

Intrinsic nonmagnetic ϕ_0 Josephson junctions in twisted bilayer graphene

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Recent experiments have demonstrated the possibility to design highly controllable junctions on magic angle twisted bilayer graphene, enabling the test of its superconducting transport properties. We show that the presence of chiral pairing in such devices manifests in the appearance of an anomalous Josephson effect (ϕ_0 behavior) even in the case of symmetric junctions and without requiring any magnetic materials or fields. Such behavior arises from a combination of chiral pairing and nontrivial topology of the twisted bilayer graphene band structure that can effectively break inversion symmetry. Moreover, we show that the ϕ_0 effect could be experimentally enhanced and controlled by electrostatic tuning of the junction transmission properties.

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Introduction. Since its discovery in 1962 [1], the Josephson effect has emerged as one of the most remarkable manifestations of quantum coherence at the macroscopic scale with multiple technological applications [2]. Material platforms on which Josephson junctions (JJs) can be fabricated do not cease to grow, ranging from conventional or unconventional superconductor tunnel junctions [3] to hybrid nanostructures including exotic materials [4]. The discovery of superconducting phases in magic angle twisted bilayer graphene (MATBG) [5] provides an additional playground for the study of the Josephson effect. In fact, the possibility to produce tunable junctions on this material by electrostatic gating has been recently demonstrated [6,7], with enabled phase control in ring-shaped configurations [8], and JJs exhibiting unconventional Fraunhofer patterns have been found [9].

This work is in parallel with worldwide efforts to clarify the origin and precise nature of unconventional superconductivity in MATBG [5,10–16]. Many theoretical studies point to the possibility of chiral ($d + id$ or $p + ip$) pairing symmetry arising as a result of electron correlations and the peculiar topology of flat bands in MATBG [17–22]. Furthermore, robust nematic behavior has been observed for a variety of twist angles in the superconducting phase of twisted bilayer systems [23–26], a phenomenon that has attracted recent theoretical interest [27–32]. The next key step in the field is thus to find reliable signatures that would help us to elucidate the pairing mechanism in MATBG.

In this Letter we address this issue showing that chiral pairing symmetry would manifest in an anomalous ϕ_0 behavior (a nonzero supercurrent in the absence of a superconducting phase difference) in monolithic MATBG JJs. We demonstrate that this effect is a consequence of the nontrivial topology of

the normal MATBG bands and thus cannot be captured by trivial models. We further show that the effect is enhanced for extended junctions and can be controlled by electrostatic gating of a middle normal region.

Before entering into the peculiarities of MATBG JJs, let us give a more general context on the topic of ϕ_0 junctions. In general, ϕ_0 behavior requires breaking time reversal symmetry (TRS) and inversion symmetry, having been predicted to appear in different systems by the combined effect of spin-orbit interactions and magnetic fields [33–41] or in junctions through magnetic metals with broken inversion symmetry [42–45]. In all these proposals the broken symmetries are linked to the spin degree of freedom and we could refer to them as “magnetic” ϕ_0 junctions.

In the case of MATBG the valley degree of freedom comes into play, providing new possibilities. In Ref. [46] it has been suggested that ϕ_0 behavior could appear in MATBG JJs with conventional s -wave pairing provided that the junction is established through a “valley-polarized” region. This mechanism has also been proposed to explain recent observations of the anomalous Josephson effect in twisted trilayer JJs [47]. By contrast, we here analyze direct junctions between two MATBG regions with chiral pairing. An important aspect of MATBG which guides our study is that the large size of its moiré pattern (>10 nm) would allow transport experiments on junctions along well-defined directions on the moiré lattice. We show that, only when the nontrivial topology of the MATBG is taken into account, ϕ_0 behavior can appear spontaneously when the junction is defined along certain directions on the moiré pattern that break translation invariance, or along any orientation when graphene’s sublattice symmetry is broken. We argue that these effects would allow one to distinguish among possible pairing symmetries in MATBG.

Qualitative description. We consider a junction defined on a bulk MATBG sample as schematically illustrated in Fig. 1(a). External gates can be used to tune the doping level on the two sides of the junction independently and each side is characterized by the presence of a generic chiral pairing.

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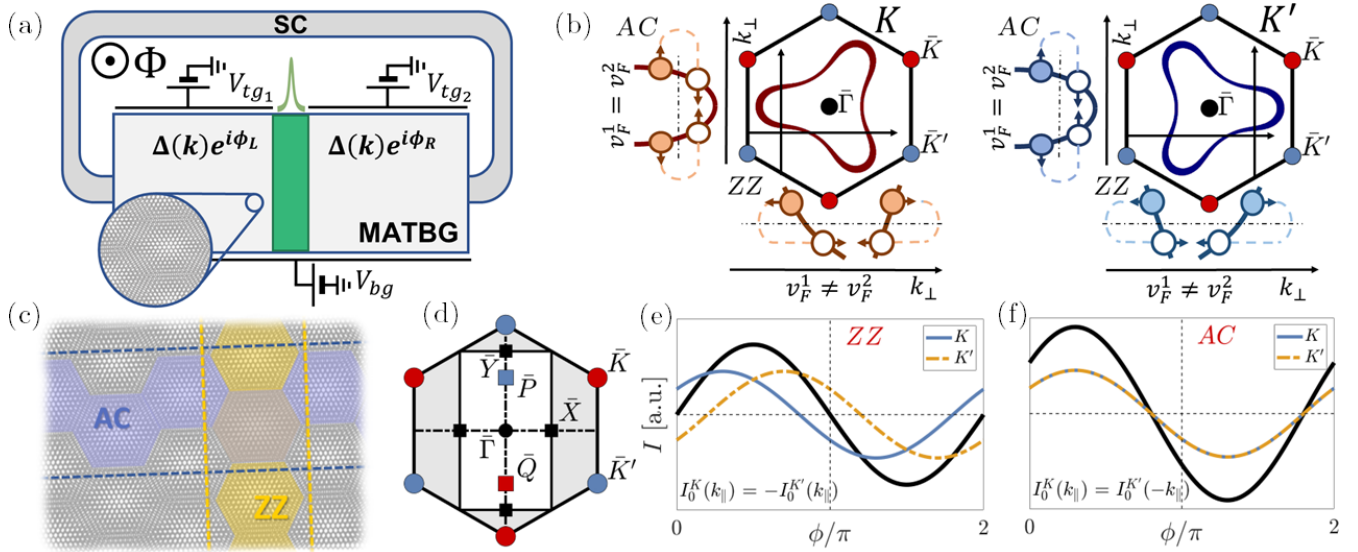


FIG. 1. (a) Scheme of a MATBG Josephson junction: Side gates control the doping level on the two sides and the middle barrier height. The junction is immersed in a loop that determines the superconducting phase difference $\phi = \phi_L - \phi_R$. (b) Typical normal-state Fermi surfaces at the K and K' valleys in the hexagonal Brillouin zones. The Andreev reflection coefficients on opposite Fermi points for a given $k_{\parallel}$ can differ either due to trigonal warping, leading to $v_F^1 \neq v_F^2$ (ZZ), or due to nontrivial topology combined with chiral pairing (AC). (c) Representation of AC and ZZ directions on the moiré lattice. (d) Hexagonal and folded rectangular Brillouin zones indicating high-symmetry points. (e), (f) Schematic CPRs at the two valleys for the AC and ZZ cases.

The barrier height, and therefore the junction transmission, can also be externally controlled. The junction is placed on a loop that enables control over the superconducting phase difference ϕ , determining the junction current-phase relation (CPR), $I_f(\phi)$.

At band fillings close to the van Hove singularities (vHSs), where superconductivity is observed, MATBG is characterized by a normal Fermi surface exhibiting trigonal warping [48,49]. Typical shapes of these surfaces on the two valleys (denoted by K and K') are shown in Fig. 1(b). The main effect of trigonal warping is to break the intravalley inversion symmetry along certain directions, i.e., $E(k_{\perp}) \neq E(-k_{\perp})$, where $k_{\perp}$ denotes the wave vectors perpendicular to the junction interface. The Fermi velocities v_F at the Fermi points for a given parallel momentum $k_{\parallel}$ thus differ. This is the case, for instance, for the junctions defined parallel to the line joining the moiré minivalleys $\bar{K} - \bar{K}'$ (i.e., $k_{\parallel} = k_y$), which we call “zigzag” (ZZ) in analogy with pristine graphene. On the contrary, junctions defined perpendicular to the $\bar{K} - \bar{K}'$ line (i.e., $k_{\parallel} = k_x$), called “armchair” (AC), are not affected by trigonal warping in the normal state [50]. However, as we discuss below, such symmetry can be broken even for AC junctions due to the combined effect of normal-state topology and pairing properties. As illustrated in Fig. 1(c), these junction orientations correspond in real space to lines joining nearest-neighbor (ZZ) or next-nearest-neighbor (AC) “sites” in the moiré pattern, which can be associated with the charge centers.

From this definition we observe that an AC junction breaks the moiré translation invariance along the interface. Periodicity along the junction interface (i.e., along $k_{\parallel} = k_x$) is recovered by cell duplication, which corresponds to folding the Brillouin zone into a rectangular one as shown in

Fig. 1(d). Consequently, in the superconducting state the Andreev bound states (ABSs) formed at such an interface result from coupling scattering states of different types on both sides, which might differ by a topological phase. While this asymmetry is a necessary condition, to obtain a net ϕ_0 behavior this phase difference should survive after $k_{\parallel}$ and valley integration.

We find that spontaneous valley supercurrents $I_f^K(\phi = 0) \neq 0$ appear in the AC case due to the combined effect of chiral pairing and nontrivial band topology, and in the ZZ due to Fermi-surface warping. However, while the ZZ currents on opposite valleys cancel each other [i.e., $I_f^K(0) = -I_f^{K'}(0)$; see Fig. 1(e)], for AC we have $I_f^K(0) = I_f^{K'}(0) \neq 0$. Consequently, the valley currents do not compensate leading to a net ϕ_0 -junction behavior [see Fig. 1(f)]. As we show below, breaking graphene’s sublattice symmetry yields an anomalous CPR also for junctions defined on a ZZ direction only in models where nontrivial band topology is taken into account.

Our main result, which we expose in detail below, is thus that the presence of chiral pairing on a nontrivial MATBG band manifests as an anomalous Josephson effect. This is always the case in AC junctions, due to the naturally broken moiré translation invariance at the interface, and can be induced on ZZ junctions by breaking graphene’s sublattice symmetry.

CPR calculations for MATBG junctions. To give a quantitative estimation of the effect discussed above, we consider a six-band model (6BM) for MATBG junctions which accounts for both the appropriate flat-band dispersions and their nontrivial (fragile) topology [51]. Our calculations are based on the recursive Green’s functions method introduced in Refs. [52–54], which we extend here to the superconducting case [see the Supplemental Material (SM) [55] where we also

include the pairing implementation for the trivial two-band model (2BM) [49]. To introduce chiral superconductivity in these models we project the pairing order parameter onto the directional $p_{\pm}$ orbitals which provide the main contribution to the flat bands below the vHS [55]. For simplicity, we only consider here intervalley, spin-singlet chiral d -wave superconductivity with zero center-of-mass momentum. From the irreducible representations of the crystal symmetry group [56] in the triangular lattice we build up the form factors for the pairing

$$\Delta_{d_{x^2-y^2}} = \frac{\Delta}{\sqrt{3}} [\cos(k_y L_y) - \cos(k_x L_x/2) \cos(k_y L_y/2)],$$

$$\Delta_{d_{xy}} = \Delta \sin(k_x L_x/2) \sin(k_y L_y/2), \quad (1)$$

where the chiral pairing is the superposition $\Delta_{d+id'} = \Delta_{d_{x^2-y^2}} + i\Delta_{d_{xy}}$, $L_{x,y}$ are the orthogonal lattice vectors in the doubled unit cell, and we set the value of Δ as roughly one order of magnitude smaller than the bandwidth. Some other configurations (e.g., chiral p wave with $S^z = 0$ or superpositions of nematic chiral orders with local s wave) could be implemented using the same approach, but have minor effects on the CPR. Notice that chiral superconductivity is a necessary ingredient to obtain ϕ_0 behavior as it breaks TRS. However, not all TRS-breaking pairings lead to the ϕ_0 effect, e.g., nodal intravalley pairings never show uncompensated valley currents for any junction configuration studied in this Letter [55]. In this sense, ϕ_0 behavior is intrinsically linked only to chiral superconductivity.

Along with pairing, we consider the effect of a sublattice symmetry-breaking perturbation (δ_S) acting over $p_{\pm}$ that can be produced by alignment with the hexagonal boron nitride (hBN) substrate [55]. We note that the nontrivial topology of the bands in the 6BM is already reflected in an anisotropic response of the superfluid weight [55]. The relevant effect of remote bands for superconductivity in MATBG has been discussed in several theoretical works [27,57–61].

The junction is defined as a smooth barrier at the moiré scale for which $k_{\parallel}$ is a good quantum number. Its effective transmission is controlled by a coefficient τ such that for a symmetric junction (SS) with the same chemical potential on both leads, $\mu_L = \mu_R = \mu_0$, we recover the bulk system at zero-phase bias and $\tau = 1$. We also consider asymmetric junctions (SS') in which $\mu_L \neq \mu_R$ and symmetric junctions through a finite-sized, heavily doped normal region (SNS) where $|\mu_L| = |\mu_R| \ll |\mu_c|$ and $\tau = 1$.

We first discuss the case of AC junctions. Figure 2(c) shows the evolution of the zero-phase supercurrent $I_J(0)$ for $\tau = 0.8$ as a function of the chemical potential, both in the case of equal ($\mu_L = \mu_R$) and unequal (μ_L fixed, varying μ_R) doping levels for the 6BM. Since the critical currents for all cases are of the same order, we normalize $I_J(0)$ to the critical current I_c^0 for $\mu_L = \mu_R = -1$ meV. The red dashed lines indicate the vHS positions.

While the most striking result is the presence of ϕ_0 behavior for a symmetric junction without external fields, the computed $|I_J(0)/I_c^0|$ values are relatively small (less than 10%) in the range of doping levels where superconductivity in MATBG is expected to appear. We observe similar values of $I_J(0)$ for symmetric junctions with p or n doping close

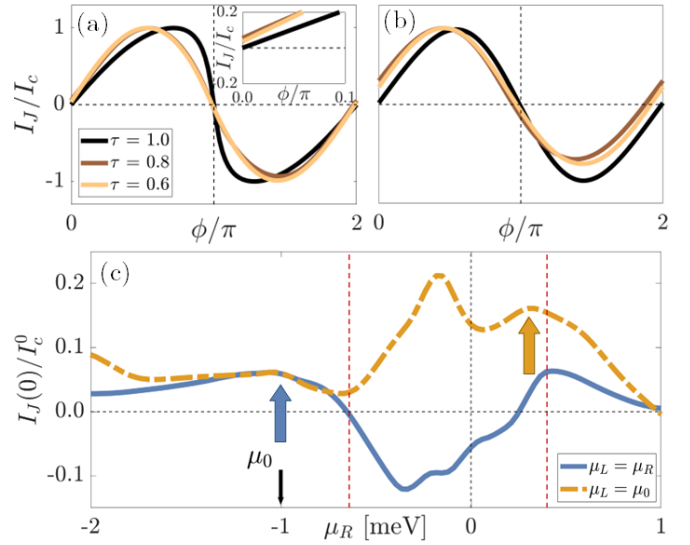


FIG. 2. (a), (b) Current-phase relation for AC junctions in the 6BM for different values of the effective transmission τ . (a) Symmetric SS junction with $\mu_L = \mu_R = -1$ meV and (b) asymmetric SS' junction with $\mu_L = -1$ meV and $\mu_R = 0.3$ meV. The inset in (a) shows a closeup of the $\phi \simeq 0$ region. (c) Zero-phase current $I_J(0)$ as a function of the chemical potential for symmetric ($\mu_L = \mu_R$) and asymmetric ($\mu_L = \mu_0$, with $\mu_0 = -1$ meV) junctions. $I_J(0)$ is normalized to the critical current I_c^0 in the symmetric case with $\mu_L = \mu_R = \mu_0$ and $\tau = 0.8$. The red dashed lines mark the position of the vHSs and color arrows indicate the situation for the CPRs in (a) and (b).

to the vHSs [see Fig. 2(c)]. The ϕ_0 effect is generally larger for asymmetric junctions of $n - p$ type, i.e., with $\mu_L < 0$ and $\mu_R > 0$. In contrast to the symmetric case, in this situation we observe no changes in the sign of $I_J(0)$ as a function of the doping level. Finally, for both symmetric and asymmetric junctions we find large nonvanishing $|I_J(0)/I_c|$ values close to charge neutrality $\mu = 0$. We notice, however, that experimental samples show no robust superconducting dome at those fillings and that the superconducting state could have different properties close to charge neutrality [11,62].

Having demonstrated the anomalous Josephson effect of short symmetric and asymmetric junctions with chiral pairings, we now show that the ϕ_0 effect can be enhanced and tuned by a middle normal region in the SNS configuration. We thus assume symmetric junctions including a finite normal central region with large doping beyond the flat bands, so that the barrier transmission is limited by the chemical potential mismatch. We show in Fig. 3 that it is possible to tune the $|I_J(0)/I_c|$ ratio by varying the doping levels [Fig. 3(a)] or the length [Fig. 3(b)] of the central region. Notice that doping the central region at the dispersive bands or increasing its length reduces the critical current by an order of magnitude with respect to the cases analyzed in Fig. 2.

Finally, we analyze ZZ junctions in Fig. 4. As discussed above, trigonal warping leads to compensated valley currents at zero-phase bias. However, when sublattice symmetry is broken, the nontrivial topology of MATBG captured by the 6BM leads to uncompensated currents and thus ϕ_0 behavior. This behavior is not observed in the trivial 2BM [55]. Similarly to

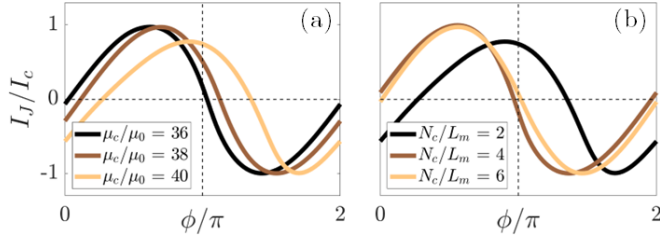


FIG. 3. Josephson effect for AC SNS junctions described by the 6BM. (a) CPR for different potential barrier heights $\mu_c = -36, -38, -40$ meV, where the length of the normal region is $N_c = 2$ (in units of moiré lattice length L_m). (b) CPR for several values of the width of the normal region $N_c = 2, 4, 6$ (approximately 25, 55, 80 nm). In all cases the chemical potentials are $\mu_L = \mu_R = \mu_0 = -1$ meV and $\tau = 1$.

AC junctions, we find larger $|I_J(0)/I_c|$ values for asymmetric junctions of $n-p$ type. We also observe a change in sign of $I_J(0)$ through the $p-p'$ to $p-n$ transition. The size of $|I_J(0)/I_c|$ also increases as the sublattice symmetry-breaking parameter δ_S value is increased [inset of Fig. 4(c)]. It is worth mentioning that we observe asymmetries between the negative and positive critical currents (superconducting diode effect [38,39,63]) in both AC and ZZ orientations for asymmetric SS' and SNS junctions. This effect is observed when the ϕ_0

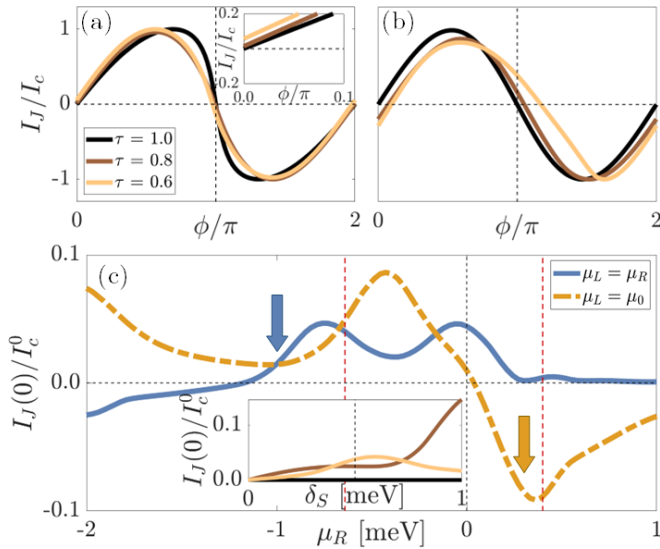


FIG. 4. (a), (b) Current-phase relation for ZZ junctions without sublattice symmetry in the 6BM for different values of the effective transmission τ . (a) Symmetric SS junction with $\mu_L = \mu_R = -1$ meV and (b) asymmetric SS' junction with $\mu_L = -1$ meV and $\mu_R = 0.3$ meV. The inset in (a) shows a closeup of the $\phi \simeq 0$ region. (c) Zero-phase current $I_J(0)$ as a function of the chemical potential for symmetric ($\mu_L = \mu_R$) and asymmetric ($\mu_L = \mu_0$, with $\mu_0 = -1$ meV) junctions. $I_J(0)$ is normalized to the critical current I_c^0 in the symmetric case with $\mu_L = \mu_R = \mu_0$ and $\tau = 0.8$. The red dashed lines mark the position of the vHSs and color arrows show the situation for the CPRs in the panels above. The inset shows the evolution of $I_J(0)$ as a function of the sublattice symmetry-breaking potential δ_S in the symmetric junction.

behavior is enhanced and could thus be electrically controlled by tuning the chemical potential and effective transmission. Let us further comment that if, additionally, valley polarization is included, e.g., by means of TRS breaking in the parent state as in Ref. [46], one would observe ϕ_0 behavior for all orientations.

Minimal model. A scattering theory that confirms and gives further insights to the previous results can be built by linearizing the bulk Hamiltonian with respect to $k_\perp$ around each Fermi point for a given $k_\parallel$ [55]. For the 6BM, Andreev reflection coefficients for the electron-to-hole processes (and their time reversed) acquire different phases $\theta(k_\parallel)$ and $\theta'(k_\parallel)$. Applying matching conditions at the interface corresponding to a single-channel junction with transmission $T_k = T(k_\parallel)$, leads to the following set of Andreev bound states [55],

$$\frac{\epsilon(k_\parallel)}{\tilde{\Delta}(k_\parallel)} = \pm \sqrt{T_k + R_k \cos^2\left(\frac{\bar{\theta}_k}{2}\right) - T_k \sin^2\left(\frac{\phi}{2} - \frac{\delta\theta_k}{2}\right)}, \quad (2)$$

where $R_k = 1 - T_k$, $\tilde{\Delta}(k_\parallel)$ is an effective gap, $\bar{\theta}_k = \theta(k_\parallel) + \theta'(k_\parallel)$, and $\delta\theta_k = \theta(k_\parallel) - \theta'(k_\parallel)$. This simple dispersion relation is valid provided that $T_k < 1$. For the AC junction we get $\theta^K(k_\parallel) = \theta^{K'}(-k_\parallel)$ (same for θ'), which yields that $I_J^K(0) = I_J^{K'}(0)$, in agreement with the full numerical results. By contrast, the warping distortion of the Fermi surface in the ZZ case results in $\theta(k_\parallel) \neq \theta'(k_\parallel)$, even for the nontopological 2BM [55]. However, in this orientation we find that $\theta^K(k_\parallel) = -\theta^{K'}(-k_\parallel)$ (same for θ') and, consequently, $I_J^K(0) = -I_J^{K'}(0)$. The ZZ valley currents are thus compensated unless sublattice symmetry is broken and fragile topology taken into account.

Conclusions. We have shown that chiral pairing in MATBG junctions would manifest in the appearance of a nonmagnetic ϕ_0 behavior, stemming from the nontrivial topology of the MATBG bands. This effect can then be used to distinguish chiral superconductivity from other mechanisms such as nodal pairing. Moreover, we illustrated how the effect is enhanced for extended junctions and controlled by electrostatic gating of a middle heavily doped normal region. Although we have focused on the $d + id$ case, other chiral symmetries such as $p + ip$ or combinations of these symmetries with s -wave pairing, would show similar behavior. The orientation sensitivity of the ϕ_0 effect could help to distinguish this different orbital character in an actual experiment. Furthermore, it would allow us to distinguish whether the anomalous behavior is due to a broken valley symmetry parent state or due to configurations involving symmetric contributions from both valleys and chiral pairing.

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