

Pairing of composite electrons and composite holes in $\nu_T = 1$ quantum Hall bilayersLuca Rüegg^{✉,*}, Gaurav Chaudhary^{✉,†} and Robert-Jan Slager

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Motivated by recent experimental indications of preformed electron-hole pairs in $\nu_T = 1$ quantum Hall bilayers at relatively large separation, we formulate a Chern-Simons (CS) theory of the coupled composite electron liquid (CEL) and composite hole liquid (CHL). We show that the effective action of the CS gauge field fluctuations around the saddle point leads to stable pairing between CEL and CHL. We find that the CEL-CHL pairing theory leads to a dominant s -wave channel in contrast to the dominant p -wave channel found in the CEL-CEL pairing theory. Moreover, the CEL-CHL pairing is generally stronger than the CEL-CEL pairing across the whole frequency spectrum. Finally, we discuss possible differences between the two pairing mechanisms that may be probed in experiments.

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Introduction. Quantum Hall (QH) bilayers exhibit many phenomena that are absent in the QH monolayers [1,2]. At the total filling fraction $\nu_T = \nu_+ + \nu_- = 1$ (+/- refer to the upper/lower layer), when the two layers have almost equal carrier densities and small layer separation d [characterized by the ratio d/ℓ , where $\ell = \sqrt{\hbar/(eB)}$ is the magnetic length], the system exhibits a remarkable exciton condensate (XC) phase [3,4]. This superfluid phase of excitons was first observed in a Josephson-like zero-bias peak of the tunneling current between the two layers [3], and was later confirmed by a vanishing counterflow Hall resistance [5] and a perfect Coulomb drag [6].

In the $d \rightarrow 0$ limit, the system can be viewed as a two-component monolayer QH system, which is an incompressible state described by the Halperin (111) wave function [7]. The enriched physics associated with the layer degree of freedom is manifested in a broken symmetry [8] and spontaneous interlayer phase coherence [4,9,10]. In the opposite limit of large interlayer separation, the two layers can be described by a compressible state of decoupled composite Fermi liquids (CFLs) [11]. The intermediate layer separation, where the possible transition between the two phases occurs, is not entirely understood. There exist many different possibilities: a potential phase transition [12–14], a composite boson (CB) exciton condensate [15], phase coexistence between CBs and CFs [16], and several interlayer pairing instabilities [17–25].

More insight into the intermediate region is provided by two recent experiments [26,27]. In an interlayer tunneling

experiment, Eisenstein *et al.* [26] report that the tunneling pseudogap is suppressed at interlayer distances larger than what is expected for the transition to the XC phase. In a temperature-dependent Coulomb drag and counterflow experiment on QH bilayers of graphene, Liu *et al.* [27] report that a significant fraction of excitons are present at temperatures higher than the transition temperature T_c associated with the formation of the XC phase. The main features of these experiments indicate a region of preformed pairs of electrons and holes which go over a smooth BCS-BEC-like crossover to the XC phase with decreasing d/ℓ . The BCS-BEC crossover picture is also supported by a large overlap between numerical exact diagonalization and a trial BCS wave function for interlayer CF electron-hole pairs [25]. Alternatively, a connection between the BEC phase and the $p + ip$ paired CFs was shown in Ref. [20] using a particle-hole symmetric Dirac composite fermion formulation [28].

In this Letter, we formulate a Chern-Simons (CS) theory of a bilayer of composite electron liquid (CEL) in one layer and composite hole liquid (CHL) in the other as schematically shown in Fig. 1(a). Microscopically, the two layers are coupled by Coulomb interactions of bare particles. Starting from a large separation limit, within the random-phase approximation (RPA) [29,30], we arrive at an effective action for the fluctuations of the CS gauge fields around their mean-field value. The CS gauge fields mediate the effective interactions between the CFs in the two layers as shown in Fig. 1(b). This approach was previously used for the bilayers where both layers are a CEL [21–24], and the emergence of pairing instabilities between the two CELs was predicated. We show that in the CEL-CHL theory the effective interactions also lead to an interlayer pairing instability (albeit in the CE-CH channel) and facilitate the formation of interlayer “composite excitons (CXs).” These CXs are thus expected to start forming at large layer separation and can be thought of as the particles that undergo the BCS-BEC crossover as the layers are brought closer. We also show that the dominant pairing occurs in the s -wave channel of the CXs, thus theoretically justifying the

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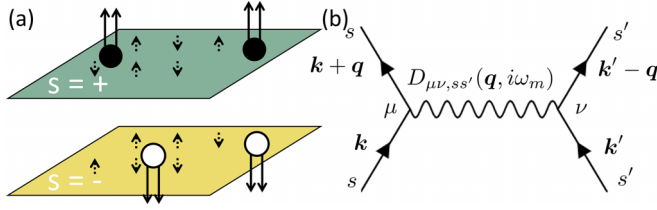


FIG. 1. (a) Schematic of the CEL-CHL bilayer system. The solid spheres with arrows are the CEs and the open spheres are CHs. The dashed arrows stand for the gauge fluctuations $\tilde{a}^s$. (b) Feynman diagram for the effective interaction between CEs and CHs mediated by the effective gauge propagator.

picture in Ref. [25]. This is in contrast to the CEL-CEL theory where p -wave pairing instability is favored [19,24,31]. This difference is rooted in the density-current coupling, which in the CEL-CEL theory breaks time-reversal symmetry (TRS) to favor a particular p -wave channel. In our CEL-CHL theory, the density-current coupling has opposite sign in the two layers and drops out in the Cooper channel, leading to an effective TRS.

Finally, we show that when the full frequency dependence of the effective interactions is taken into account, the s -wave CEL-CHL pair is more tightly bound than the p -wave CEL-CEL pair, thus favoring a BCS-BEC crossover picture in this theory. Furthermore, when the two layers have slightly mismatched densities, i.e., $\nu_T = (\frac{1}{2} - \delta) + (\frac{1}{2} + \delta)$, the two theories have different pair-breaking mechanisms, which also favors larger pair binding energies in the CEL-CHL theory. This insight can be possibly used in experiments to further confirm BCS-BEC crossover of CXs.

CS theory. For the CEL-CHL CS theory, we first need to consider the theory of holes in the lowest Landau level (LLL). For our purpose, we consider the Lagrangian theory of holes in the LLL formulated in Ref. [32] (after setting $\hbar = c = e = 1$),

$$\mathcal{L}_{\text{hole}} = h^\dagger(i\partial_t - A_t + \mu_h)h - \frac{1}{2m_h}h^\dagger(i\nabla - \mathbf{A})^2h - V(h) + \frac{1}{2\pi}A_t\nabla \wedge \mathbf{A}, \quad (1)$$

with the Coulomb interaction potential

$$V(h) = \frac{1}{2} \int d^2\mathbf{y} h^\dagger h \frac{1}{|\mathbf{x} - \mathbf{y}|} h^\dagger h, \quad (2)$$

where h is the hole field, A is the external vector potential, μ_h is the hole chemical potential, and m_h is the hole effective mass. Importantly, the hole field has opposite charge in the external gauge field A compared to the electron field. The CS term associated with the external field in the Lagrangian incorporates the effects of the filled LLL. A functional variation of the action with respect to the external field A yields

$$n_e + n_h = \frac{B}{2\pi}, \quad (3)$$

$$(\mathbf{j}_e)_i + (\mathbf{j}_h)_i = \frac{\epsilon_{ij}}{2\pi} E_j, \quad (4)$$

where ϵ_{ij} is the Levi-Civita symbol. The first equation means that electrons plus holes fill the LLL. The second equation is

the Hall conductivity. While, at the level of the Hamiltonian, the hole theory in the LLL can be obtained by a simple particle-hole transformation on the electron theory, in the Lagrangian incorporation of the extra CS term above puts important physical constraints. This extra CS term does not change the Hamiltonian [33].

We transform the holes h into composite holes (CHs) χ by attaching two magnetic flux quanta to each [see Fig. 1(a)]. In the Lagrangian, this is achieved by including the dynamical gauge field a [32],

$$\begin{aligned} \mathcal{L}_{\text{CH}} = & \chi^\dagger(i\partial_t + a_t - A_t + \mu_\chi)\chi - V(\chi) \\ & - \frac{1}{2m_\chi}\chi^\dagger(i\nabla + \mathbf{a} - \mathbf{A})^2\chi + \frac{1}{2\pi}A_t\nabla \wedge \mathbf{A} \\ & - \frac{1}{4\pi}a_t\nabla \wedge \mathbf{a}, \end{aligned} \quad (5)$$

where the interaction potential is the same as above since the density of holes and CHs is the same. The equation of motion of the dynamical gauge field stems from the second CS term and reads $\nabla \wedge \mathbf{a} = 4\pi\chi^\dagger\chi$, which again has an opposite sign compared to the case of the CF field ψ and its associated dynamical gauge field $\nabla \wedge \mathbf{a} = -4\pi\psi^\dagger\psi$. The Halperin-Lee-Read theory and hence the CF Lagrangian break the particle-hole symmetry [11]. The CHL theory described by the above Lagrangian cannot be obtained by a simple particle-hole transformation of the CEL Lagrangian. Instead, as argued in Ref. [32], this CHL at $\nu = \frac{1}{2}$ is a topologically distinct state to the CEL at $\nu = \frac{1}{2}$. Experimental findings in Ref. [34] indicate that the correct CFL in the LLL is the one of the minority charge carriers, i.e., CEL below $\nu = \frac{1}{2}$ and CHL above $\nu = \frac{1}{2}$. In the bilayer at $\nu_T = 1$, charge disorder may lead to locally imbalanced layer densities, thus enabling domains that are described by CEL in one layer and CHL in the other layer. With this overarching picture, the interlayer pairing between these two topologically distinct CFLs is our main motivation.

For our bilayer system, we take CFs as the degrees of freedom in one layer and CHs in the other, and accordingly denote $\psi_{+/-} = \psi/\chi$. The total Euclidean Lagrangian is [35]

$$\begin{aligned} \mathcal{L} = & \sum_{s=+/-} \left[\psi_s^\dagger(\partial_\tau + ia_\tau^s + siA_\tau - \mu_s)\psi_s \right. \\ & + \frac{1}{2m_s}\psi_s^\dagger(i\nabla + \mathbf{a}^s + s\mathbf{A})^2\psi_s + \frac{si}{4\pi}a_\tau^s\nabla \wedge \mathbf{a}^s + V(a^s) \left. \right] \\ & + \frac{i}{2\pi}A_\tau\nabla \wedge \mathbf{A} + V_{+-}(a^+, a^-), \end{aligned} \quad (6)$$

with the interactions

$$V(a^s) = \frac{1}{2} \int \frac{d^2\mathbf{y}}{(4\pi)^2} \nabla \wedge \mathbf{a}^s \frac{1}{|\mathbf{x} - \mathbf{y}|} \nabla \wedge \mathbf{a}^s, \quad (7)$$

$$V_{+-}(a^+, a^-) = \int \frac{d^2\mathbf{y}}{(4\pi)^2} \nabla \wedge \mathbf{a}^+ \frac{1}{\sqrt{|\mathbf{x} - \mathbf{y}|^2 + d^2}} \nabla \wedge \mathbf{a}^-. \quad (8)$$

In the expressions for the interactions above we already enforce the equations of motion for the dynamical gauge fields.

Effective interaction. The Lagrangian in Eq. (6) possesses the mean-field solution $a^s = -sA$. We denote fluctuations in the dynamical gauge fields around this saddle point as $\tilde{a}^s$. Furthermore, in the Coulomb gauge, the spatial part can be written as $\tilde{\mathbf{a}}(\mathbf{q}, i\omega_m) = \tilde{a}_1(\mathbf{q}, i\omega_m)\hat{z} \wedge \hat{\mathbf{q}}$, where $\omega_m = 2\pi mT$ is a bosonic Matsubara frequency. Within the RPA, the effective action for the CS gauge field fluctuations up to second order is [21–24,35]

$$S_{\text{eff}} = \frac{T}{2} \sum_{\omega_m} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \times \sum_{\mu\nu, ss'} \tilde{a}_\mu^s(\mathbf{q}, i\omega_m) D_{\mu\nu, ss'}^{-1}(\mathbf{q}, i\omega_m) \tilde{a}_\nu^{s'}(-\mathbf{q}, -i\omega_m), \quad (9)$$

where $D(\mathbf{q}, i\omega_m)$ is the gauge propagator and μ, ν are temporal and spatial coordinates denoted by 0 and 1, respectively. We are interested in the low-energy, long-wavelength modes, i.e., the regime $\frac{\omega_m}{\epsilon_F} \ll \frac{q^2}{k_F^2} \ll 1$ and $qd \ll 1$. The most singular terms in the gauge propagator are then [35]

$$D_{11,ss}(\mathbf{q}, i\omega_m) \approx -\frac{1}{\chi q^3} \left(\frac{m_+ m_-}{2\pi(4\pi)^2} q + \frac{m_+ m_- k_F}{(2\pi)^3} \frac{|\omega_m|}{q} \right),$$

$$D_{11,s(-s)}(\mathbf{q}, i\omega_m) \approx \frac{1}{\chi q^3} \frac{m_+ m_-}{2\pi(4\pi)^2} q, \quad (10)$$

where $\chi = \frac{m_+ m_- k_F}{2(2\pi)^4} \frac{|\omega_m|}{q^3} + (\frac{m_+ + m_-}{192\pi^4} + \frac{2dm_+ m_-}{(4\pi)^4})$. These are interpreted as current-current correlations. Importantly, the intra- and interlayer terms are equally singular.

After summing over the spatiotemporal indices, the CS gauge field mediates an effective interaction within the CEL/CHL, and between the CEL and the CHL, given by their respective matrix elements [24,35]

$$V_{ss'}^{\text{eff}}(\mathbf{k}, \mathbf{k}', \mathbf{q}, i\omega_m) = -ss' \frac{(\hat{\mathbf{q}} \wedge \mathbf{k})(\hat{\mathbf{q}} \wedge \mathbf{k}')}{2m_s m_{s'}} \times \left[q^2 \left(\frac{m_+ + m_-}{6\pi m_+ m_-} + \frac{d}{4\pi} \right) + \frac{|\omega_m| k_F}{q \pi} \right]^{-1}. \quad (11)$$

The dominant contribution in the interlayer Cooper channel $V_c(\mathbf{k}, \mathbf{q}, i\omega_m) = V_{+-}^{\text{eff}}(\mathbf{k}, -\mathbf{k} - \mathbf{q}, \mathbf{q}, i\omega_m) + V_{-+}^{\text{eff}}(\mathbf{k}, -\mathbf{k} - \mathbf{q}, \mathbf{q}, i\omega_m)$ is attractive. Thus, to this point, the pairing instability in the CEL-CHL theory is similar to the one obtained for the CEL-CEL pairing theory [21–24]. Our starting theory in Eq. (6) differs from the CEL-CEL theory, not only via the additional CS term of the external gauge field, but also regarding the charges of the CFs. However, in the mean-field solution, the attached flux cancels the external field for the electrons and holes alike. The remaining fluctuations near the mean field take both positive and negative values. Thus, the electrons and holes experience like charges in their respective CS gauge field fluctuations and resultant like charge currents. Since the dominant pairing contributions stem from the current-current coupling, to this level, there is yet no difference between CEL-CEL and CEL-CHL pairing. Further analysis shows that this attraction can be associated with the out-of-phase interlayer current fluctuations $b_1^- = \frac{\tilde{a}_1^+ - \tilde{a}_1^-}{\sqrt{2}}$ [35]. As we will see below,

the difference between the two theories appears in the density-current coupling of the CS gauge fields, which leads to different pairing symmetries.

Pairing symmetry. Within Eliashberg theory [36], the inverse Green's function in the Nambu space spanned by $[\psi_+(\mathbf{k}), \psi_-(\mathbf{k}), \psi_+^\dagger(-\mathbf{k}), \psi_-^\dagger(-\mathbf{k})]^T$ is

$$G^{-1}(\mathbf{k}, i\epsilon_n) = \begin{pmatrix} (i\epsilon_n Z_n^s - \xi_k^s) \delta_{ss'} & \hat{\phi}_n(\mathbf{k}) \\ \hat{\phi}_n(\mathbf{k})^\dagger & (i\epsilon_n Z_n^s + \xi_k^s) \delta_{ss'} \end{pmatrix}, \quad (12)$$

with Z_n^s the quasiparticle residue, $\hat{\phi}_n(\mathbf{k})$ the anomalous self-energy, $\xi_k^s = \frac{k^2}{2m_s} - \mu_s$, and $\epsilon_n = 2\pi(n+1)T$ a fermionic Matsubara frequency. Z_n^s and $\hat{\phi}_n(\mathbf{k})$ get corrections from the exchange and Cooper channel interaction, respectively. Going beyond the dominant current-current terms, these are written as [24,35]

$$V_{ex}(\mathbf{k}, \mathbf{q}, i\omega_m) = -\frac{1}{2} \sum_{\mu\nu, ss'} \begin{pmatrix} 1 & -i\frac{\hat{\mathbf{q}} \wedge \mathbf{k}}{m_{s'}} \\ i\frac{\hat{\mathbf{q}} \wedge \mathbf{k}}{m_s} & \frac{(\hat{\mathbf{q}} \wedge \mathbf{k})^2}{m_s m_{s'}} \end{pmatrix}_{\mu\nu} \times D_{\mu\nu, ss'}(\mathbf{q}, i\omega_m) [\delta_{s,+} \delta_{s',+} + \delta_{s,-} \delta_{s',-}], \quad (13)$$

and

$$V_c(\mathbf{k}, \mathbf{q}, i\omega_m) = \frac{1}{2} \sum_{\mu\nu, ss'} \begin{pmatrix} 1 & i\frac{\hat{\mathbf{q}} \wedge \mathbf{k}}{m_{s'}} \\ i\frac{\hat{\mathbf{q}} \wedge \mathbf{k}}{m_s} & -\frac{(\hat{\mathbf{q}} \wedge \mathbf{k})^2}{m_s m_{s'}} \end{pmatrix}_{\mu\nu} \times D_{\mu\nu, ss'}(\mathbf{q}, i\omega_m) [\delta_{s,+} \delta_{s',-} + \delta_{s,-} \delta_{s',+}]. \quad (14)$$

The interlayer density-current interactions determine whether the $\pm l$ pair angular momentum states are degenerate as they are the only terms sensitive to the direction of the inserted flux quanta [24,35]. Their contribution to the Cooper channel

$$\frac{i}{2} \hat{\mathbf{q}} \wedge \mathbf{k} \left[\frac{D_{10,+}(q, i\omega_m) + D_{01,-}(q, i\omega_m)}{m_+} + \frac{D_{10,-}(q, i\omega_m) + D_{01,+}(q, i\omega_m)}{m_-} \right] \quad (15)$$

vanishes in the low-energy regime because the density-current couplings have opposite signs in the two layers [35]. The density-current term originates from the CS term because it couples the spatial and the temporal components of the gauge field. Since the CEL and the CHL have the same charge with respect to their respective gauge fluctuations, they have the same charge current in response to the gauge fluctuations. However, since the CEL and the CHL have mean-field charge densities of opposite sign, the density-current correlators in the two layers have equal and opposite contributions which cancel to restore an effective TRS. This effective TRS ultimately leads to the degenerate $\pm l$ pairing channels. This is in contrast to the CEL-CEL theory. In the CEL-CEL pairing scenario, since the charge density as well as the charge currents in the two identical CFLs have the same sign, they add up to break the TRS [24].

Having shown that $\pm l$ channels are degenerate in the CEL-CHL pairing, we next analyze the relative strength of

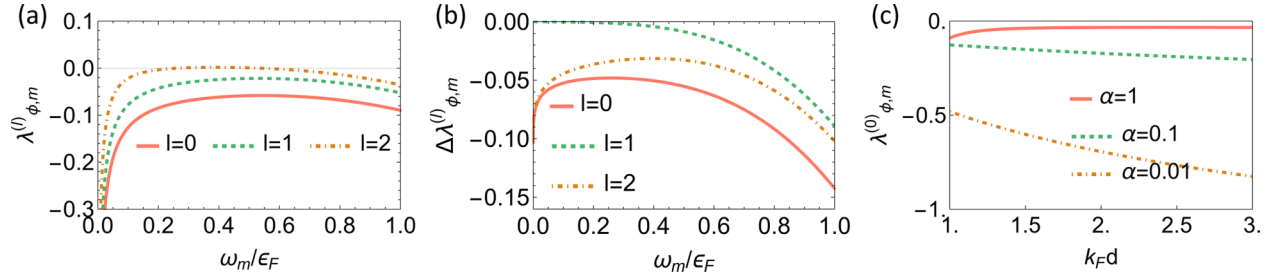


FIG. 2. (a) Effective couplings $\lambda_{\phi,m}^{(l)}$ as functions of the frequency. The integrals in Eq. (16) diverge for small q both for zero and finite frequencies. This is cured by introducing the cutoff $q_c = 10^{-5}k_F$. We set $k_F d = 1$. (b) Difference between the effective couplings for CEL-CEL and CEL-CHL pairing $\Delta\lambda_{\phi,m}^{(l)} = \lambda_{\phi,m}^{\text{CEL-CHL}(0)} - \lambda_{\phi,m}^{\text{CEL-CEL}(l)}$, with $k_F d = 1$. (c) Effective coupling $\lambda_{\phi,m}^{(0)}$ when varying the Fermi wave vector k_F and keeping distance d , effective mass m^* , and frequency ω_m constant parametrized by $\alpha = 2m^*\omega_m d^2$.

different $|l|$ channels. For this purpose, we make the effective mass approximation $m_+ = m_- = m^*$ which stands on good theoretical [32] and experimental [37] footing. We expect our main results to be unchanged by small variations in the effective masses. Under this approximation, Eq. (15) vanishes for any momentum and frequency [35]. To assess the stability of the pairing we consider the effective coupling constants

$$\lambda_{Z,m} = \int \frac{d^2q}{(2\pi)^2} \delta(\xi_{k+q}) V_{ex}(\mathbf{k}, \mathbf{q}, i\omega_m),$$

$$\lambda_{\phi,m}^{(l)} = \int \frac{d^2q}{(2\pi)^2} \delta(\xi_{k+q}) V_c(\mathbf{k}, \mathbf{q}, i\omega_m) \left(1 + \frac{q}{k_F} e^{il(\theta_q - \theta_k)}\right)^l, \quad (16)$$

where θ_q is the polar angle of $\mathbf{q}$, and we assumed that the pairing happens only at the Fermi surface, i.e., $|\mathbf{k}| = k_F$. In the limit of zero frequency and long wavelengths, the effective couplings are approximated by their singular terms [24,35]

$$\lambda_{Z,0} \approx \frac{m^*}{k_F} \int_0^{2k_F} \frac{dq}{(2\pi)^2} \left(\frac{k_F^2}{16\pi^3 q^2 \chi_0} + \frac{\frac{k_F^2}{12\pi^3 m^* \chi_0} - \frac{k_F^2 \chi'}{16\pi^3 \chi_0^2}}{q} \right),$$

$$\lambda_{\phi,0}^{(l)} \approx \frac{m^*}{k_F} \int_0^{2k_F} \frac{dq}{(2\pi)^2} \left(-\frac{k_F^2}{16\pi^3 q^2 \chi_0} + \frac{\frac{dk_F^2}{16\pi^3 \chi_0} + \frac{k_F^2 \chi'}{16\pi^3 \chi_0^2}}{q} \right), \quad (17)$$

with $\chi_0 = \frac{m^*}{96\pi^4} + \frac{2d(m^*)^2}{(4\pi)^4}$ and $\chi' = \frac{1}{144\pi^4} - \frac{2(m^*)^2 d^2}{(4\pi)^4}$. Because the leading term in the anomalous self-energy is negative, the CEL-CHL pairing is stable. This attractive interaction is mediated via the out-of-phase current fluctuations. The next-order term corresponds to the repulsive in-phase current-current interactions [24,35].

The terms distinguishing the different pairing channels enter only at q^0 order:

$$\lambda_{\phi,0}^{(l)} - \lambda_{\phi,0}^{(0)} \approx \frac{m^*}{k_F} \int_0^{2k_F} \frac{dq}{(2\pi)^2} \frac{4l^2}{128\pi^3 \chi_0}. \quad (18)$$

It is immediately clear from this that the favored pairing happens in the s -wave channel. This can be explained by the fact that the Cooper channel interactions are isotropic. Using analytic expressions for the propagators [23,24] and numerically performing the integrals in Eq. (16) [35], we

obtain the effective couplings [see Fig. 2(a)]. It is obvious that $l=0$ is the strongest pairing channel for all frequencies.

CEL-CHL vs CEL-CEL pairing. Next, we compare the relative stability of the pairs in the two theories, i.e., s -wave CEL-CHL pairs (CXs) to the CEL-CEL Cooper pairs. In Fig. 2(b), we therefore consider the difference between the leading CEL-CHL coupling $\lambda_{\phi,m}^{\text{CEL-CHL}(0)}$ and the three strongest CEL-CEL couplings $\lambda_{\phi,m}^{\text{CEL-CEL}(l=0,1,2)}$. The CEL-CHL s -wave channel dominates at all frequencies. Thus, after performing the Matsubara frequency sum, the CEL-CHL pairing theory leads to CXs that are more tightly bound than the Cooper pairs in the CEL-CEL theory. This suggests that CXs are more likely to preform in a large region before the exciton condensation [26,27]. Therefore, when viewed from this theory, the QH bilayer system has a stronger tendency to undergo BCS-BEC crossover than the CEL-CEL theory. The CEL-CEL theory possibly has a stronger tendency towards a first-order phase transition [38,39]. We mention in passing that at zero-frequency the coupling constants for the dominant $l=1$ CEL-CEL pairing channel are degenerate with the $l=0$ channel of CXs.

We also compare the response of the two theories to a slight density mismatch in the two layers, while keeping the total filling factor fixed, i.e., when the filling factors are $\nu_T = (\frac{1}{2} - \delta) + (\frac{1}{2} + \delta)$. Since each bare particle is attached to two fluxes, in each layer the external field is not exactly canceled by the flux when it is moved away from half filling of the LLL. This remnant field is often interpreted to lead to a new set of Landau levels to explain the fractionalization (“ Λ levels” of the composite fermions) [40]. Apart from the Λ levels, the density imbalance also changes the Fermi wave vector of the CFL ($k_F = \sqrt{4\pi n}$) [11]. In the following discussion, we assume that density imbalance is small enough that the individual layer is still away from a fractional state. Thus, we can ignore the effect of Λ levels and in the isolated CFL, the change in the k_F remains the only effect of density imbalance. In the CEL-CEL theory, while one of the layers is depleted, the density in the other layer increases, and as a result k_F also decreases in one layer and increases in the other layer. This leads to pair-breaking phenomena similar to the Pauli-pair breaking in conventional superconductors, where the difference in the energy of the two opposite spin electrons that form the pairs leads to eventual suppression of the

superconductivity when the magnetization energy overcomes the condensation energy [41,42].

In contrast, in the CEL-CHL theory, the layer with the increased electron density is described by the CHL [32]. Thus, the CH and CE densities are equal [35]. As a result, the density imbalance of the bare particle leads to an identical depletion of the two layers in the CF description. Thus, Pauli-like pair breaking phenomena are absent in this theory. However, since k_F decreases with the density depletion, the effective coupling constants $\lambda_{\phi,m}^{(l)}$ usually decrease for small frequencies [see Fig. 2(c)]. This can lead to a different mechanism for the suppression of CX formation with respect to the density imbalance. Notice in the CEL-CEL theory, this mechanism is also present along with the Pauli-pair breaking. Thus, the CXs are also more robust to small density imbalances compared to the CEL-CEL Cooper pairs. The relatively stronger robustness of the CXs to density imbalance has some support in experimental findings [43,44]. Thus, a more detailed experimental study with density-imbalanced states along with related numerical studies can be used to further verify CX formation.

Conclusions. We have studied the pairing of CEL and CHL in the quantum Hall bilayer at $\nu_T = \frac{1}{2} + \frac{1}{2}$ by analyzing fluctuations in the CS gauge field around the mean-field solution within the RPA. These fluctuations mediate interactions between the CEL and the CHL. In the interlayer Cooper channel,

these interactions are attractive and lead to a stable CEL-CHL pairing with $l = 0$ symmetry due to an effective TRS. This yields a microscopic understanding for numerical results [25], where an s -wave CEL-CHL pairing trial wave function was found to have the best overlap with exact diagonalization results. The CXs in our theory are formed in the large layer separation limit, which appears to be in agreement with the experimental findings of preformed pairs before the transition to the XC [26,27]. A possible future direction of research is to reevaluate the puzzling features in the zero-bias tunneling anomaly from the CEL-CHL theory [45–47]. It is crucial to mention that the pairs in our theory are pairs of CEs and CHs, which must be contrasted with the pairs of bare electrons and holes. It is the latter that condense to form an XC in closely separated bilayers. We leave this relation between the condensate of CXs and the bare excitons, along with the theory of the full BCS-BEC crossover regime, for future investigations.

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