and charge e) is a salient characteristic of fractionalization. On the mean-field level, the complex Kondo hybridization $V \sim \sum_\alpha \langle c_\alpha f_\alpha^\dagger \rangle$ [which has charge $(e, -e_*)$] condenses in an hFL. Similarly, in mixed-valence systems a spinless, charge $(e, -e_*)$ slave boson [50,51] b may be invoked to represent the physical fermionic creation operator $d_\alpha = b f_\alpha$ on partially filled orbitals with infinite Hubbard repulsion; see Fig. 1(a). Loosely speaking, b annihilates the charge and the spinon f_α the spin of the local d_α electron. The slave boson condensate $\langle b \rangle \propto V$ is adiabatically connected to the Kondo hybridization V of the local-moment system. The QPT in Fig. 1 separates a fractionalized Fermi liquid [52,53] (FL*, b uncondensed, with unit valence/star of David) from the Higgs phase (condensed $\langle b \rangle \propto V$, hFL) [39,40] with a large Fermi surface. Given the apparent analogy to the superconducting Δ , in this Letter we study vortex phases of V .

Real space vortices in the slave boson condensate were first considered in the context of cuprates [54–56]. Topological defects in Kondo lattice systems were also discussed over the years, including vortices bound to visons [57] in Kondo-Kitaev systems, as well as vortices and skyrmions in more richly structured theories of two-channel Kondo lattices [58–60]. Most important for this work are the discussion of vortex solutions of the hybridization in Kondo lattice systems [61–64], which were applied to antiferromagnetic QPTs. Less

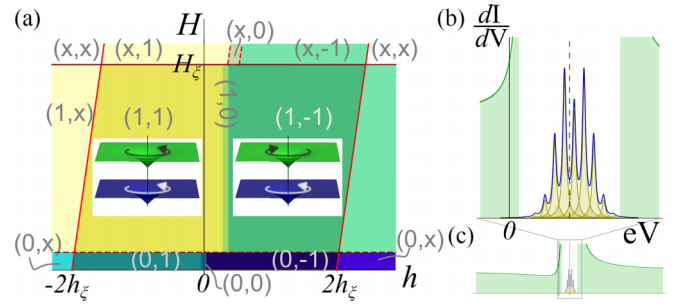


FIG. 2. (a) Phase diagram at fixed $T < T_c$ in the plane spanned by H and h . Here, we chose $e^2/e_*^2 = 10^{-4}$, $\xi/\xi_K = 1.5$, and $\kappa = 2 = \kappa_K$. The labels (ζ, ζ_K) denote the vorticity if ζ, ζ_K is an integer, while the label x denotes the absence of the corresponding order parameter. In most of the parameter space superimposed vortex-vortex/vortex-antivortex are energetically favorable (see illustration in the insets) and these regimes tend to cover all of the parameter space $H \lesssim H_\xi$, $|h| \lesssim 2h_\xi$ as κ_K, κ are further increased. (b) Tunneling density of states in the center of a Kondo vortex core; (c) the same at a larger scale.

directly connected to the physics of this Letter are vortices of V in momentum space, which occur, e.g., in theories of β -YbAlB₄ [65].

Ginzburg-Landau theory. The free energy density of the “order parameters” and (emergent) gauge fields is $f = f_{SC} + f_K + f_{MW} + f_{mw}$, where

$$f_{SC} = \alpha |\tilde{\Delta}|^2 + \beta U(\tilde{\Delta}) + \frac{|(-i\nabla - 2e\mathbf{A})\tilde{\Delta}|^2}{2m_{SC}}, \quad (1a)$$

$$f_K = \alpha_K |V|^2 + \beta_K |V|^4 + \frac{|(-i\nabla - e\mathbf{A} + e_*\mathbf{a})V|^2}{2m_K}, \quad (1b)$$

$$f_{MW} = \frac{\mathbf{B}^2 - 2\mathbf{B} \cdot \mathbf{H}}{8\pi}, \quad f_{mw} = \frac{\mathbf{b}^2 - 2\mathbf{b} \cdot \mathbf{h}}{8\pi}. \quad (1c)$$

Setting $\hbar = 1$, we define fundamental flux quanta $\Phi_0 = \pi/e$ and $\Phi_K = \pi/e_*$. The parameters $\alpha \sim (T - T_c)$ and $\alpha_K \sim (T - T_K)$ define the mean-field transitions. Boldface vectors live in physical 3D space. The dispersion of both V and $\tilde{\Delta} = (\Delta_{d_{xy}}, \Delta_{d_{x^2-y^2}})$ may be highly anisotropic with predominant 2D kinetics, but for notational simplicity this is disregarded. Note that Eq. (1) is applicable to both Kondo and mixed-valence systems alike; still we use the subscript K (for “Kondo”) throughout. For 4Hb-TaS₂, we consider $U(\tilde{\Delta}) = |\tilde{\Delta}|^4 + (\tilde{\Delta}^2)(\tilde{\Delta}^2)^*$, which is minimized by $\tilde{\Delta} \propto (1, i)^T$ corresponding to $d + id$ symmetry. It is important to emphasize that the proposed mechanism of magnetic memory is independent of the pairing symmetry. Consistently, in Figs. 1 and 2 we omit the vector on $\tilde{\Delta}$, as if s -wave superconductivity was considered. The electromagnetic terms f_{MW}, f_{mw} contain external fields $\mathbf{H}, \mathbf{h}$ and are a functional of internal fields $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{b} = \nabla \times \mathbf{a}$, where upper (lower) case letters refer to electromagnetic (emergent) gauge fields. The parameters of the Ginzburg-Landau functional, Eq. (1), define coherence length $\xi = 1/\sqrt{2m_{SC}|\alpha|}$, $\xi_K = 1/\sqrt{2m_K|\alpha_K|}$ and penetration depth $\lambda = \sqrt{m_{SC}/(2e)^2\pi\Delta^2}$, $\lambda_K = \sqrt{m_K/(2e_*)^2\pi V^2}$ for both superconducting and heavy fermion “order parameters,” where mean-field values of V, Δ are implicitly understood. We remark in passing that, for Cooper pair and slave boson

condensation alike, true spontaneous symmetry breaking is disallowed by Elitzur's theorem [40,66] and, beyond mean field, T_K denotes a crossover temperature.

Microscopics. While the form of Eq. (1) is imposed by symmetry arguments and the charges of matter fields, important physical properties, including the presence of the type-II hFL, follow from the values of parameters. Using a mean-field slave boson theory we microscopically derive those values; see Sec. S3 of Ref. [67]. We concentrate on the vicinity of the QPT [52,53,68,69] in Fig. 1(c) and approach the hFL from the FL* side, i.e., at temperatures $T \ll t_f$ much below the bandwidth of f_α fermions. To be concrete, we consider a purely 2D motion of an electronlike pocket of c_α electrons, which is smaller than the holelike pocket of f_α fermions (corresponding Fermi momenta $p_c < p_f$); see Fig. 1(b). Both pockets are treated in a parabolic band approximation and we control the theory in a large- N expansion. On the mean-field level, where $t_f \sim J_H$ is given by the Heisenberg exchange interaction, the emergent gauge coupling $1/e_*^2$ is stabilized by time-reversal-even four-spin interactions (coupling constant J_4), while the field $\mathbf{h}$ is microscopically generated [44] by the time-reversal-odd three-spin interaction $J_3 \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3)$ around the corners of a triangle of side length a . The most important result of the microscopic calculations is the smallness of $e^2/e_*^2 \sim J_4 a / 137c \ll 1$, with c the speed of light, and the fact that $\kappa_K \sim \sqrt{J_4 t_c} / J_H$, where t_c , i.e., the bandwidth of conduction electrons, is large. As J_4 approaches J_H from below near Kondo/Mott delocalization, $\kappa_K \gg 1$ and $e^2/e_*^2 \sim v_f / 137c$, where v_f is the speed of f fermions.

Inhomogeneous field configurations. In the following, we study the properties of inhomogeneous order parameter solutions and whether they minimize Eq. (1) (cf. Sec. S1 of Ref. [67]). Below the superconducting transition temperature, $T < T_c$ vortices of winding ζ (ζ_K) may occur in $\Delta(V)$, while for $T_c < T$ only vortices in V occur. By means of the minimal coupling, vortices induce a magnetic total flux Φ_{tot} (total emergent flux ϕ_{tot}) through the system

$$\begin{pmatrix} \Phi_{\text{tot}} \\ \phi_{\text{tot}} \end{pmatrix} = \begin{cases} \begin{pmatrix} \zeta \Phi_0 \\ (\zeta - 2\zeta_K) \Phi_K \end{pmatrix}, & T < T_c, \\ \frac{2\zeta_K}{1+(e^2/e_*^2)} \begin{pmatrix} \Phi_0 \frac{e^2}{e_*^2} \\ -\Phi_K \end{pmatrix}, & T_c < T. \end{cases} \quad (2)$$

For the solution above T_c , note that the combination $e_* \mathbf{B} + e \mathbf{b}$ is blind to the vortex and takes a homogeneous finite (vanishing) value in an (without an) external field (Sec. S2 of Ref. [67]). At the same time, a vortex in V implies a quantized flux of the orthogonal linear combination $e \mathbf{B} - e_* \mathbf{b}$. In the experimentally relevant case $e \ll e_*$, the flux very unevenly distributes between emergent and physical gauge fields. Using microscopic calculations, we find an extremely small magnetic field $B_{\text{max}} \lesssim \Phi_0 e^2 \ln(\kappa_K) / (4\pi e_*^2 \lambda_K^2) \sim 10^{-6} (J_H/t_c) \text{T}$ in the center of V vortices (Sec. S3 of [67]), where $J_H/t_c \lesssim 10^{-2}$. We remark in this context that the scale at which magnetization is excluded in published scanning SQUID experiments [28] is of the order of 10^{-7}T , which is about two orders of magnitude below the measured signal of superconducting vortices.

We next present the energy of a vortex relative to the homogeneous condensate in the limit of large $\kappa_K \gg 1$ and e_*^2/e^2 and to leading order in applied fields H, h . For $T < T_c$ we further concentrate on the limit of large $\kappa = \lambda/\xi$ and comparable λ, λ_K (for other limits see Sec. S1 of Ref. [67])

$$\frac{\Delta F}{L} = \begin{cases} \frac{\Phi_K}{4\pi} \left[\zeta_K^2 h_{\xi_K} \frac{\ln(\kappa_K)}{2\kappa_K^2} - (\zeta - 2\zeta_K)(H - h) \right] \\ + \frac{\Phi_0}{4\pi} \left[\zeta^2 H_{\xi} \frac{\ln(\kappa)}{2\kappa^2} - \zeta H \right], & T < T_c, \\ \frac{\Phi_K}{4\pi} \left[\zeta_K^2 h_{\xi_K} \frac{\ln(\kappa_K)}{2\kappa_K^2} - 2\zeta_K \left(\frac{e}{e_*} H - h \right) \right], & T_c < T. \end{cases} \quad (3)$$

The reference magnetic fields are $H_{\xi} = 1/(2e\xi^2)$, $h_{\xi_K} = 1/(2e_*\xi_K^2)$; they determine $H_{c,2}$ and $h_{c,2}/2$. The (ζ, ζ_K) values minimizing Eq. (3) are illustrated in Fig. 2(a). Below T_c and at a finite field, nonzero $\zeta = |\zeta_K| \neq 0$ are favorable, signaling the attraction of distinct vortex types. This concludes the main ingredients for the type-II hFL scenario of magnetic memory.

Tunneling into type-II hFLs. As highlighted above, the characteristic inhomogeneous magnetic signal of V vortices in type-II hFLs may be too weak to be experimentally accessible. We therefore consider another probe, namely tunneling spectroscopy/microscopy of hFL vortex cores.

Qualitatively, the vanishing hybridization function inside the vortex core implies line defects of FL* character. At the semiclassical level, the tunneling density of states (DOS) inside the vortex core is therefore small as compared to the hFL far away from the vortices (it is given by the bare mass of conduction electrons rather than the heavy mass). Moreover, in the hFL phase, electrons of the tip can tunnel into both c and f fermions [70]. While hFLs display a rich and characteristic tunneling spectrum due to multiple interfering tunneling channels, semiclassically these features disappear in the vortex core.

Quantum effects in the tunneling spectra of type-II hFL are expected to be strongest at a finite bias voltage corresponding to the energy E_{gap} around which the hybridization gap opens. Here, the DOS vanishes outside the vortex cores (called “Kondo insulator” when E_{gap} coincides with the Fermi energy E_F), but bound states are expected to be trapped inside the core [62]. Using the approximation of parabolic bands employed above and generalizing a classical calculation by Caroli *et al.* [71], we demonstrate in Sec. S5 of [67], the existence of bound states at energies $E_l = lE_{\text{mg}}$, where $l \in \mathbb{Z} + 1/2$. Note that the minigap $E_{\text{mg}} = V_{\text{gap}}^2 \sqrt{t_c/J_H} / J_H$ in terms of the size of the indirect gap $V_{\text{gap}} \sim V \sqrt{J_H/t_c}$ contains an additional large ratio of masses $\sqrt{t_c/J_H}$ leading to clearly spaced subgap states. We also determine the wave function of these solutions [67]. The most important feature is a suppressing factor $\sqrt{J_H/t_c}$ of the c -electron component of the wave function as compared to the f -electron component of the wave function. In the tunneling DOS, which is proportional to the wave function squared, this is the quantum manifestation of the small mass associated with the FL*-esque $V = 0$ physics inside the vortex core. In Figs. 2(b) and 2(c) we present the tunneling DOS (arbitrary units) in the vortex core superimposed with the signal of scattering states (or, equivalently, tunneling in the absence of a vortex [70]). Note the asymmetry of the signal, which is due to unequal roles

played by c and f fermions. A sufficiently sharp tip has strong support with the central subgap states only. Here we use a tip radius $\sim 3a$, $E_{\text{mg}}/V_{\text{gap}} = 1/10$, and a broadening of peaks $\Gamma = 0.2E_{\text{mg}}$.

Of course, this discussion of tunneling signatures assumes that the picture of the slave boson condensate survives to energies E_{gap} away from the Fermi energy, which is expected to hold if $|E_{\text{gap}} - E_F| \ll T_K$.

Discussion. In summary, we studied the type-II hFL phase, in which external magnetic fields induce vortex lattices in the slave boson condensate $V \propto \langle b \rangle$. An important task for future theoretical studies is the stability of this effect with respect to quantum and thermal fluctuations. The Lagrangian governing such fluctuations above T_c is essentially Eq. (1b) and mathematically equivalent to type-II superconductors. By analogy one therefore naively infers topologically stable vortex configurations and a robust type-II hFL, even in the presence of fluctuations. However, the dynamics of emergent gauge fields is more subtle and governed by a compact U(1) lattice gauge theory of which Eq. (1c) only contains the leading terms. In contrast to the type-II superconductor, this allows for nonperturbative monopole configurations with a Boltzmann weight exponentially small in gauge coupling. Thus we expect the

vortex decay rate to vanish to all orders in perturbation theory, but exponentially slow decay processes due to monopoles may limit the type-II hFL scenario to sufficiently fast experimental cycles.

While the type-II hFL theory is designed for mixed-valent 4Hb-TaS_2 the concept applies to fractionalized theories of the Hubbard model, too, including those with non-Abelian gauge groups. Such topological field configurations provide an interesting solid-state realization of cosmic strings and hedgehogs in Higgs condensates [72,73]. We argue that the type-II heavy Fermi liquid provides a particularly compelling scenario for the magnetic memory effect in 4Hb-TaS_2 , which in contrast to other scenarios neither assumes spontaneous (time-reversal) symmetry breaking above T_c nor the existence of local moments in the 1T layers.

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